Energizing Turbulence Closures In Ocean Models

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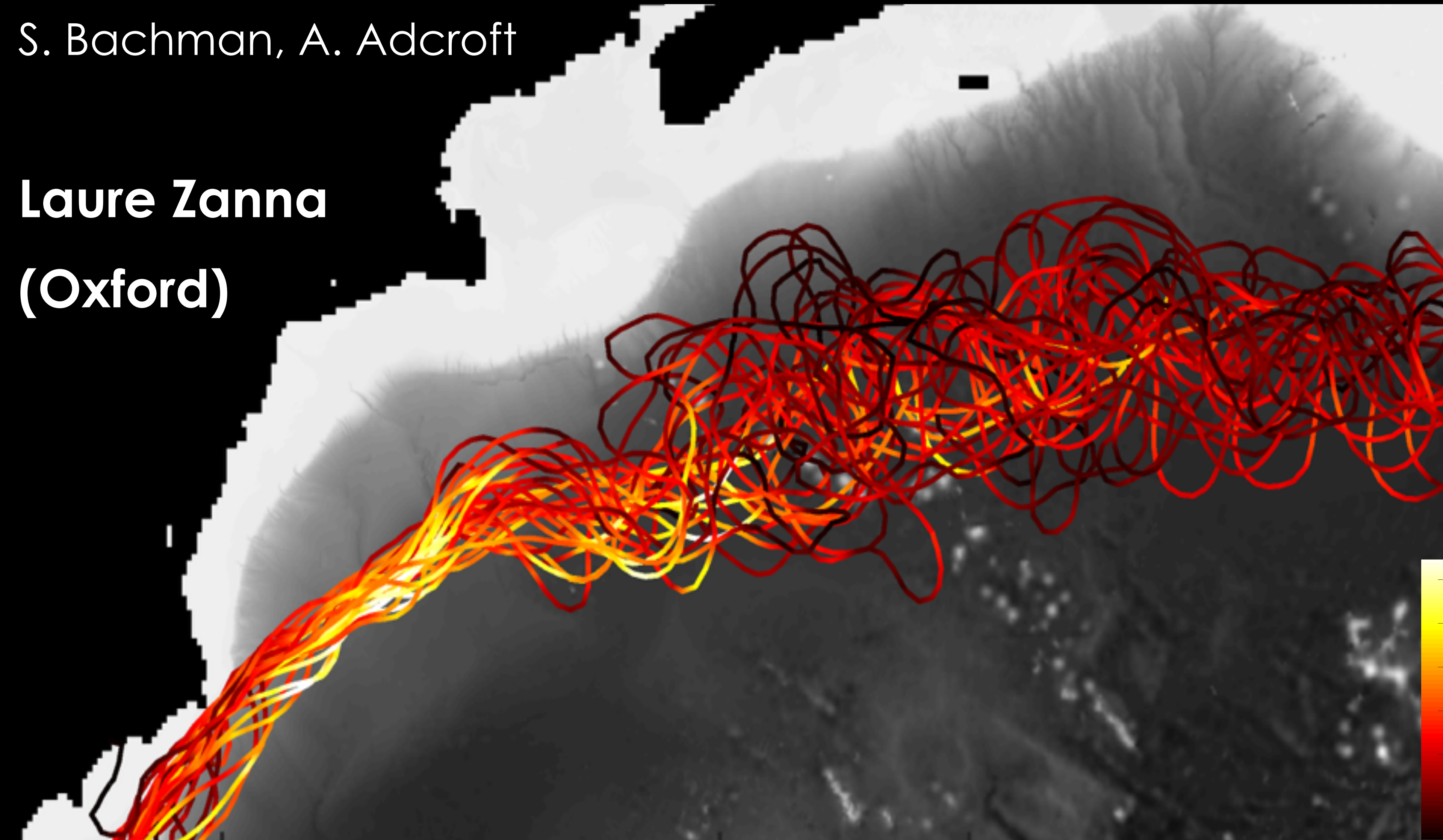


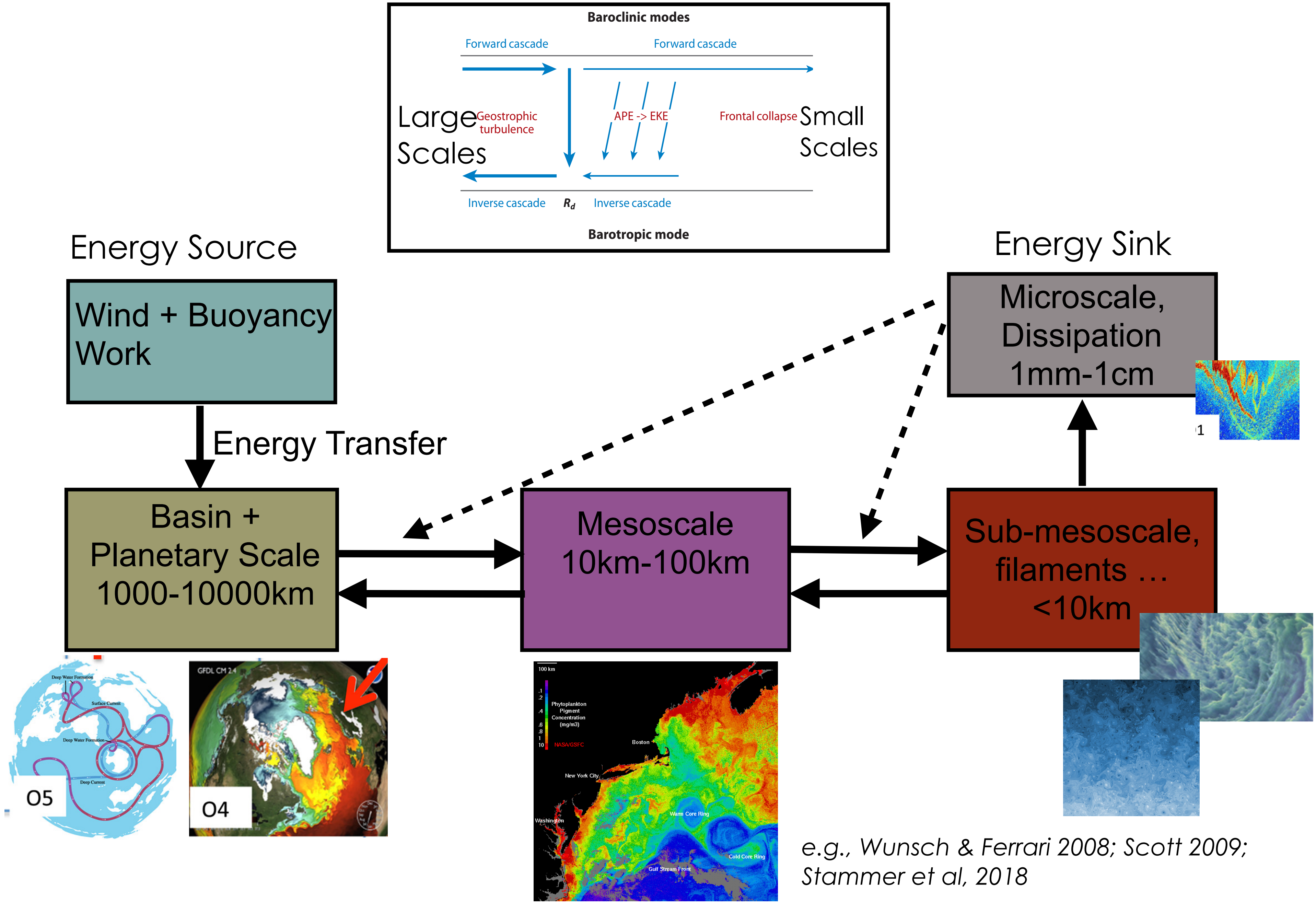
Outline

- Motivation: energy cycle & parametrizations

- Recent Avenues for Mesoscale Eddy Parameterizations & Energy Sources/Sinks with a focus on
  - Mesoscale to large-scale interactions & a new PV-based/ momentum parameterization based on non-newtonian flow tensor
  - A new momentum parameterization based on non-newtonian flow tensor

