Measuring the Fate of Kinetic Energy Injected into the Ocean Mesoscale

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Introduction

Aim: To develop new analysis tools that can diagnose properties of mesoscale dynamics, without being affected by the complexities of the real ocean.

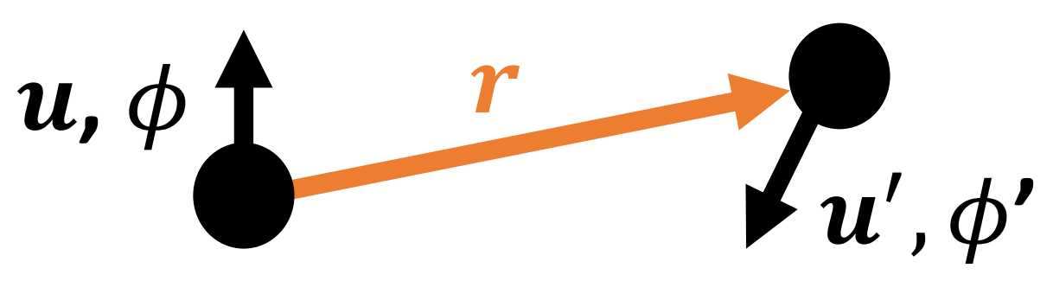
Ocean dynamics at the mesoscale are turbulent and play a critical role in transporting properties between locations and across scales within the ocean. While there are several ways of observing mesoscale dynamics, it is not possible to get a complete picture of the mesoscale dynamical field from present observations.

Typically, the current spatial and temporal coverage of many mesoscale data sets is sparse, and analysis techniques often assume that turbulence is isotropic, homogeneous, and stationary – which is not true of the real ocean.

Structure Functions

Structure functions depend on spatial correlations of variables. They can tell us the type of turbulence, the distribution of energy across scales, and how energy is moved between scales (the spectral fluxes).

There are two particularly useful statistics to diagnose spectral fluxes: **advective** structure functions ($\overline{\delta \boldsymbol{u} \cdot \delta \boldsymbol{A_u}}$ where $\boldsymbol{A_u} = (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u}$; *Pearson et al., 2021*) and **third-order** structure functions ($\overline{\delta u_L (\delta \boldsymbol{u} \cdot \delta \boldsymbol{u})}$)



$$u_L = \boldsymbol{u} \cdot \hat{\boldsymbol{r}}$$
$$\delta\phi = \phi' - \phi$$

The table below shows theoretical relations between several structure functions and the **inverse energy cascade rates** (ϵ_μ) and **downscale enstrophy cascade rates** (η_ν) of 2D turbulence. Similar relations exist for quasi-geostrophic turbulence.

Structure function	Enstrophy cascade η_ν	Inverse energy cascade $\epsilon_\mu > 0$
$\delta \boldsymbol{u} \cdot \delta \boldsymbol{\mathcal{A}_u}$	$\frac{1}{2}\eta_\nu r^2$	$2\epsilon_\mu$
$\delta u_L \delta u_L \delta u_L$	$\frac{1}{8}\eta_\nu r^3$	$\frac{3}{2}\epsilon_\mu r$
$\delta u_L(\delta \boldsymbol{u} \cdot \delta \boldsymbol{u})$	$\frac{1}{4}\eta_\nu r^3$	$2\epsilon_\mu r$
$\nabla_r \cdot [\delta \boldsymbol{u}(\delta \boldsymbol{u} \cdot \delta \boldsymbol{u})]$	$\eta_\nu r^2$	$4\epsilon_\mu$

Grey boxes indicate relations that **do not assume isotropy** in their derivation

What can structure functions tell us about ideal mesoscale dynamics?

