1. Introduction
Do paleo-data have the power to constrain the past ocean circulation and property distributions? Here we design an idealized experiment to test our predictive ability to reconstruct 3D ocean properties by considering paleoceanographic-like data. We attempt to reconstruct the known, modern-day global distributions by using a state estimation method that combines a kinematic tracer transport model with observations that have paleoceanographic characteristics. This test is a prerequisite to being able to reconstruct the past ocean circulation.

2. Experimental Design
Define a suite of modern-day gridded tracer distributions [1,2,3] as the "truth" or REFERENCE:

- SPARSE experiment: "Observe" δ18O, δ13C, Cd/Ca at all 492 sites.
- PROXY experiment: "Observe" δ18O, δ13C, Cd/Ca at all 492 sites.
- SPARSE+PROXY experiment: "Observe" δ18O, δ13C, Cd/Ca at all 492 sites.

To assess the performance of the reconstruction method, add one additional case:
- OI: "Observe" δ18O at the 492 sites and reconstruct with an optimal interpolation/objective mapping method.

Use a steady-state estimation method [3] to reconstruct global property fields in the following scenarios:
- SPARSE experiment: "Observe" δ18O, δ13C, Cd/Ca at all 492 sites.
- PROXY experiment: "Observe" δ18O, δ13C, Cd/Ca at all 492 sites.
- SPARSE+PROXY experiment: "Observe" δ18O, δ13C, Cd/Ca at all 492 sites.

3. Reconstructed deep ocean δ13C structure

Average δ13C between 2 and 3 km depth for the 4 experiments (background color) and the observations in this depth range (colored squares). Compare to the true distribution in the left hand column of the poster. Paleo-data that are either sparse or proxy data types provide information to reconstruct the large-scale structure, but data that have both characteristics (i.e., sparse proxies) cannot.

4. Deep ocean δ13C errors

Average δ13C RMS error as a function of horizontal distance (x axis) and vertical distance (y axis) from a given observation for the following experiments: (top left) SPARSE, (top right) PROXY, (bottom left) SPARSE+PROXY, and (bottom right) OI. Hotter colors represent the reconstruction being closer to the truth. The contour interval is 0.05‰ VPDB. The SPARSE experiment, for example, shows the influence of observations over 10000 horizontal km and 2 vertical km.

5. Reconstructed Atlantic δ13C structure

Atlantic Ocean zonally-averaged δ13C (background colors) and the observations in this basin (colored squares). Compare to the true distribution in the left hand column of the poster. The SPARSE and PROXY experiments are generally successful, but the SPARSE+PROXY experiment is not.

6. Atlantic δ13C errors

Atlantic Ocean zonally-averaged δ13C errors (background colors) and the pointwise observational mxffffffff in this basin (colored squares). In the more realistic SPARSE+PROXY experiment, errors are large in the Southern Ocean and the Arctic Ocean. Large errors in the surface ocean may be alleviated with the inclusion of planktonic foraminiferal data.

7. Vertical error stats

Vertical profile of the root-mean-square error of the reconstruction (left) at the observational sites and (right) at every grid point. The metrics are diagnosed for the SPARSE (red), PROXY (cyan), PROXY+SPARSE (green), and OI (black) experiments. The 0.05‰ error level (vertical dashed gray line, both panels) is the noise level in the observations. The "zero skill" error level (rightmost dashed gray line, right panel) is included for context.

The SPARSE and PROXY experiments reconstruct the withheld data everywhere below 1500 meters depth.

8. Influence function of an observation

δ13C RMS error as a function of horizontal distance (x axis) and vertical distance (y axis) from a given observation for the following experiments: (top left) SPARSE, (top right) PROXY, and (bottom right) OI. Hotter colors represent the reconstruction being closer to the truth. The contour interval is 0.05‰ VPDB. The SPARSE experiment, for example, shows the influence of observations over 10000 horizontal km and 2 vertical km.

9. Summary
- Global property distributions can be reconstructed from data as sparse as that from the LGM.
- Inference of past ocean properties is limited, however, with a sparse dataset of 3 LGM proxy data types (δ18O, δ13C, Cd/Ca).
- A state estimation method has skill at large spatial scales and outperforms objective mapping/optimal interpolation.
- Reconstruction of the past overturning circulation is likely more difficult than the 3D property distributions [4,5].

Acknowledgments
The authors thank Tom Chalk and Carl Wunsch for comments on the manuscript and Tor Eldevik, Steve Jayne, Peter Huybers, Louise Kelling, Lorraine Lisiecki, Carlye Peterson, and Pelle Robbins for discussions. This work was supported by NSF grants 1124880 (G.G.) and 1125422 (H.J.S.) and the WHOI Ocean and Climate Change Institute (G.G.).

References
1. Schmittner et al., Biogeosci. Discuss., 2013.

This work: