

# The Impact of Sub-Grid Heterogeneity on Air-Sea Turbulent Heat Flux in Coupled Climate Models

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contributed equally to this work.

**Key Points:**

- The influence of sub-grid, defined as ocean mesoscales, heterogeneity on turbulent air-sea heat flux is quantified

## The Impact of Sub-Grid Heterogeneity on Air-Sea Turbulent Heat Flux in Coupled Climate Models

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# Eddy-Mean formalism:

**A:** vector of inputs needed to compute heat flux

$$\partial_t \theta + \mathbf{u} \cdot \nabla \theta = \frac{Q_{net}(\mathbf{A})}{\rho_0 c_p} + \kappa \nabla^2 \theta$$

Filter in time or space (as suited to the problem)

$$\theta = \bar{\theta} + \theta'$$

Mean

Variability/Eddy

$$\partial_t \bar{\theta} + \nabla \cdot (\bar{\mathbf{u}} \bar{\theta}) = \frac{\overline{Q_{net}(\mathbf{A})}}{\rho_0 c_p} + \kappa \nabla^2 \bar{\theta}$$

$$\partial_t \theta' + \nabla \cdot (\mathbf{u} \theta)' = \frac{Q'_{net}(\mathbf{A})}{\rho_0 c_p} + \kappa \nabla^2 \theta'$$

➔  $\partial_t \bar{\theta} + \nabla \cdot (\bar{\mathbf{u}} \bar{\theta}) = \frac{Q_{net}(\bar{\mathbf{A}})}{\rho_0 c_p} + \kappa \nabla^2 \bar{\theta} + (\nabla \cdot (\bar{\mathbf{u}} \bar{\theta}) - \nabla \cdot (\bar{\mathbf{u}} \bar{\theta})) + \left( \frac{\overline{Q_{net}(\mathbf{A})}}{\rho_0 c_p} - \frac{Q_{net}(\bar{\mathbf{A}})}{\rho_0 c_p} \right)$

➔  $\partial_t \bar{\theta} + \nabla \cdot (\bar{\mathbf{u}} \bar{\theta}) = \frac{Q_{net}(\bar{\mathbf{A}})}{\rho_0 c_p} + \kappa \nabla^2 \bar{\theta} + \boxed{\nabla \cdot \mathbf{F}^*} + \boxed{\frac{Q^*}{\rho_0 c_p}}$

Resolvable

Needs parameterization

$$\boxed{\nabla \cdot \mathbf{F}^*}$$

**Almost all ocean sub-grid parameterizations:** Mesoscale eddies (Gent-McWilliams, Solomon-Redi), Submesoscale eddies (Fox-Kemper, Bodner, etc), Boundary layer (KPP, EPBL etc), Shear mixing, Overflows, Bottom boundary layers, Symmetric instability ...

$$\boxed{\frac{Q^*}{\rho_0 c_p}}$$

**Sub-grid air-sea fluxes:** Missing? (Gustiness literature from atmospheric point of view comes the closest to addressing this). Bulk formulae established using MOST and ~1 hour averaging probably don't work for all scales.

- This is a very rich field of study with lot of literature.**
- What drives the variability of SST? (... Bishop et al 2017, Small et al 2020)
  - What drives the variability of heat fluxes? (... Small et al 2019)
  - Impact of heat flux variability on ocean energetics? (... Bishop et al 2020, Guo et al 2022)
  - ...

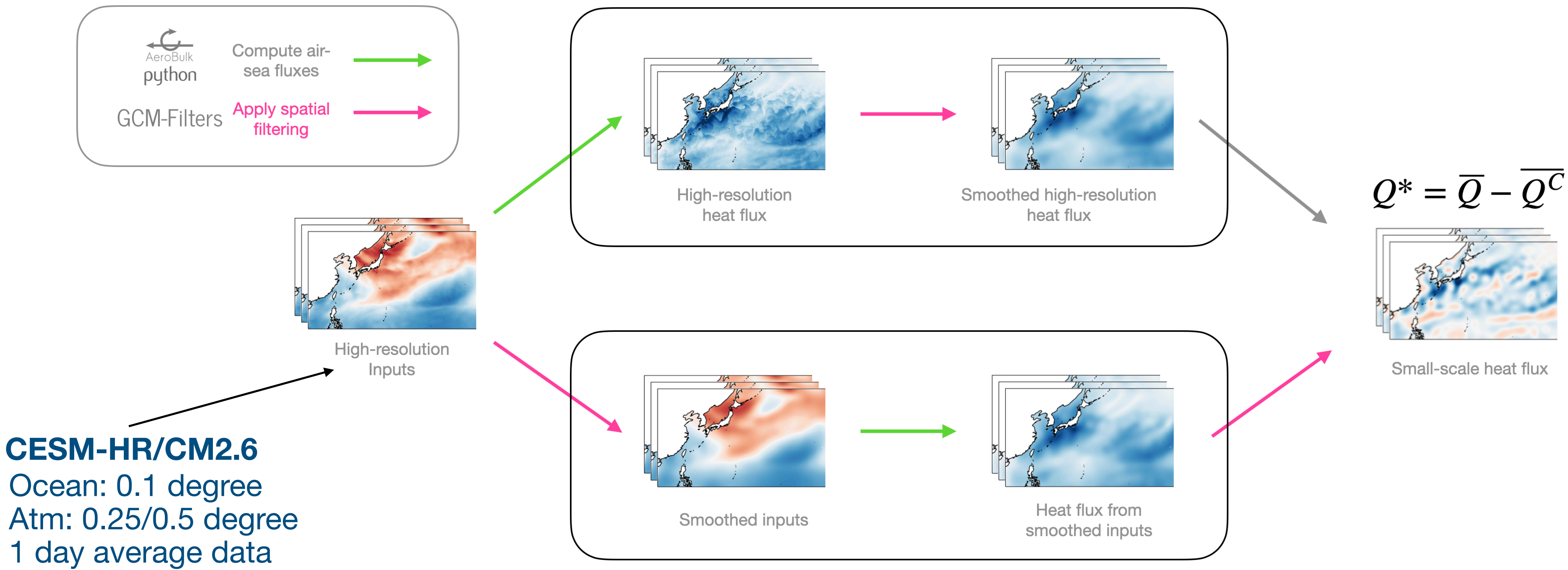
Note:  $Q^* \neq Q'$

We have a lot of HR coupled model data.  
What if we just diagnose  $Q^*$  from it?

$\overline{(\cdot)}$  • Gaussian filter -> roughly match spectra from  
• a low res model, removes mesoscale eddies.

$$Q_{turb}^* = \overline{Q_{turb}(\mathbf{A})} - \overline{Q_{turb}(\overline{\mathbf{A}})}$$

$$\mathbf{A} = [\mathbf{u}_o, \mathbf{u}_a, \theta_o, \theta_a, q_a, p_a]$$



In our thinking there is a big (buried) assumption (for this work to have any usefulness) that the bulk flux formulae are appropriate to use at the grid of the HR coupled model.

However, this is no worse than assuming that bulk formulae are applicable in coarse coupled model; in fact maybe it is less worse.

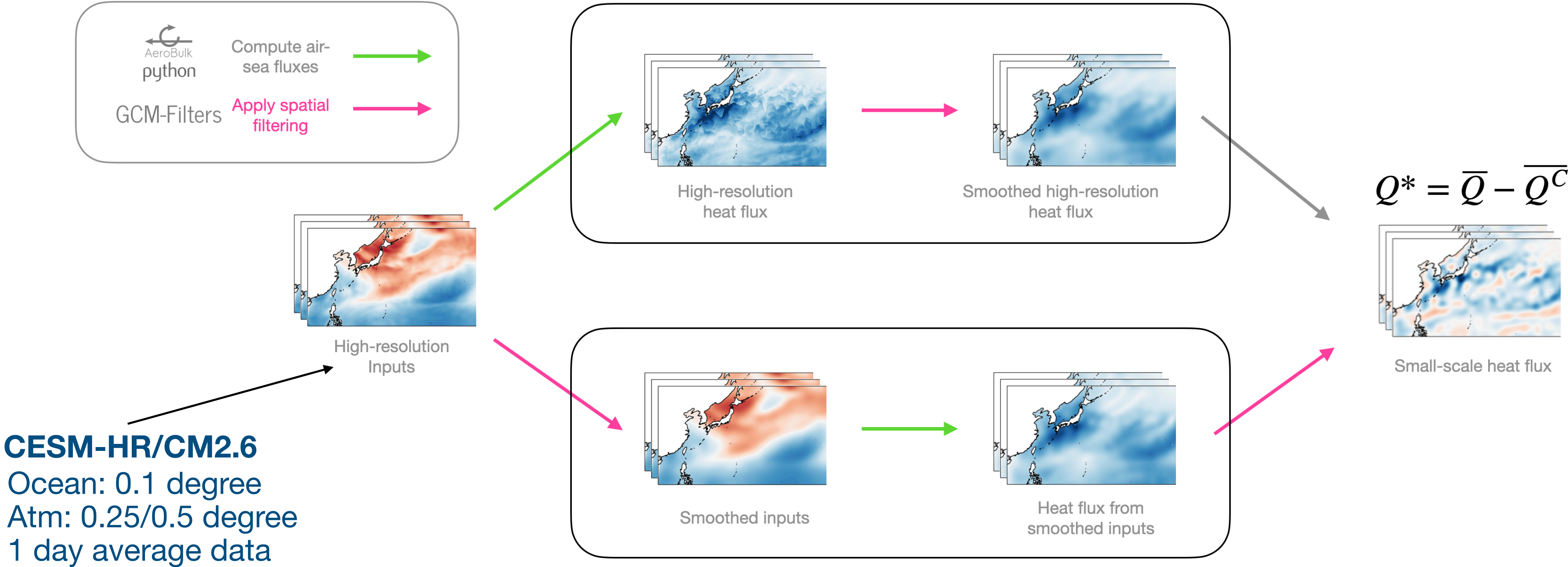


We have a lot of HR coupled model data.  
What if we just diagnose  $Q^*$  from it?