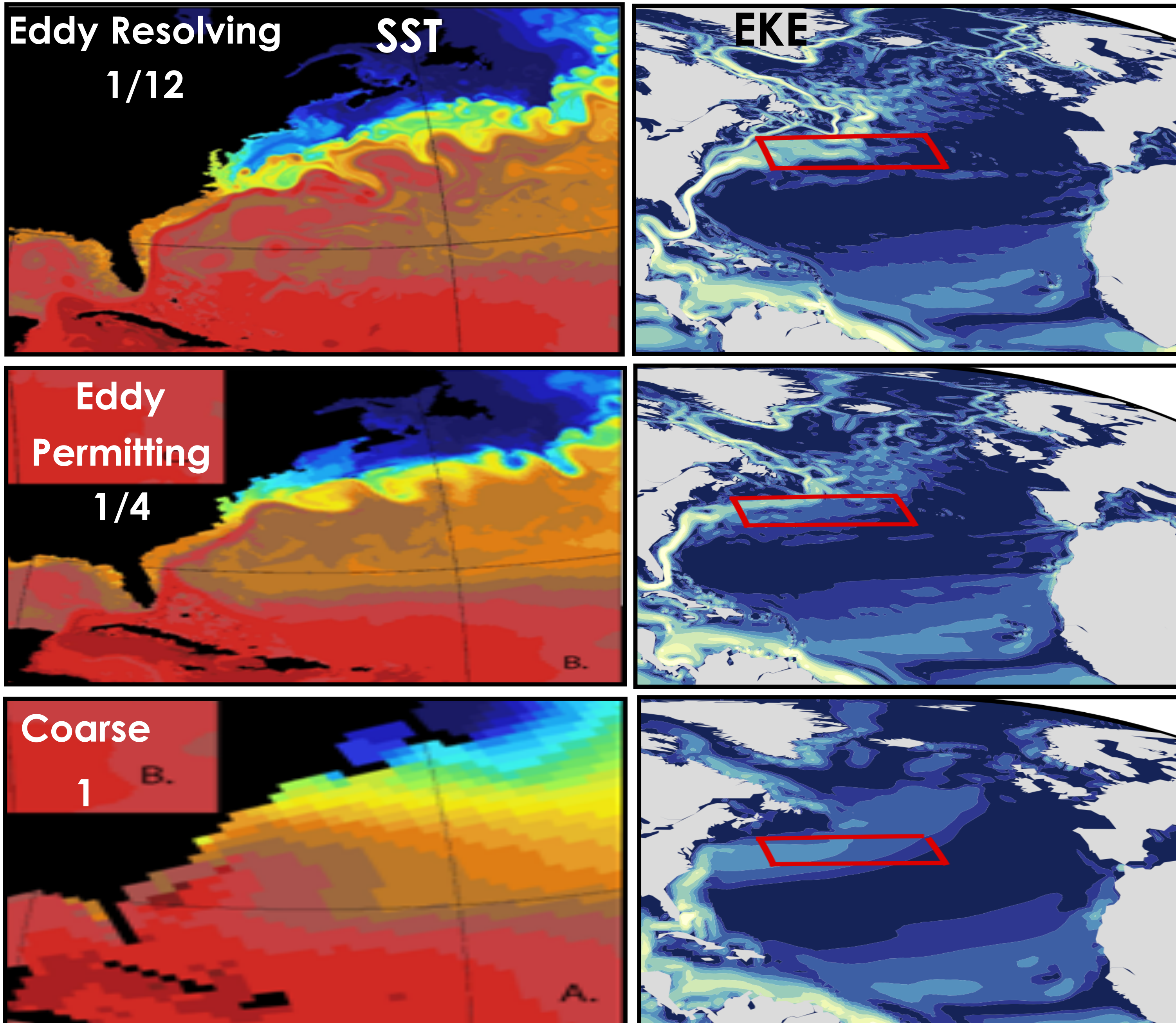
- Summary & Possible Avenues for 2019+

Energy Cycle, Reservoirs & Scales



e.g., Wunsch & Ferrari 2008; Scott 2009; Stammer et al, 2018

Symptoms of missing energy in models

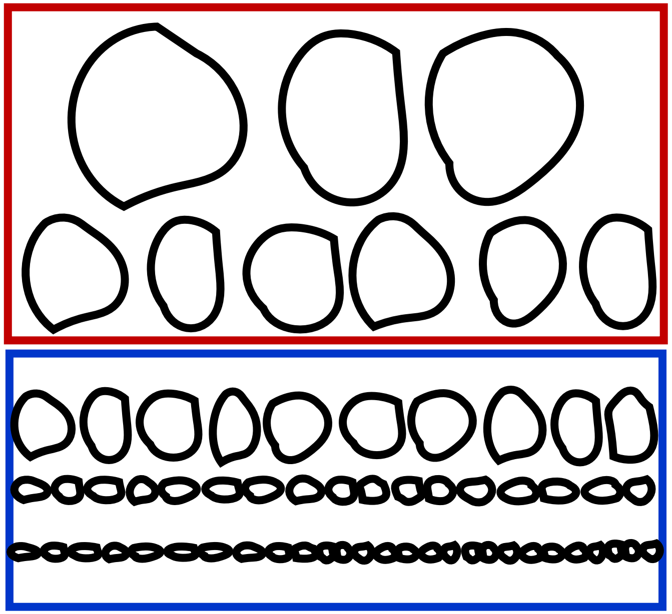


High Resolution Model= "truth", turbulent, sharp gradients, filaments, eddies

Lower Resolution Models: weak jets, little/no eddies & variability

The Parameterization/Closure Problem

- Including unresolved processes at low computational cost



$\overline{(\ )}$ = slow- /large-scale fluctuations

> grid-box size

$(\ )'$= fast/small-scale (eddy) fluctuations

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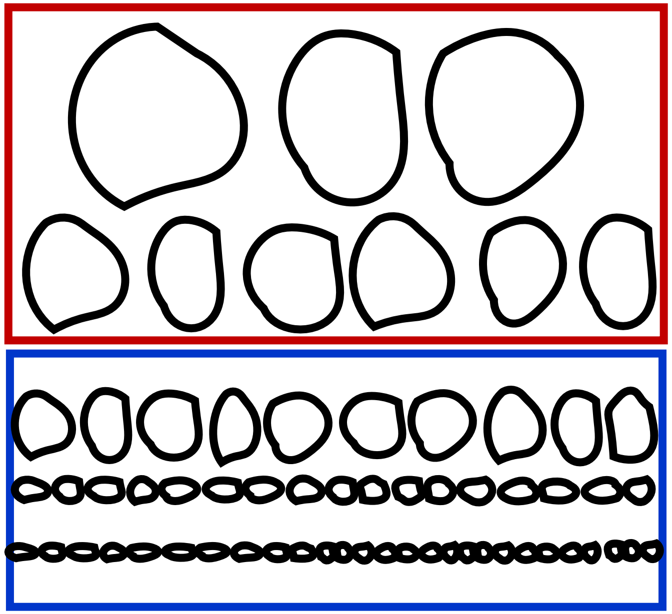

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- E.g., momentum (the same applies to buoyancy equation)

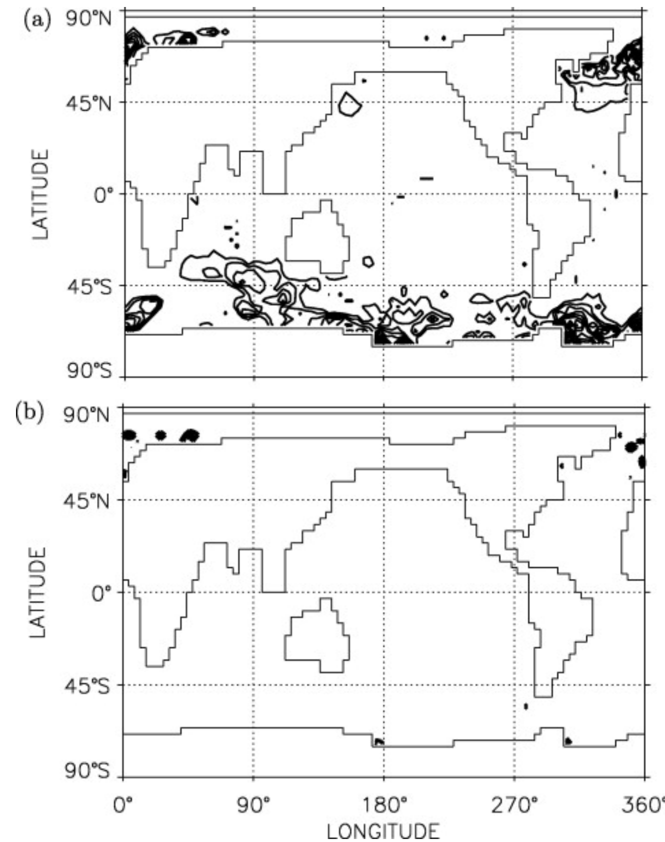
$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} - f\overline{v} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x} + \frac{\overline{F}_x}{\rho_0} - \frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y}$$

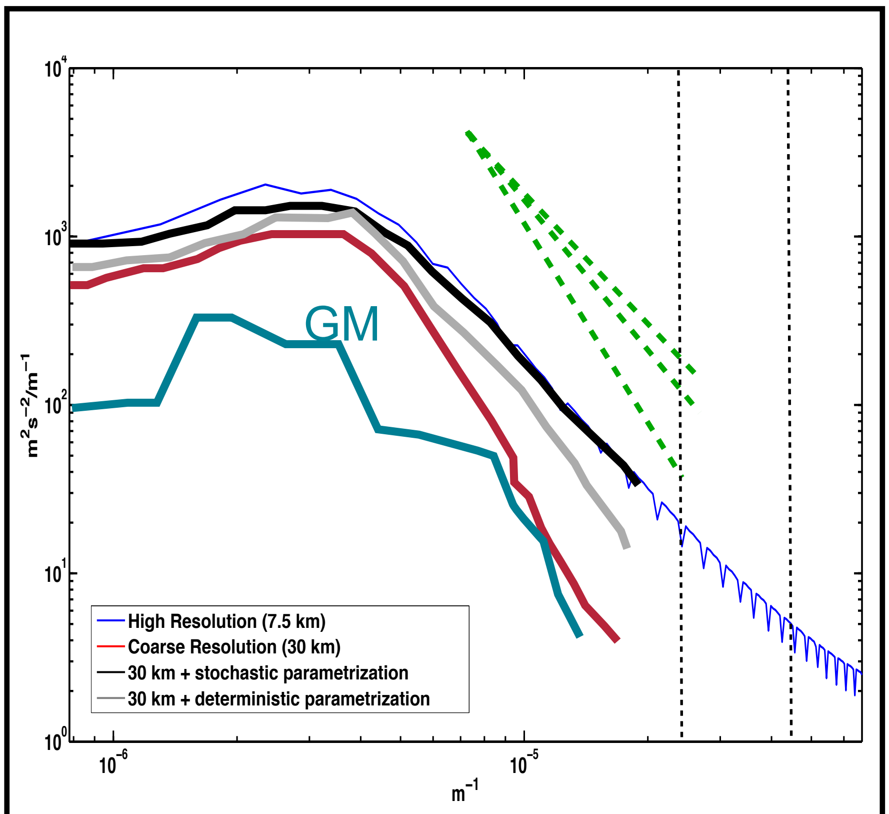
$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} - f\overline{v} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x} + \frac{\overline{F}_x}{\rho_0} + \boxed{\textbf{Turbulence closure}}$$

Buoyancy Closure / Gent-McWilliams (1990)

- **Mimics** baroclinic instability, flattening of isopycnal, net sink of APE

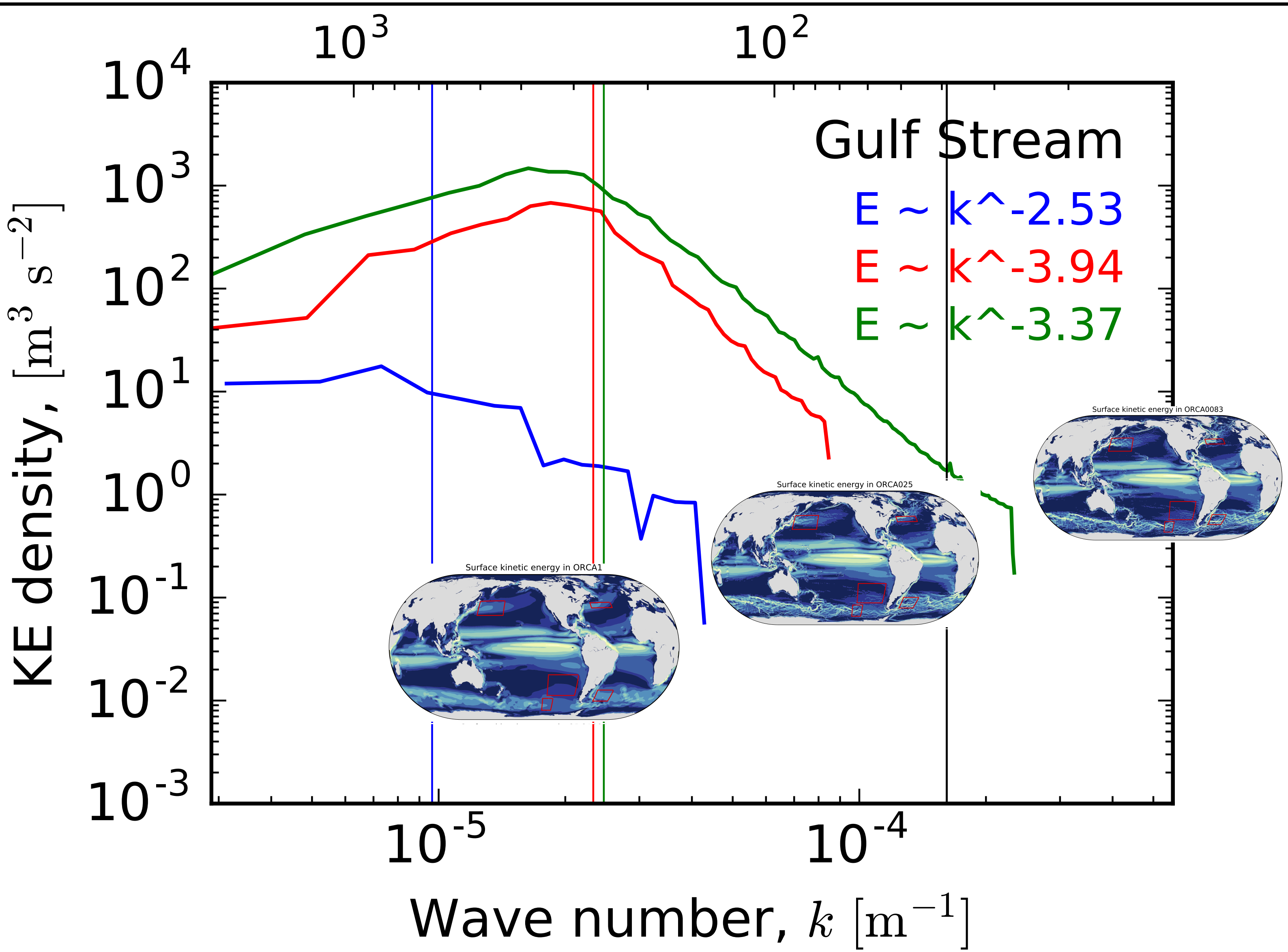
Gent et al, 1995





➡ *Large improvements to large-scale circulation, especially Southern Ocean & density distribution, stratification, eliminated spurious convection*

