Transport by Coherent Lagrangian Vortices

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Outline

1. What are eddies?
2. Global database of coherent Lagrangian vortices
3. Relating coherent structures to eddy flux (altimetry)
4. Coherent Lagrangian vortices in models

Papers

2014-2018

- Abernathey, R. and G. Haller, 2018: *Transport by Lagrangian Vortices in the Eastern Pacific*. J. Phys. Oceanogr., **48**, 667–685

2016-2018

- Tarshish, N., R. Abernathey, C. Zhang, C. O. Dufour, I. Frenger, and S. M. Griffies, 2018: *Identifying Lagrangian coherent vortices in a mesoscale ocean model*. Ocean Modelling, **130**, 15–28

2017-2019

- Sinha, A., Balwada, D., Tarshish, N., and Abernathey, R., 2019: *Modulation of Lateral Transport by Submesoscale Flows and Inertia Gravity Waves*. Journal of Advances in Modeling Earth Systems, **11**.

In progress

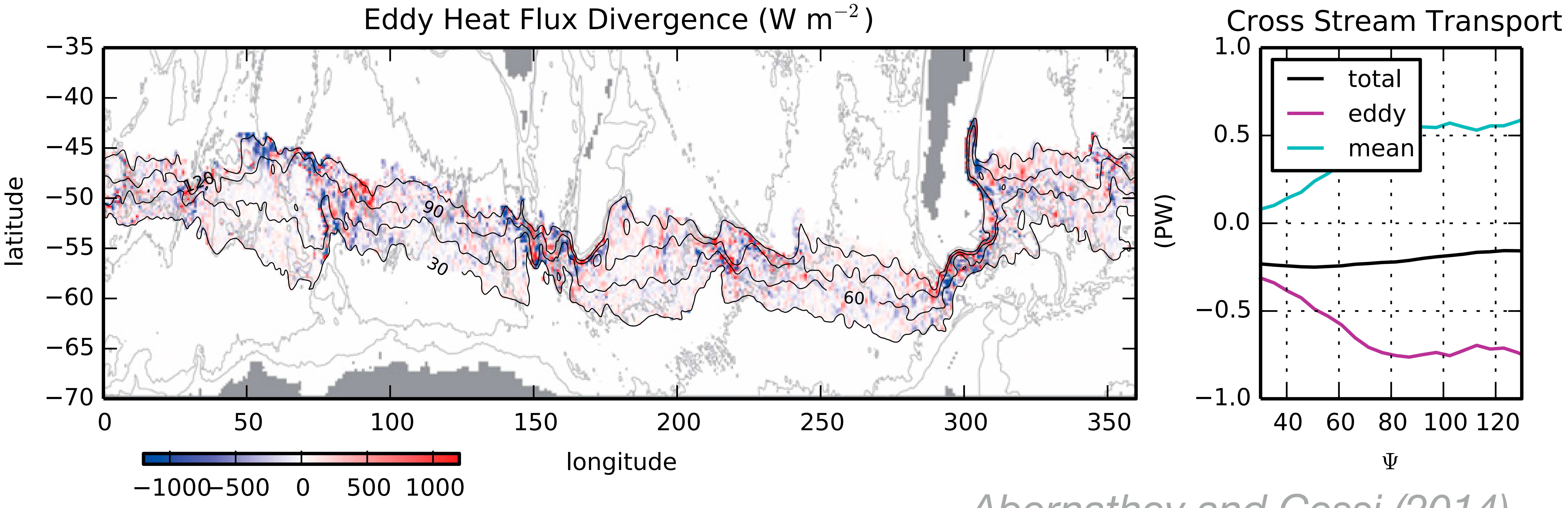
- Zhang, W., C. Wolfe, and R. Abernathey, 2019: *Role of Coherent Eddies in Potential Vorticity Transport in Quasigeostrophic Turbulence*. JFM
- Tongya, L and R. Abernathey, 2019: *Comparing 3D Lagrangian and Eulerian Eddies in a Primitive Equation Ocean Model*.
- Abernathey et at. *A Global Atlas of Coherent Lagrangian Vortices*.