Double averaging, like  
dealiasing of spectral methods

$$Q_{turb}^* = \overline{Q_{turb}(\mathbf{A})} - \overline{Q_{turb}(\overline{\mathbf{A}})}$$

$$\mathbf{A} = [\mathbf{u}_o, \mathbf{u}_a, \theta_o, \theta_a, q_a, p_a]$$



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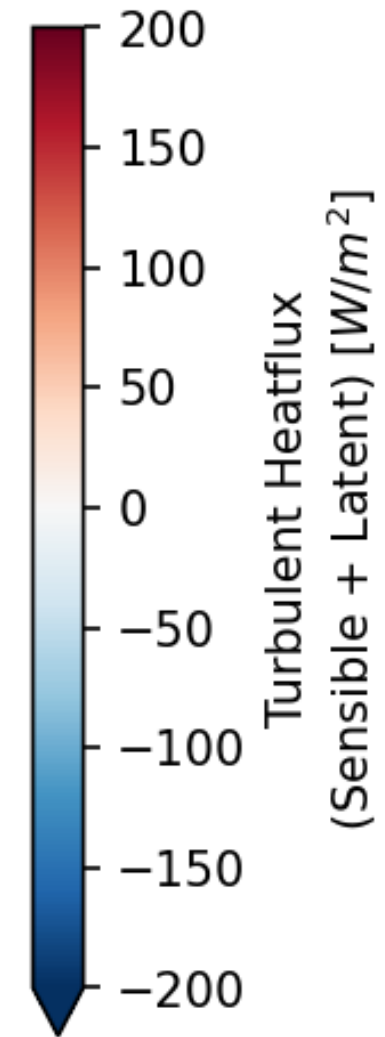
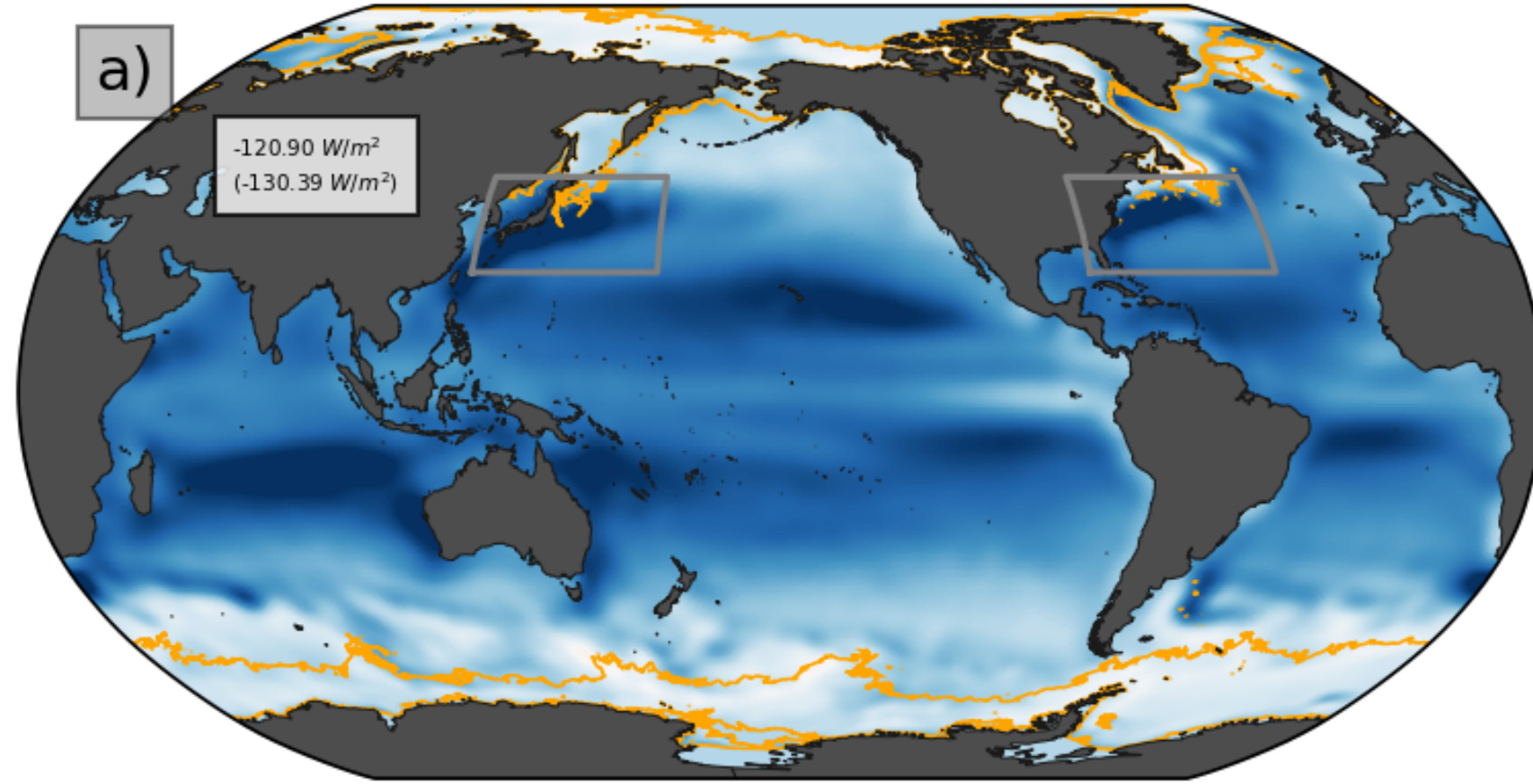
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# 20 Year Average $Q^*$

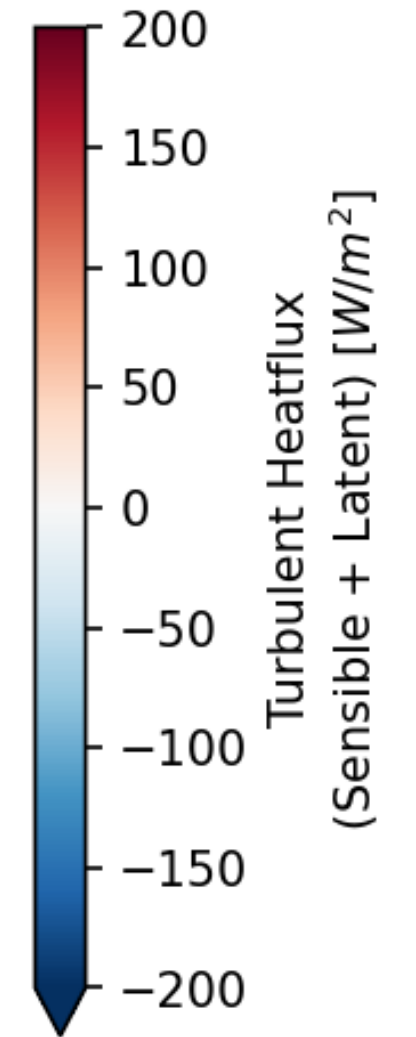
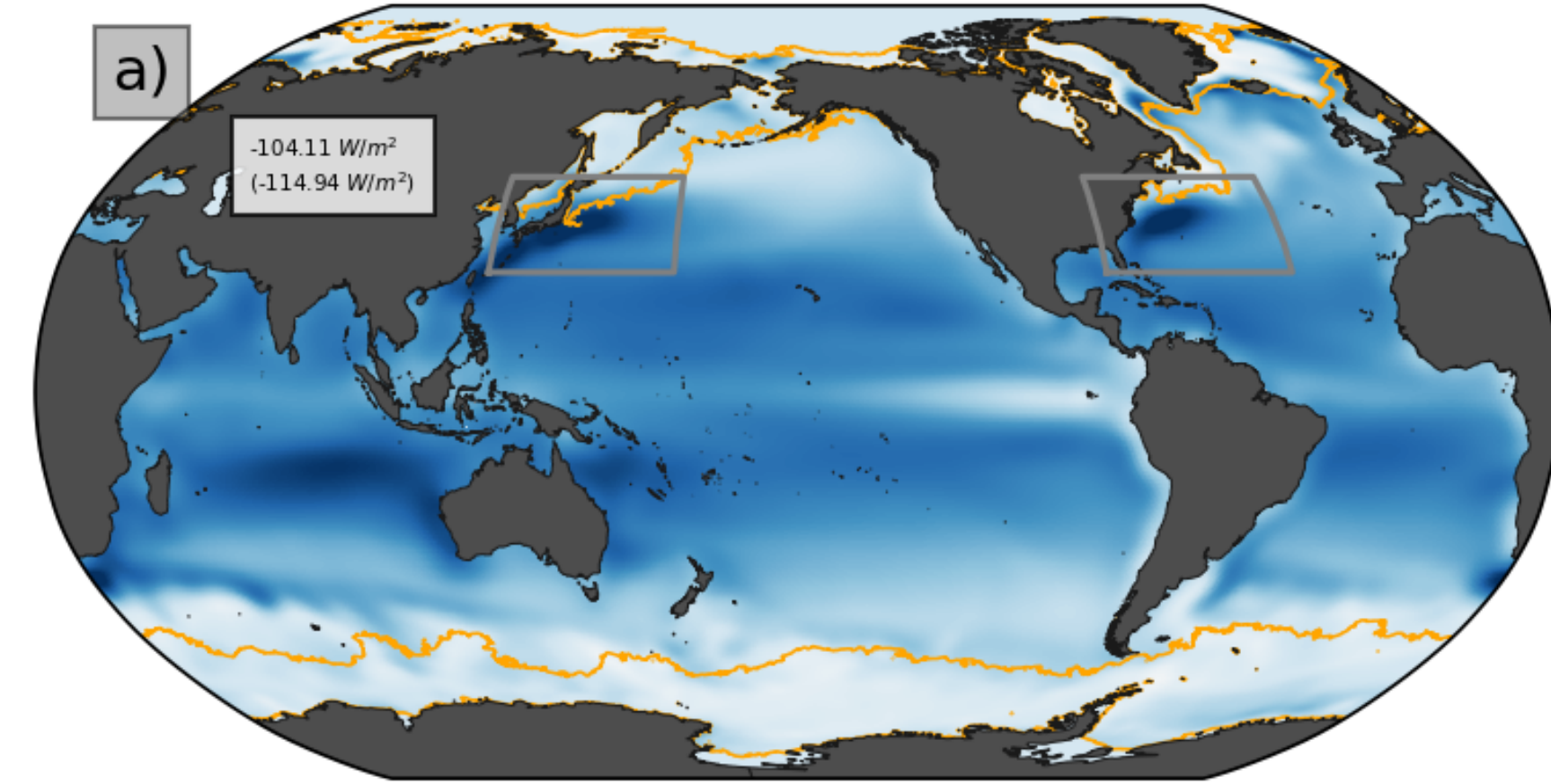
CESM-HR