Effects of Resolution & Sub-Grid Parametrization

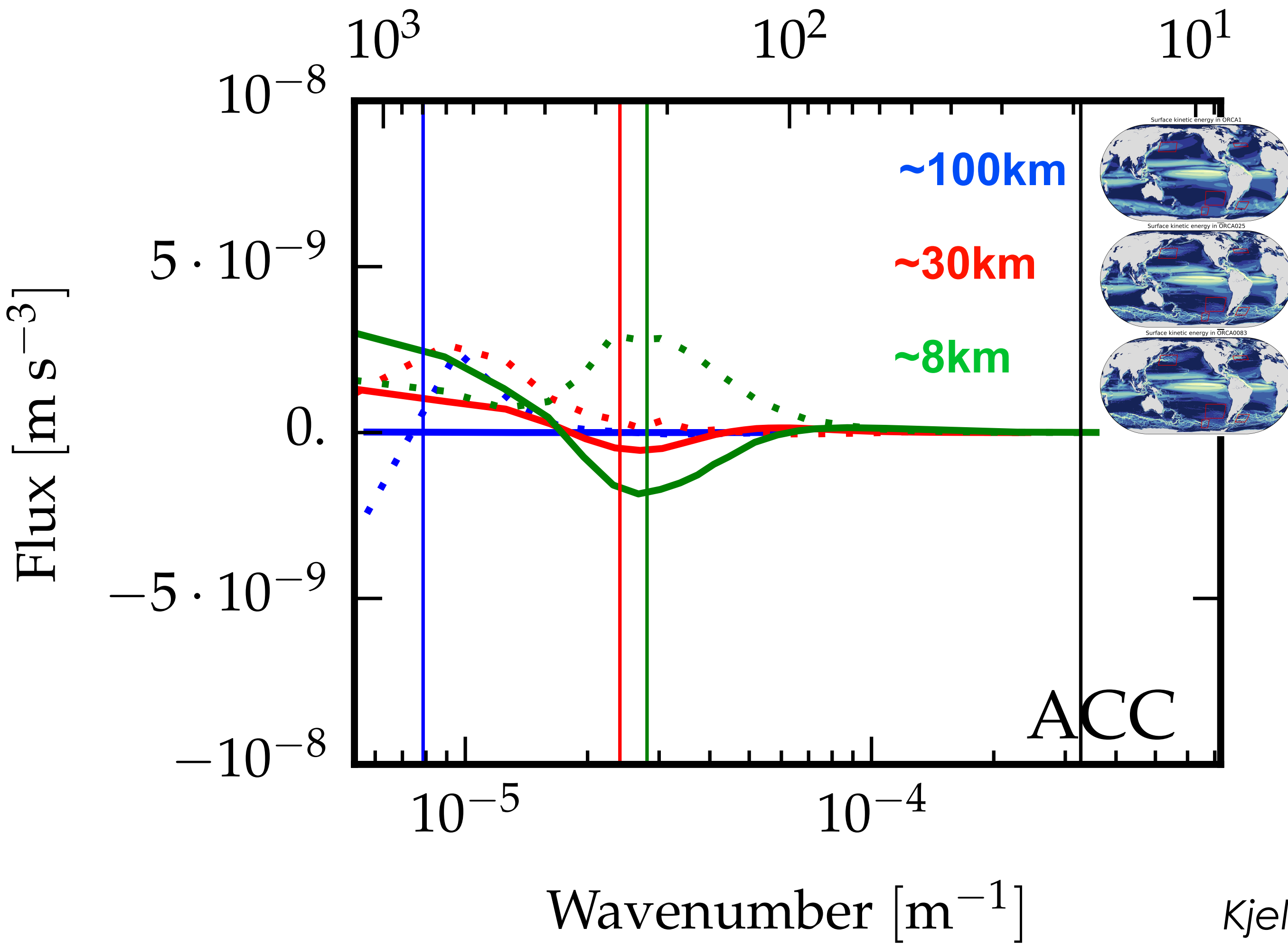


Kjellsson & Zanna, 2017

see also Pearson et al, 2017 for impact of sub-grid parametrizations

Energy Cascade

- Reduced transfer of energy towards the large scale (solid lines) *(Kraichnan 67, Leith 68, Charney 71)* at lower resolution
- Reduced conversion of APE to KE (dashed lines) at low resolution



Kjellsson & Zanna, 2017

Improving the Energy Cycle + Scale-Interaction

Various avenues *(not necessarily independent from each other)*

- Prognostic Eddy (kinetic +/or potential) Energy Equation in 2D or 3D

(e.g., Cessi 2008, Eden & Greatbatch, 2009, Adcroft & Marshall, 2010)

$$\overline{E}_{\text{eke}} = \overline{(u'^2 + v'^2)}/2 \qquad \rho_0 \frac{d\overline{E}_{\text{eke}}}{dt} = -\nabla \cdot (\text{fluxes}) + \overline{S} - g\overline{\rho' w'} - \rho_0 \epsilon_{\text{eke}}$$

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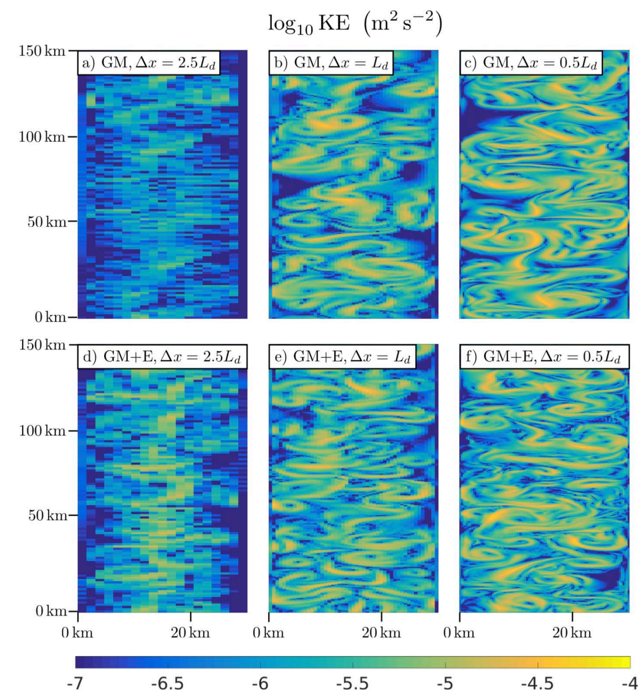
- Exchange of energy between reservoirs and/or scales: conversion of eddy energy into the mean flow *(e.g., Marshall et al 2017, Jansen et al 2015, Bachman 2019)*



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- Momentum closures for scale interaction
  - ➡ **Holm et al 2008**: Lagrangian Averaged Navier Stokes-alpha model
  - ➡ **Berloff 2005:** Stochastic Reynolds stresses
  - ➡ **Porta Mana & Zanna 2014:** Non-Newtonian Stress to parameterize turbulent fluxes by capturing both the inverse energy cascade & momentum fluxes

Our Approach: Reynolds Stresses ~ Non-Newtonian Flow

- **Eddy forcing =** Non-Newtonian / Rivlin-Ericksen Forcing *(Rivlin Ericksen 1955, Rivlin 1957)*

$$\frac{\partial \overline{u}}{\partial t} + \boxed{\overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y}} - f\overline{v} = -\frac{1}{\rho_0}\frac{\partial \overline{p}}{\partial x} + \frac{\overline{F}_x}{\rho_0} + \boxed{\nu \nabla^n \mathbf{u}} + \boxed{\text{Mesoscale eddy forcing}}$$

$$\kappa \, \nabla \cdot \mathbf{A}_2$$

Strain Tensor $\quad \mathbf{A}_1 = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$

Rivlin-Ericksen Tensor $\quad \mathbf{A}_2 = \frac{D\mathbf{A}_1}{Dt} + \nabla \mathbf{u}^T \mathbf{A}_1 + \mathbf{A}_1 \nabla \mathbf{u} + \mathbf{A}_1^2$

Mana & Zanna 2014, Anstey & Zanna 2017, Zanna et al 2017, Bachman et al 2018, Kjjellson et al, (In Prep)

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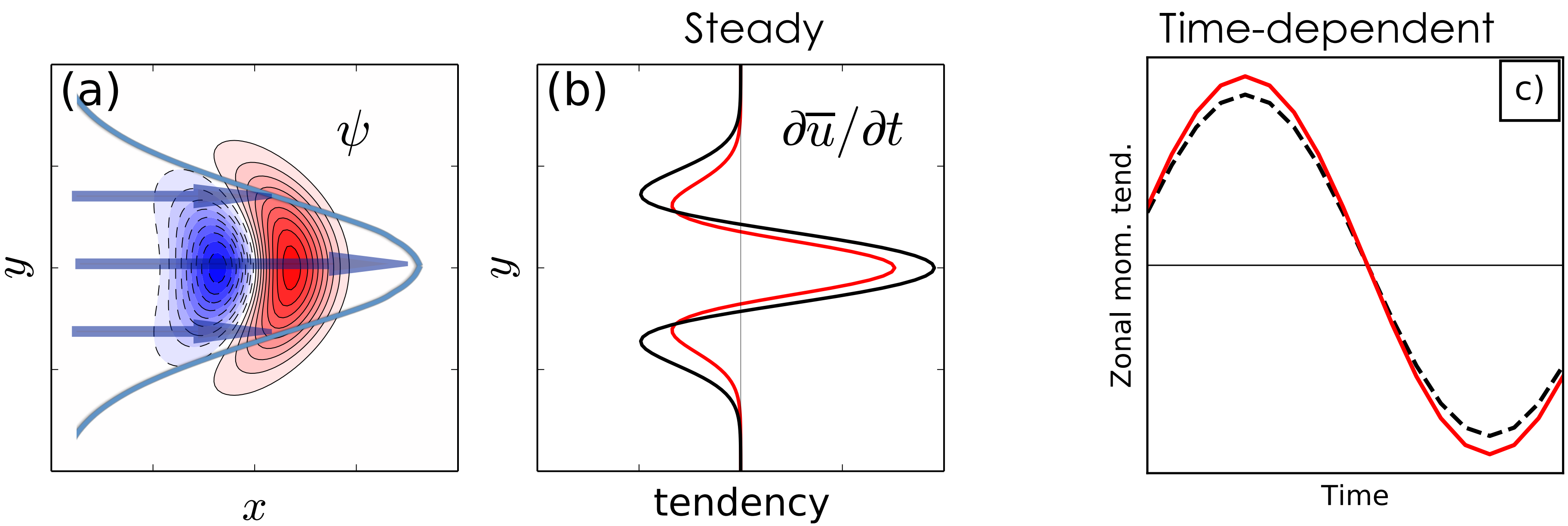
- **Why use non-Newtonian stresses as a representation of Reynolds stresses?**
  - Nonlinear & not exclusively down-gradient
  - Inject/redistribute energy + modify viscosity
  - Other properties (flow-aware): depends on the shear/strain rate, related to the local flow instabilities …

Mana & Zanna 2014, Anstey & Zanna 2017, Zanna et al 2017, Bachman et al 2018, Kjjellson et al, (In Prep)

Eddy Momentum Fluxes

➡ jet rectification & sharpening via upgradient momentum fluxes *(Starr 1963, Shutts 1986)*



Black = eddy momentum fluxes

$$-\frac{\partial\overline{u'u'}}{\partial x}-\frac{\partial\overline{u'v'}}{\partial y}$$

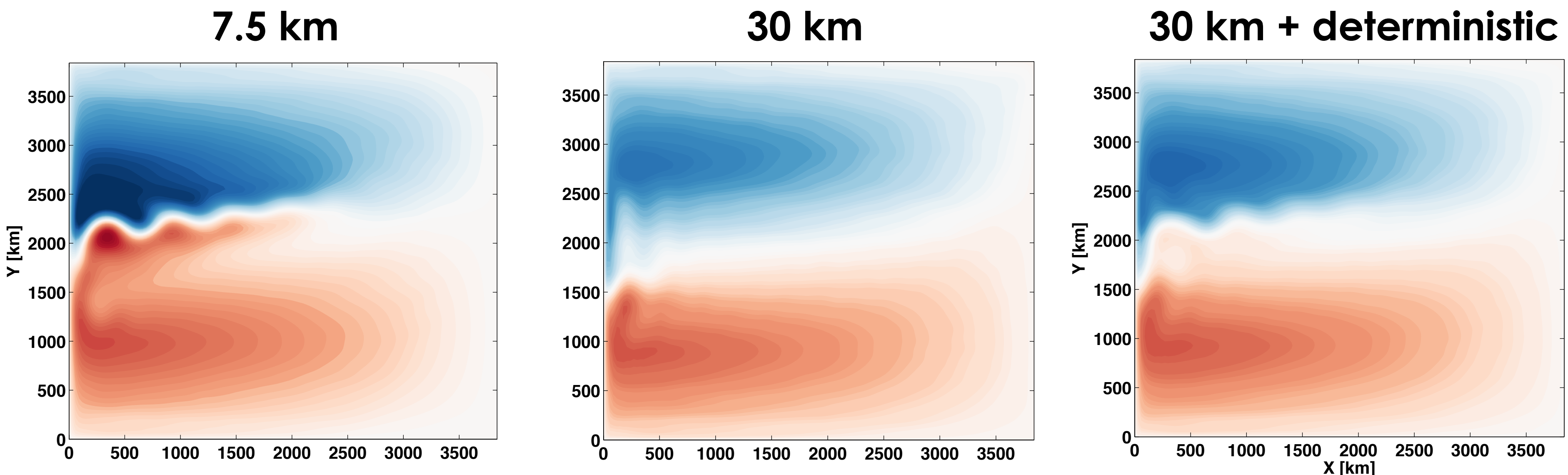
Red = Non-Newtonian fluxes

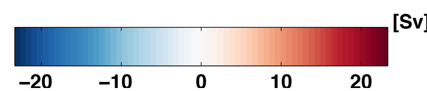
$$\nabla\cdot\mathbf{A}_2$$

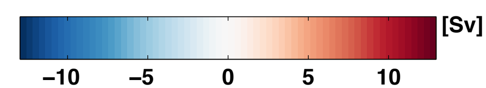
with $\mathbf{A}_2=\frac{D\mathbf{A}_1}{Dt}+\nabla\mathbf{u}^T\mathbf{A}_1+\mathbf{A}_1\nabla\mathbf{u}$