Eddy Heat Fluxes

Advection equation: $$\frac{D\theta}{Dt} = \frac{\partial\theta}{\partial t} + \boldsymbol{v} \cdot \nabla\theta$$

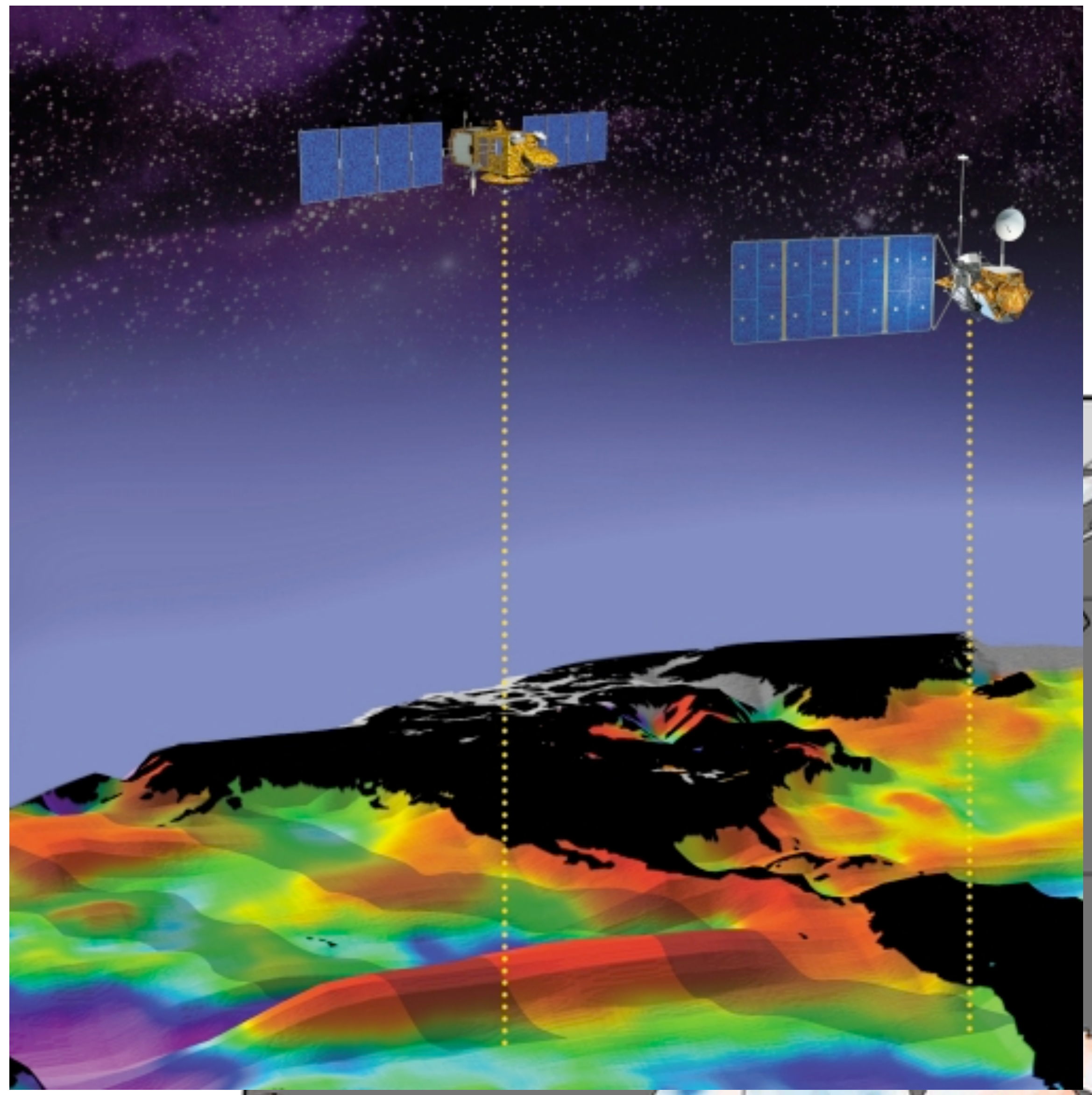
Reynolds average: $$\frac{\partial\overline{\theta}}{\partial t} + \overline{\boldsymbol{v}} \cdot \nabla\overline{\theta} = \nabla \cdot (\overline{\boldsymbol{v}'\theta'})$$ ← **"eddy flux"**

$\overline{(\ )} =$ time average $\qquad (\ )' =$ fluctuation / "eddy"