GeophysicalFlows.jl (*Constantinou et al., 2021*) was used to simulate forced-dissipative, anisotropic 2D turbulence (Fig. 3). In these simulations, **advective structure functions could diagnose:**

- **Energy cascade rate to large scales (ϵ_μ)**
  (magnitude of structure function)
- **Injection scale of energy (external forcing)**
  (wavenumber of oscillations; perhaps related to *Xie et al., 2021*)
- **Enstrophy cascade rate to small scales (η_ν)**
  (magnitude of alternatively-scaled structure function)

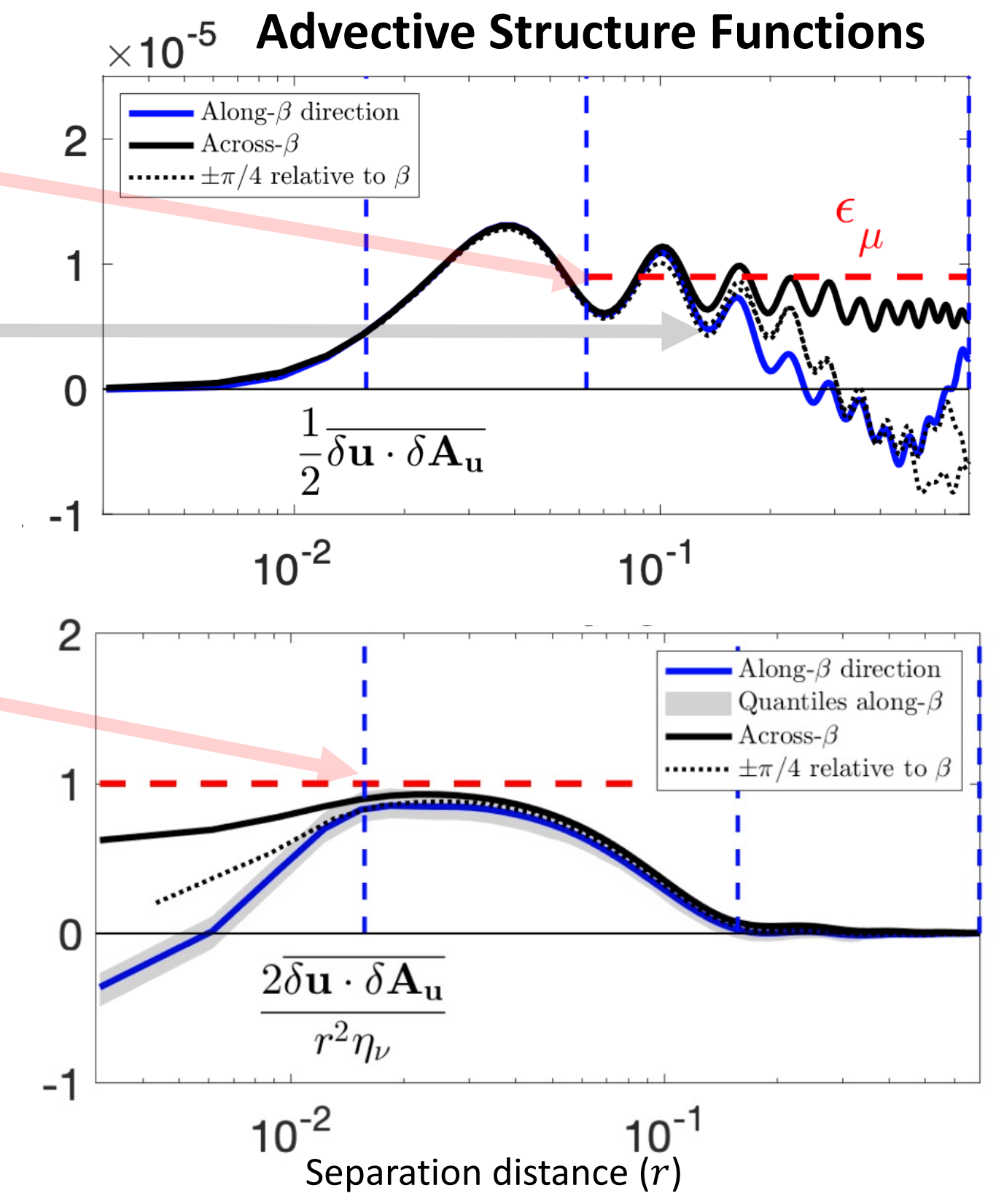


Figure 1. **Top:** Advective structure function versus separation distance (r) scaled to estimate the inverse energy cascade rate (red dashed line shows model-diagnosed cascade rate). Line styles denote different orientations of separation used for the structure function calculation (see Fig. 3). **Bottom:** Advective structure function scaled to estimate the downscale enstrophy cascade rate (red dashed line shows model-diagnosed cascade rate). In both figures the blue dashed lines, from left to right, denote a minimum scale of the enstrophy cascade, the forcing (energy injection) scale, and the largest scales of the inverse kinetic energy cascade respectively.