Large Scale Flux  $\overline{Q^C}$

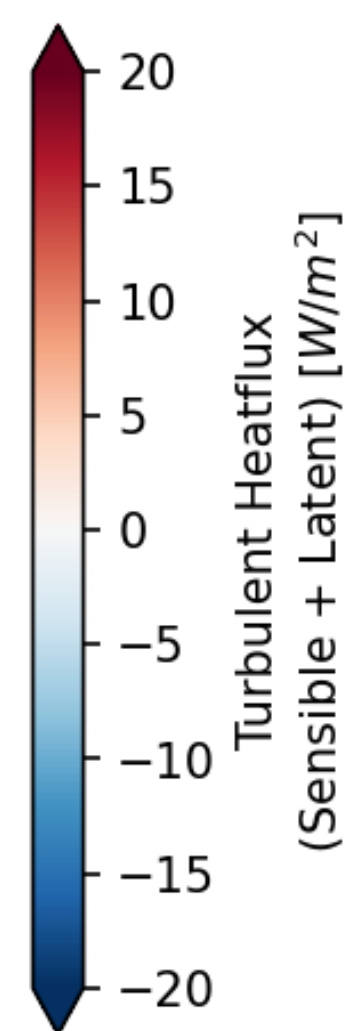
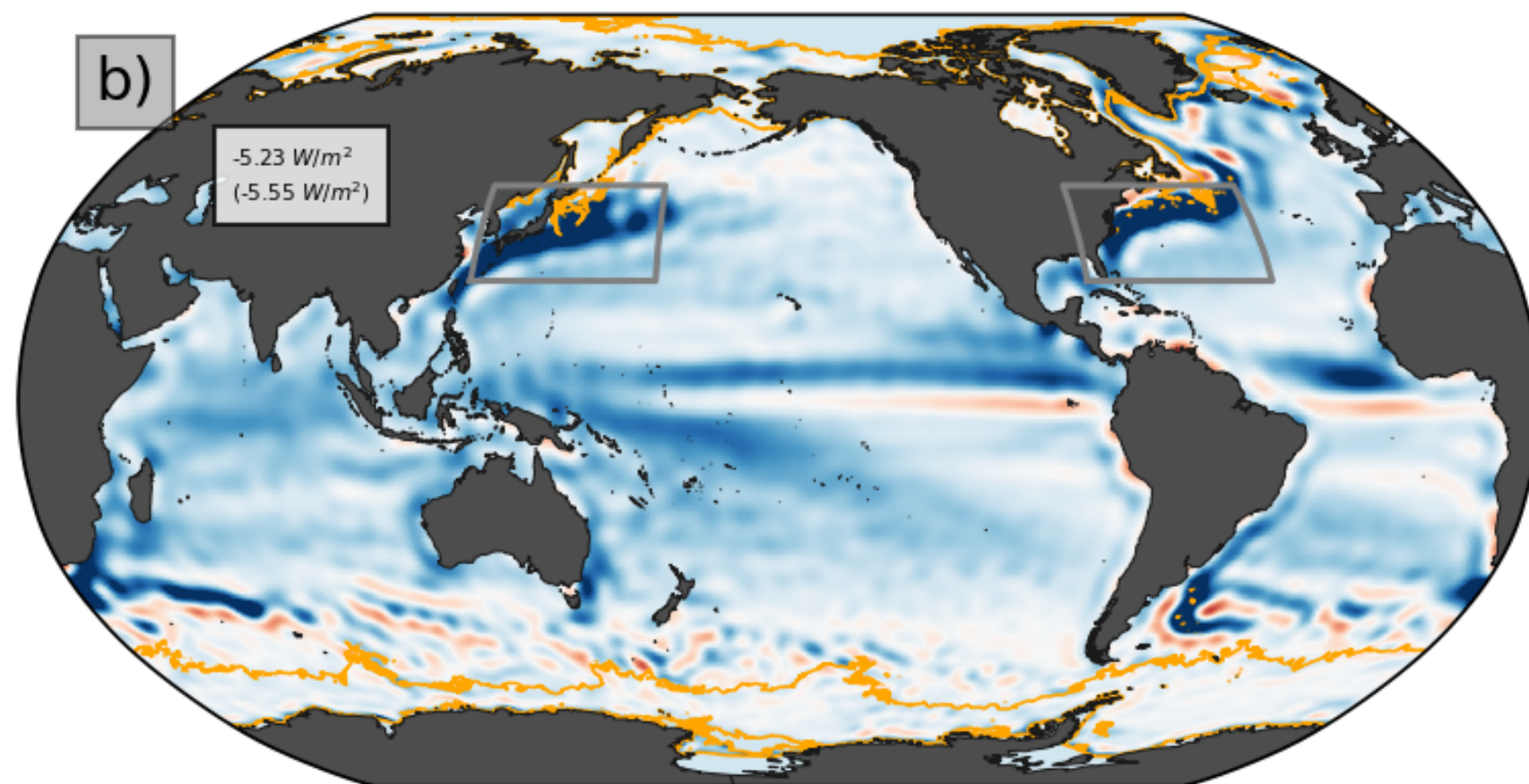


CM2.6

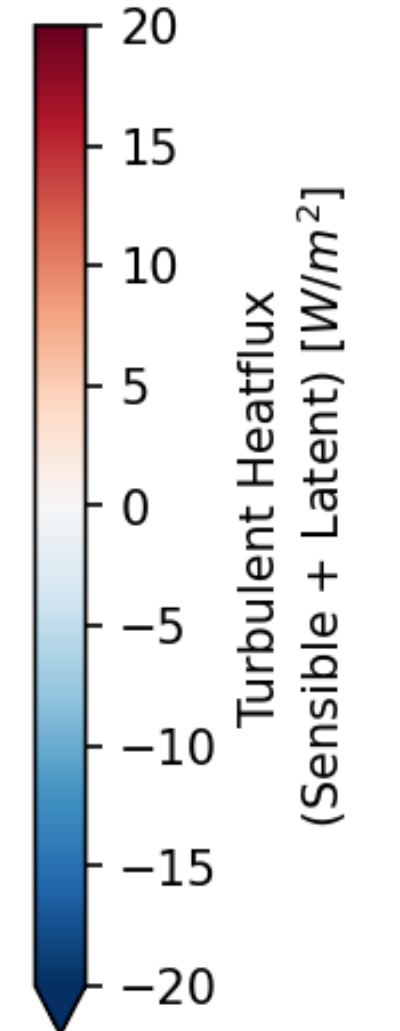
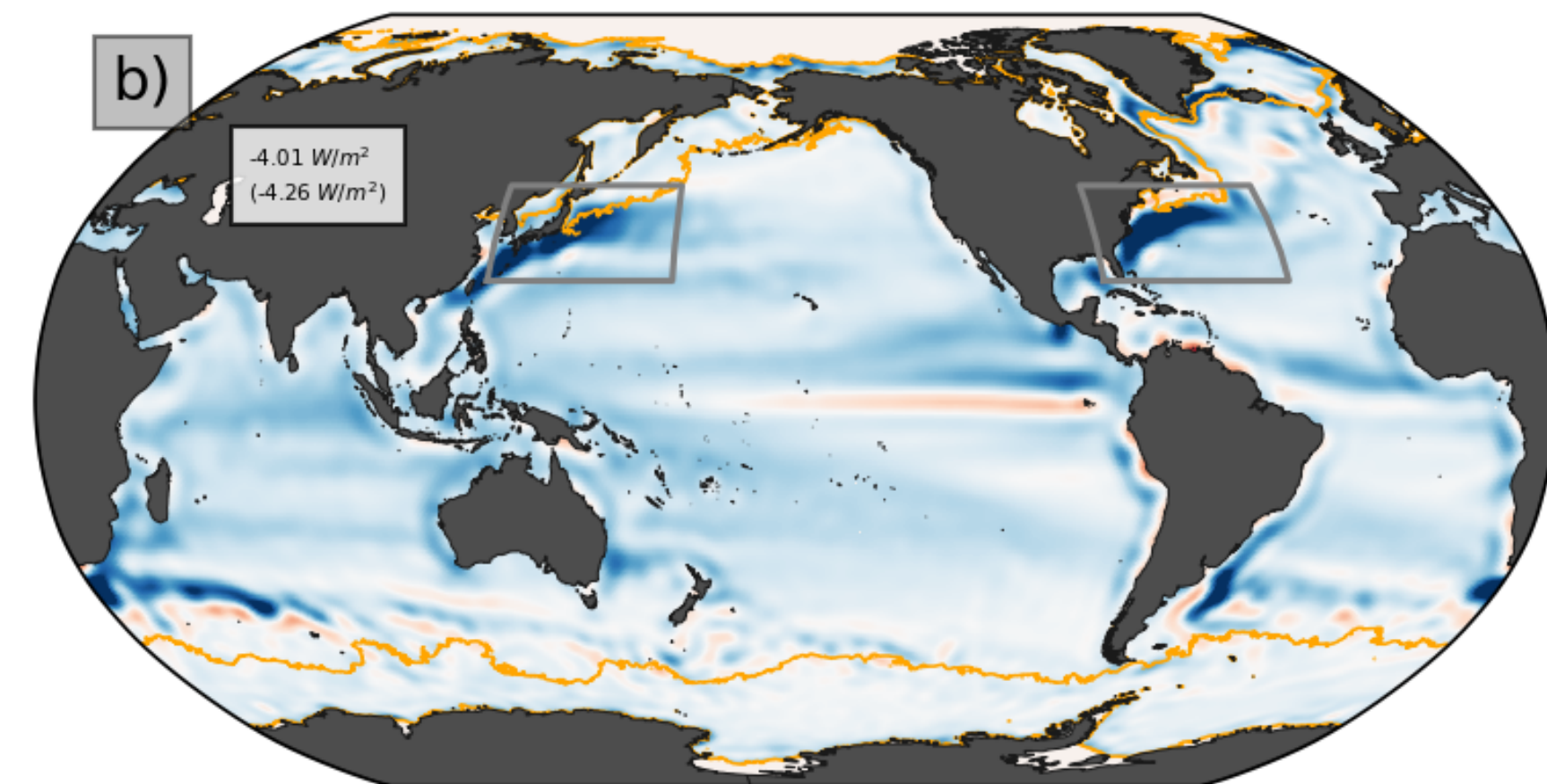
Large Scale Flux  $\overline{Q^C}$