Anstey & Zanna, 2017; Kjellsson et al, In Prep

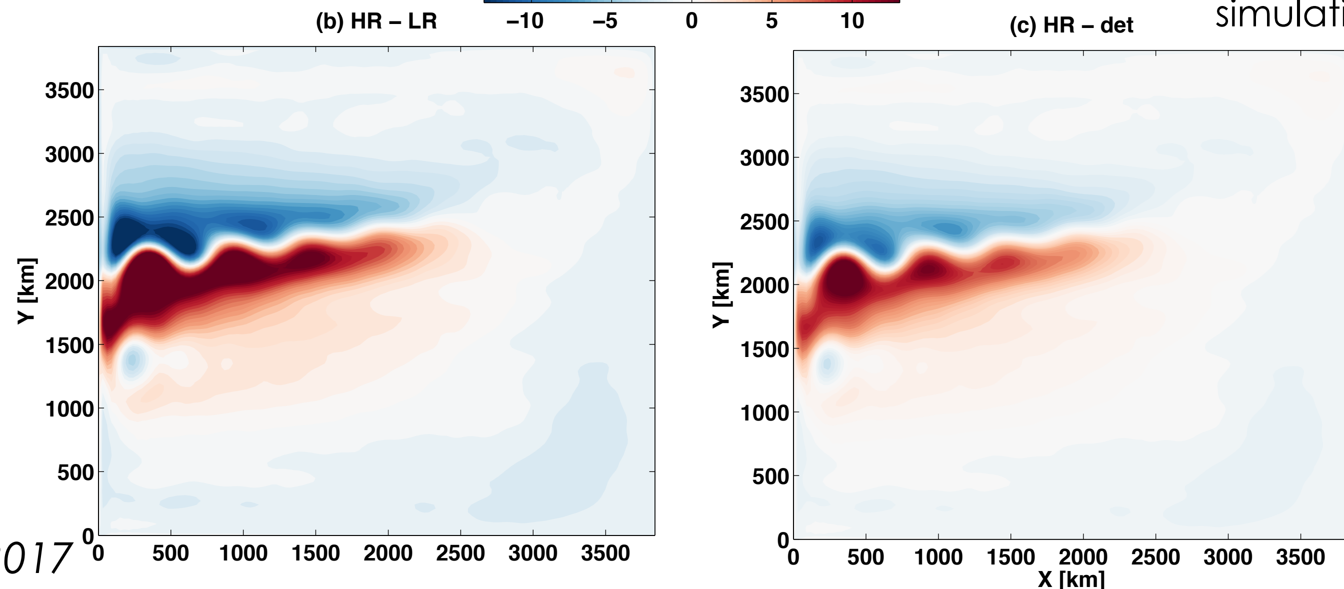
Time-Streamfunction & Error in a QG model







(numerically stable simulation)



Zanna et al, 2017

Energy Transfer

- Impact of parametrization: to compensate for the loss of energy from viscosity & kappa is scale-aware

Bachman et al, In Prep

$$\frac{\partial E_K}{\partial t} = -\kappa\nu\left(\nabla\overline{\psi}\cdot\nabla\tilde{\psi}\right) \qquad \tilde{\psi} = \left(1-\kappa\nabla^2\right)^{-1}\nabla^n\overline{\psi}.$$

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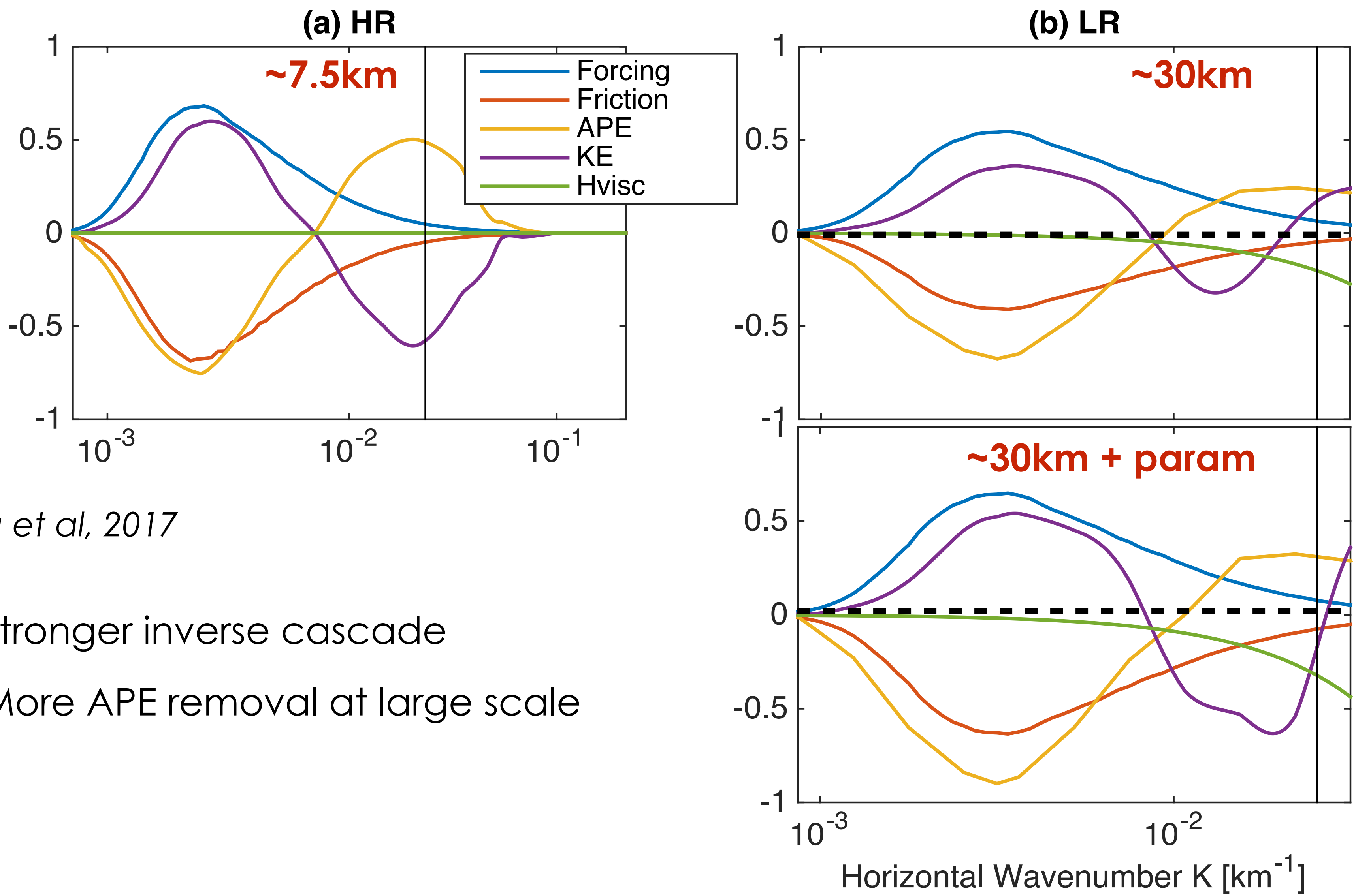

Zanna et al, 2017

- Stronger inverse cascade
- More APE removal at large scale

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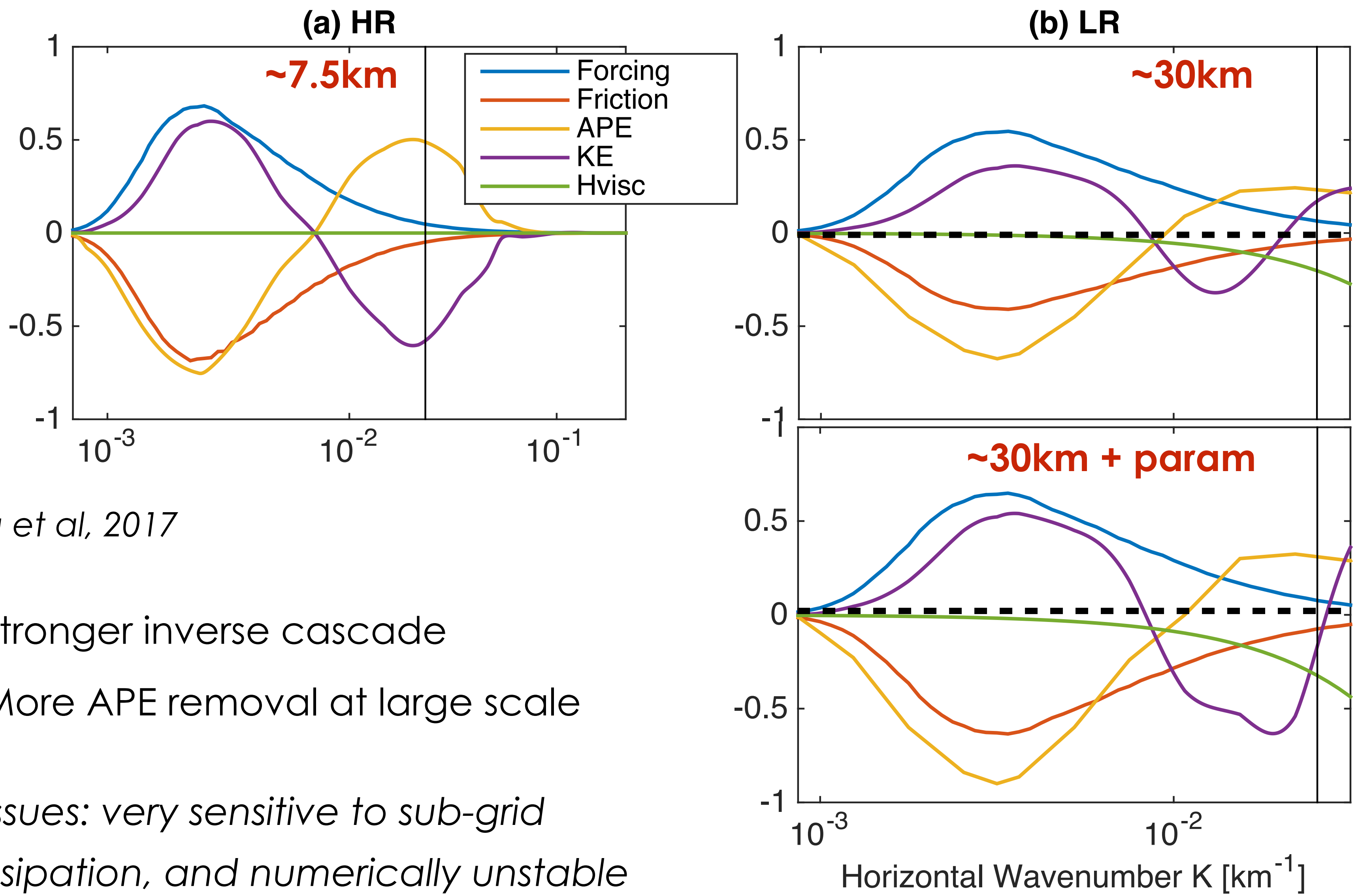


Zanna et al, 2017

- Stronger inverse cascade
- More APE removal at large scale
- *Issues: very sensitive to sub-grid dissipation, and numerically unstable*