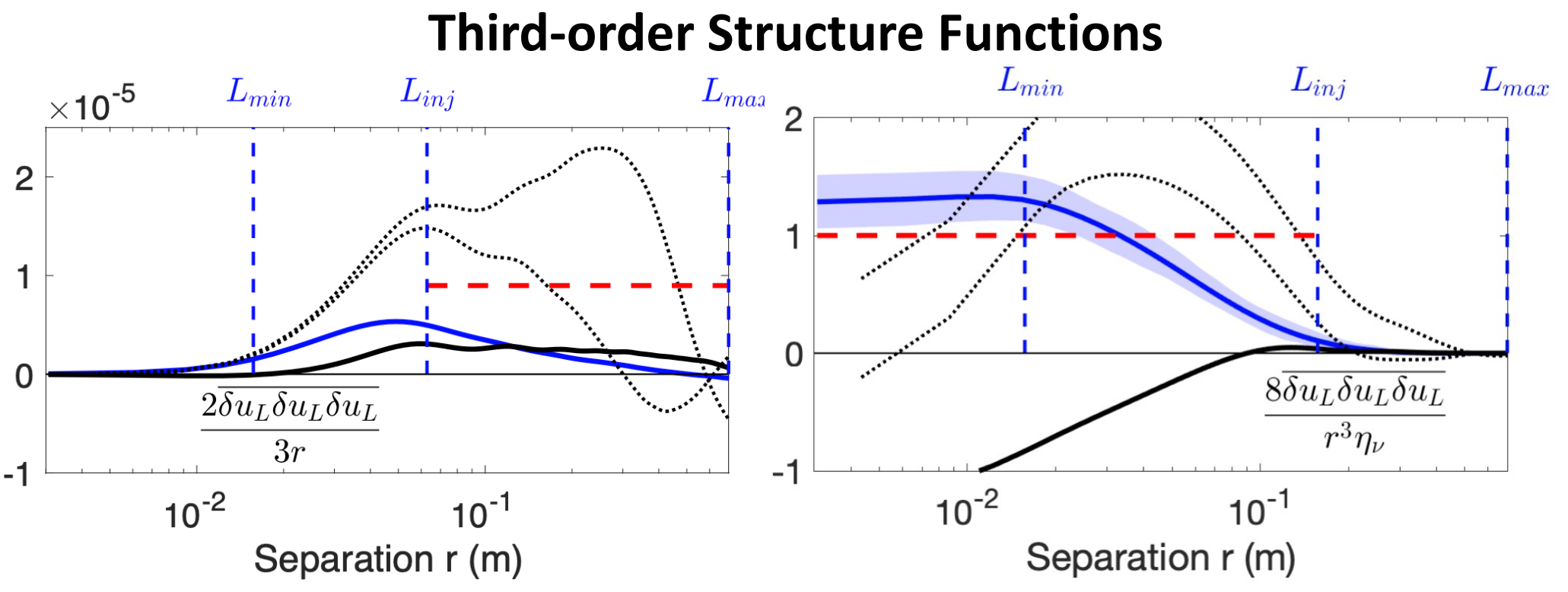


Figure 2. As in Figure 1 but now using traditional third-order structure functions to estimate the cascade rates of energy (left) and enstrophy (right)