Small Scale Flux  $Q^*$



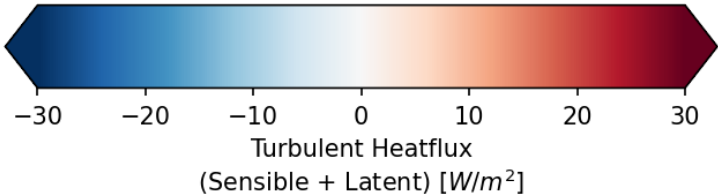
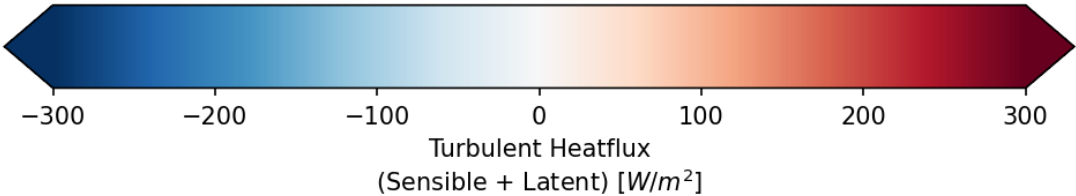
Small Scale Flux  $Q^*$



- In long averages  $Q^*$  10-20% of the large scale flux.
- Both models produce similar results.



