Parameterizing Mesoscale Eddy Effects in Large-scale Ocean Models

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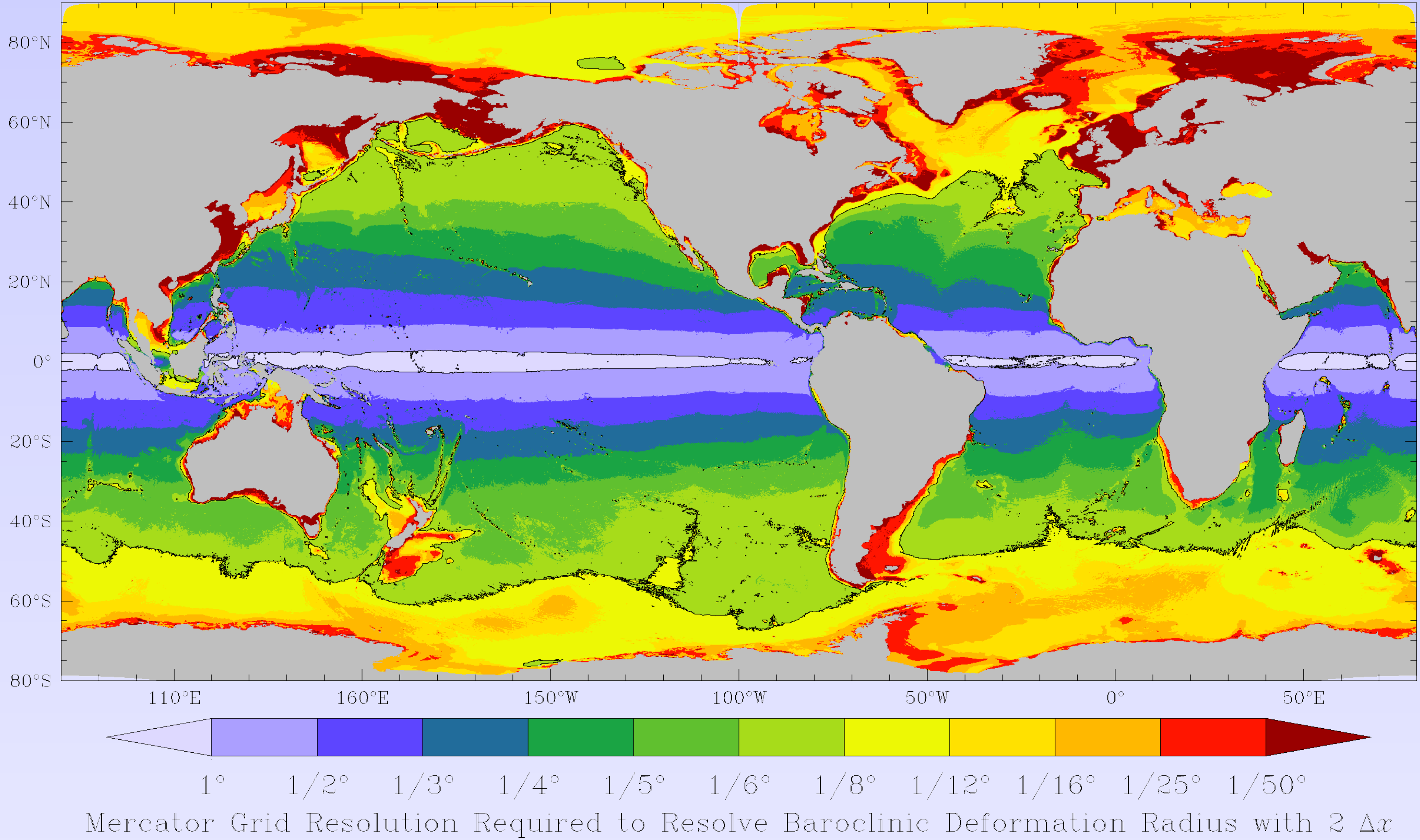
with contributions from Malte Jansen, Angélique Melet, Alistair Adcroft, and Matthew Harrison

Developments in Parameterizing Eddies

- Know where to parameterize eddies (and where not to)
  - Use a Resolution Function (Hallberg, Ocean Mod., 2013)
- Along-isopycnal tracer diffusion can coexist with explicit eddies (unlike isopycnal height / G-M diffusion)
- Make the most of resolution…
  - Backscatter energy (Jansen & Held, Ocean Mod., 2014)
- Predict the unresolved eddy intensity dynamically
  - Mesoscale Eddy Kinetic Energy parameterization
- [Add vertical structure to eddy parameterizations & solve an elliptic equation for eddy transport streamfunction (Danabasoglu & Marshall, 2007; Ferrari et al. 2010; Abernathey et al. 2013; …)]

Mercator/Tripolar Resolution Required to Admit 1st Baroclinic Deformation Radius

$$L_{Def} = \sqrt{c_g^2 / \left(f^2 + 2\beta c_g\right)}$$



Coupled Climate Model (CM2.5): ~7 years/day on 6,000 Processors at 1/4° res.
Cost goes as $(\text{Resolution})^3$; Saturated throughput goes as $(\text{Resolution})^{-1}$.

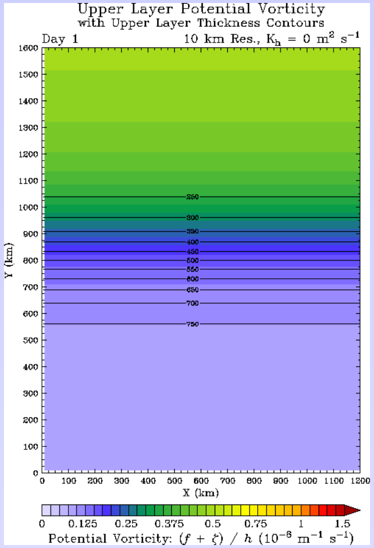
Phillips model of baroclinic instability

2-layer stacked shallow water model. (See textbook discussion in Pedlosky, 1987.)

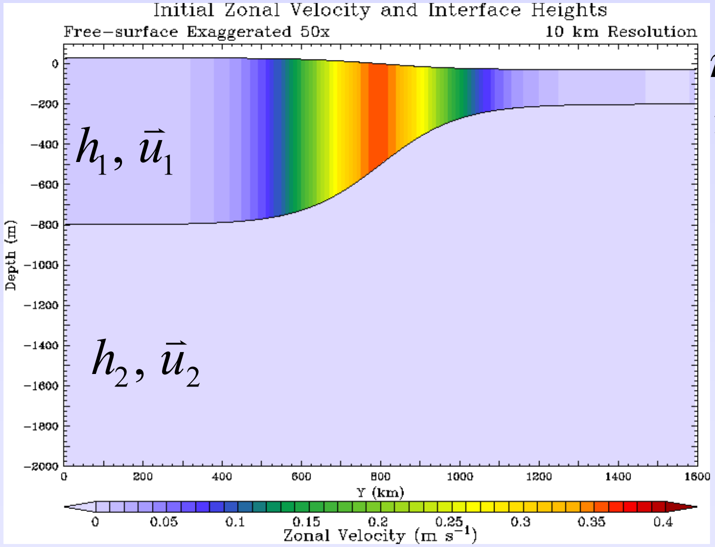
$$\frac{\partial \vec{u}}{\partial t} + \left(f + \hat{k}\cdot\nabla\times\vec{u}\right)\times\vec{u} = -\nabla\left(M + \tfrac{1}{2}\|\vec{u}\|^2\right) - \nabla\cdot\vec{\vec{T}} - c_D\|\vec{u}\|\delta_{k2}\vec{u} \qquad M_1 = g\eta_{1/2}$$

$$\frac{\partial h_1}{\partial t} + \nabla\cdot\left(h_1\vec{u}\right) = \gamma\left(\overline{\eta_{3/2}}^{x} - \eta_{3/2,\mathrm{Ref}}\right) - \nabla\cdot\left(K_h\nabla\eta_{3/2}\right) \qquad M_2 = M_1 + \frac{g\Delta\rho}{\rho_0}\eta_{3/2}$$

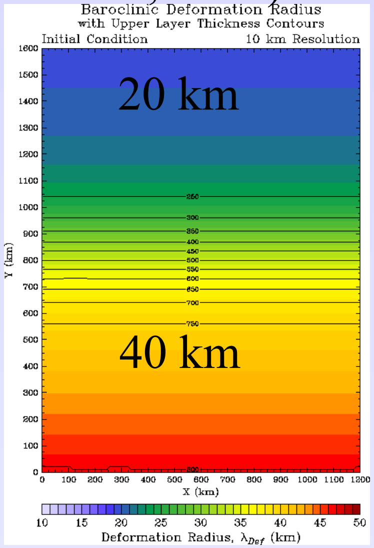
$$\frac{\partial h_2}{\partial t} + \nabla\cdot\left(h_2\vec{u}\right) = -\gamma\left(\overline{\eta_{3/2}}^{x} - \eta_{3/2,\mathrm{Ref}}\right) + \nabla\cdot\left(K_h\nabla\eta_{3/2}\right)$$

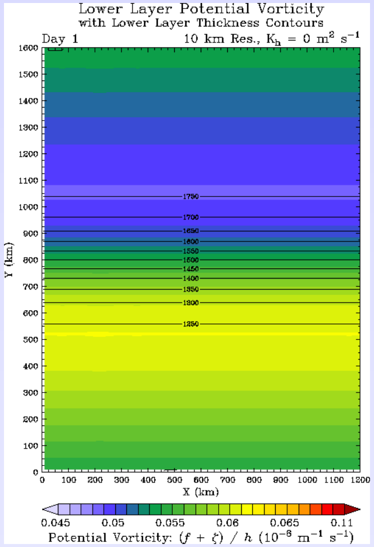


- Exhibits instabilities with peak growth rates at wavelengths of $2\pi\lambda_{def}$.
- Restoring of zonal-mean interface height reenergizes the flow.
- Strong bottom drag prevents barotropization. (Arbic & Flierl, 2006)