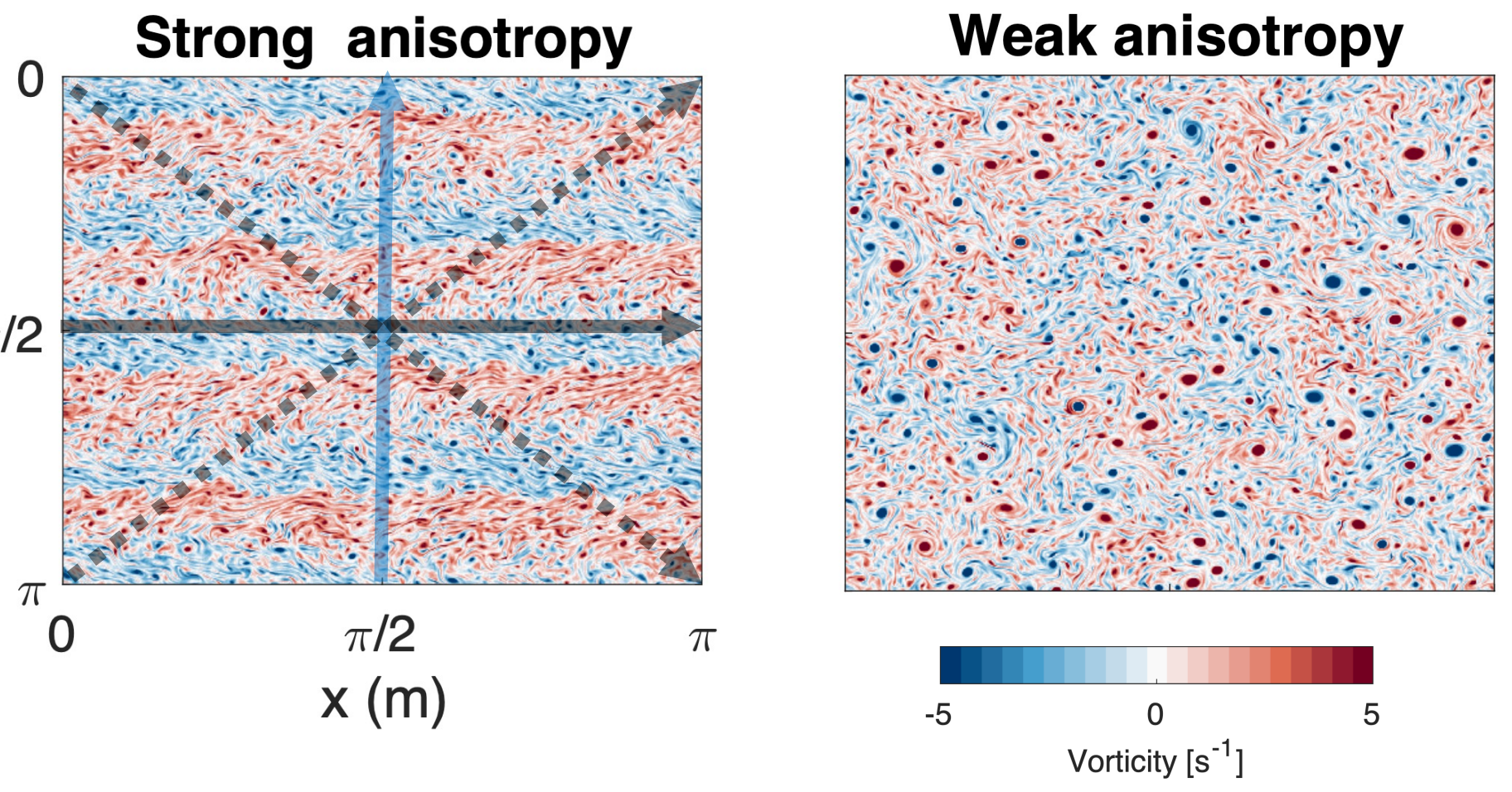


Figure 3. Snapshots of vorticity in simulations of 2D turbulence with varying degrees of anisotropy. Arrows in the left panel denote different structure function calculation directions (see Figs. 1 & 2)

In contrast to ***third-order*** structure functions, new ***advective*** structure functions work even in strongly anisotropic flows and have faster convergence (not shown) due to flow-derivative constraints (vorticity & incompressibility; see Fig. 4)