Energy Constraint in a primitive equation model

- Depth-averaged Mesoscale Eddy Kinetic Energy Equation *(e.g., Marshall & Adcroft 2010)*

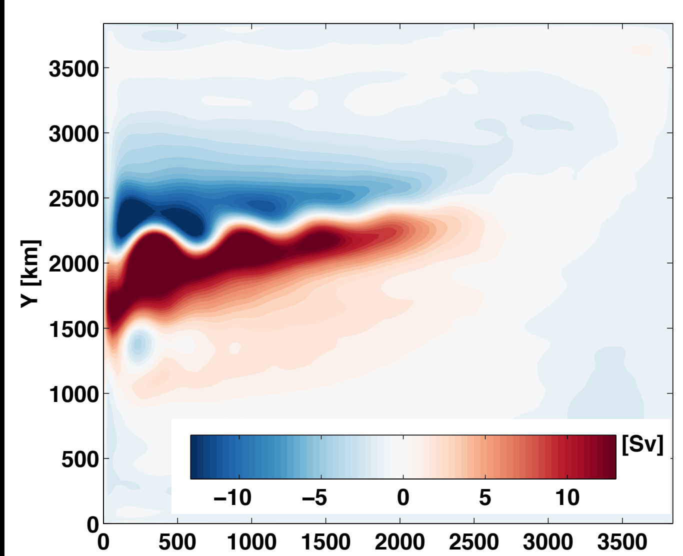
$$\frac{\partial E}{\partial t} = \hat{E}_b - \lambda E + \nabla \cdot \gamma_M \nabla E + \gamma_{non-Newt} \hat{E}_{non-Newt}$$

- Energy-constrained (& still scale-aware) κ

$$\kappa = \alpha_{nonnewt} E T^2_{non-newt}$$

- **Reduction of model bias in transport by 80%**

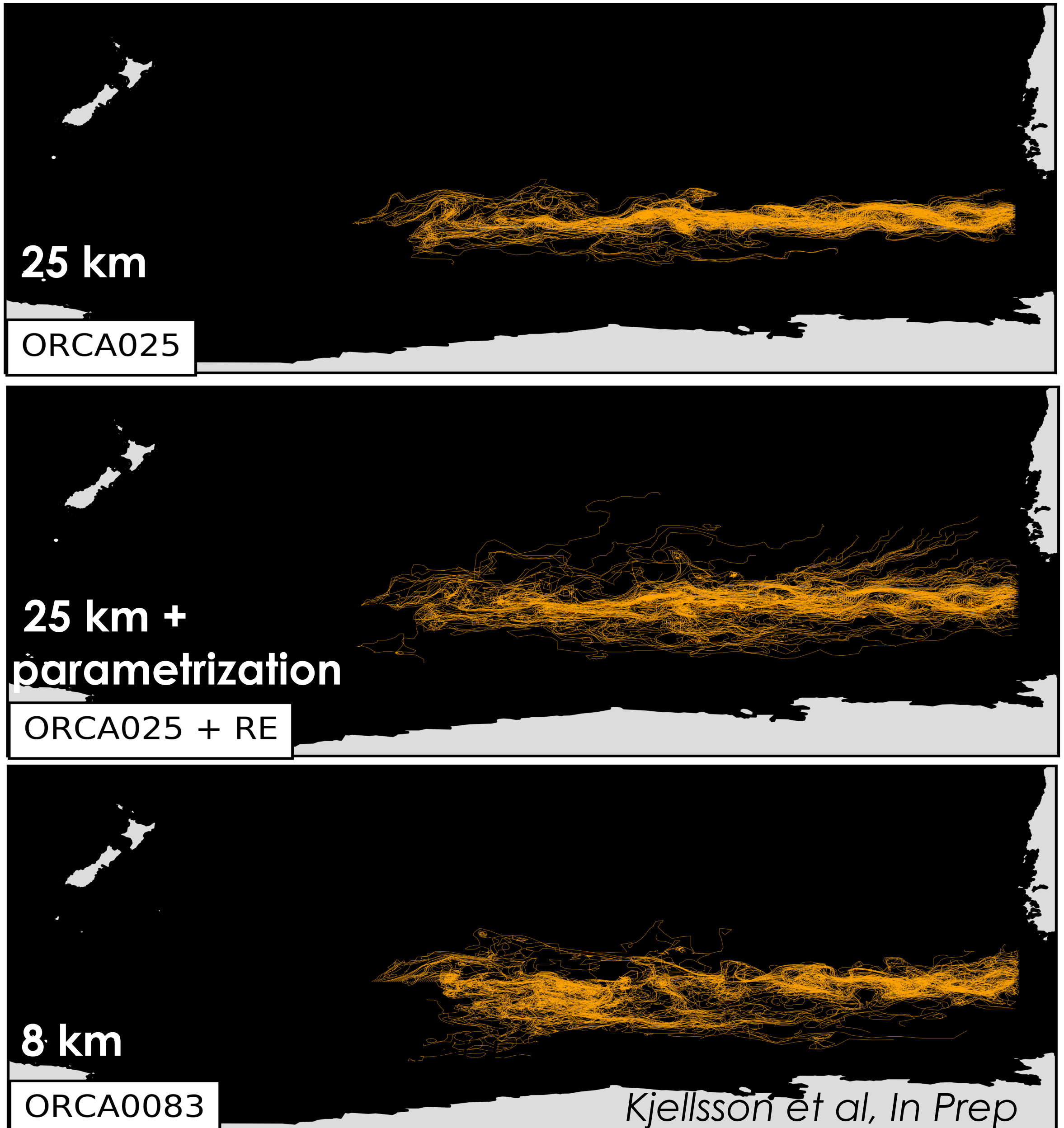
MOM6: HR- parametrised run



Zanna, Adcroft et al, In Prep

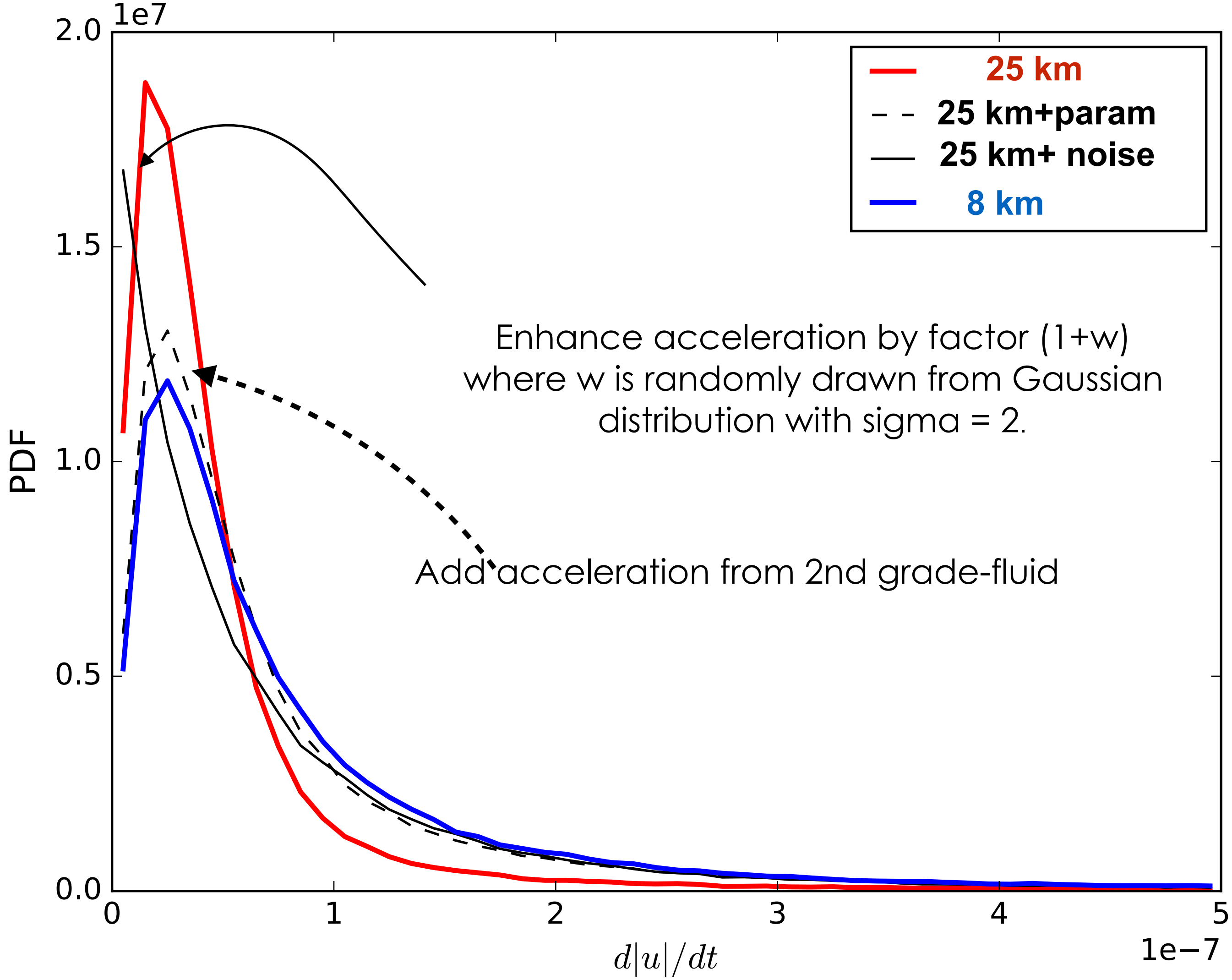
Lagrangian Modelling: Trajectories

200 particle trajectories



Lagrangian perspective: dispersion

- PDFs of absolute accelerations



Kjellsson et al, In Prep

Concluding Remarks

- Resolution & sub-grid parametrisation are breaking the energy cycle
- Many (new & old) avenues to close/improve the energy cycle in models
  - Conversion of energy between scales, reservoirs (and location)
  - **Non-Newtonian closure (PV closure)**: re-injects some of the energy lost by viscosity, scale- and flow- aware; improvements in jet dynamics, energetics, and mixing/dispersion/diffusivities

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  - Validating, testing all schemes in realistic models & impact …
  - Theory + diagnostics with models/obs (e.g., momentum + buoyancy, role of dissipation, topography, barotropic vs. baroclinic eddy energy…)
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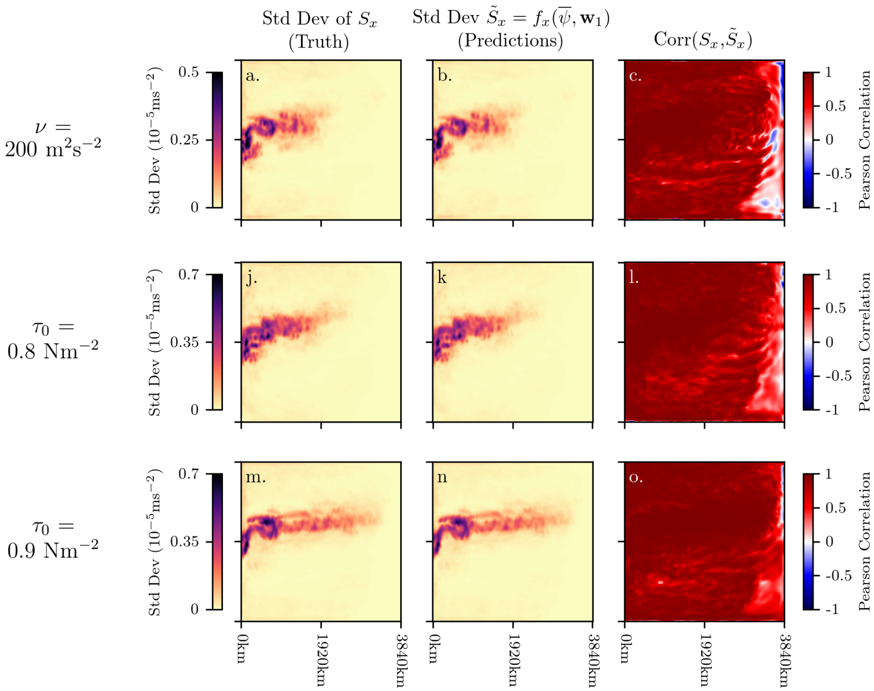
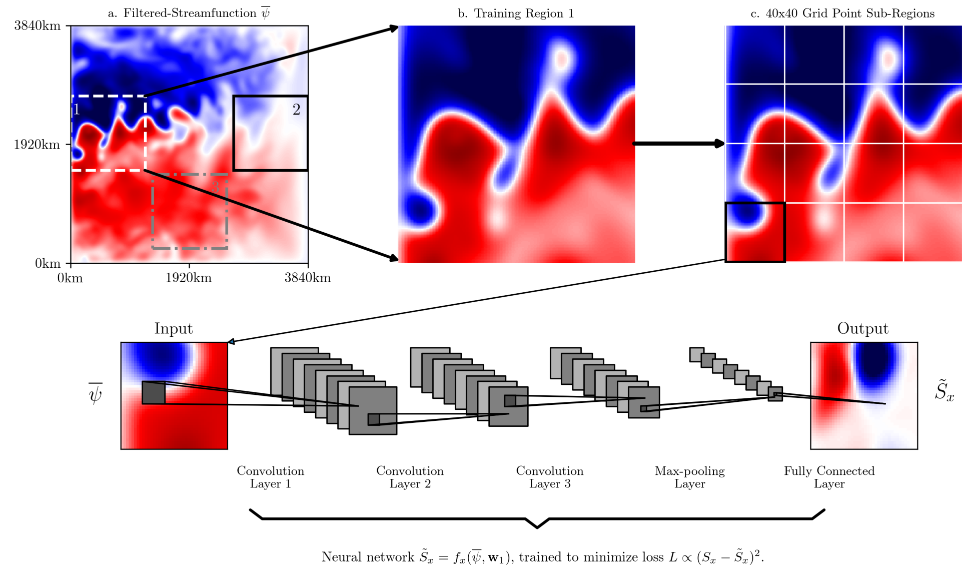
- And thinking (way?) outside the box ?

