Coupling between AMOC variability and wind-driven gyres

Michael Spall
David Nieves
WHOI

1. How is variability in AMOC propagated from the high latitude North Atlantic to the Southern Ocean?

2. How do wind-driven gyres influence AMOC variability?
   - steady winds may alter buoyancy-forced variability
   - time-dependent winds may introduce variability to MOC
Develop ideas with a single layer reduced gravity model
(later compared with a two layer shallow water numerical model)

Force the model with a specified inflow at the northern boundary with mean $S$
a sinusoidal variation $\Delta S$, frequency $\omega_f$

$$\psi_N = -S \left[ 1 + \frac{\Delta S}{S} \sin(\omega_f t) \right], \quad y = L_N$$

Outflow is assumed to be in geostrophic balance with zonal layer thickness
variation from the eastern to the western boundary

$$\psi_S = -\frac{g'}{2f_s} \left[ h_E^2 - h_W^2 \right], \quad y = -L_S$$

Similar formulations have been used by Johnson and Marshall (2002a, b; 2004)
Consider a mass budget for the entire basin as a balance between inflow, outflow, diapycnal mixing, and storage:

\[ \frac{d\bar{h}^{x,y}}{dt} = -\frac{1}{A} \int_0^{L_x} h_{uv} \left|_{y=L_N}^{y=-L_S} \right. dx - \frac{1}{\gamma} \left( \bar{h}^{x,y} - H_{\gamma} \right) \]

A=area=L_x(L_N+L_S)  \quad \gamma \text{ is diapycnal mixing time scale}  \quad H_{\gamma} \text{ is target layer thickness}

The key assumption is that the layer thickness on the eastern boundary is the same as the basin averaged layer thickness, the balance then simplifies to:

\[ \frac{d\bar{h}^{x,y}}{dt} = -\psi_N \frac{g'}{A} - \frac{g'}{2f_S A} \left[ \left( \bar{h}^{x,y} \right)^2 - h_{SW}^2 \right] - \frac{\bar{h}^{x,y} - H_{\gamma}}{\gamma} \]

This assumption can be shown to be valid for time scales exceeding the basin crossing time scale for baroclinic Rossby waves.
**Analytic solution for adiabatic case (γ=0)**

Average layer thickness

\[ \bar{h}^{x,y} = H \tanh \left( \frac{t - c_0}{2\tau} \right) + h' \left( \sin \theta + \sin(\phi)e^{-t/\tau} \right) \]

mean thickness

\[ H = \sqrt{2fS/g' + h_{SW}^2} \]

\[ \theta = \omega_f t - \phi \]

\[ \phi = \tan^{-1}(\omega_f \tau) \]

Perturbation amplitude

\[ h' = \frac{\tau \Delta S/A}{\sqrt{1 + (\omega_f \tau)^2}} \]

Adjustment time scale

\( (= \text{volume/transport}) \)

\( = \text{the basin-crossing time scale at } y=L_S \)

\[ \tau = \frac{A_0S}{g'H} \]

The transport through the southern boundary can then be calculated as:

\[ \psi_S = -S \left( 1 + \frac{Ah'}{S\tau} \sin(\theta) + \frac{Ah'^2}{4S\tau H} (1 - \cos(2\theta)) \right) \]

Note that the variability in transport at \( L_S \) decreases for increasing \( \omega_f \tau \)
Comparison between theory and reduced gravity numerical model

5 year periodic forcing
vary $g'$

Numerical layer model : solid lines
Theory : dashed lines
Generally close agreement

Outflow at southern boundary has much weaker amplitude than inflow at northern boundary – the difference represents storage in the basin interior
It is helpful to nondimensionalize the equations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>$L = L_x$</td>
</tr>
<tr>
<td>velocity</td>
<td>$U = (g'H)^{1/2}/3$</td>
</tr>
<tr>
<td>time</td>
<td>$T = 3L_x/(g'H)^{1/2}$</td>
</tr>
<tr>
<td>thickness</td>
<td>$H = (2f_s\Psi_N/g')^{1/2}$</td>
</tr>
</tbody>
</table>

yields 4 nondimensional numbers

- $\mu = L_x^2/3L_d^2$
- $\lambda = L_S/L_x$
- $\alpha = L_N/L_S$
- $P = T_f/T$

for the North Atlantic

- $\lambda \approx 0.6$
- $\mu = \mathcal{O}(10^2)$
- $\alpha = \mathcal{O}(1)$

Thickness anomaly

$$\overline{h'} = \left(1 + \left[\mu \lambda^2 \left(1 + \alpha \frac{2\pi}{P}\right)^2\right]\right)^{1/2}$$

Transition from weak variability to strong variability is at approx $P > \mu \lambda^2$

( The red star is roughly North Atlantic basin with forcing period of 20 years )
Compare with delay-equation solutions of Johnson and Marshall (2002a, b; 2004)

\[ \bar{\chi}' = \frac{1/2}{\left(1 + \left[ \mu \lambda^2 (1 + \alpha) \frac{2\pi}{P} \right]^2 \right)^{1/2}} \]

Nondimensional form of Johnson and Marshall coupled delay equations

\[ h_e = \lambda \bar{c} \left[ -1 + \left( 1 + \frac{\Psi_N + 2\lambda S}{\lambda^2 \bar{c}^2} \right)^{1/2} \right] \]

\[ \bar{c} = \int c \, dy \approx 4\mu^{1/2} \quad S = \int h_e (t - \mu / c) c \, dy \]

Theory captures basic structure of the more complete solution but in closed form
a) Variability of MOC at southern boundary is large for low frequency forcing \((P > \mu \lambda^2)\)

b) Mass exchange between the western boundary current and the interior is large for high frequency forcing

c) Latitude range of exchange between boundary current and interior varies – it is equatorially trapped for high and very low frequency forcing and broadly distributed for intermediate frequency forcing
How might wind forcing alter these results?

- Double gyre wind stress in two layer primitive equation model (20 km grid)
- MOC is forced in both upper (northward) and lower (southward) layers
- Wind-driven gyres introduce region of homogenized PV
- Seek to understand:
  -- altered communication between the interior and the western boundary currents?
  -- time-dependent winds introduce time dependence in MOC?
**time-dependent MOC (20 year period)**

with and without steady wind

log(spectral energy) of meridional transport in layer 2 interior (\(x > 500\) km) as a function of latitude with and without steady winds

Wind-driven gyres alters frequency distribution:

- introduces high frequency at gyre/gyre boundary (\(y=4000\) km) and subtropical (\(y=2500\) km)
- enhances low frequency at northern and southern limits of subtropical and subpolar gyres (also perhaps slightly in the southern hemisphere)

- **Wind does not alter the frequency of the total MOC, just the zonal distribution**
Steady MOC with time-dependent wind (20 year period)

Strong wind pulls water in from the north and reduces outflow to the south in layer 2 (and vice versa) with slight phase lag

Results from adjustment of region of homogenized PV under the Gulf Stream to changing winds

PV (thickness) anomaly propagates westward, down western boundary, across the equator, and poleward along the eastern boundary – where it alters the MOC
Summary

• Analytic solution derived for mass storage and export of mid-depth MOC

• The deep western boundary current is very leaky at frequencies less than the basin crossing time scale

• Results are broadly consistent with previous results of Johnson and Marshall and provide complimentary viewpoint

• Steady wind-driven mid-latitude gyres alter the low frequency pathways of MOC

• Time-dependent winds at mid-latitudes introduce low-frequency variability into the MOC extending to high latitudes in both hemispheres