$$\eta_{1/2} = \eta_{3/2} + h_1$$
$$\eta_{3/2} = -D + h_2$$

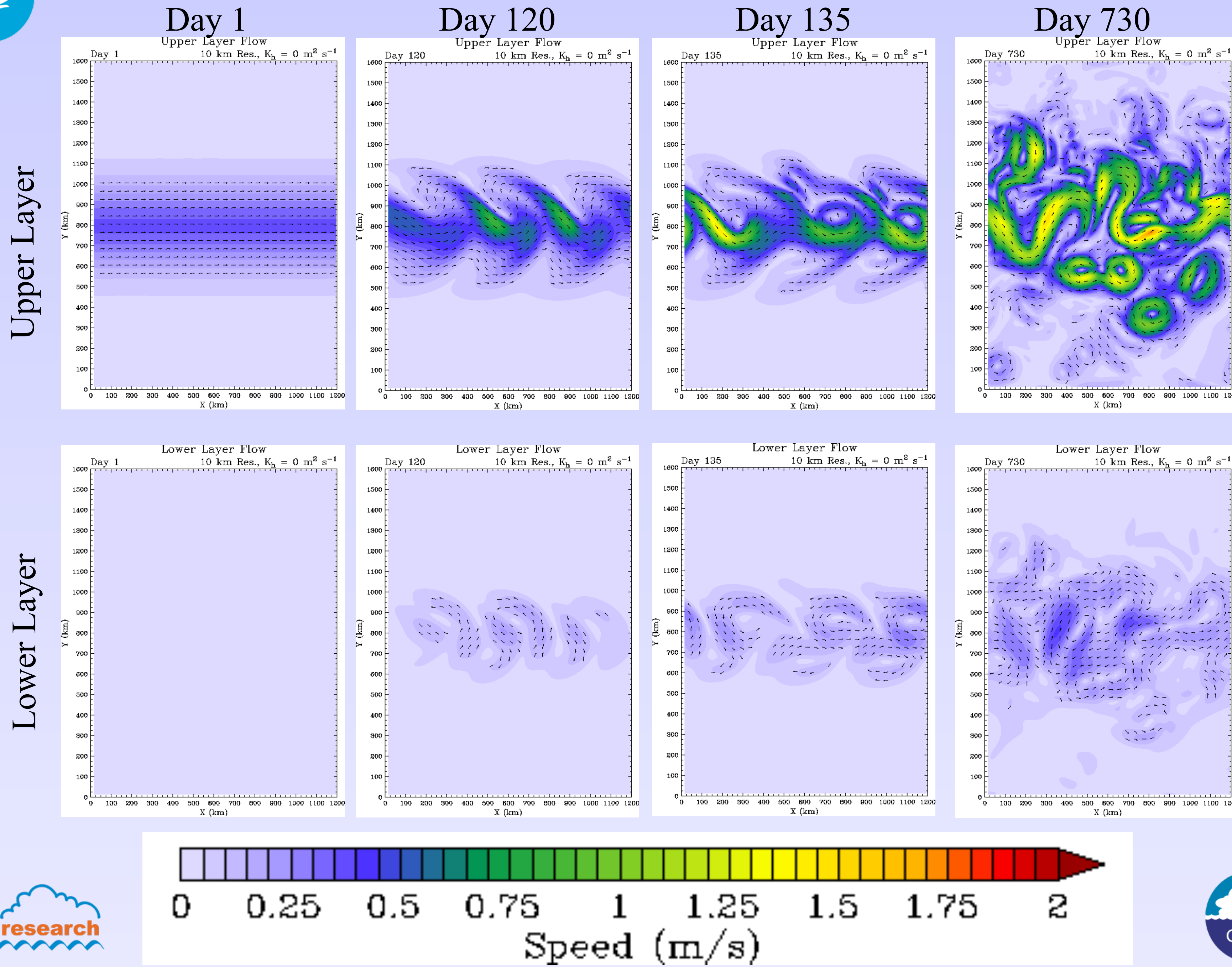




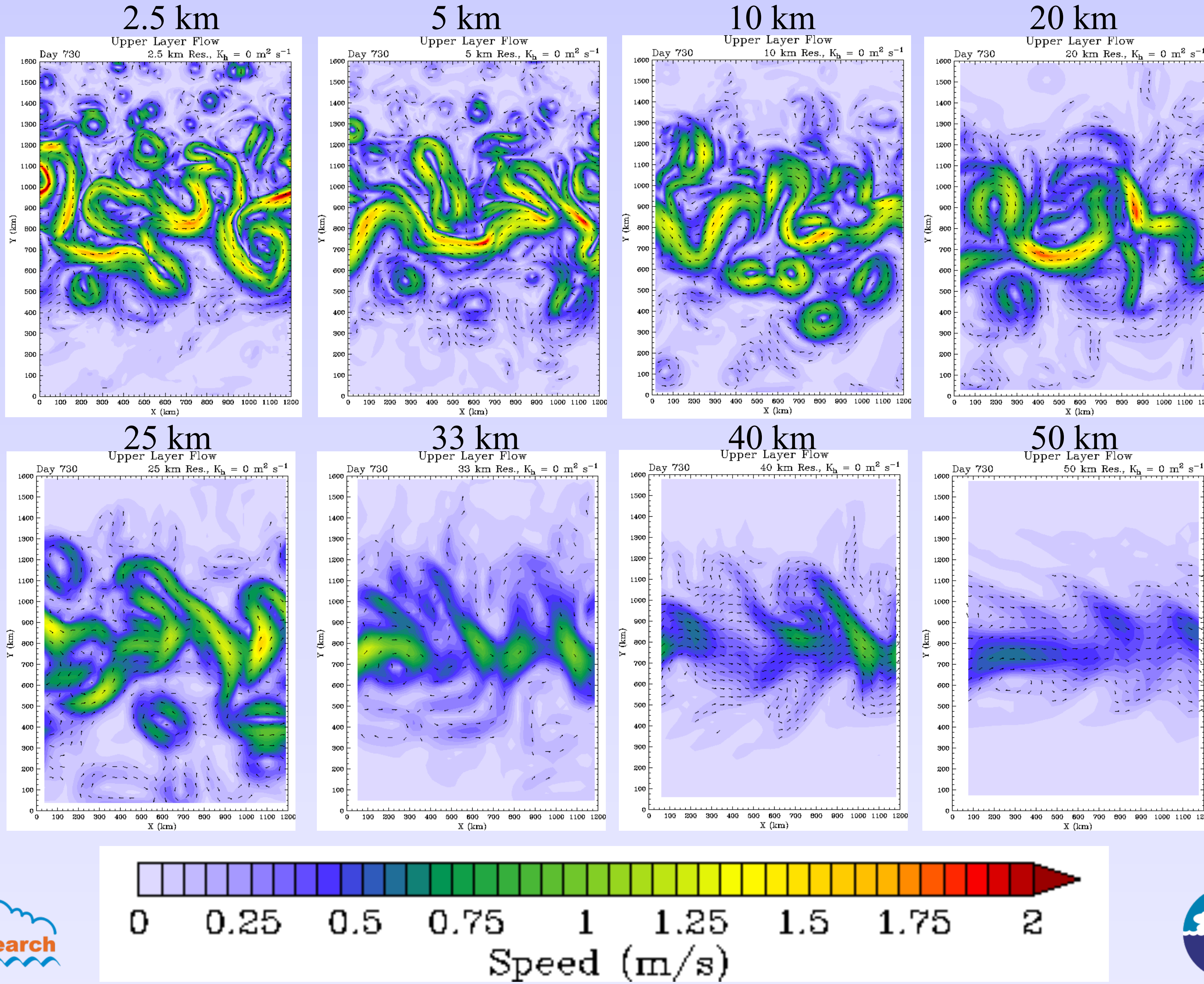
$$\lambda_{Def} = \frac{1}{|f|}\sqrt{\frac{g' h_1 h_2}{h_1 + h_2}}$$

Ref: Hallberg, Ocean Mod., 2013

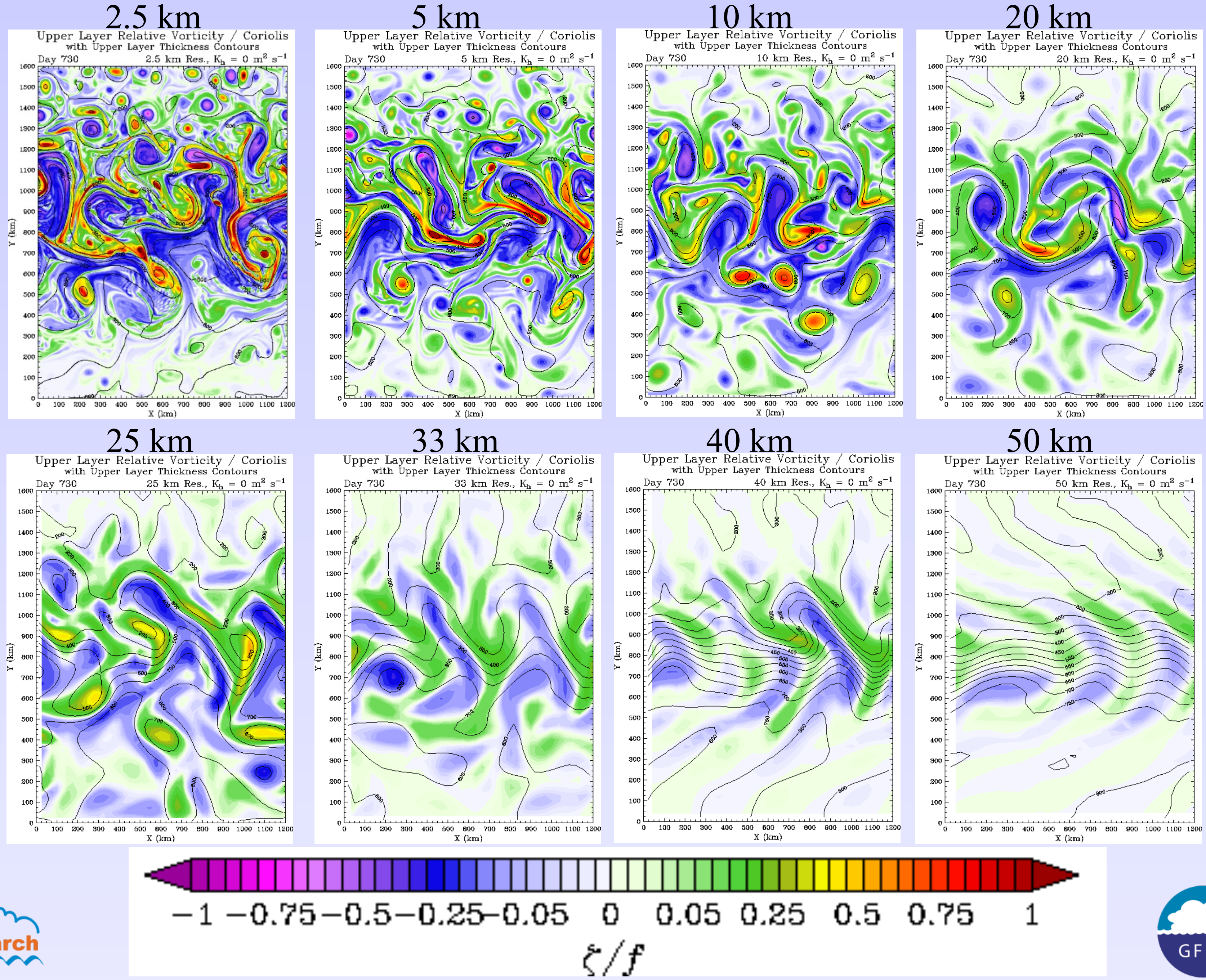
10 km Resolution, $K_h = 0$



Upper Layer Flow at Various Resolutions, Day 730, $K_h = 0$



Upper Layer Relative Vorticity / Coriolis at Various Resolutions, Day 730, $K_h = 0$