# Some daily snapshots

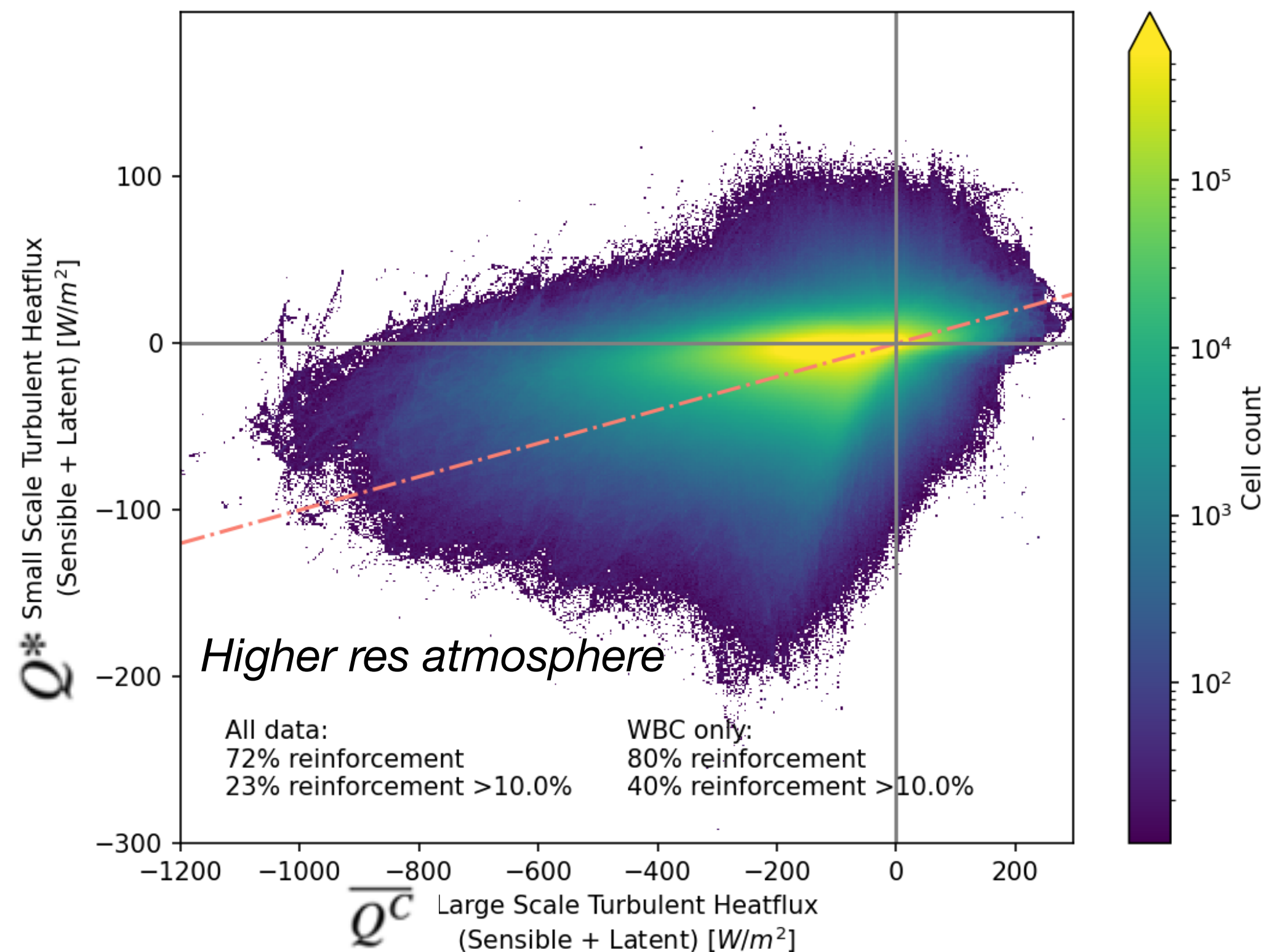


CM2.6

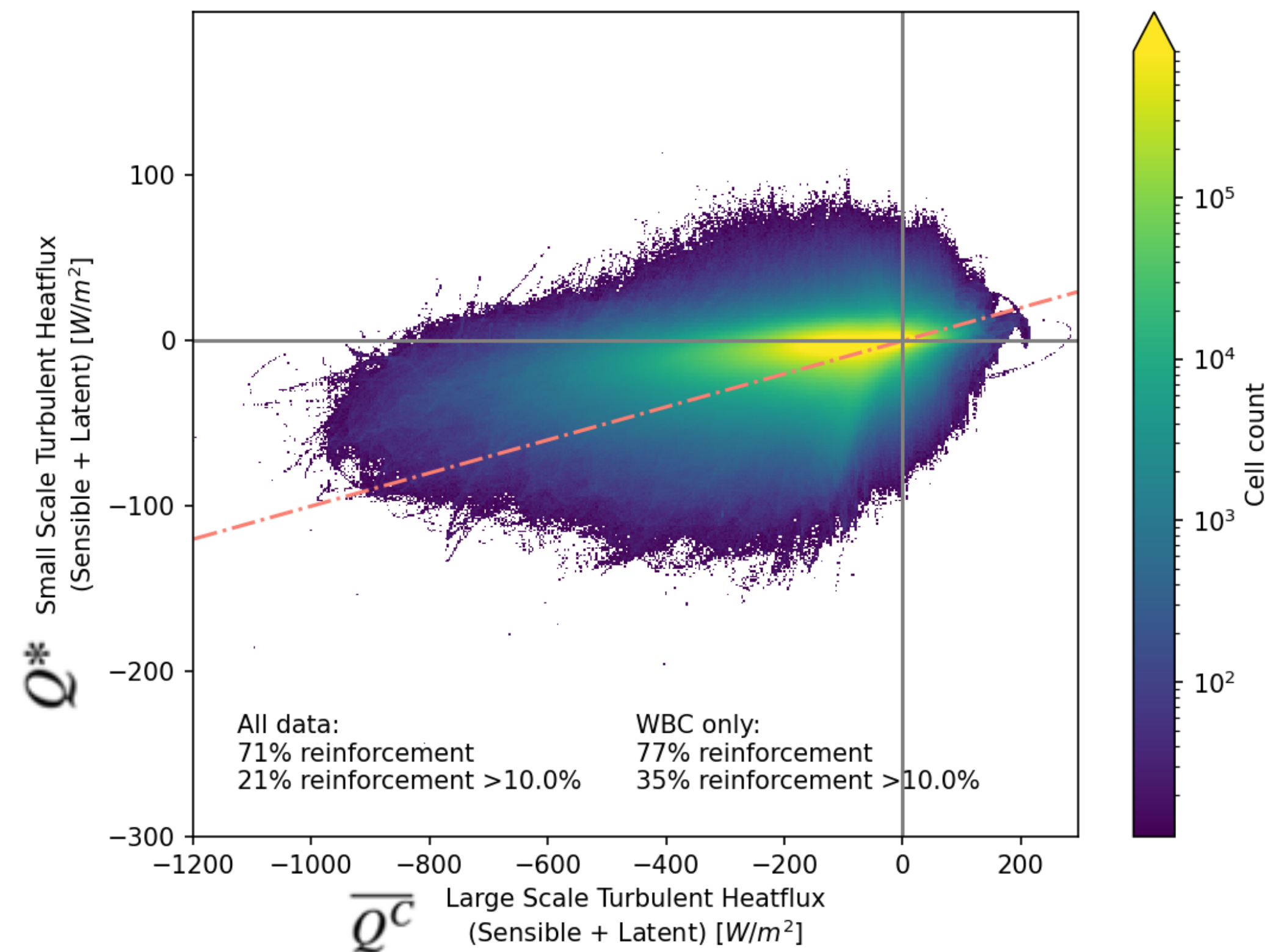


# Daily $Q^*$

CESM-HR



CM2.6

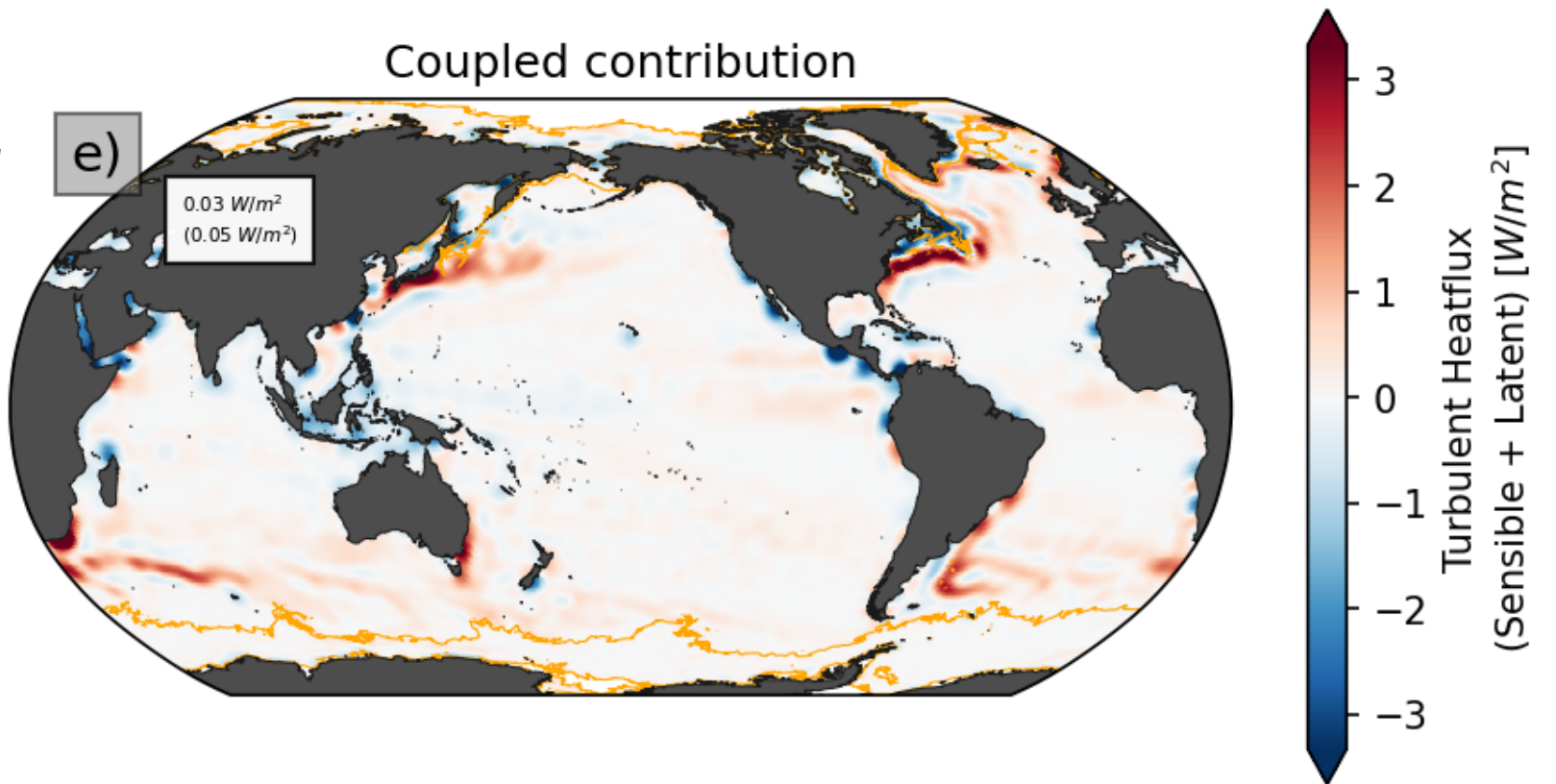
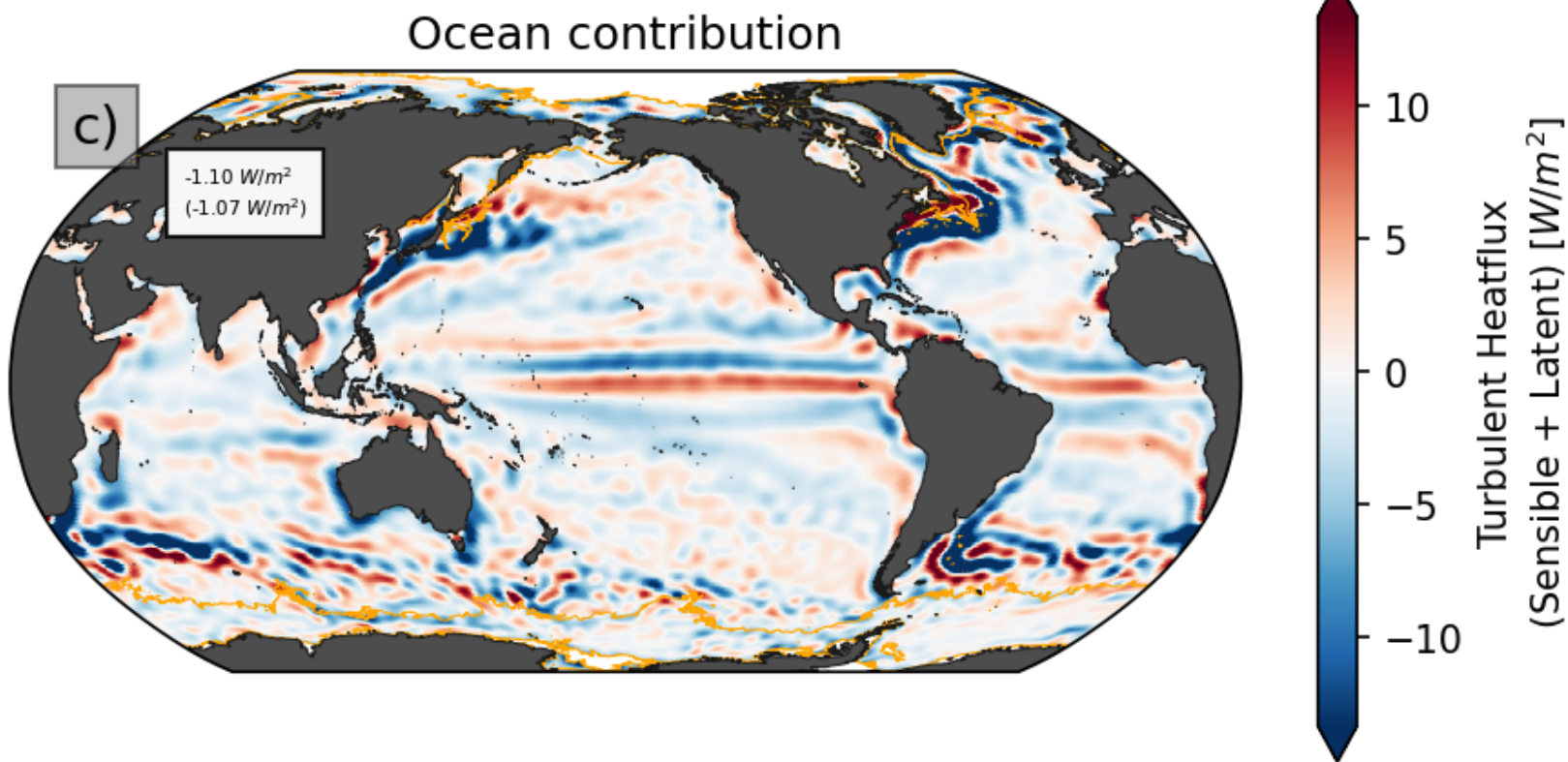
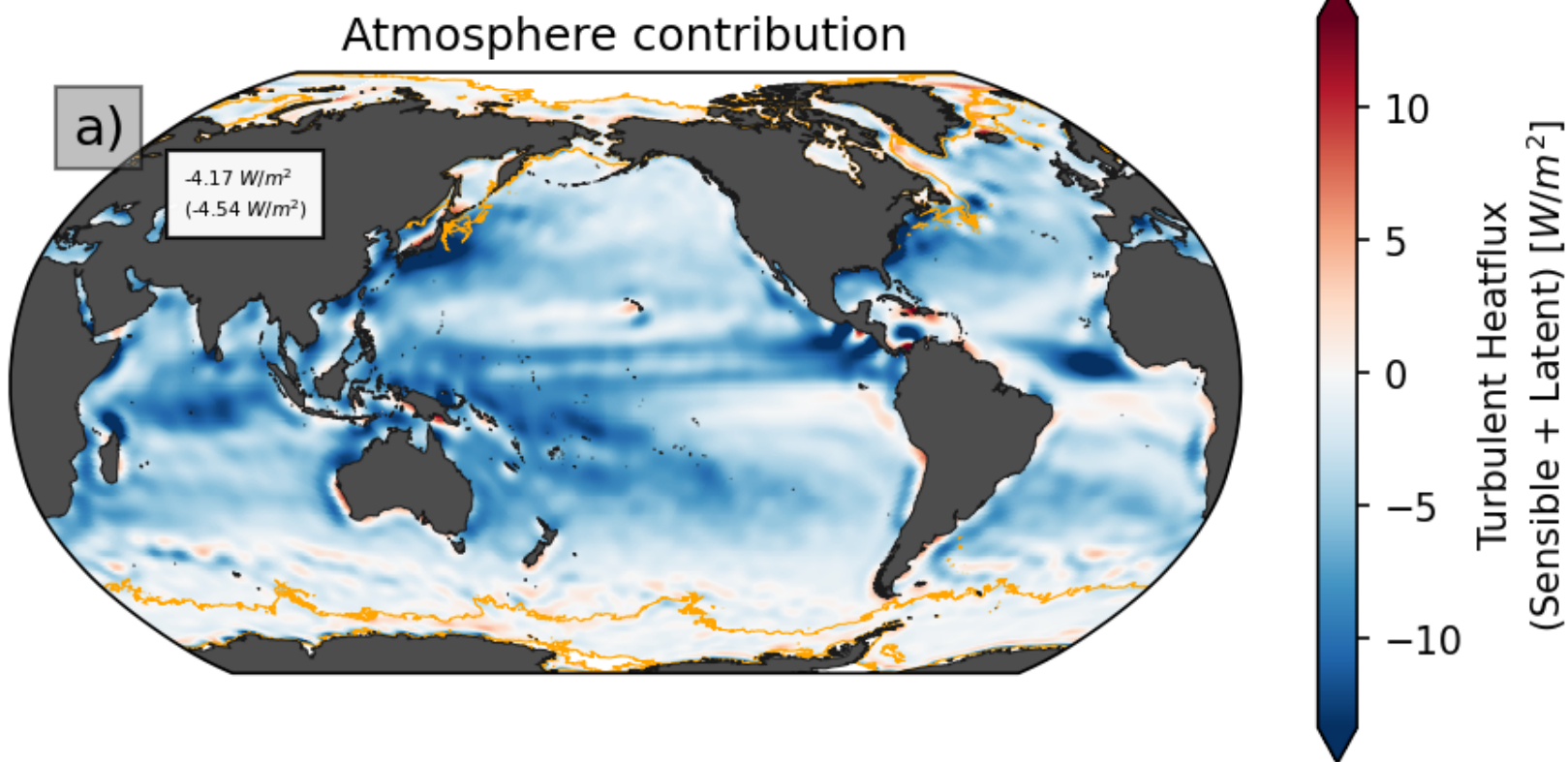
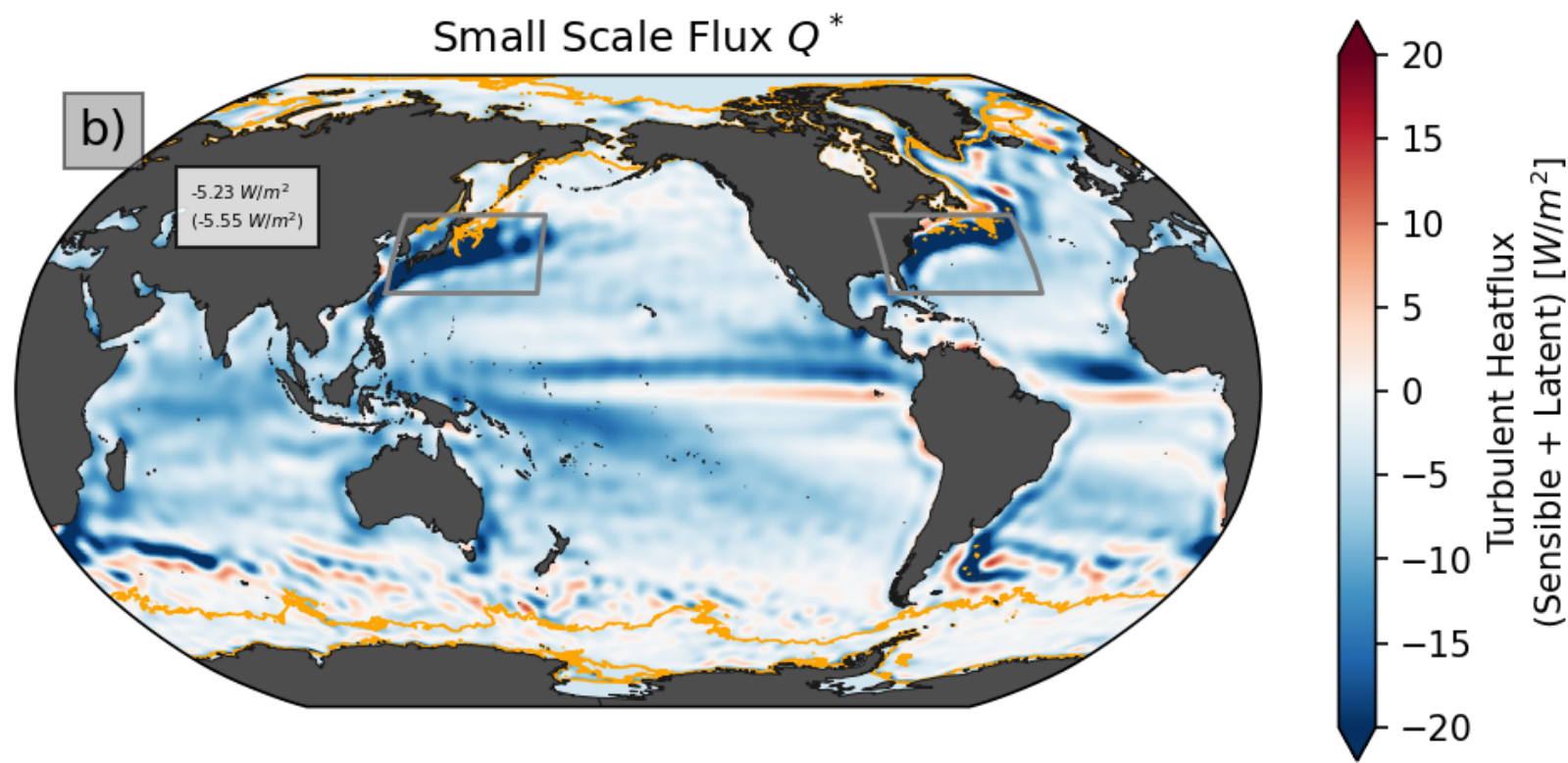


- Generally  $Q^*$  goes in the same direction as flux computed from coarse variables.
- $Q^*$  can get quite large in many situations (e.g. WBCs), fluxes  $O(100 W/m^2)$ .



What contributions come from different fluids?

20 Year Averages



Total sub-grid

$$Q^* = \overline{Q(\mathbf{A}_o, \mathbf{A}_a)} - \overline{Q(\overline{\mathbf{A}_o}, \overline{\mathbf{A}_a})}$$

Atmospheric sub-grid

$$Q^{*,a} = \overline{Q(\mathbf{A}_o, \mathbf{A}_a)} - \overline{Q(\mathbf{A}_o, \overline{\mathbf{A}_a})}$$

Oceanic sub-grid

$$Q^{*,o} = \overline{Q(\mathbf{A}_o, \mathbf{A}_a)} - \overline{Q(\overline{\mathbf{A}_o}, \mathbf{A}_a)}$$