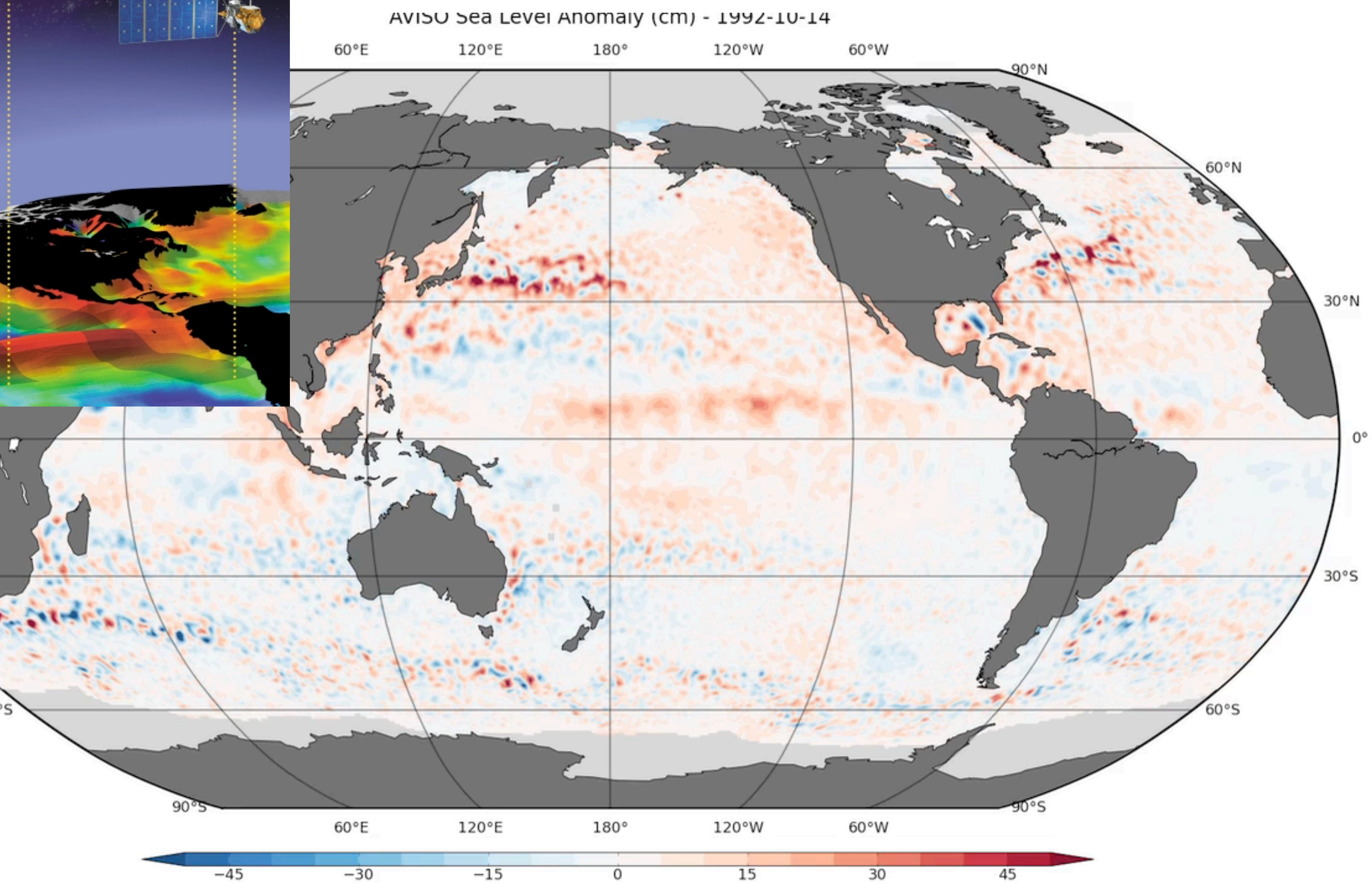
Abernathey and Cessi (2014)

Satellite Altimetry



$$\hat{\boldsymbol{k}} \times \boldsymbol{v}_g = -\frac{g}{f}\nabla\eta$$

ssh is streamfunction for surface geostrophic flow



Sea-surface-height eddies (Chelton et al., 2011)