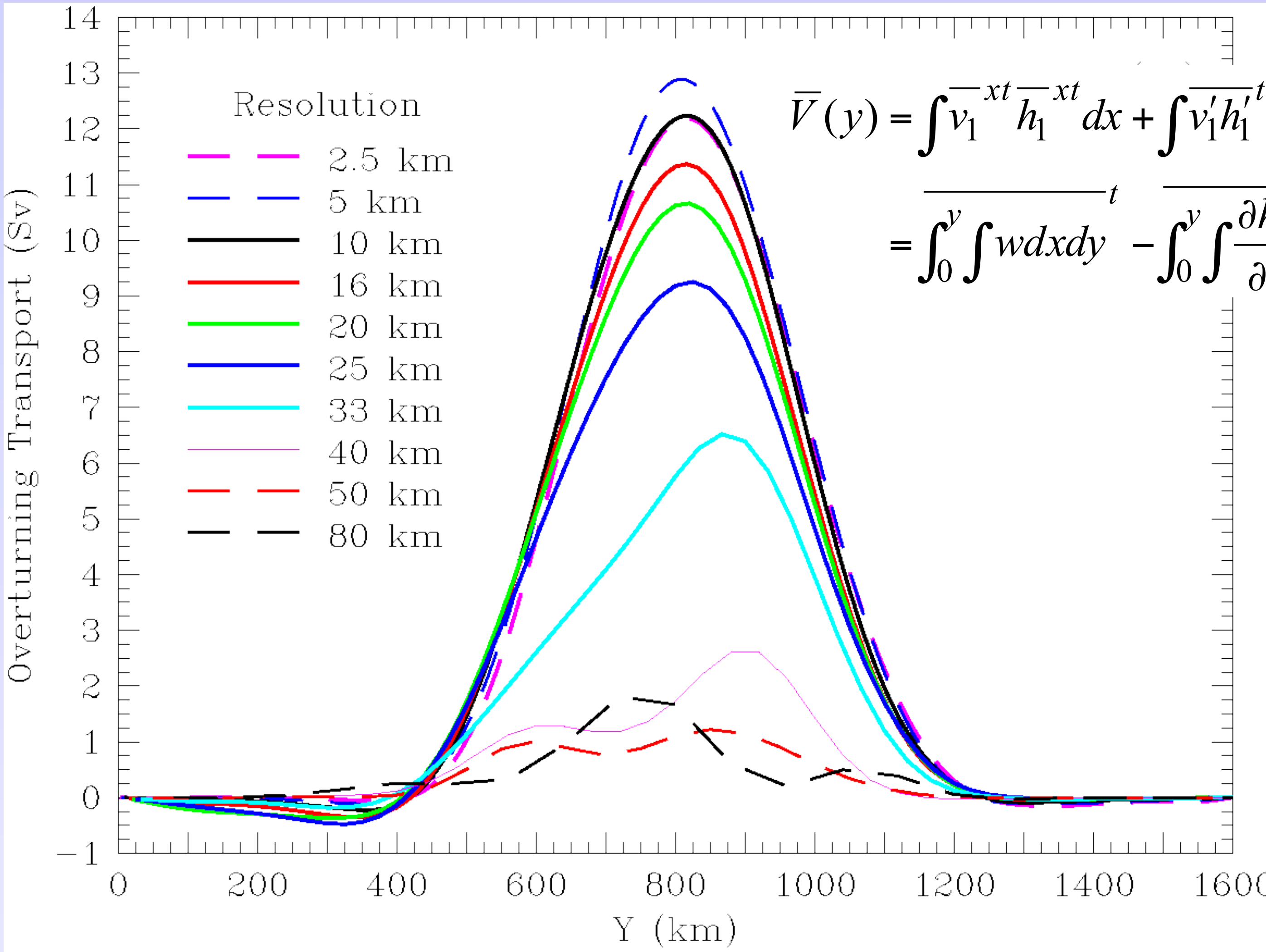


Effects of Resolution on Overturning Transport

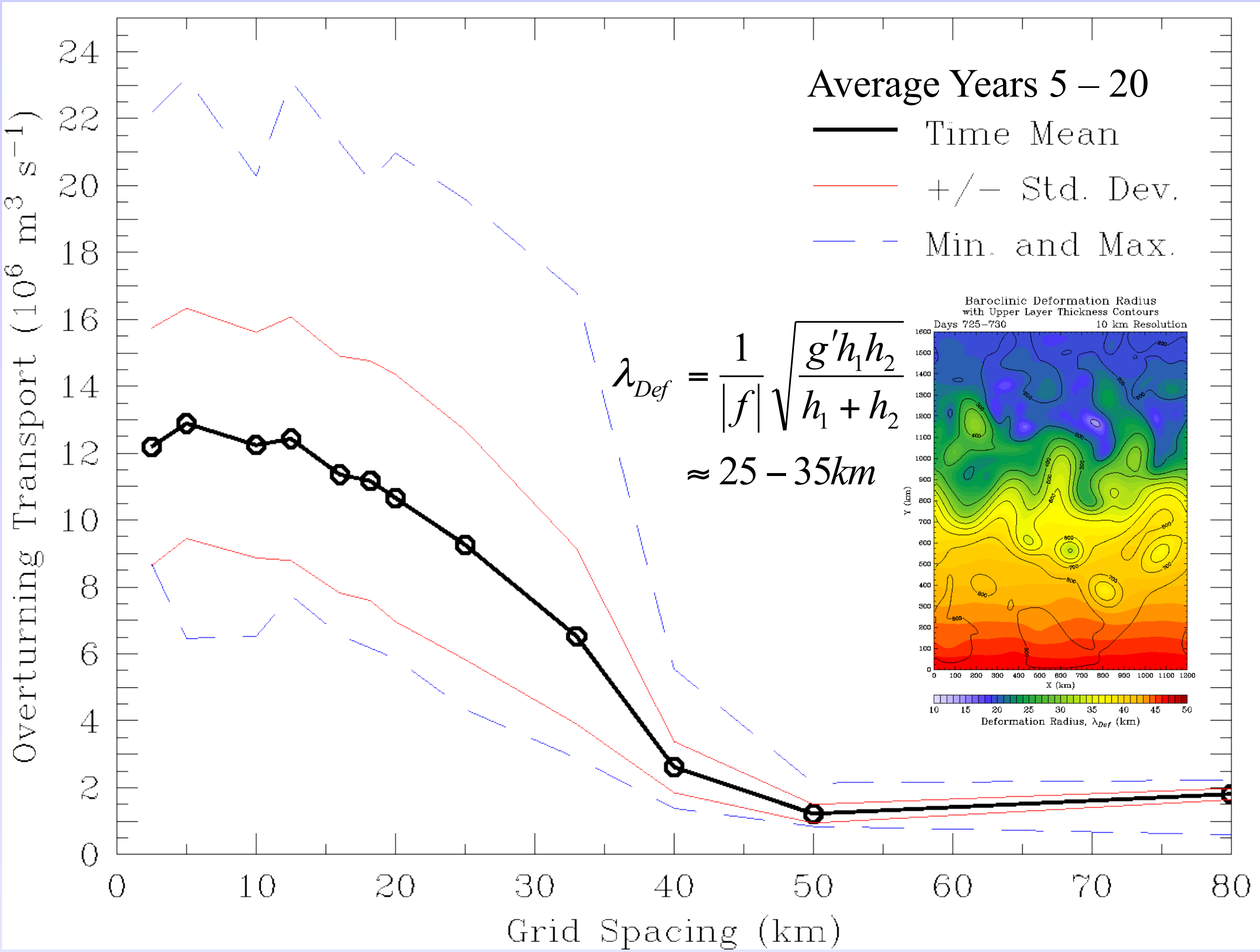
$$\overline{V}(y) = \overline{\int v_1 h_1 dx}^{t}$$

$$\overline{V}(y) = \int \overline{v_1}^{xt}\,\overline{h_1}^{xt} dx + \int \overline{v_1' h_1'}^{t} dx$$

$$= \overline{\int_0^y \int w\,dx\,dy}^{t} - \overline{\int_0^y \int \frac{\partial h_1}{\partial t}\,dx\,dy}^{t} \approx \overline{\int_0^y \int w\,dx\,dy}^{t}$$



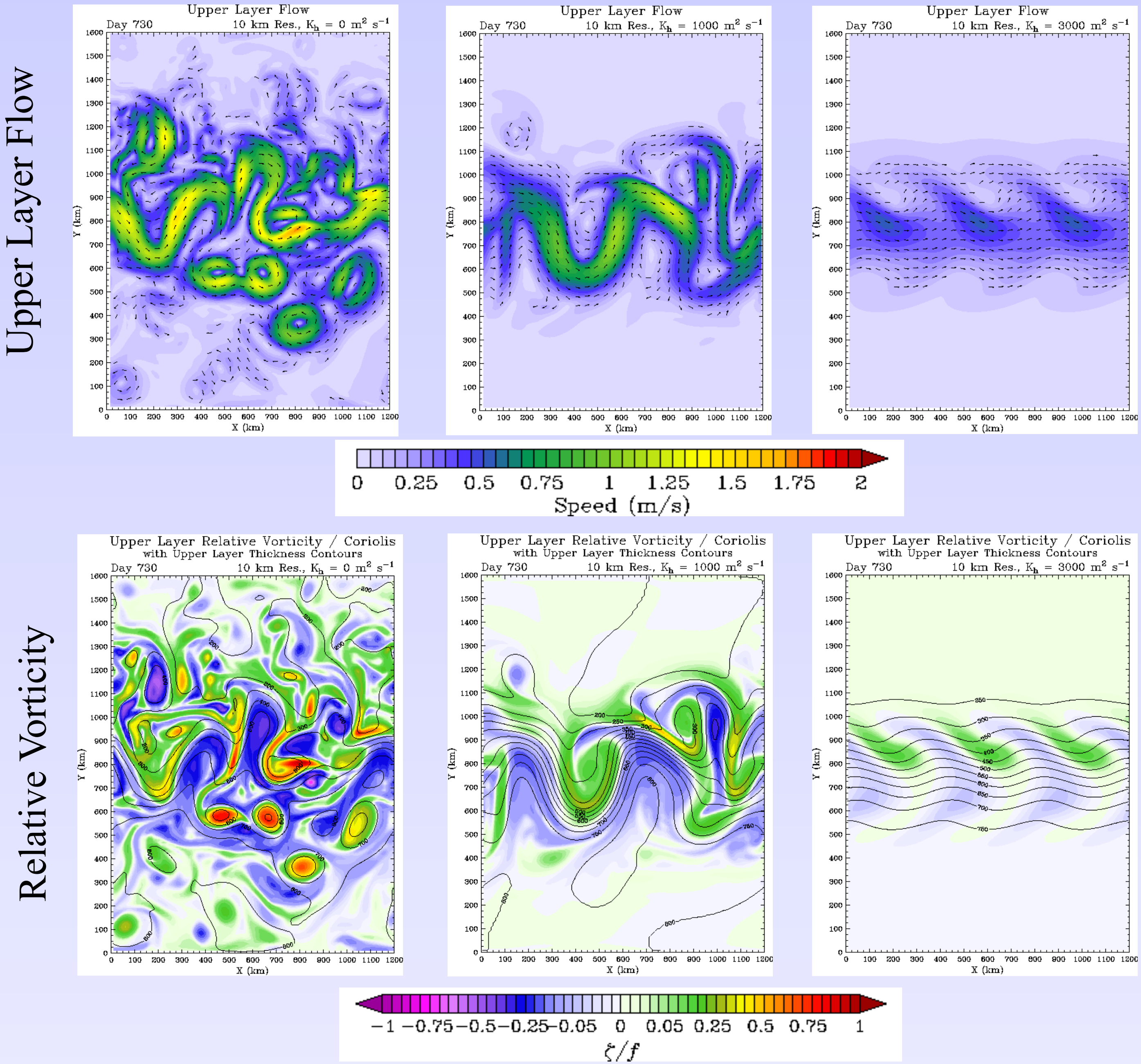
Effects of Resolution on Eddy-Driven Overturning



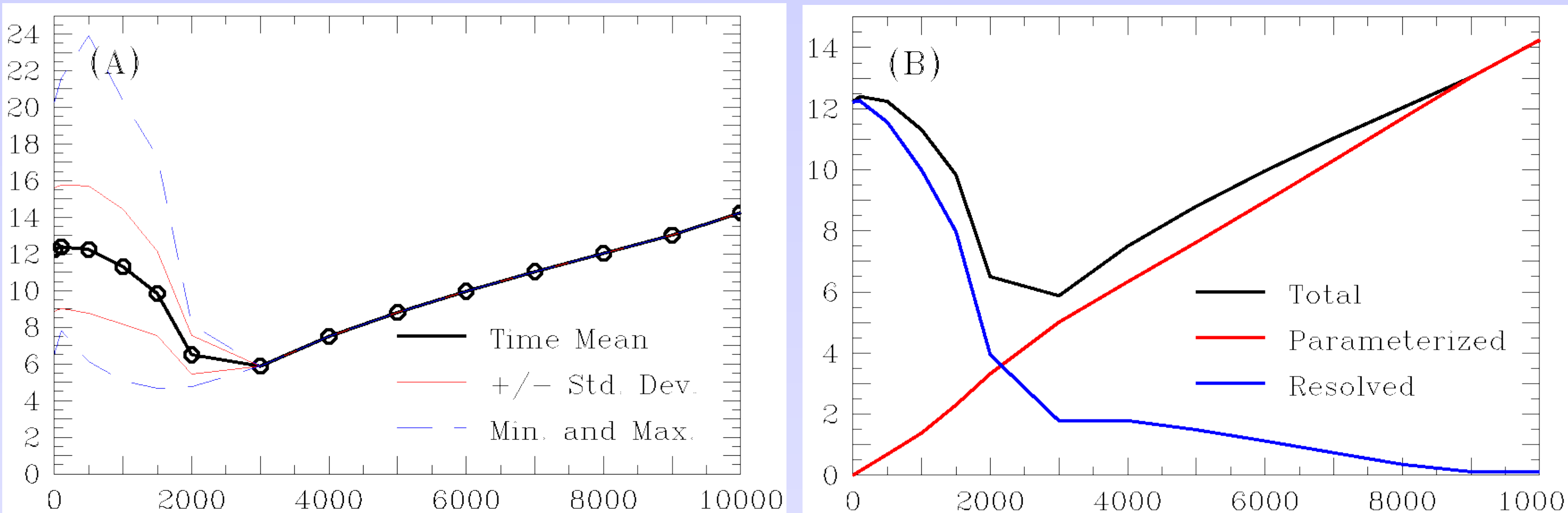
Baroclinic eddy effects are absent or need to be parameterized when the deformation radius is poorly resolved.