Residual/“coupled” sub-grid

$$Q^{*,coupled} = Q^* - Q^{*,a} - Q^{*,o}$$

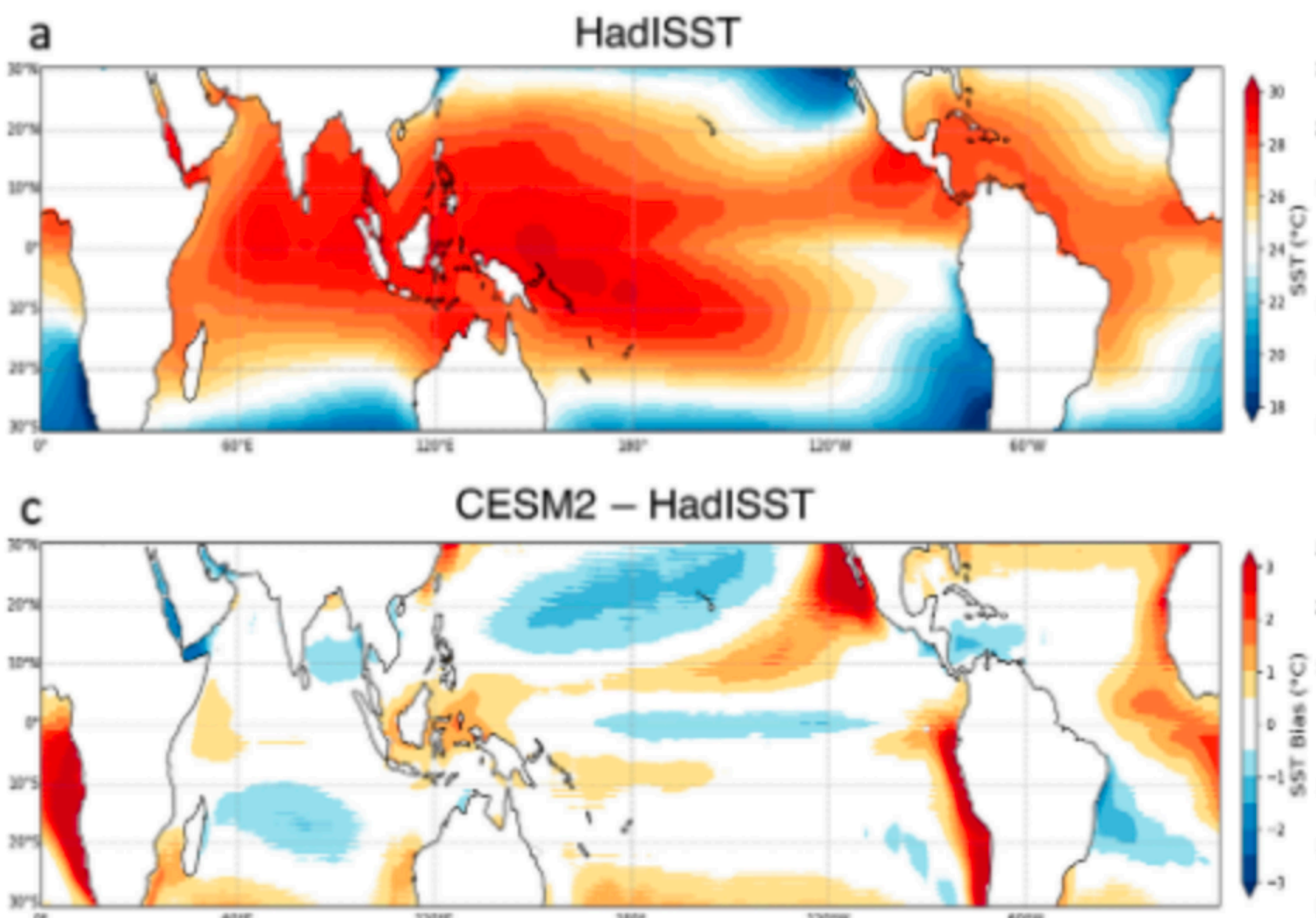


# Is this important?

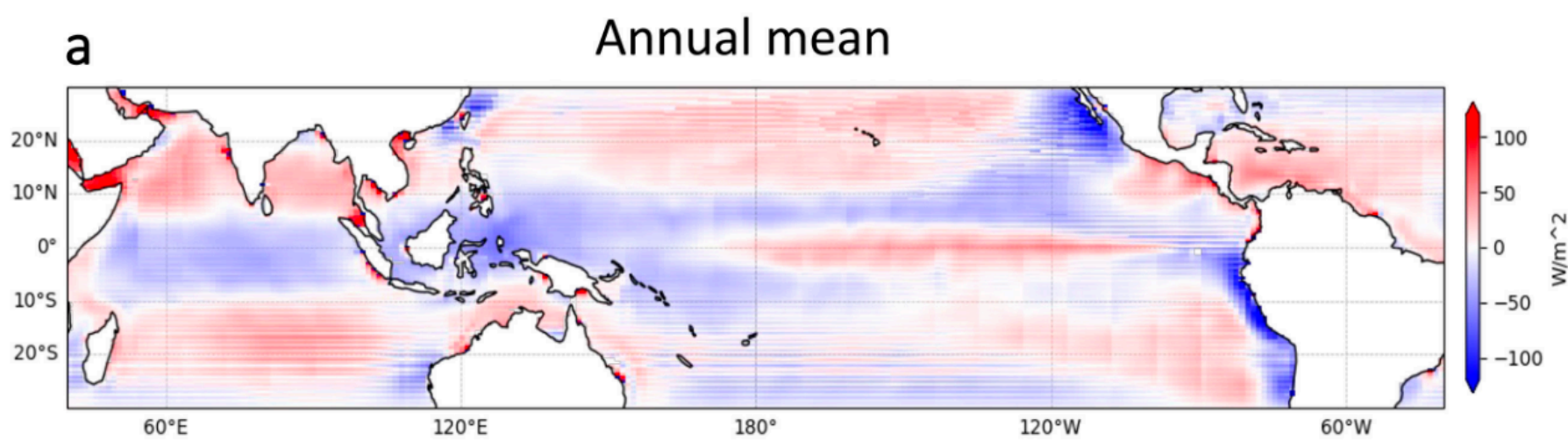
Probably depends on who is asking.

- $O(100\text{W/m}^2)$  fluxes seem important, atleast for regional aspects.
- Oceanic contribution seems to project well onto mean biases.

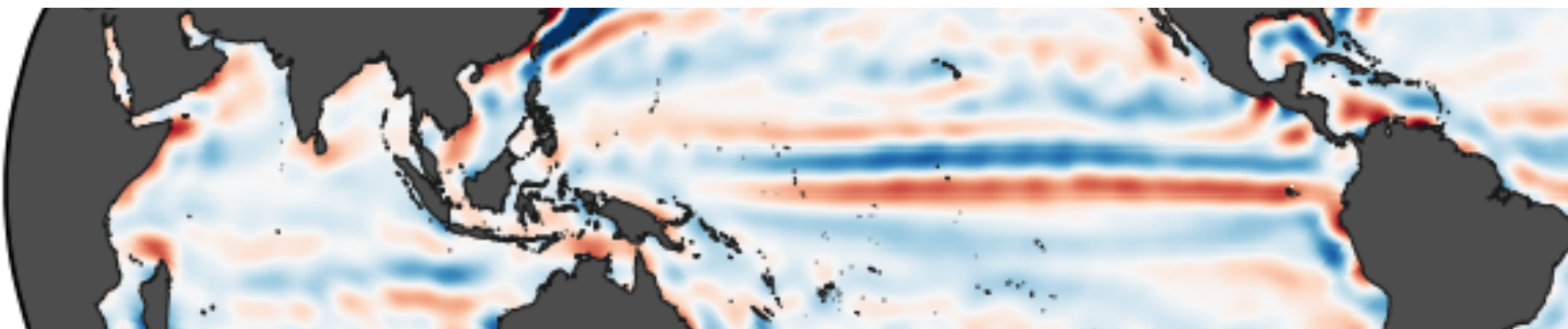
Model bias



Flux correction needed to reduce bias



Our diagnosed mean sub-grid flux patterns



# What's next?

- Do results change as we increase ocean model resolution?
- Can we figure out a way to represent  $Q^*$  only in terms of large scale variables? / Build a parameterization?



Prani Nalluri  
PhD Student

$$Q^* = \overline{Q(\mathbf{A}_o, \mathbf{A}_a)} - \overline{Q(\overline{\mathbf{A}_o}, \overline{\mathbf{A}_a})} \approx f_\theta(\overline{\mathbf{A}_o}, \overline{\mathbf{A}_a})$$

## Data-Driven Equation Discovery of Ocean Mesoscale Closures

Laure Zanna<sup>1,2</sup>  and Thomas Bolton<sup>2</sup> 

## Stochastic-Deep Learning Parameterization of Ocean Momentum Forcing

Arthur P. Guillaumin<sup>1</sup>  and Laure Zanna<sup>1</sup> 

## A Data-Driven Approach for Parameterizing Ocean Submesoscale Buoyancy Fluxes

Abigail Bodner<sup>1</sup>, Dhruv Balwada<sup>2</sup>, and Laure Zanna<sup>1,3</sup>

Design and implementation of a data-driven parameterization for mesoscale thickness fluxes