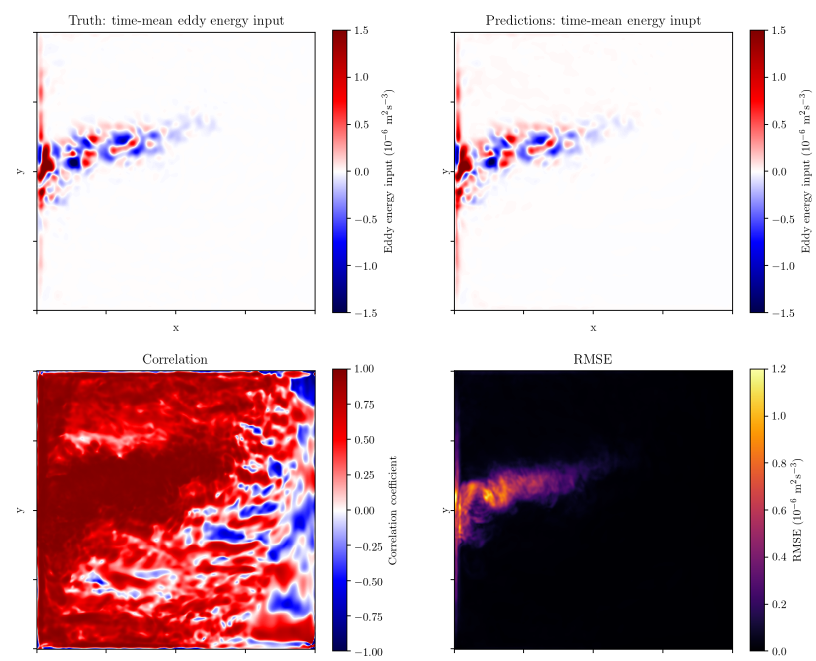
Machine Learning & Eddy Parametrization

- Generalisation to different dynamical regime





- Predicting eddy energy probabilistically/stochastically



Bolton & Zanna, 2019; Zanna et al 2018, 2019

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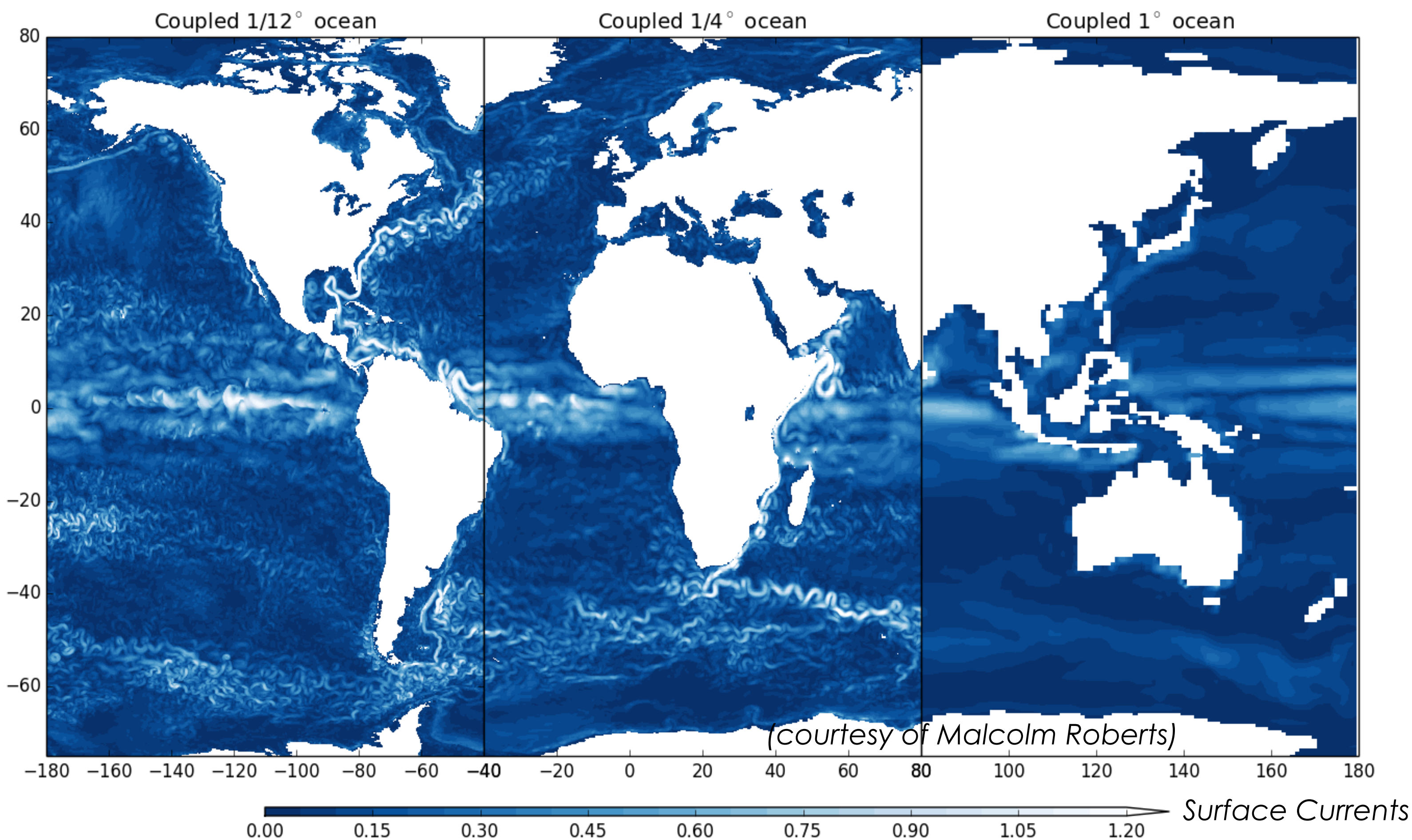
Ocean Models & Resolution

Resolution

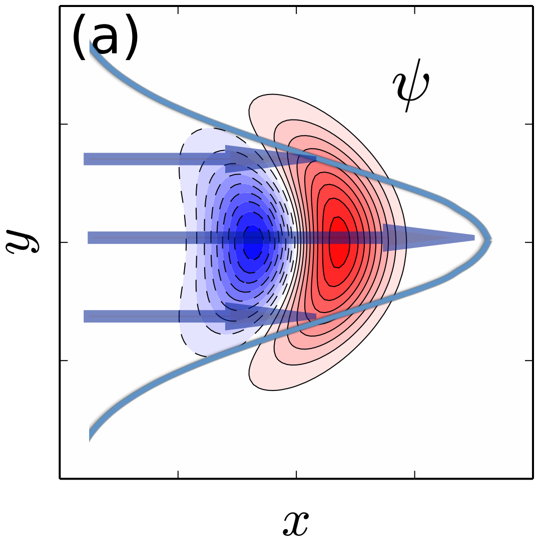
Timestep



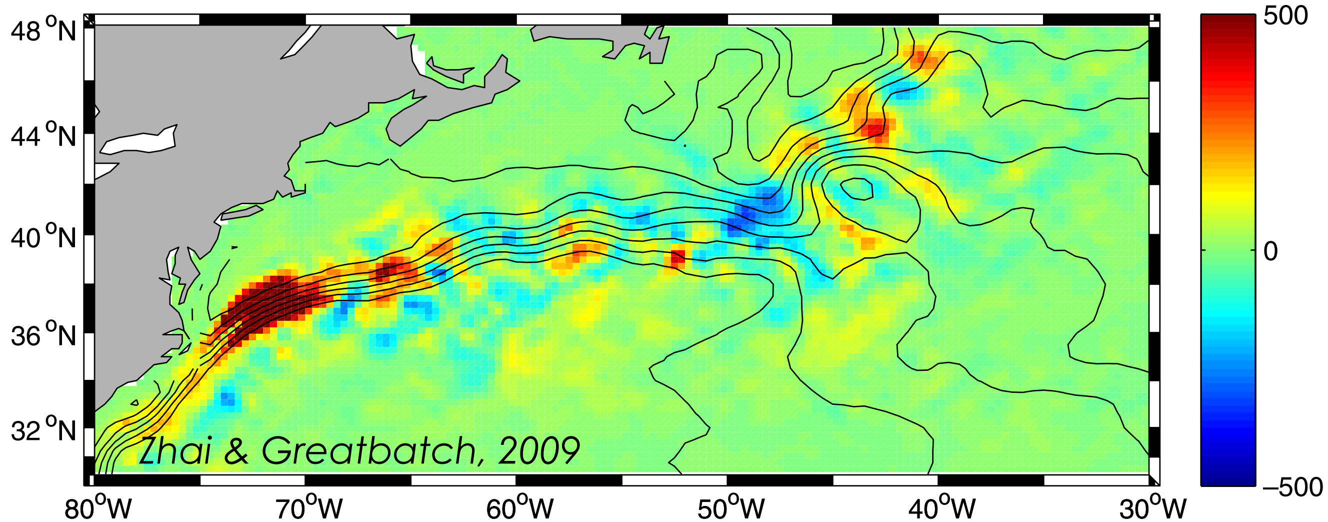
Computational cost

Eddy Momentum Fluxes

➡ jet rectification & sharpening via upgradient momentum fluxes *(Starr 1963, Shutts 1986)*



Eddy momentum flux from altimetry (~15 years)