Effects of Interface Height Diffusion (G-M mixing) at 10 km Resolution, Day 730

K_h=0 m^2 s^{-1} | K_h=1000 m^2 s^{-1} | K_h=3000 m^2 s^{-1}



Interface height diffusion is much more effective at suppressing eddies than at parameterizing their effects!



Overturning Transport (10^6 m^3 s^{-1})

Interface Height Diffusivity (m^2 s^{-1}) Interface Height Diffusivity (m^2 s^{-1})

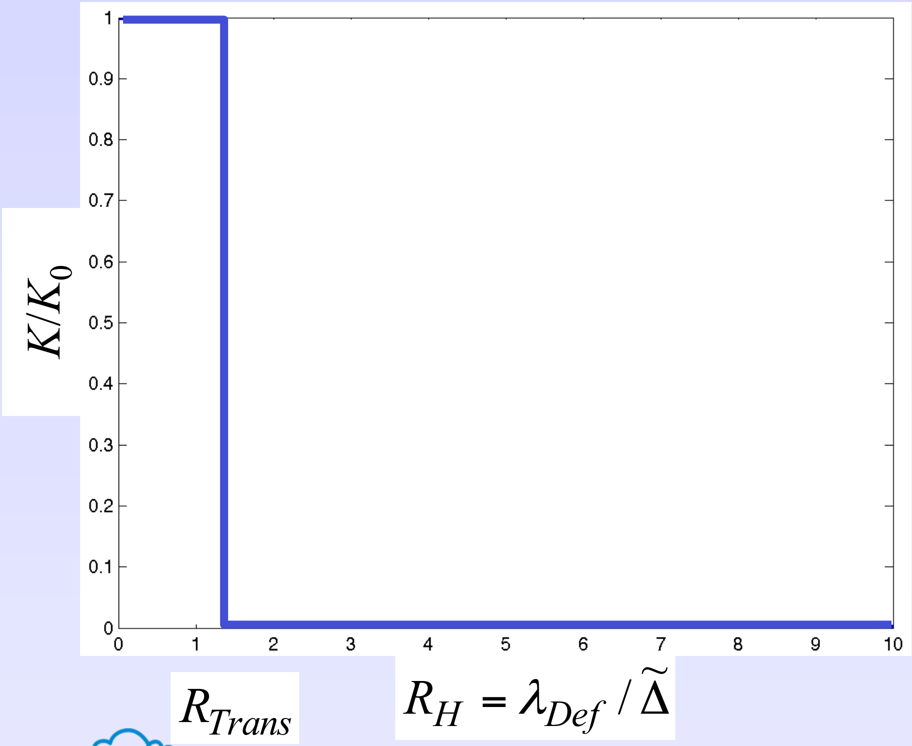
$$\bar{V}(y) = \left(\int \overline{v_1}^{xt}\, \overline{h_1}^{xt}\, dx + \int \overline{v_1' h_1'}^{t}\, dx \right) + \int \overline{K_h \frac{\partial \eta_{3/2}}{\partial y}}^{t} dx$$

The best solution for the Phillips problem:

Abruptly disable the Laplacian interface height diffusivity once the ratio of the grid spacing to the deformation radius reaches R_{Trans}

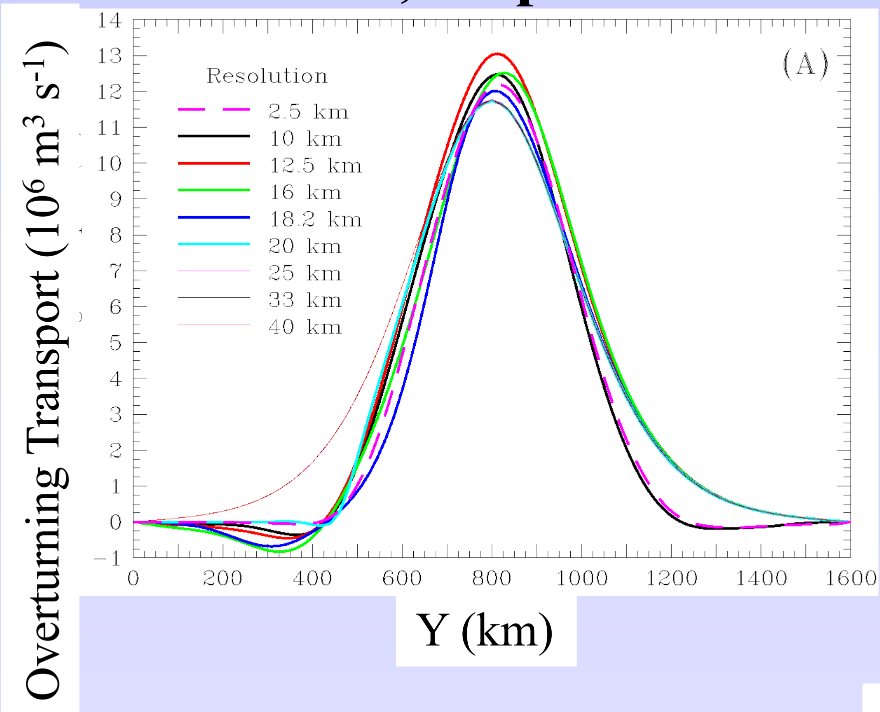
$$K = \begin{cases} K_0 & \lambda_{Def} / \tilde{\Delta} < R_{Trans} \\ 0 & \lambda_{Def} / \tilde{\Delta} \ge R_{Trans} \end{cases} \qquad \lambda_{Def} = \sqrt{\frac{{c_{g1}}^2}{f^2 + 2\beta c_{g1}}} \qquad \tilde{\Delta} = \sqrt{\left(\Delta x^2 + \Delta y^2\right)/2}$$

This works reasonably for the Phillips problem for any R_{trans} >~ 2!

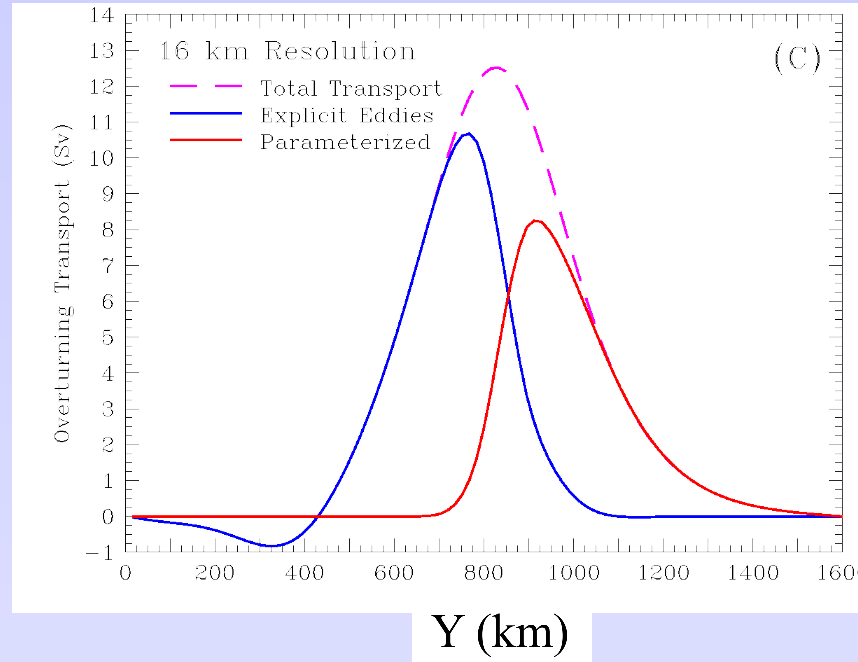


Step Resolution Function

Total Overturning at Various Resolutions, Step Function



Parameterized vs. Explicit Eddy Transports, 16 km Res.

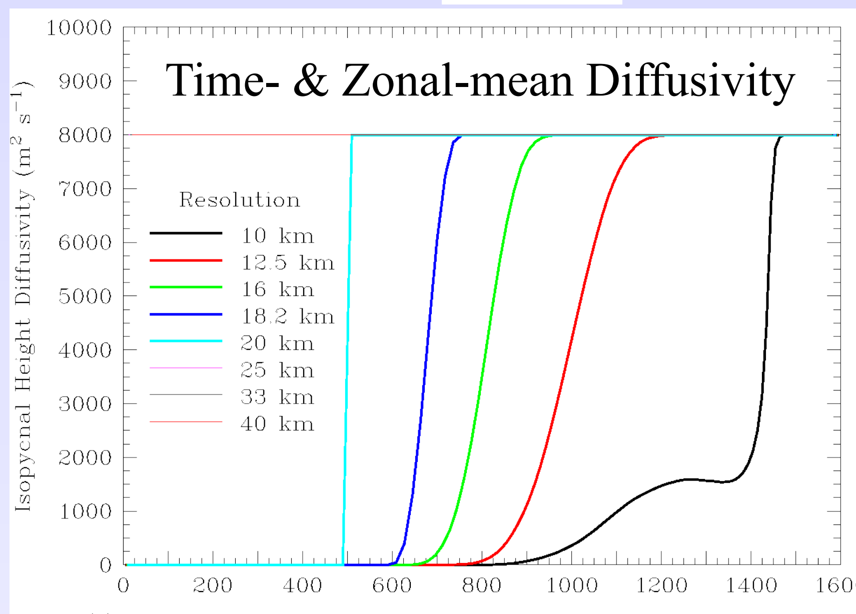


Strongly unstable case

$$K = \begin{cases} K_0 & \lambda_{Def} / \widetilde{\Delta} < R_{Trans} \\ 0 & \lambda_{Def} / \widetilde{\Delta} \geq R_{Trans} \end{cases}$$

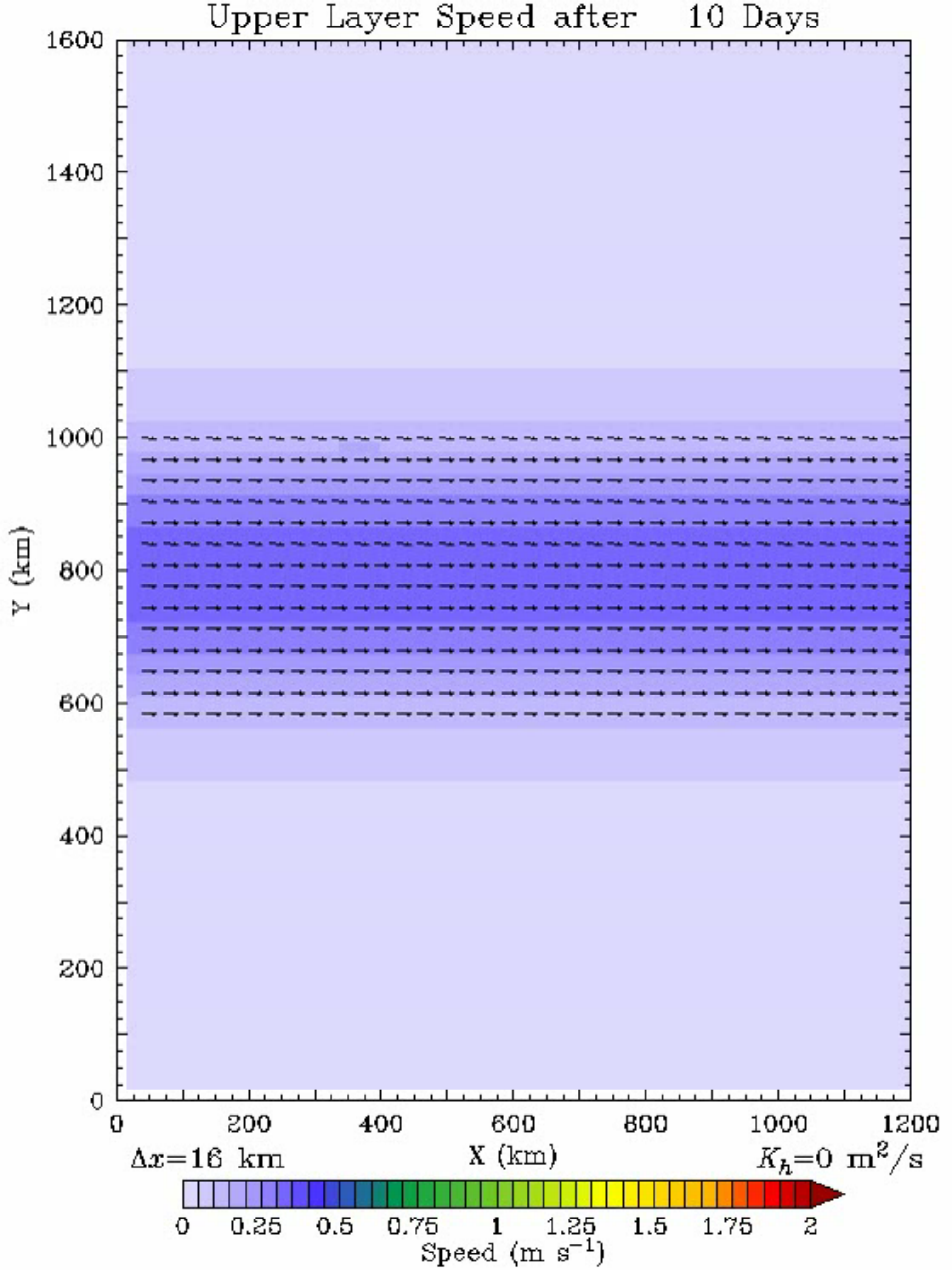
$$R_{Trans} = 2 \quad K_0 = 8000\ m^2 s^{-1}$$

(Parameterize K_0 later with MEKE.)