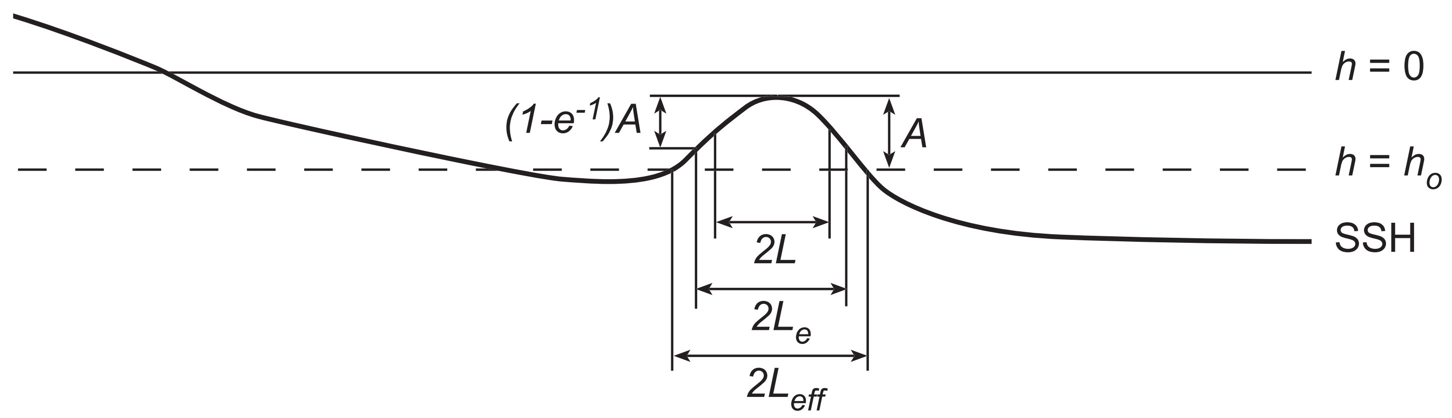
1. The SSH values of all of the pixels are above (below) a given SSH threshold for anticyclonic (cyclonic) eddies.

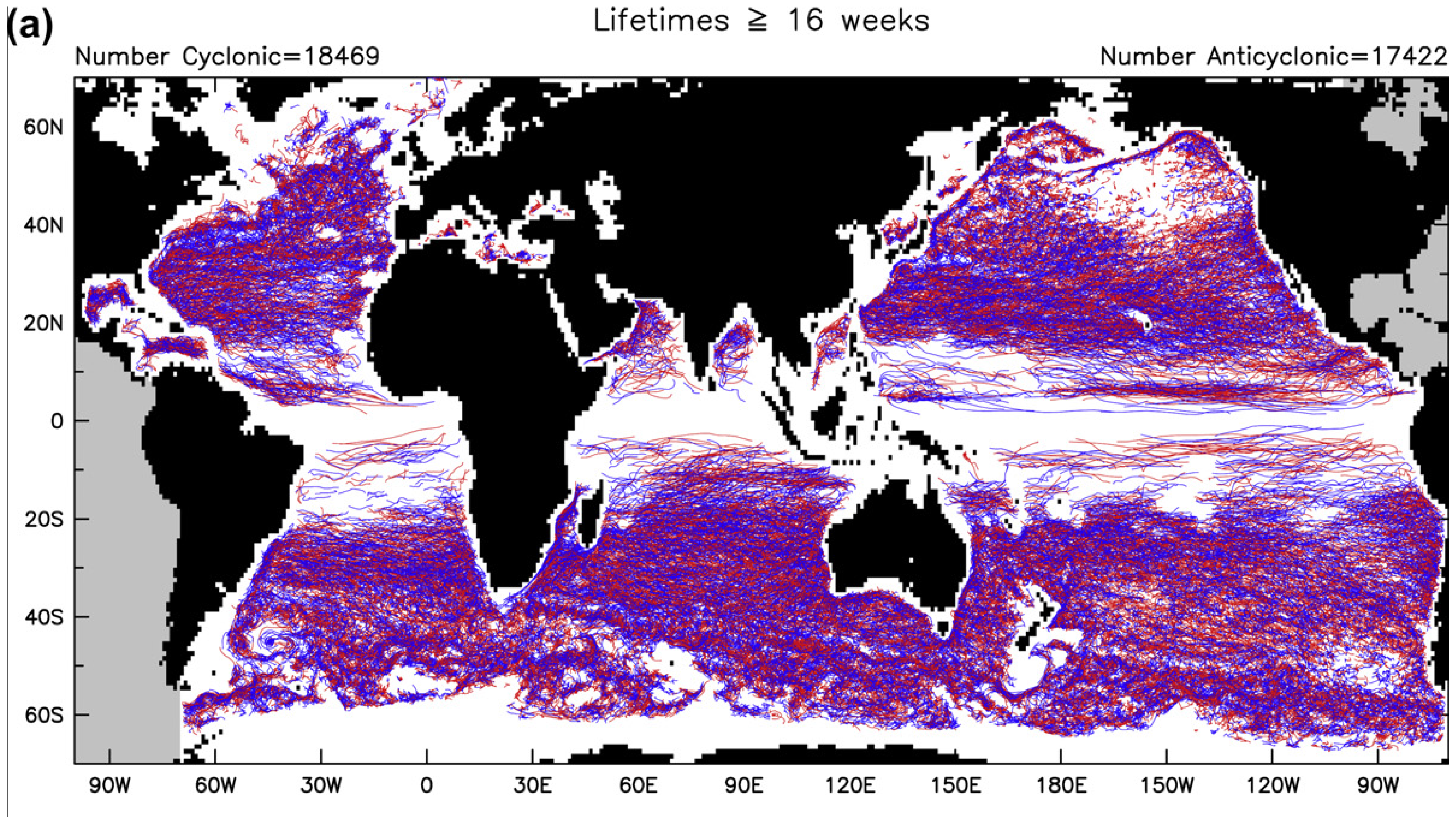
2. There are at least 8 pixels and fewer than 1000 pixels comprising the connected region.