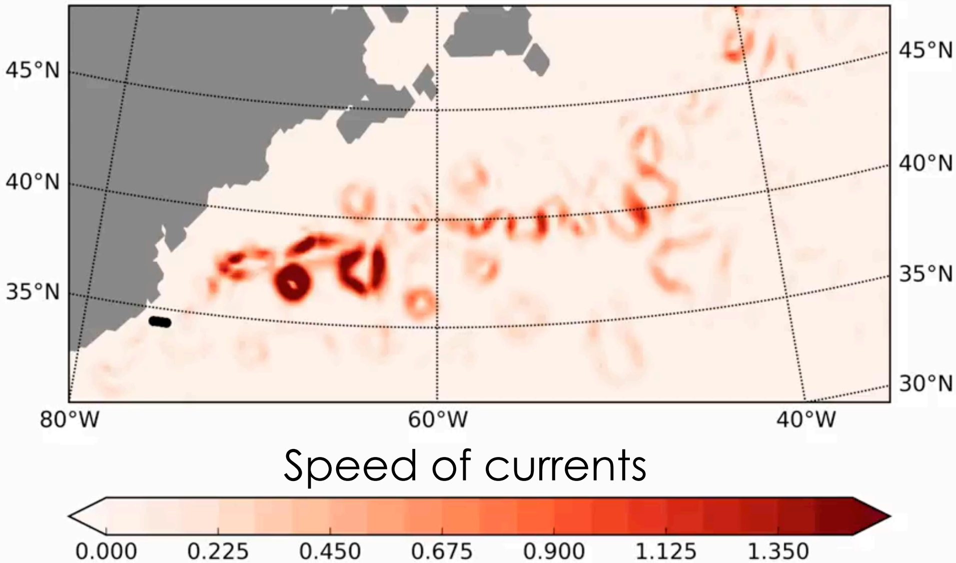


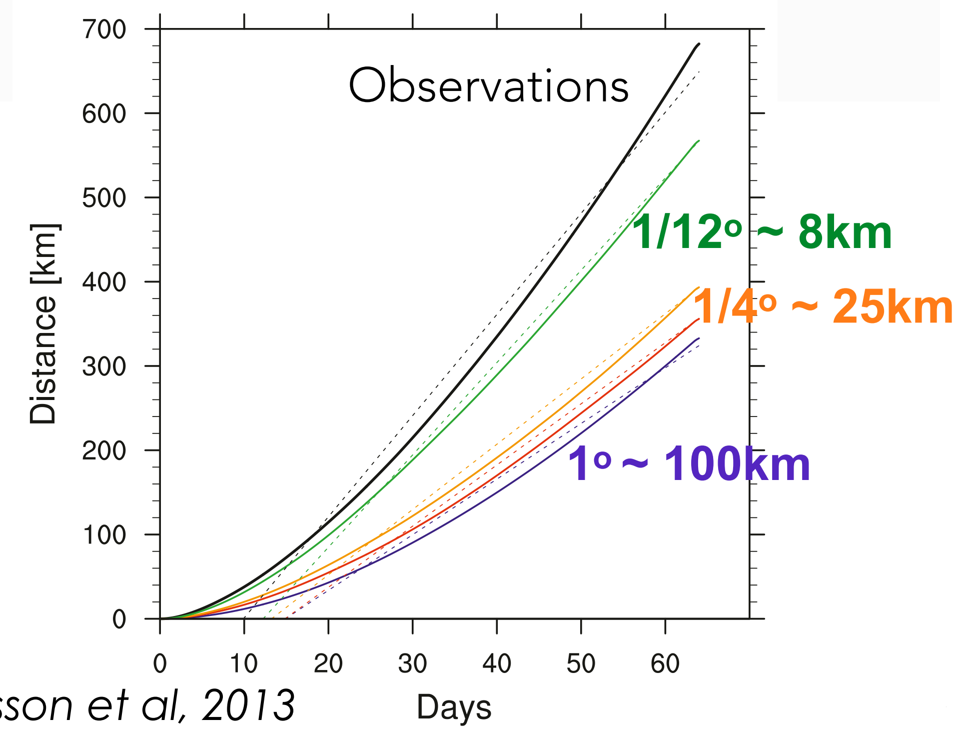
eddy momentum fluxes

$$-\frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y}$$

Anstey & Zanna, 2017

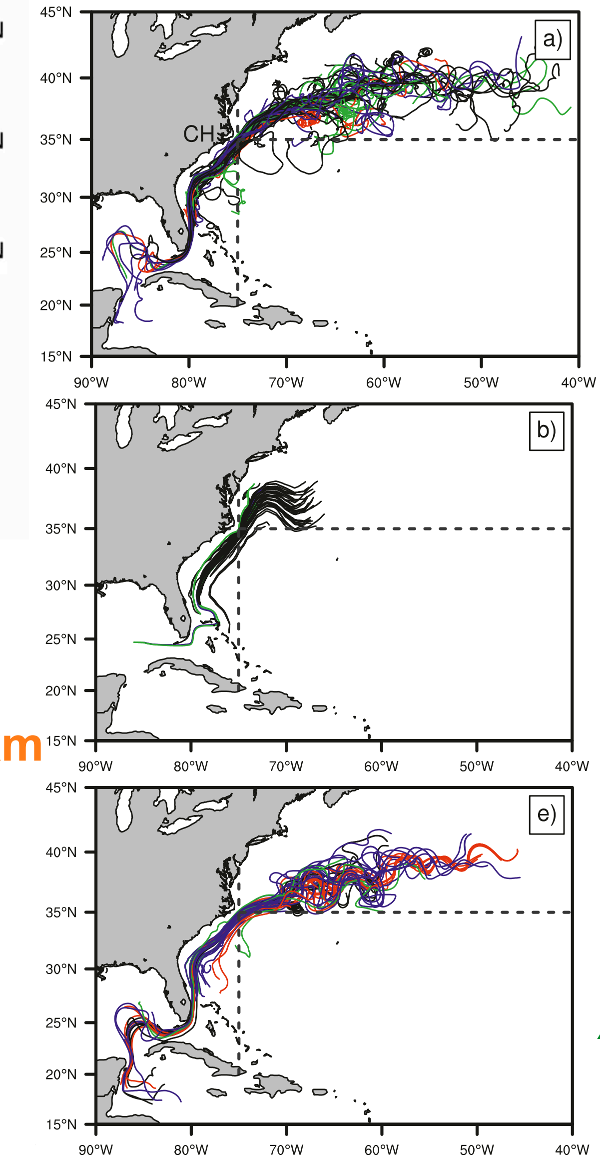
Lagrangian perspective: displacement (Brownian Motion)





Nilsson et al, 2013

Particles traced in Gulf Stream using AVISO (1/4°) velocities.



Observations

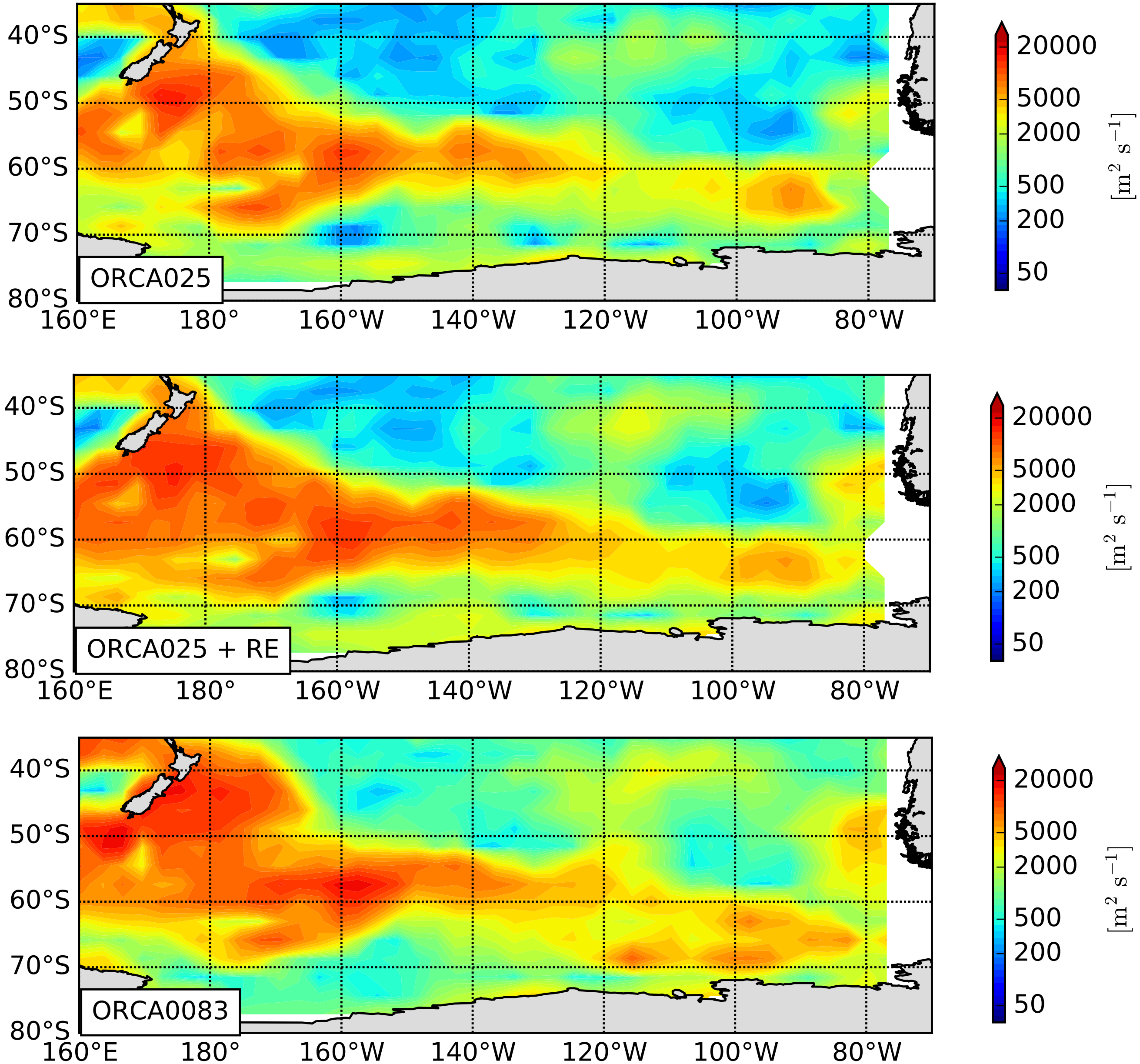
Model:

1° ~ 100km

Model:

1/12° ~ 8km

Lagrangian Modelling: Absolute Diffusivity



Dispersion

$$< D^2(t) >=< |\mathbf{x}(t) - \mathbf{x}(t_0)|^2 >$$

Velocity Autocorrelation

$$R(\tau) = \frac{1}{\sigma^2 T} \int_0^T \mathbf{u}'(t+\tau) \cdot \mathbf{u}'(t)\, dt$$

Absolute Diffusivity:

$$K_A(t) = \frac{1}{2} \frac{d < D^2(t) >}{dt}$$

$$\propto \int_0^t R(\tau)\ d\tau.$$

Turbulence ~ Non-newtonian Stress in Idealised Ocean Model

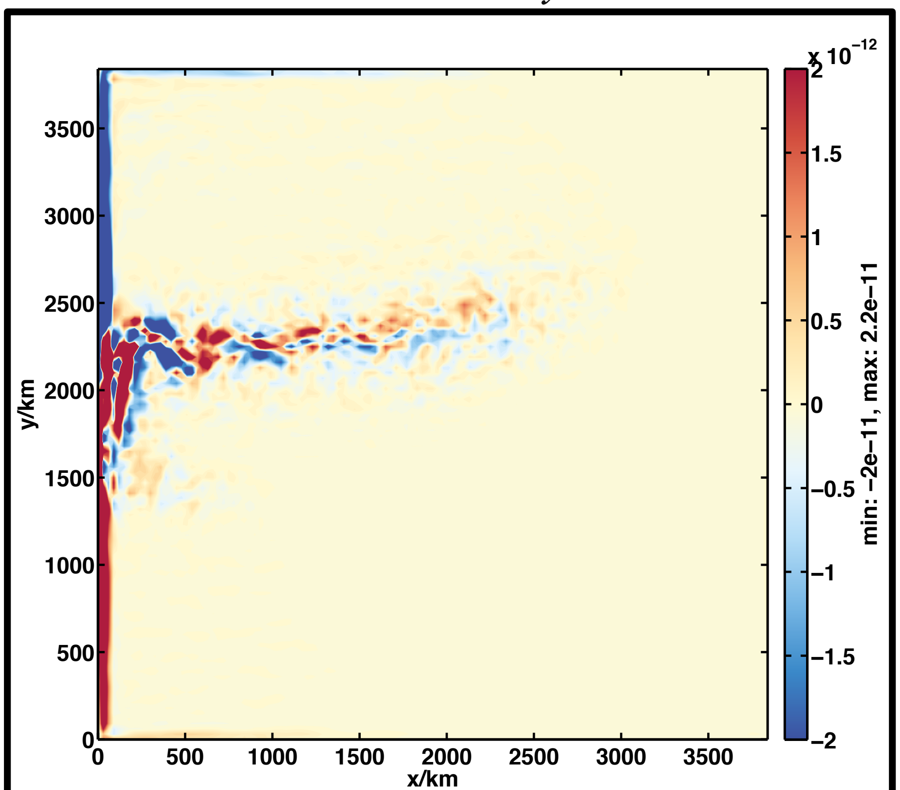
Missing Turbulent Forcing which depends on eddy (sub-grid) scales