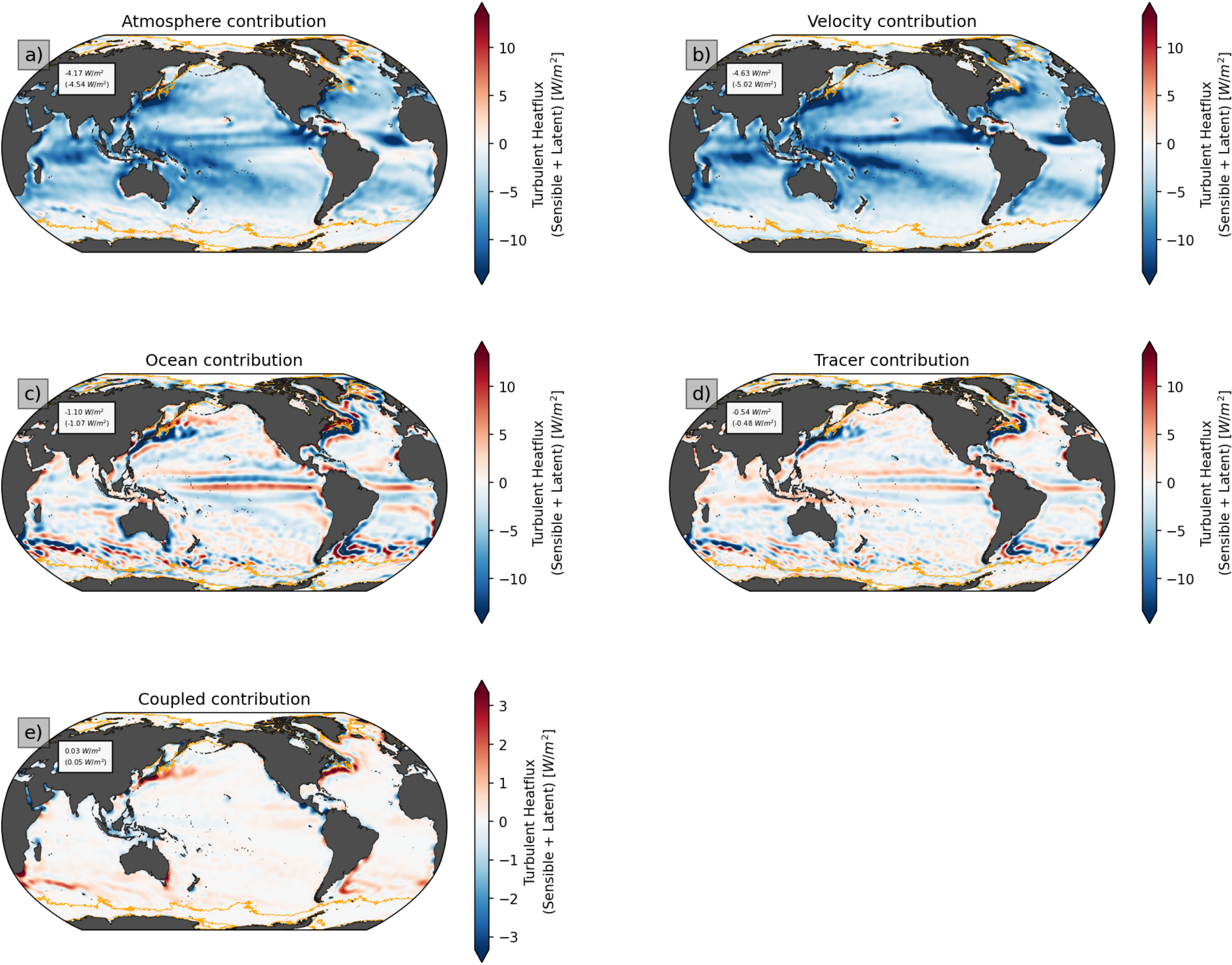
Dhruv Balwada<sup>1</sup>, Pavel Perezhogin<sup>2</sup>, Alistair Adcroft<sup>3</sup>, and Laure Zanna<sup>4</sup>



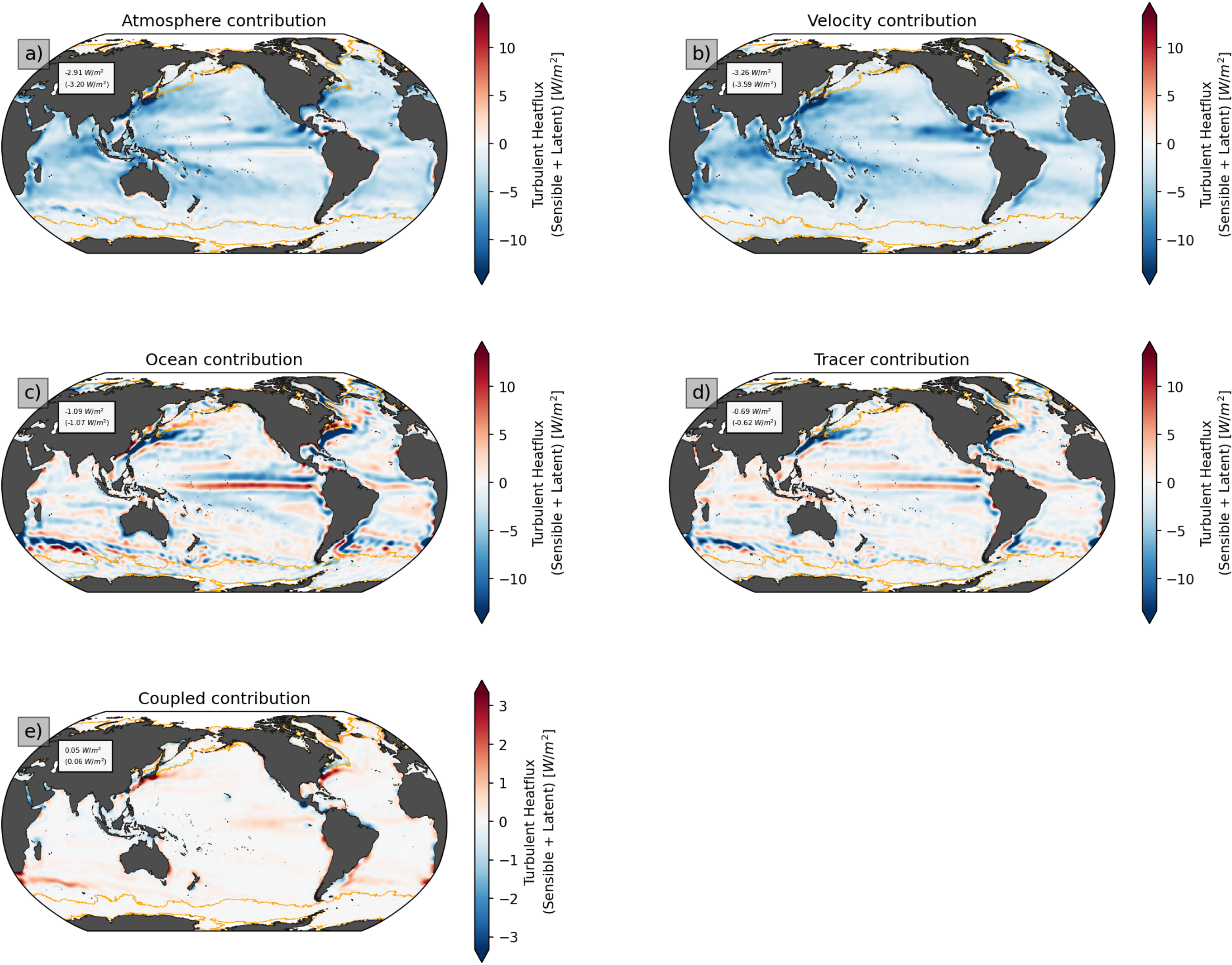
# Summary:

- The influence of sub-grid, defined as ocean mesoscales, heterogeneity on turbulent air-sea heat flux is quantified. Often sub-grid variability enhances air-sea fluxes.
- This effect systematically cools the ocean by about 4 W/m<sup>2</sup> in the global, with large O(100)W/m<sup>2</sup> spatiotemporal variations -> May be quite relevant regionally
- Some of the patterns of sub-grid heat fluxes project appropriately on to the model biases
- Atmospheric contribution from wind heterogeneity cools the ocean, while oceanic contribution from sea surface temperature heterogeneity can heat or cool
- ML or other approaches may provide appropriate ways to upscale bulk formulae fluxes.
  - Not uncommon: Fox-Kemper et al submesoscale restratification parameterization was derived for a single front (~O(1)km), and then upscaled to coarse model grid based on observed density spectra.
- Should bulk flux averaging time scales be recalibrated?

**Extras**







### 5.1. Some intuition into partial filtering

To provide a some intuition into what partially filtering into ocean/atmos or tracer/vel means, it is helpful to consider a simpler non-linearity of the type  $N = AB$  (instead of the full bulk formula  $Q$ ). Here  $A$  and  $B$  may be any two model variables, for example velocity and temperature. According to our notation we have  $N^c = \overline{A} \overline{B}$  is the large-scale non-linearity response, and  $N^* = \overline{N} - \overline{N^c} = \overline{AB} - \overline{\overline{A} \overline{B}}$  is the contribution of small-scale variability. Note that  $N^* \neq \overline{A'B'}$  (using  $A = \overline{A} + A'$ ), since the filter we use is not a Reynold's decomposition and also includes Leonard and cross terms (Germano, 1986). We may define partially averaged contributions as  $N^{*,A} = \overline{AB} - \overline{\overline{A} B}$  and  $N^{*,B} = \overline{AB} - \overline{\overline{A} \overline{B}}$ . So we get:  $N^{*,A} = \overline{A'B} + \overline{A'B'}$ ,  $N^{*,B} = \overline{AB'} + \overline{A'B'}$ , and  $N^* = \overline{AB'} + \overline{A'B} + \overline{A'B'}$ . Hence  $N^{*,A-B} = N^* - N^{*,A} - N^{*,B} = -\overline{A'B'}$ , which comes purely from the small-scale correlations. Also note that both  $N^{*,A}$  and  $N^{*,B}$  have  $\overline{A'B'}$  in them.

In the context of heat flux formulae,  $N^{*,A-B} = -\overline{A'B'}$  (which is  $Q^{*,O-A}$ ) roughly corresponds to the part of the heat flux that results over an oceanic warm filament due to the anomalously warmed atmosphere or anomalously fast winds above it, rather than the large anomalous cooling flux of the warm filament that might take place just because of a large-scale cool atmosphere above the filament (which would be accounted for by  $Q^{*,O}$ ). In fact, note that the impact of this correlated part (equivalent to  $\overline{A'B'}$ ) would be present in both  $Q^{*,O}$  and  $Q^{*,A}$ .