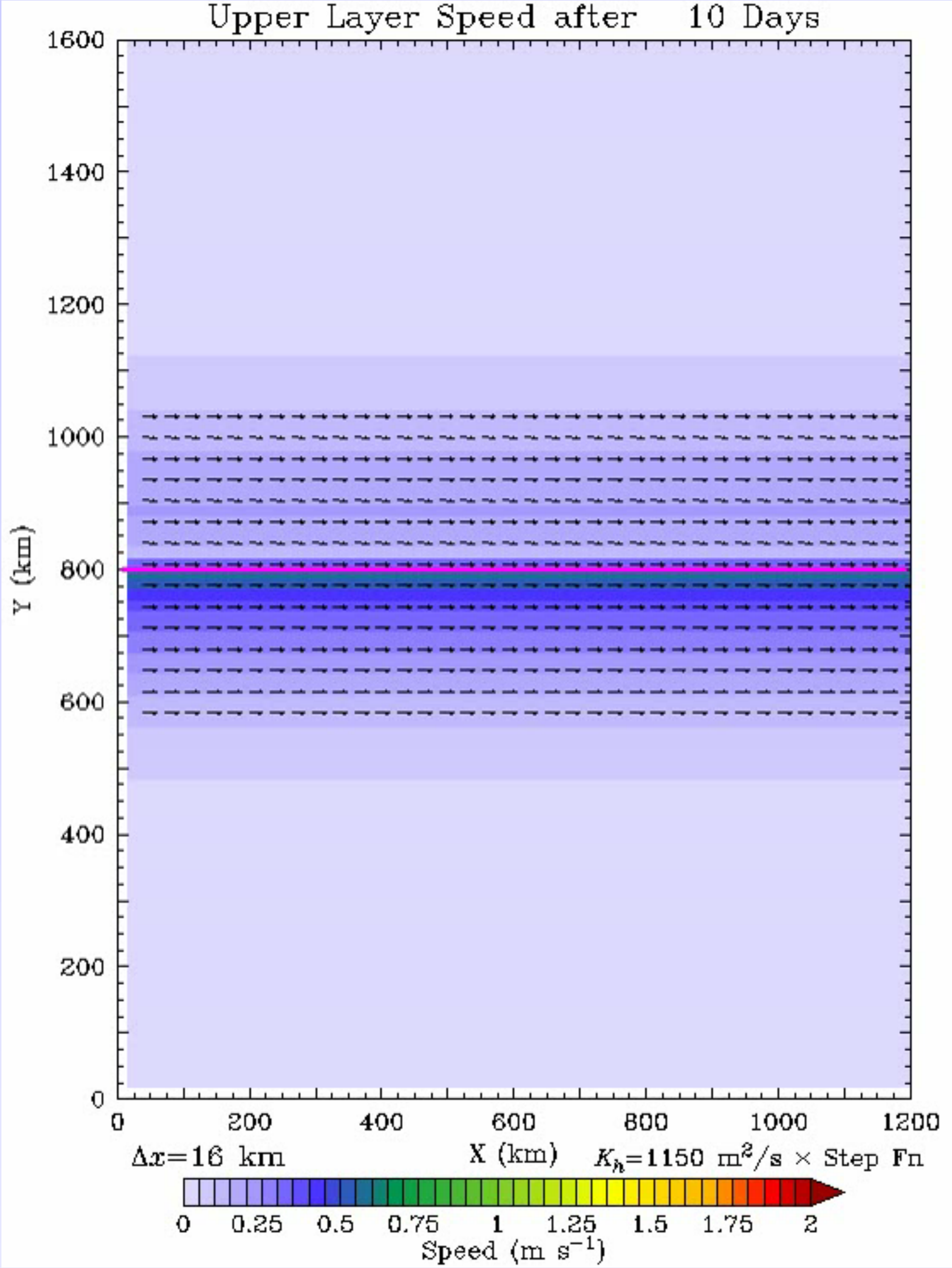
Speed with no interface height diffusivity

16 km resolution



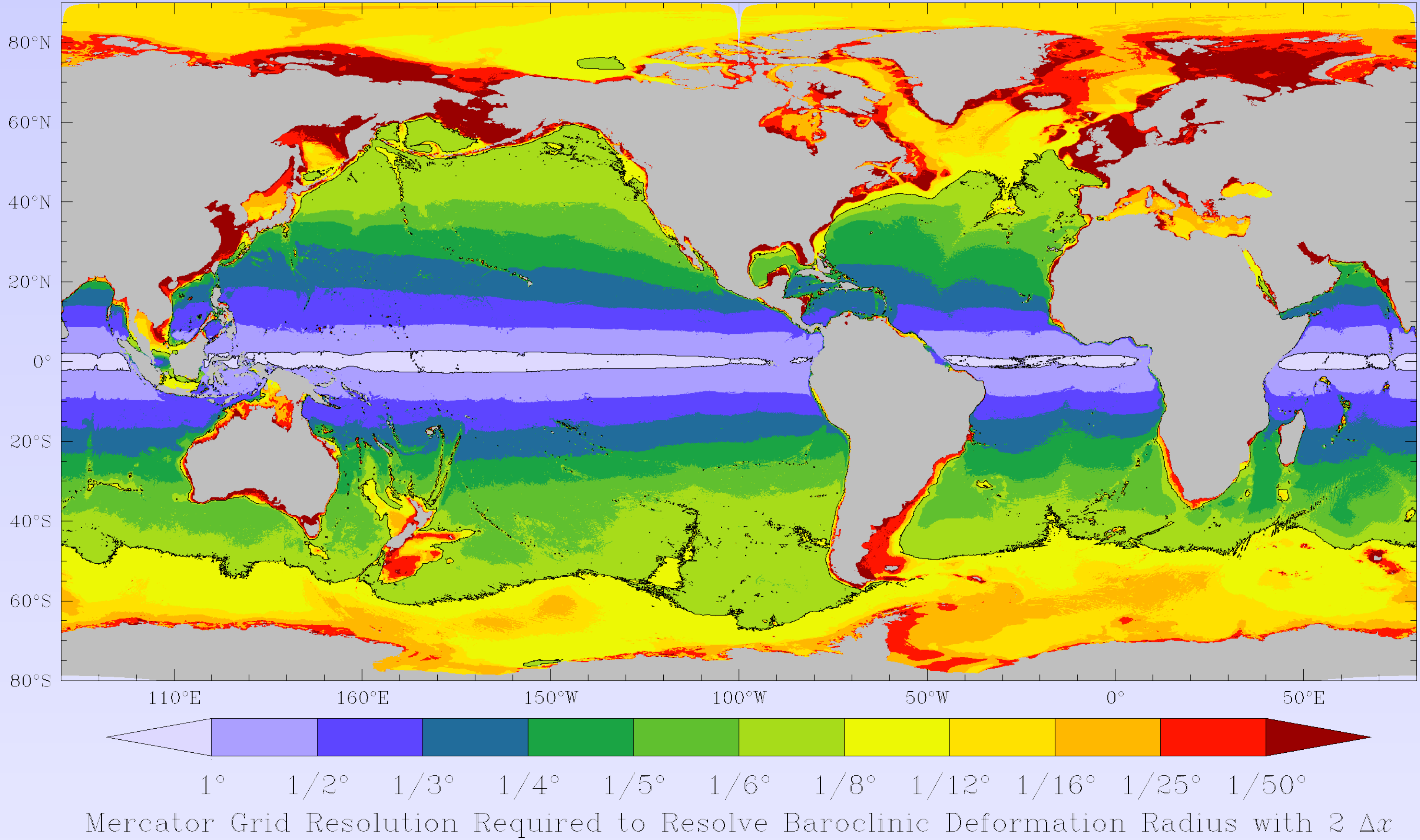
Speed with step-function Resolution Function applied to interface height diffusivity

16 km resolution



Mercator/Tripolar Resolution Required to Admit 1st Baroclinic Deformation Radius

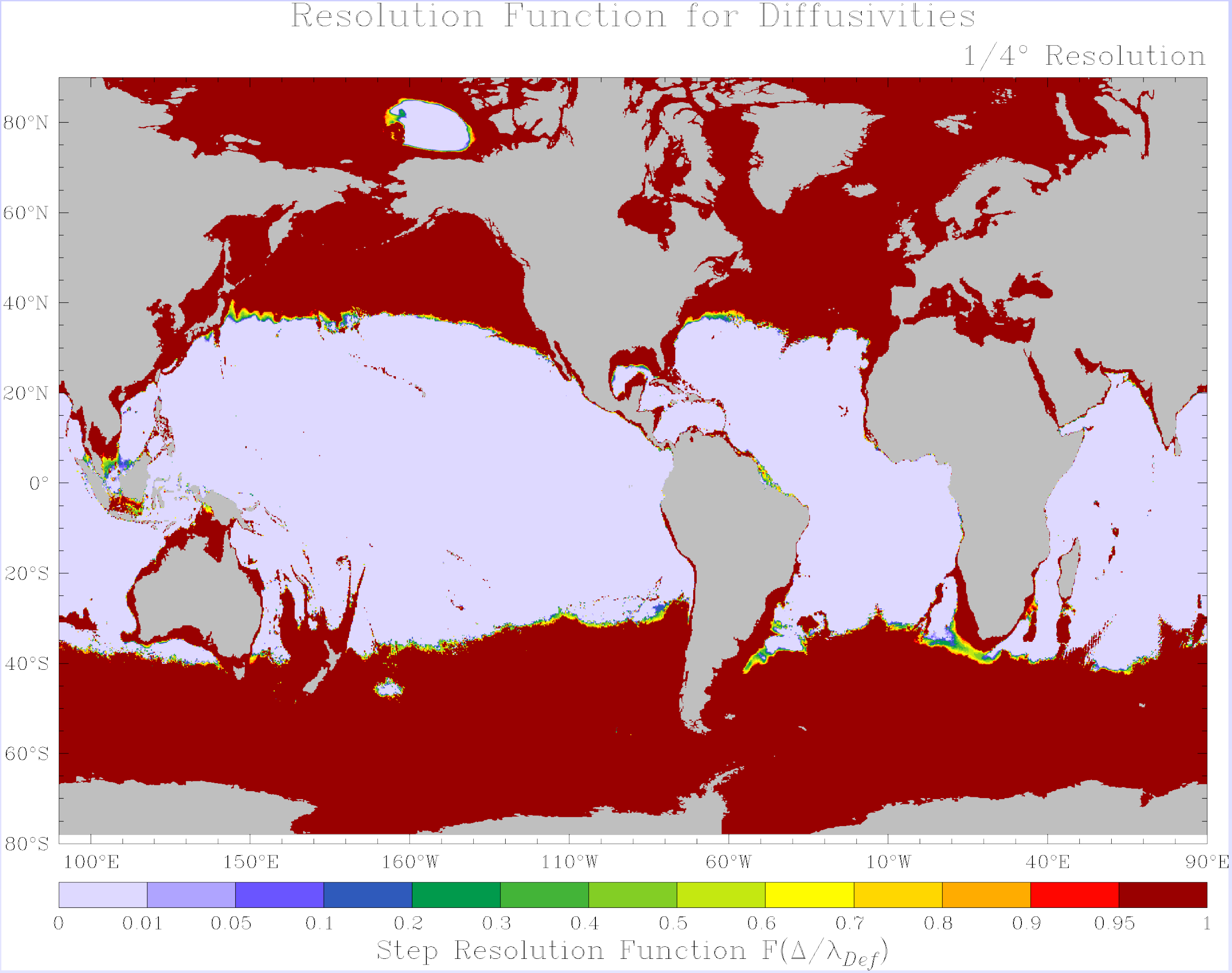
$$L_{Def} = \sqrt{c_g^2 / \left(f^2 + 2\beta c_g\right)}$$



Coupled Climate Model (CM2.5): ~7 years/day on 6,000 Processors at 1/4° res.