3. There is at least one local maximum (minimum) of SSH for anticyclonic (cyclonic) eddies.

4. The amplitude of the eddy is at least 1 cm (see below).

5. The distance between any pair of points within the connected region must be less than a specified maximum.





Two Paradigms of “Eddy Flux”

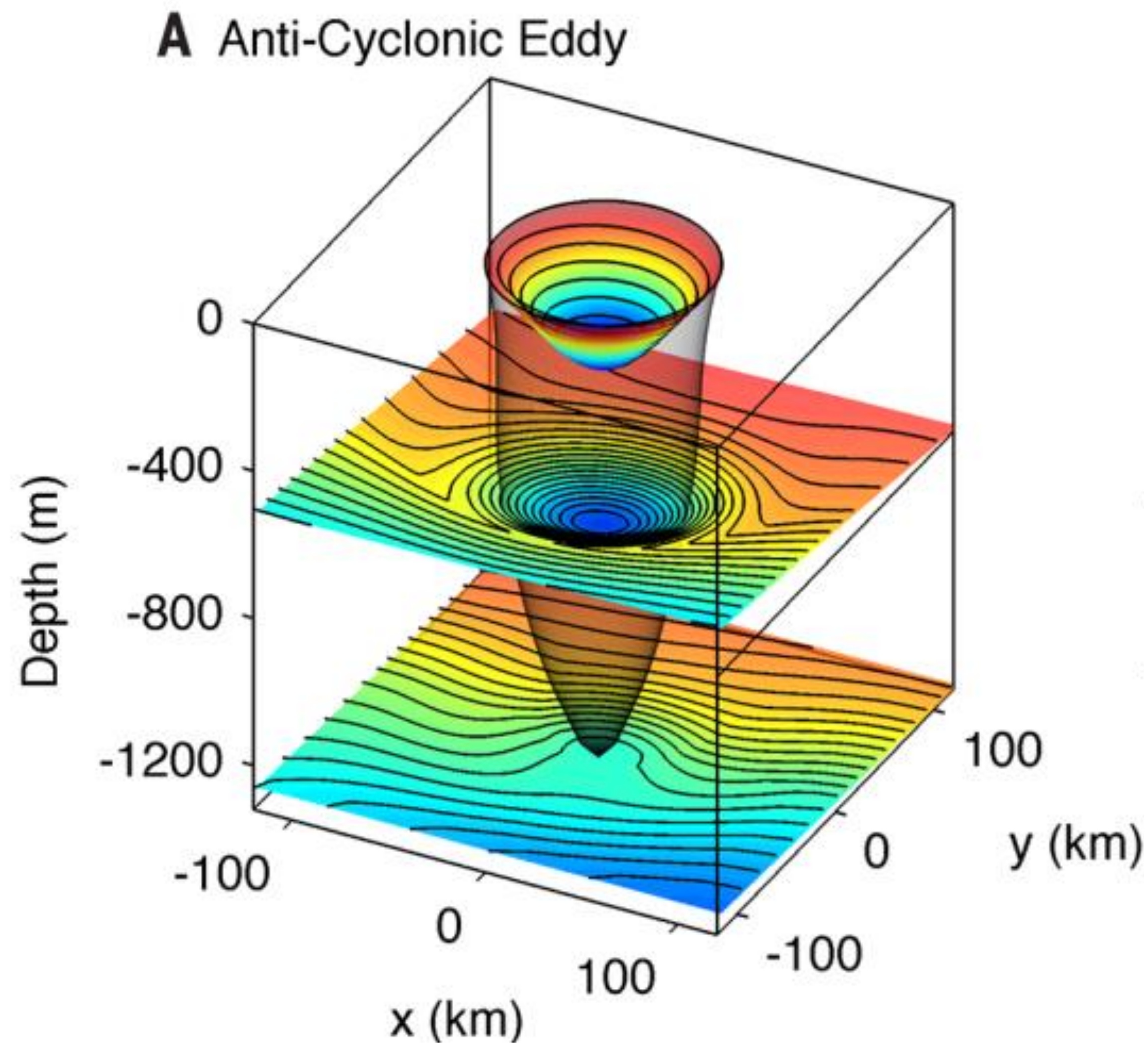
Reynolds Flux

$$\overline{v'\theta'} \qquad \overline{u'v'}$$

- Statistical
- Eulerian
- Flux due to correlations btw. fluctuating velocity and tracer
- Atmospheric science, linear theory, eddy parameterization, model diagnostics

Coherent Eddy

- Countable
- Lagrangian(?)
- Flux due to motion of coherent structure
- Dynamical systems, observational physical oceanography
- Non-local transport: Challenging to parameterize



Can we infer eddy fluxes from coherent structures?