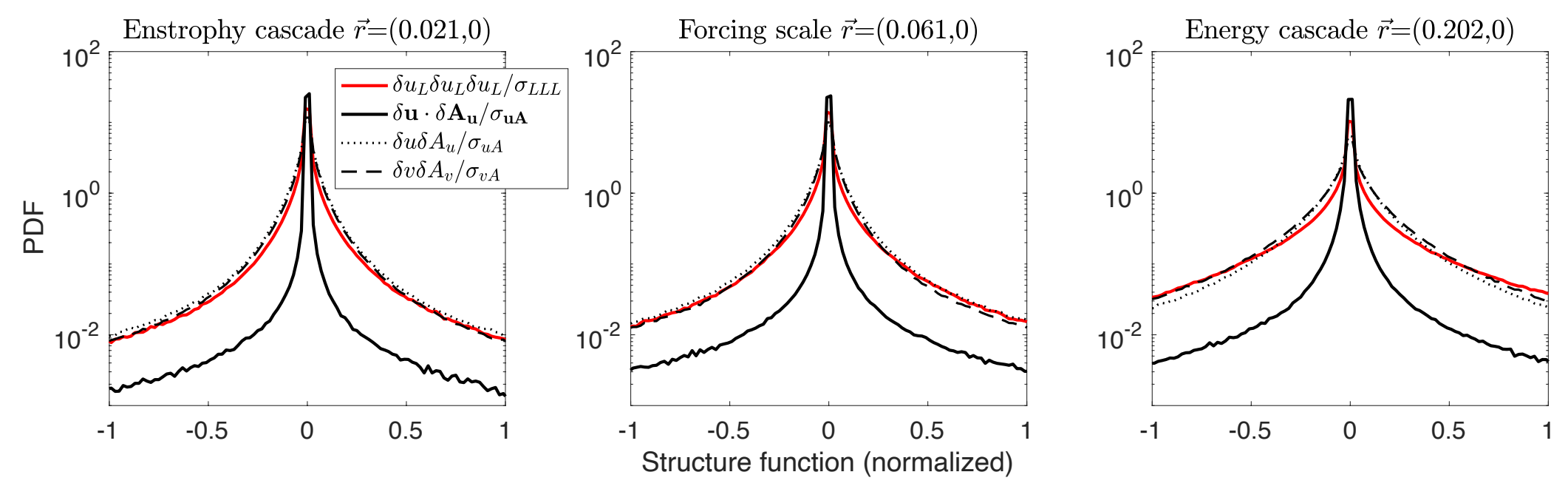


Figure 4. PDFs of structure function values within a snapshot of the weakly anisotropic simulation at three different separation distances associated with the (left) enstrophy cascade, (middle) forcing scale and (right) inverse energy cascade. The intermittency of turbulence leads to comparable, wide-tailed, distributions for the third-order structure function and the two components of the advective structure function. However, the advective structure function does not show as much intermittency because the dynamical constraints on derivatives couples the two components of this term so much of their intermittency cancels.

Some pros and cons of Advective Structure Functions (SFs)

Pros

Advective SFs (Fig. 1) can diagnose:

- Direction & rate of spectral cascade (including overlapping and non-inertial cascades)
- Scale of forcing (energy injection)

They can utilize irregular/gappy data

Unlike third-order SFs, advective SFs converge quickly and can be applied to anisotropic flows