Cost goes as (Resolution)^3; Saturated throughput goes as (Resolution)$^{-1}$.

Annual Mean Step Resolution Function for ¼° Model



$R_{Trans} = 1.41$

Marginal instability requires $R_{Trans} \geq\sim 2$

R_{Trans} can be smaller (explicit eddies over more area) for strong instability.

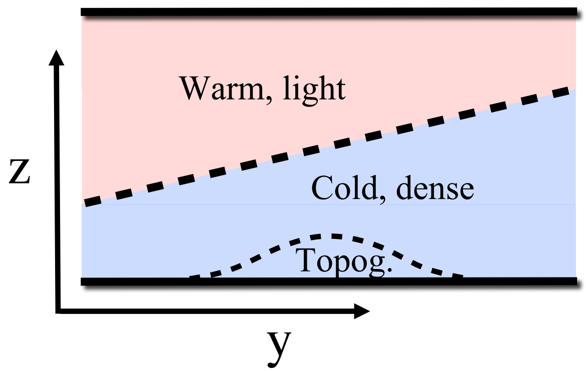
Along-isopycnal Tracer Diffusion vs. Isopycnal Height (or Gent-McWilliams) Diffusion

- Interface height diffusion suppresses baroclinic eddies; along-isopycnal tracer diffusion does not.
  - Parameterized and resolved tracer mixing can coexist.
  - Eddy mass transport is either explicit or parameterized - not both!
- Parameterized eddy watermass transport (e.g., G-M) should be disabled abruptly (via a step Resolution Function) when and where eddies are resolved.
- Eddy tracer diffusivities can (should?) be scaled away smoothly, and can represent mixing by unresolved scales even when some eddy scales are resolved.
  - Use a gradually varying resolution function?

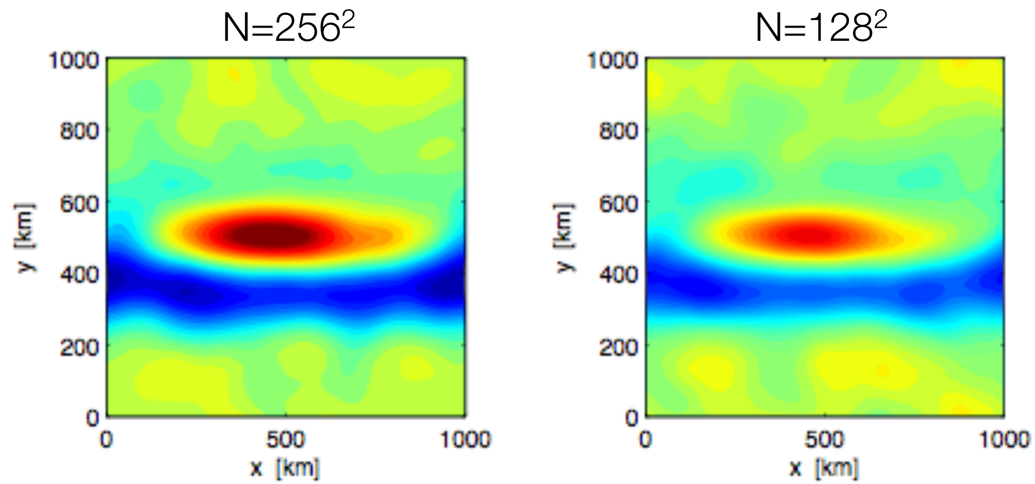
Degradation of Mean Flow at Eddy-Permitting Resolution

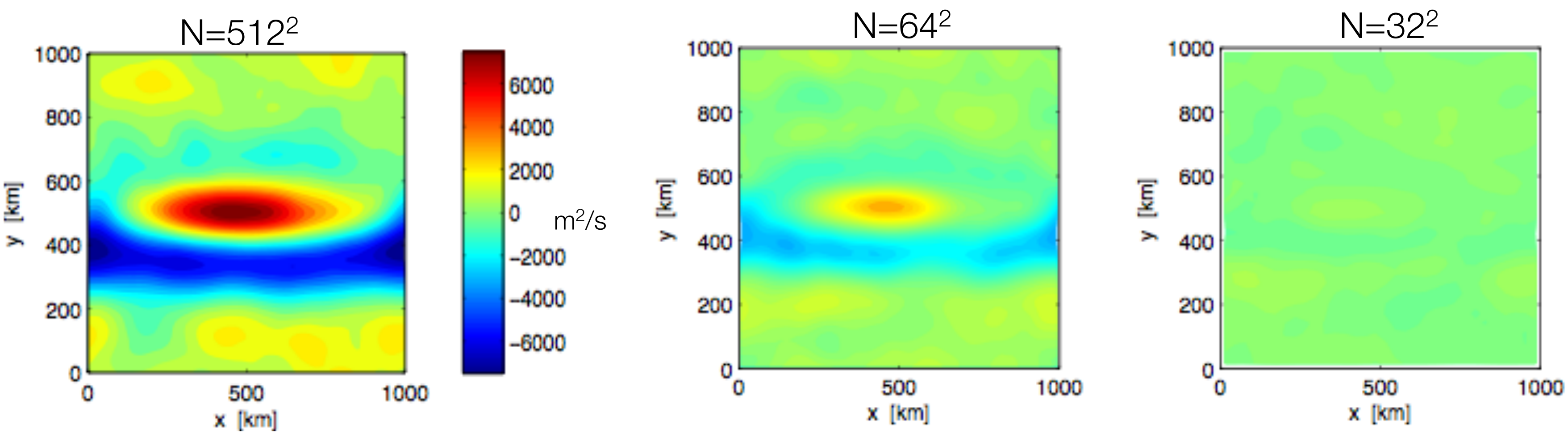
Ref: Jansen and Held (Ocean Modelling, 2014)

Baroclinically unstable mean flow in a quasigeostrophic model



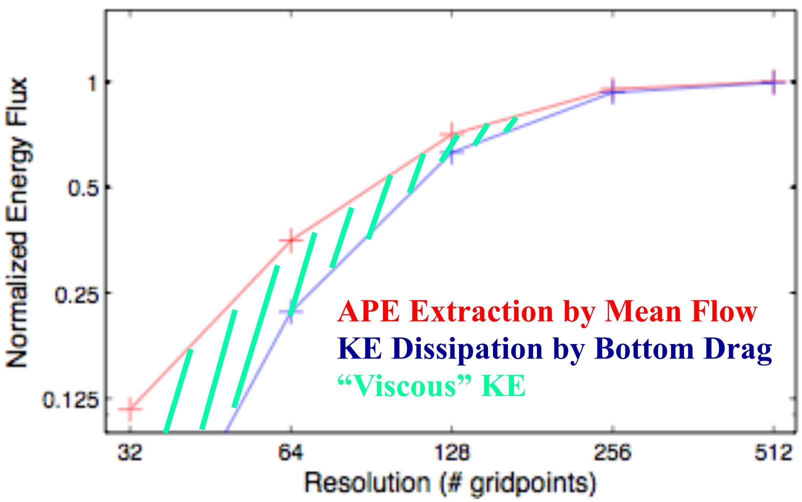
Mean Flow (Lower Layer Streamfunction) at Various Resolutions





Backscatter of Small-Scale Dissipated Energy

Ref: Jansen and Held (Ocean Modelling, 2014)

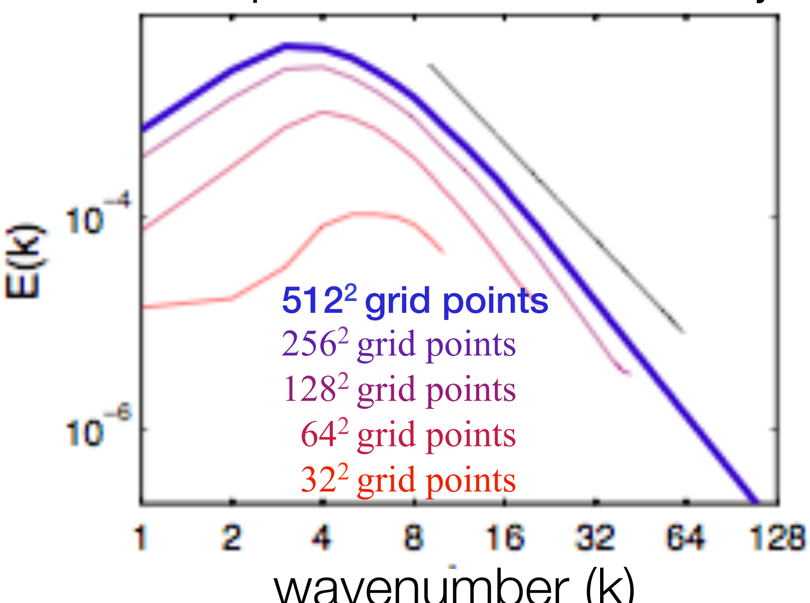


Add anti-viscous Laplacian forcing such that 90% of the energy dissipated near the grid scale is returned:

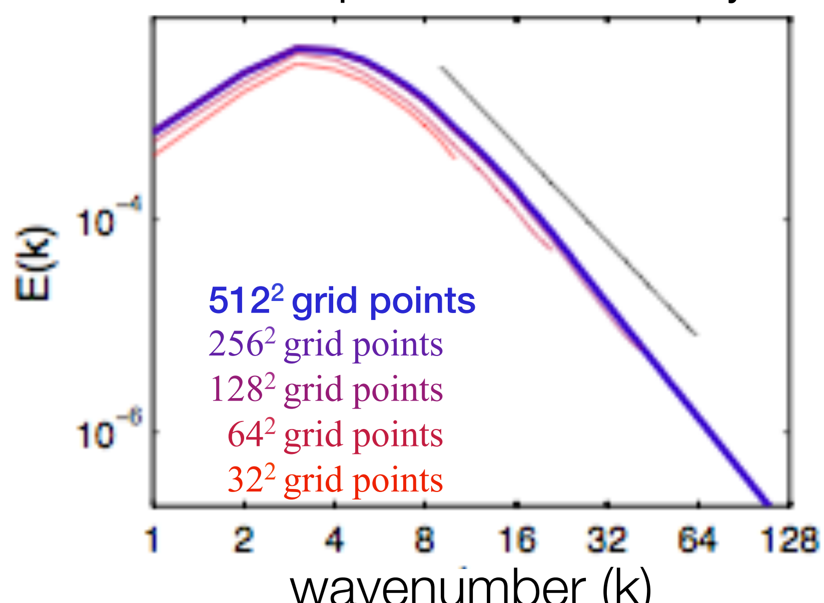
$$\frac{D\vec{u}}{Dt} + \ldots = -A_2 \nabla^2 \vec{u} - A_4 \nabla^4 \vec{u}$$

$$A_2 \int \left(\vec{u} \cdot \nabla^2 \vec{u} \right) dV = -0.9\, A_4 \int \left(\vec{u} \cdot \nabla^4 \vec{u} \right) dV$$