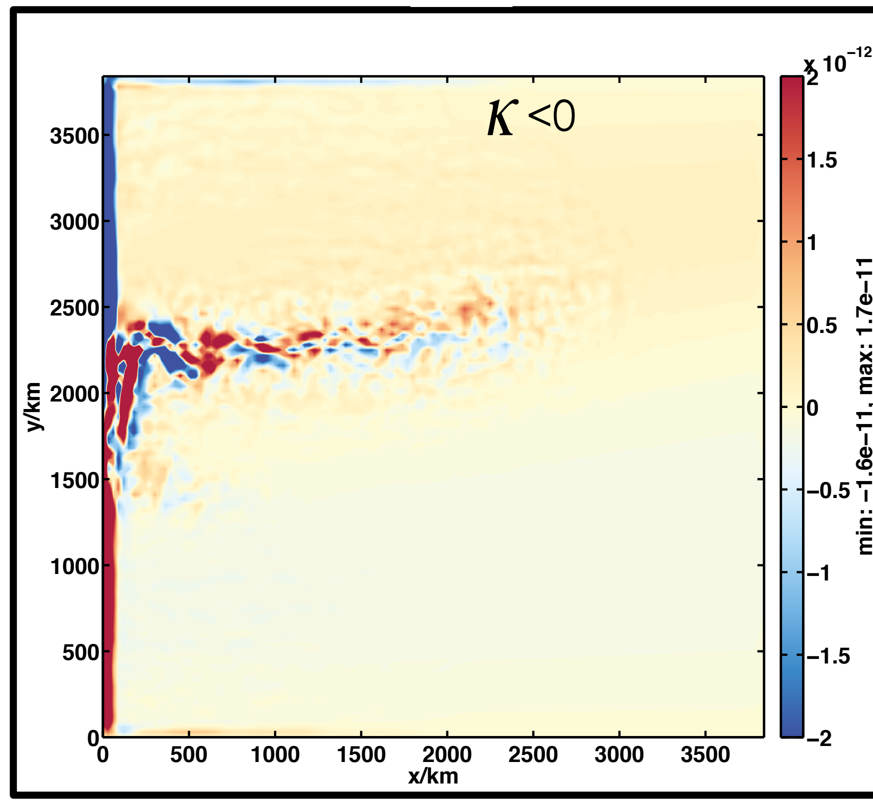
Parametrization as a function of the coarse-grained resolved scales

vorticity forcing related to

$$-\frac{\partial \overline{u'u'}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y}$$

$$\kappa \nabla \cdot \mathbf{A}_2$$



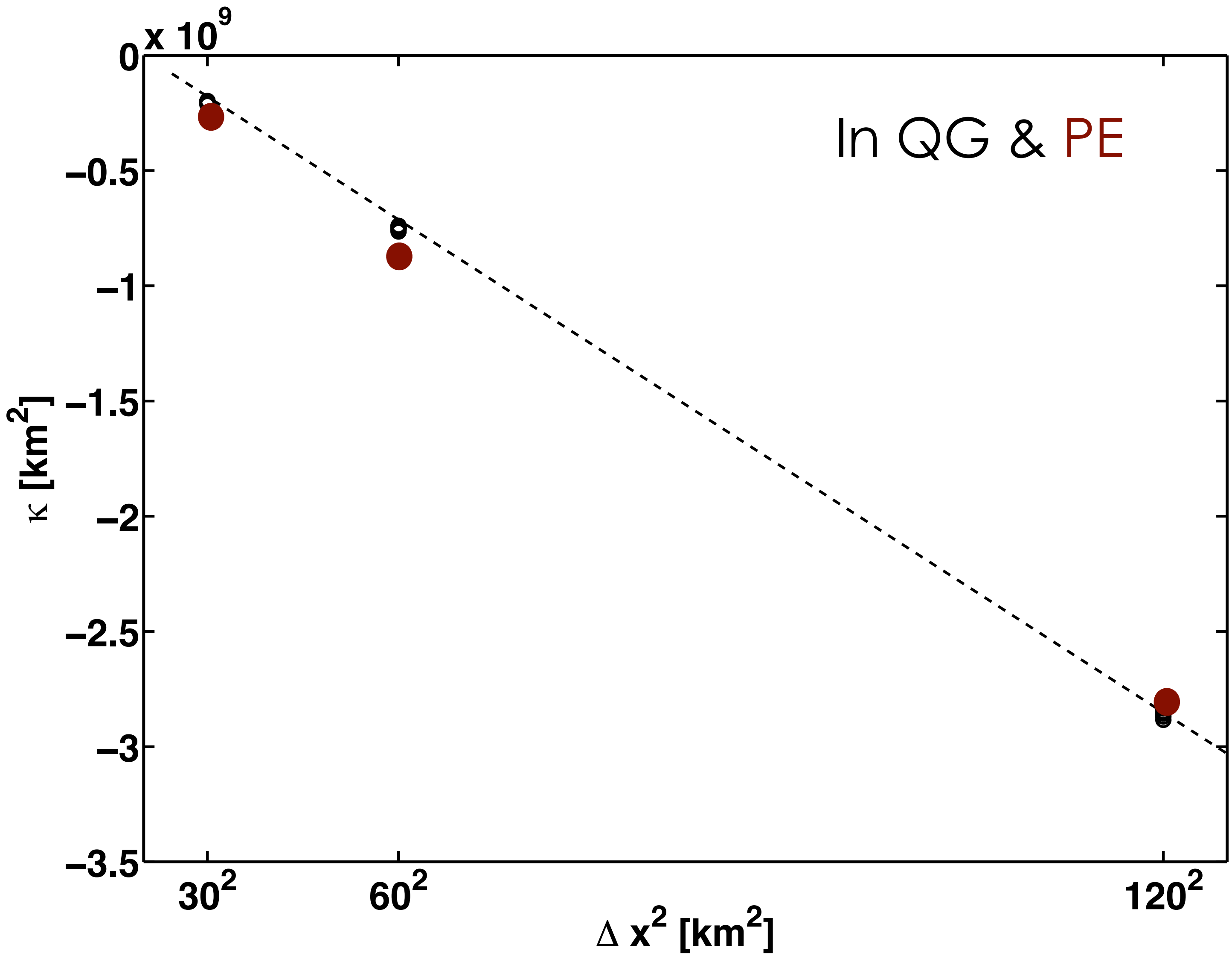


(both based on coarse-graining from 7.5km to 30km horizontal resolution)

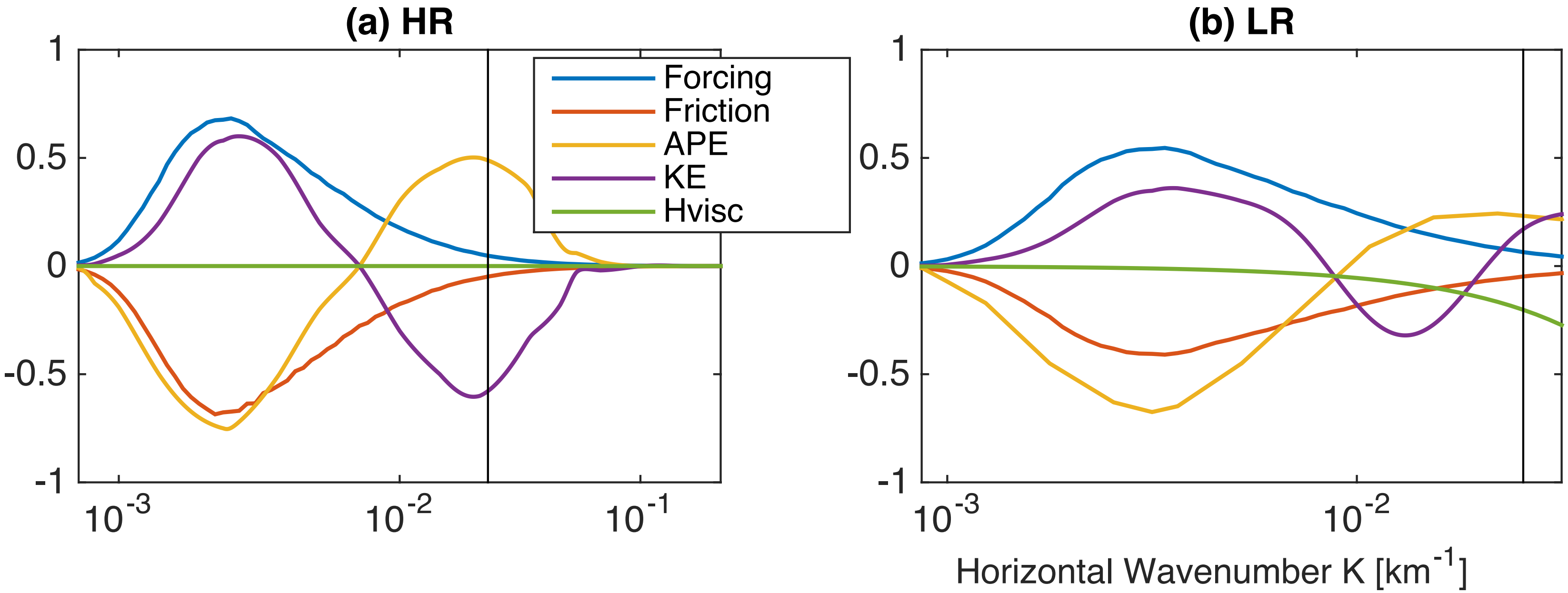
Mana & Zanna, 2014

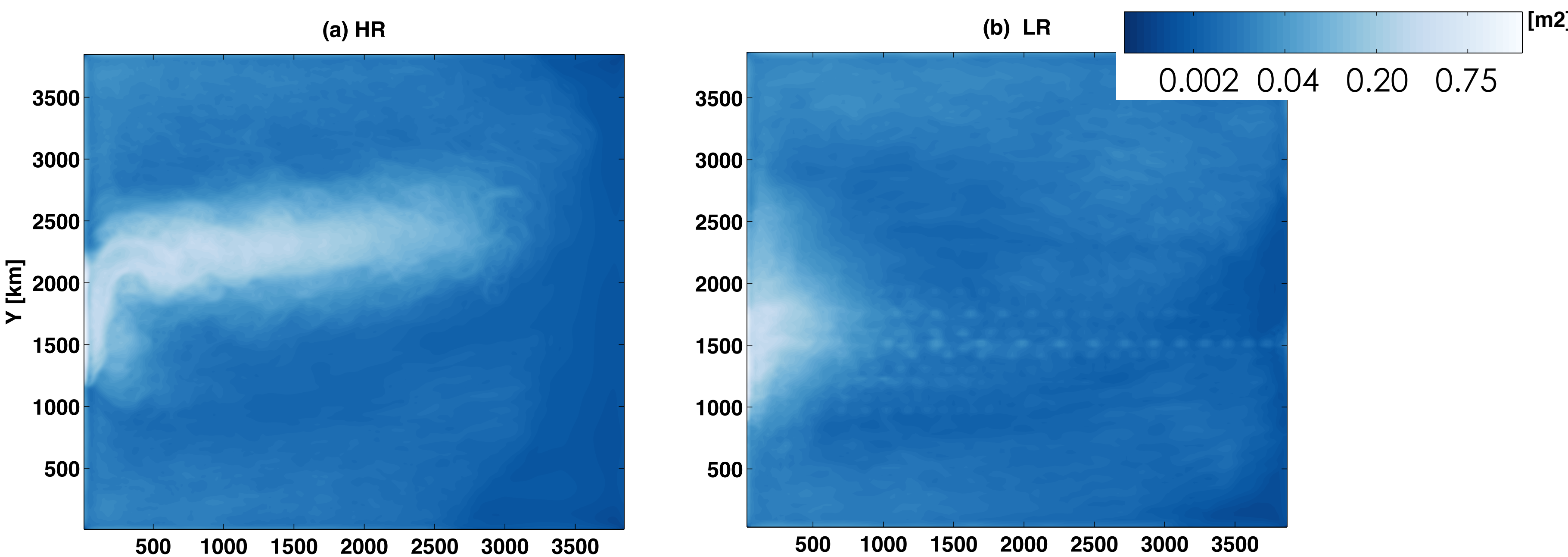
Coefficient = length^2

- Scales **only** with the coarse resolution grid box size $\kappa \sim \Delta x^2$



Spectral Transfer of Total Kinetic Energy





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