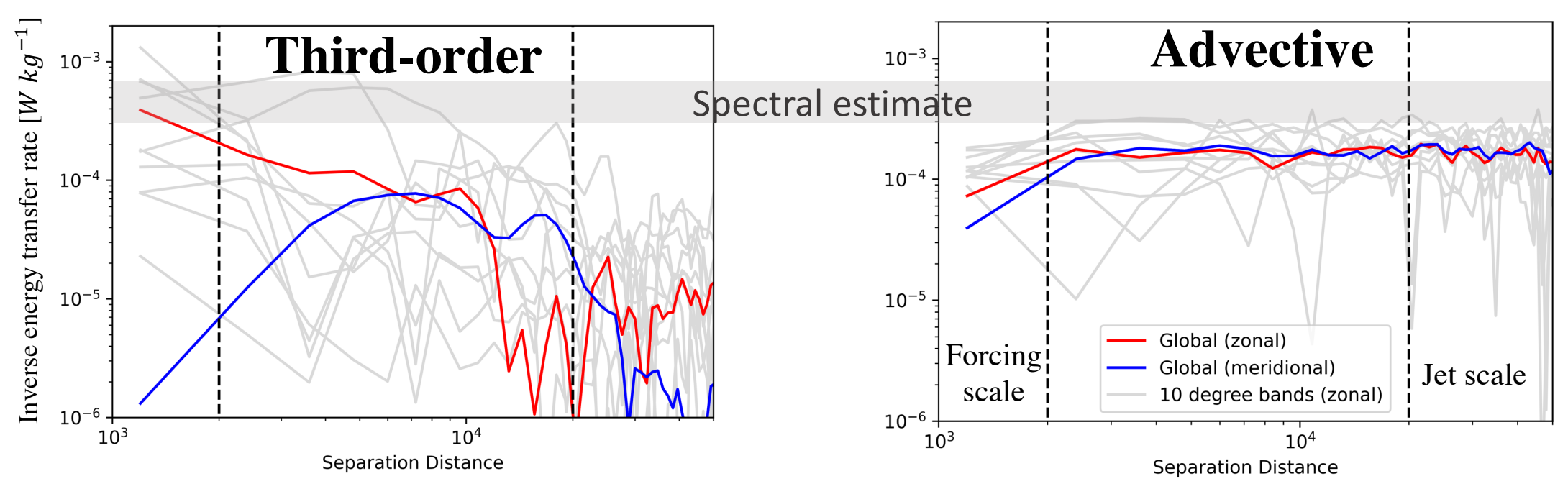
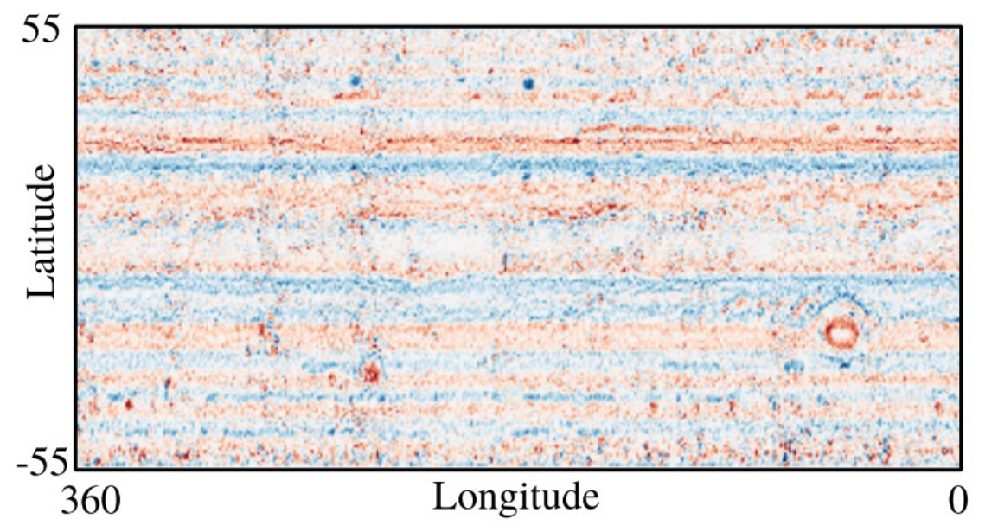


Figure 5. Estimates of the inverse cascade rate of kinetic energy in Jupiter's atmosphere using (left) third-order structure functions and (right) advective structure functions. Statistics were calculated using a satellite (Cassini) day-snapshot of Jupiter's upper atmospheric motion. Red and blue lines show structure functions calculated with zonal and meridional separations respectively. Grey lines show the spatial variability of the zonal structure function (one line for each 10-degree-latitude band). The grey shading denotes a spectrally-derived flux estimate from Young et al., 2017. A snapshot of the vorticity field is shown below.

Cons

Advection term requires local derivatives

Heterogeneity effects are an open question



What about more complex systems?

More realistic systems generally will not exhibit two inertial cascades (inverse and direct) within two distinct ranges of spatial scale. It is likely that cascades **will not be inertial** (i.e., constant flux at all scales) and there **may be multiple properties being cascaded** with distinct cascade dynamics.