Kinetic Energy Spectra
Dissipative Biharmonic Viscosity

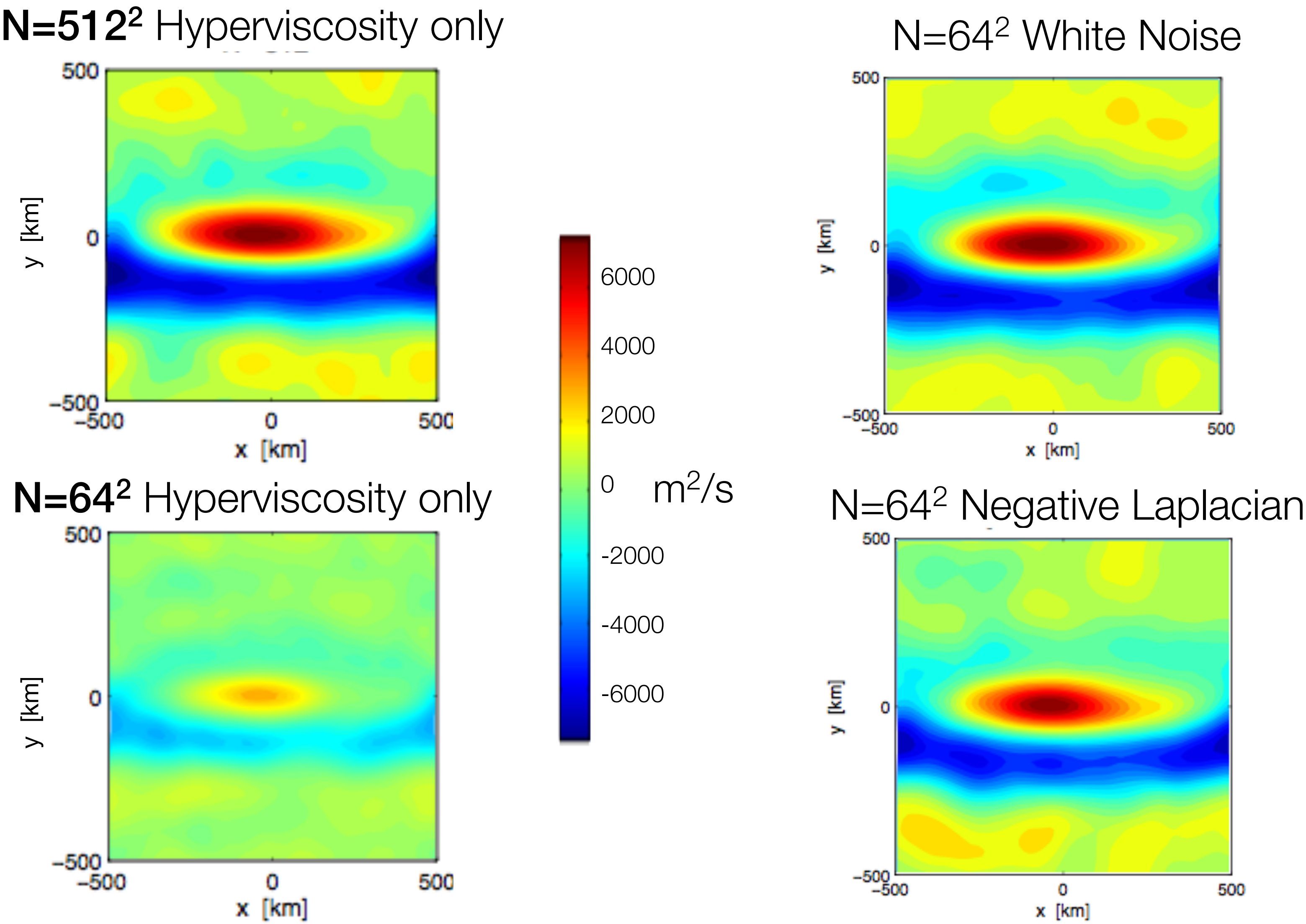


Kinetic Energy Spectra
Add Laplacian Anti-Viscosity



Mean Flow With and Without Energy Backscatter

Ref: Jansen and Held (Ocean Modelling, 2014)



Backscatter allows eddy effects to be explicitly modeled at lo wer resolution, and hopefully use smaller values of R_{Trans}

MEKE and Resolution Function Scaling

Resolution function scaling seems particularly promising when combined with a parameterize to diagnose the unresolved Mesoscale Eddy Kinetic Energy (Cessi, 2008; Eden et al., 2008; Marshall & Adcroft, 2010; Melet et al., 2014; Jansen et al., 201X):

$$\frac{\partial E}{\partial t} = Src - \gamma E - \frac{c_d \left\| u_{bot} \right\|}{H} E + \frac{1}{H} \nabla \cdot \left(H \kappa_E \nabla E \right)$$

$$Src = \frac{(1?)}{H} \sum_{k=1}^{K} g'_k \kappa_{Int} \left\| \nabla \eta_k \right\|^2 - \frac{(0.001?)}{H} \sum_{k=1}^{K} h_k u_k \cdot \nabla \tau_{Visc}$$

$$\kappa_{MEKE} = (0.3?) \min\left(\lambda_D, \sqrt{\Delta x^2 + \Delta y^2}, L_{Other} \right) \sqrt{2E}$$

$$\kappa_{Int} = F(R_H)(\kappa_{MEKE} + \kappa_{Background})$$

E : Unresolved Mesoscale Energy/Mass in Column

MEKE is self-regulating; the MEKE-parameterized mixing changes the resolved state to reduce MEKE's source of energy.

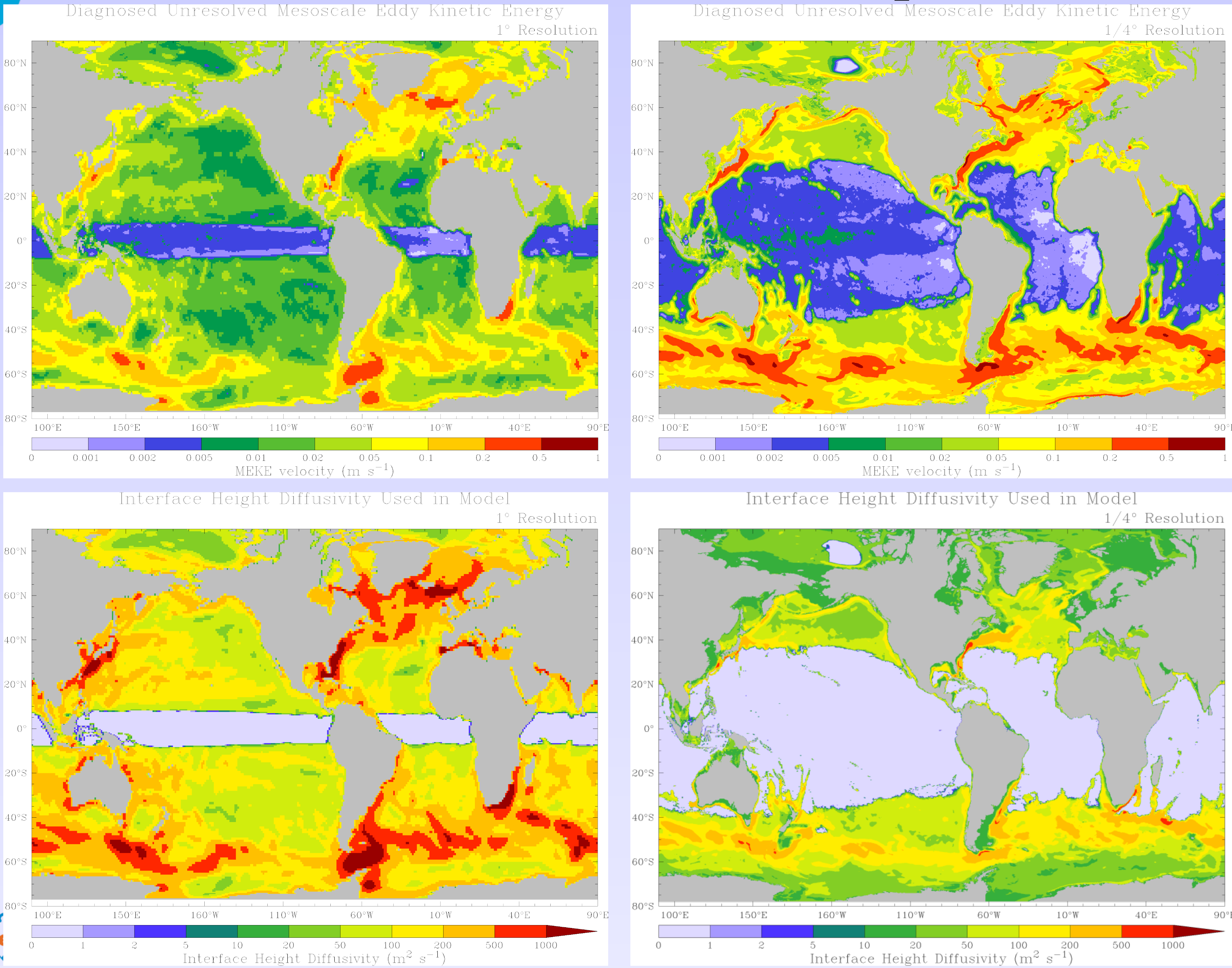
MEKE in 2 Global Models with Step Function $F(R_H)$

1°

1/4°

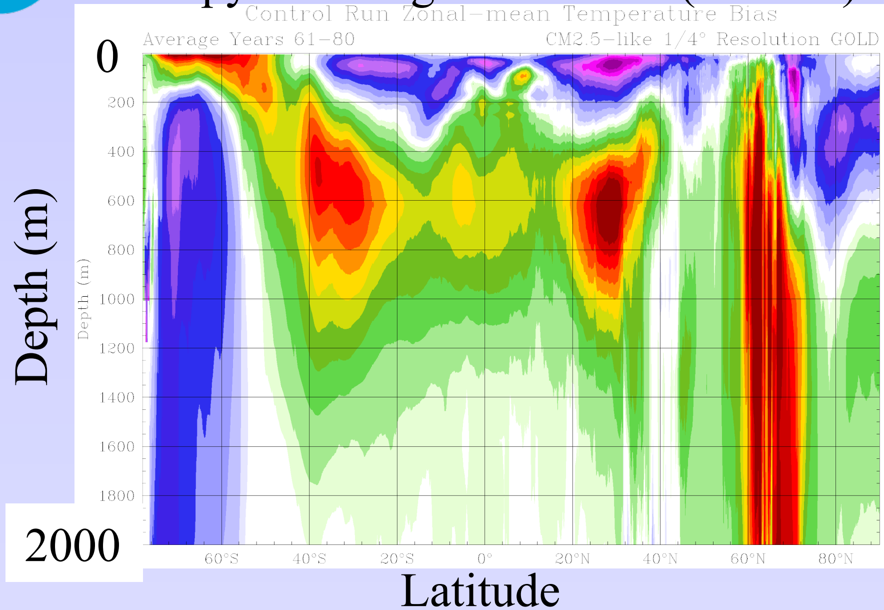
MEKE

K_H

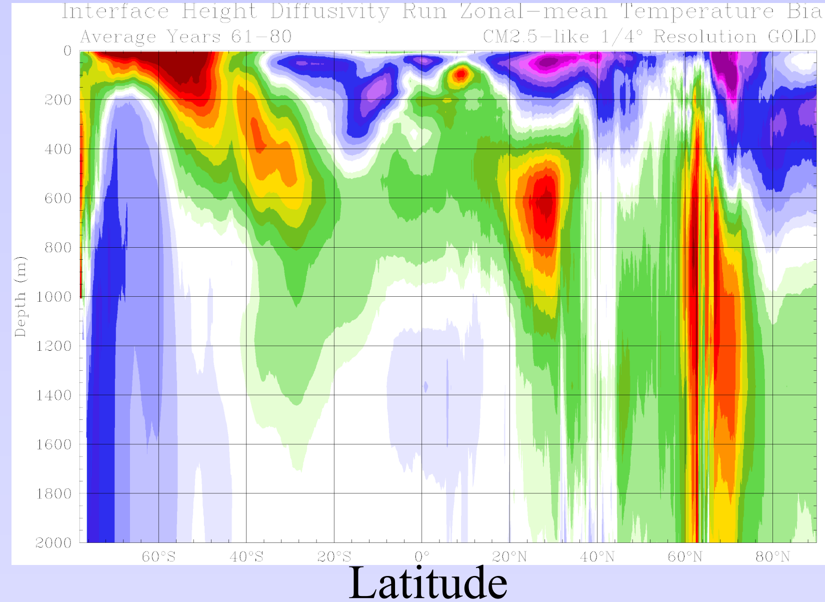


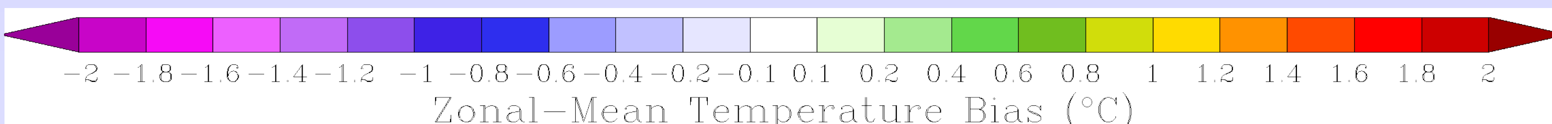
Temperature Bias Reduction in $^{1}/_{4}°$ Coupled Model

No Isopycnal Height Diffusion (Control)



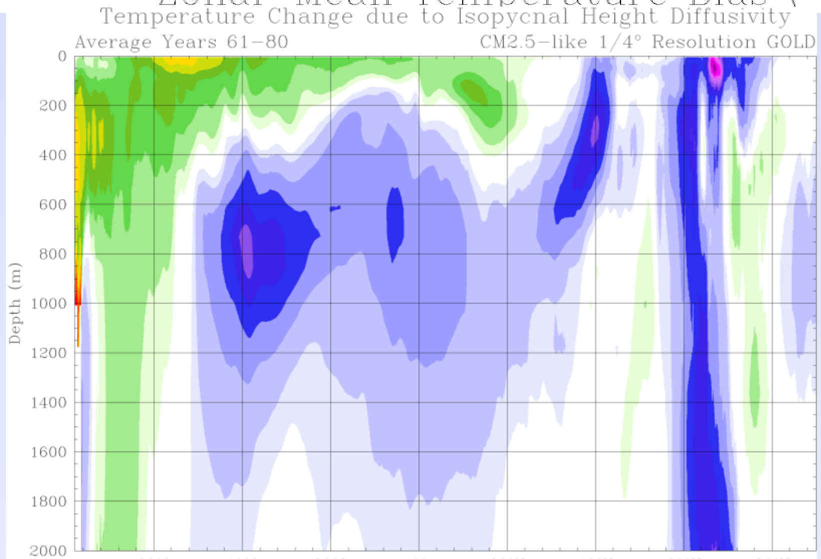
With Res. Fn. & Isopycnal Height Diffusion





Change in Bias

Same scale as above



Averages over years 61-80

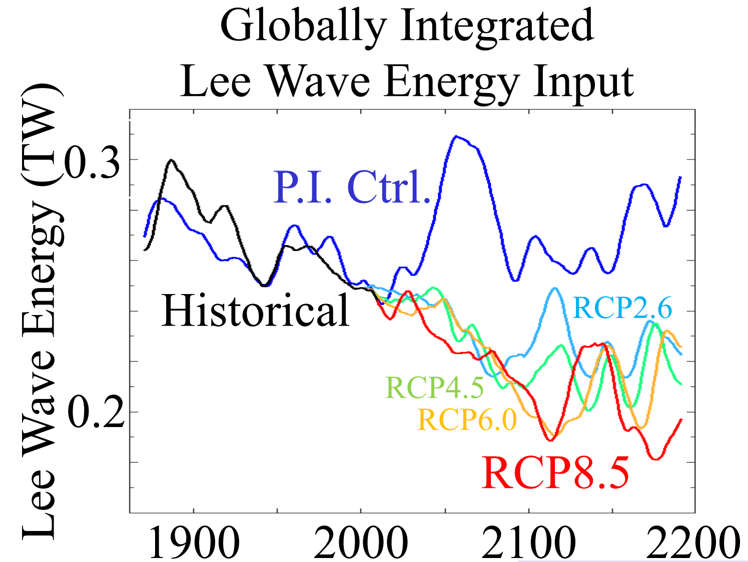
- Atmosphere not retuned
- Parameterization strength not optimized
- Vertically uniform diffusivities

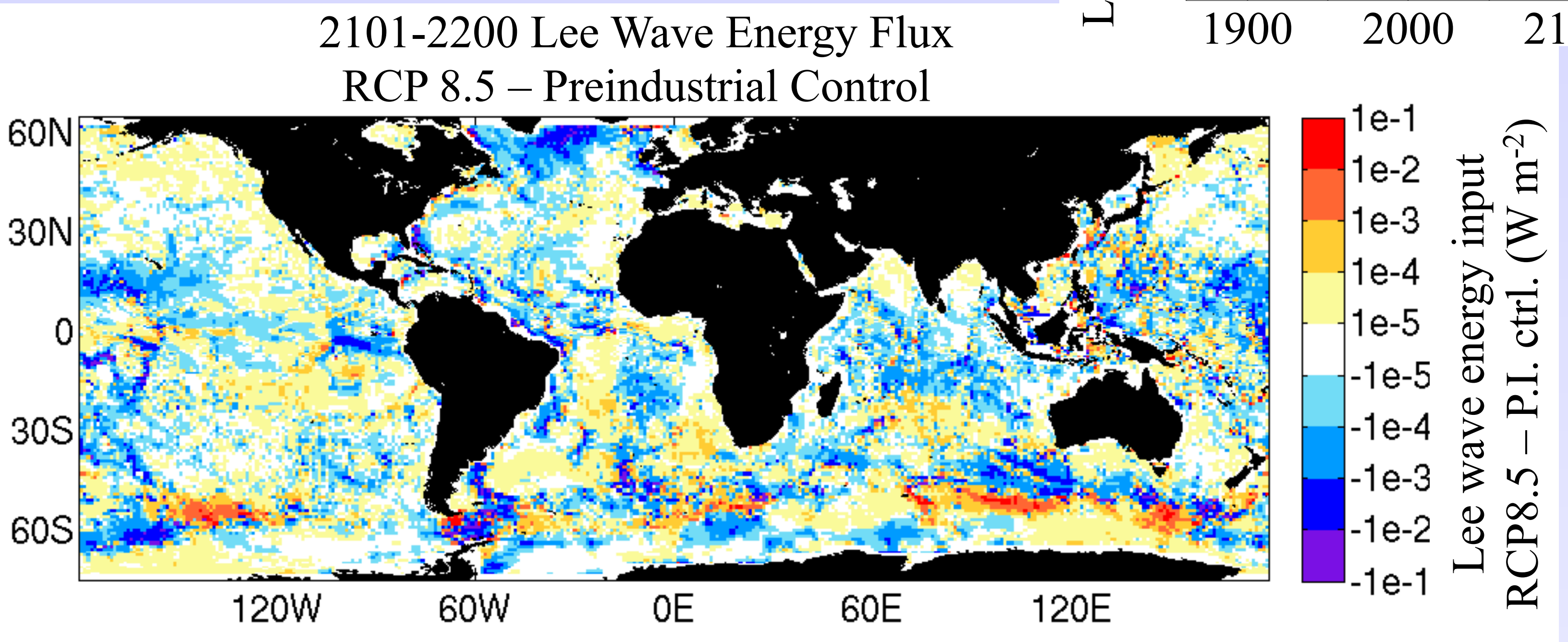
Changing Energy Fluxes into Lee Waves

Ref: Melet et al, 2014, *J. Climate* (in press)

Globally, energy input into eddy- and mean-flow driven lee waves decreases with a warming climate due to weaker near-bottom flows…

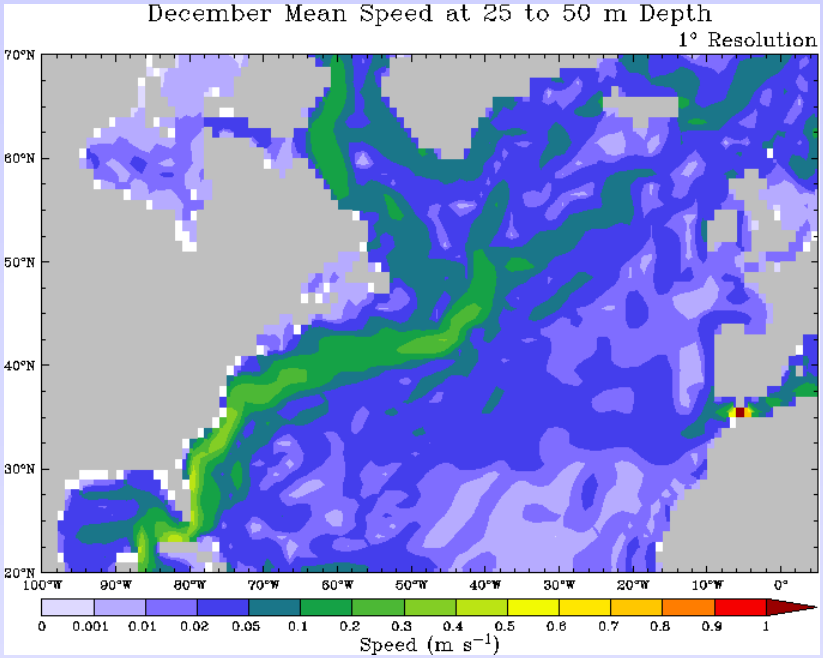
But lee-wave energy input to the Southern Ocean increases due to stronger eddies & mean flow…

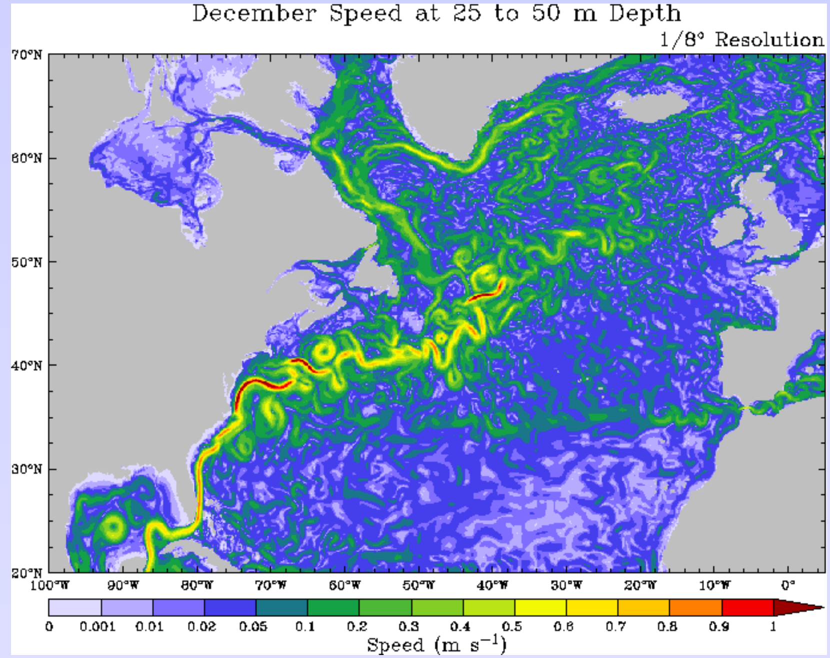




The Gulf Stream in 2 Global Models with Resolution Function Scaling and MEKE

1°





$^{1}/_{8}$°

- With MEKE scaled via a Resolution Function, these global ocean models differ only in their choice of grid-spacing and time-step
- Backscatter gives plausible explicit eddy effects at lower resolutions
- Climate projections of heat and carbon update require that eddy parameterizations respond appropriately to an evolving ocean state