Surface quasigeostrophic (SQG) flow provides a convenient tool to incorporate these flow complexities in an idealized geophysical system. We simulate decaying SQG turbulence, where there is an inertial cascade of buoyancy variance from large- to small-scales, representing frontogenesis, and an overlapping inverse cascade of kinetic energy with a scale-varying spectral flux (i.e., **it is not an inertial cascade**)

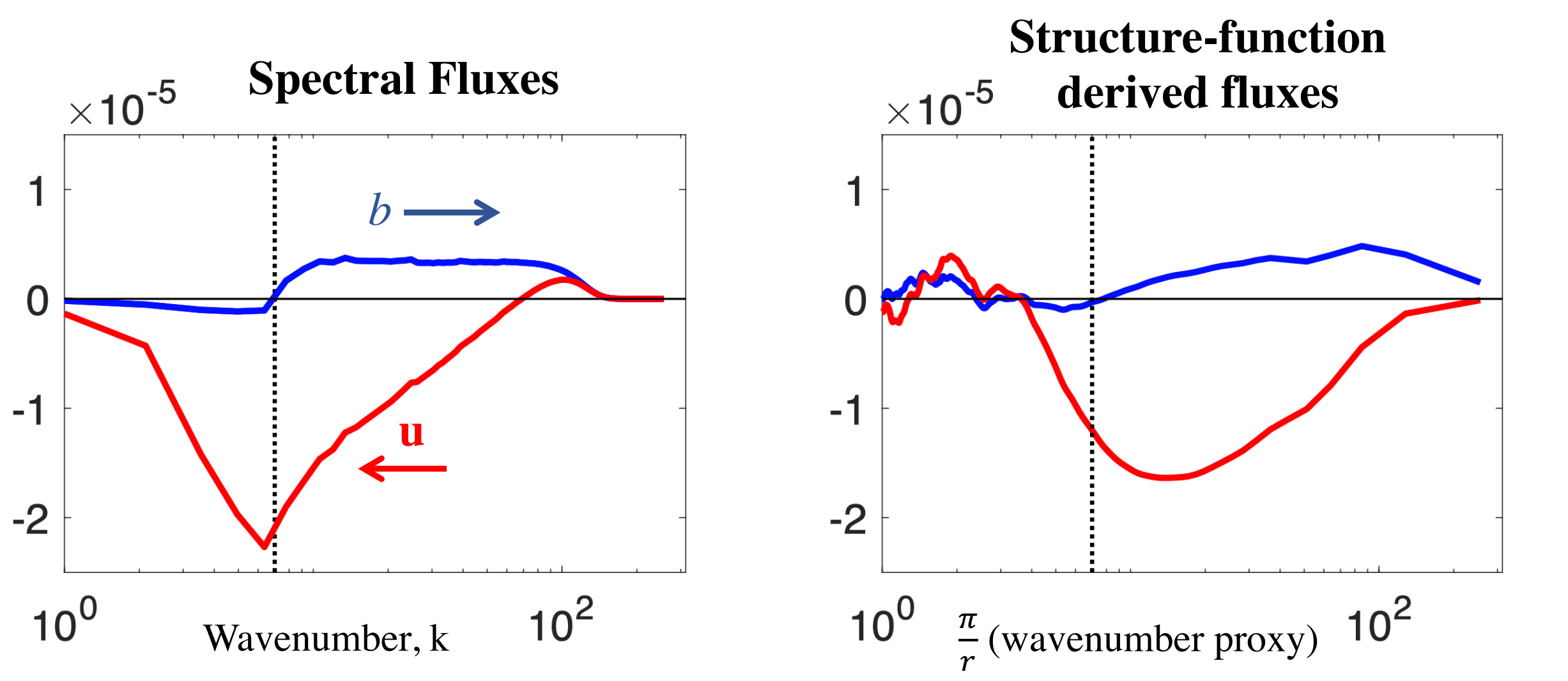


Figure 6. Comparison of (left) spectral fluxes and (right) estimates of these fluxes using structure functions in a decaying SQG turbulence simulation. The structure functions are able to reflect the strength and direction of both cascades simultaneously. It should be noted that the power spectra of velocity variance and buoyancy variance are equivalent to each other in SQG flow, so the red/blue differences above reflect the effect of two different advection operators acting on otherwise identical power spectra.

Conclusions

- An array of structure functions provide useful tools for mesoscale-ocean analysis (we only discussed two structure functions here).
- New **advective** structure functions can be applied to highly anisotropic idealized turbulence to diagnose cascade rates and energy injection scales.
- **Advective** structure functions converge to the correct result faster than **third-order** structure functions.
- Ongoing work to extend these methods for increasingly realistic ocean flows, quantify sampling/convergence properties of different structure functions, apply these tools to various observational data sets, and create an open-source *Python* toolbox for structure function analysis.

References

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