nature COMMUNICATIONS

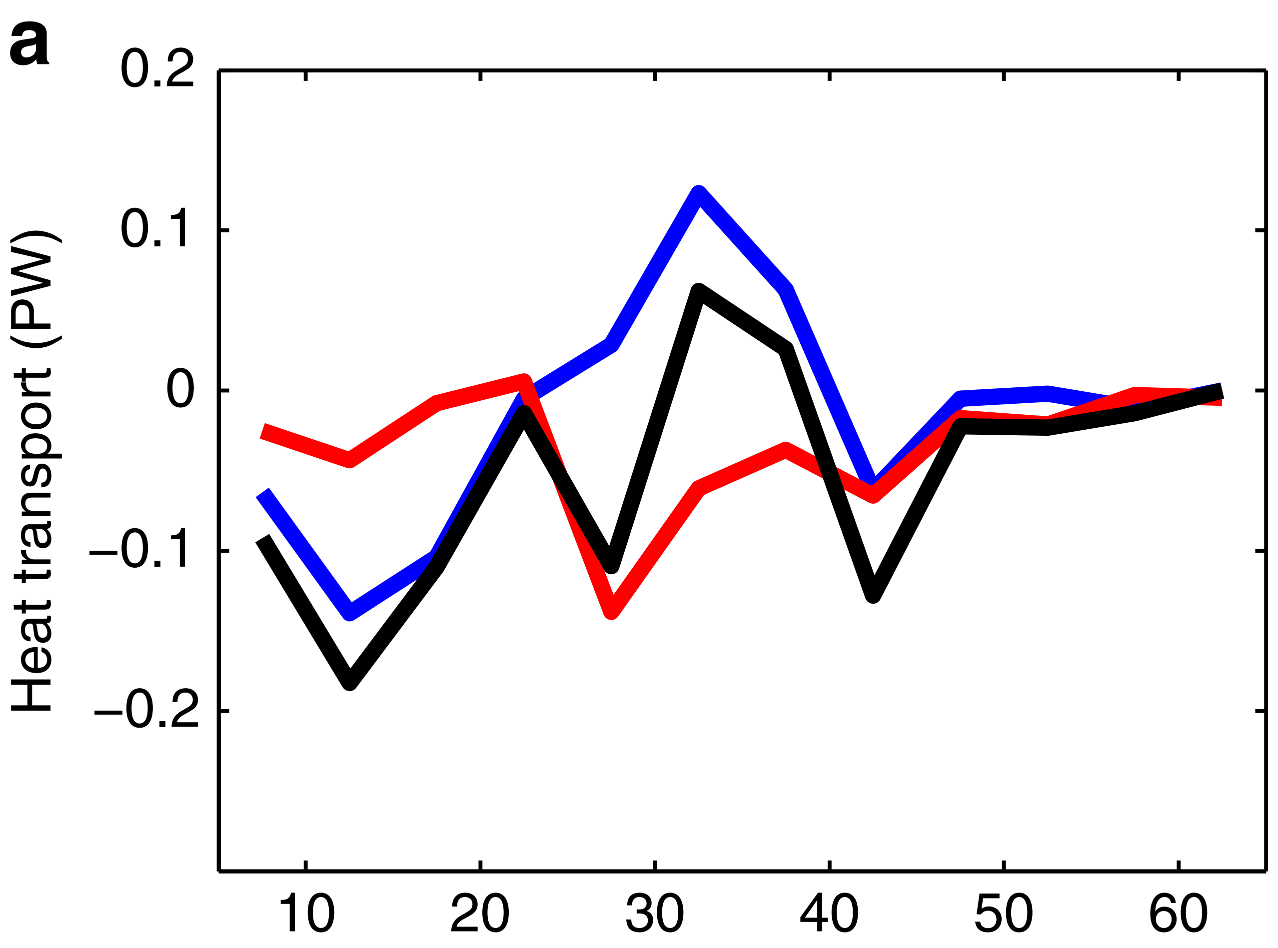
ARTICLE

Received 23 Jun 2013 | Accepted 22 Jan 2014 | Published 18 Feb 2014

DOI: 10.1038/ncomms4294

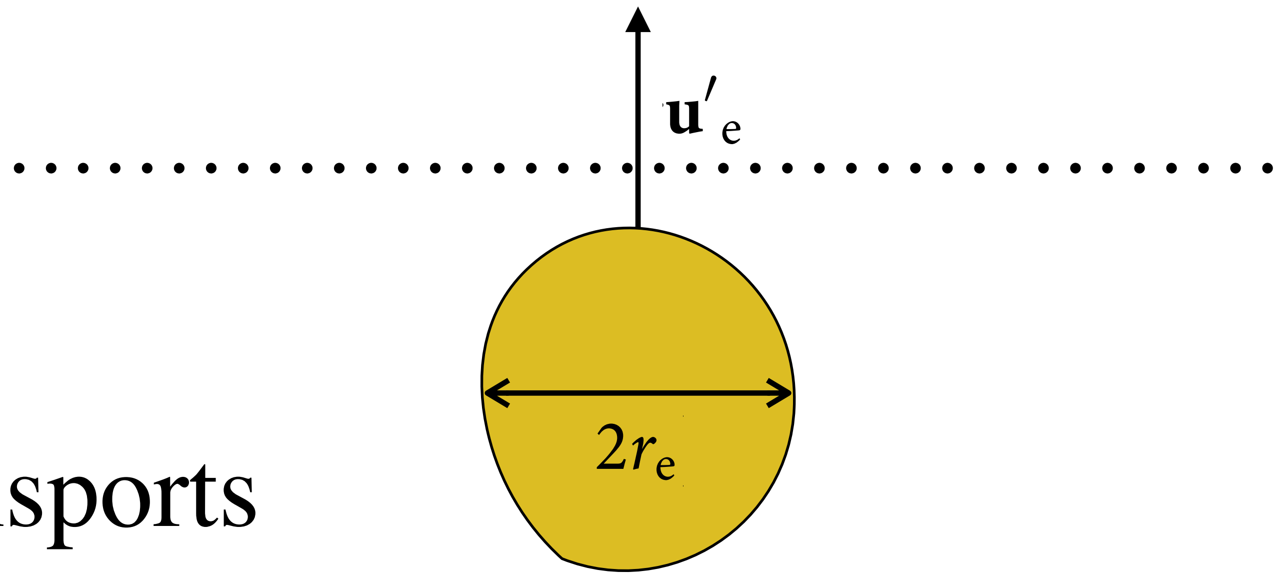
Global heat and salt transports by eddy movement

Changming Dong[1,2,3], James C. McWilliams[2], Yu Liu[3] & Dake Chen[1]



$$\mathbf{Q}_{\rm eh} = s\mathbf{u}'_{\rm e} \int dz \rho_{\rm o} C_{\rm po} (2r_{\rm e}) T'_{\rm e},$$

$$\mathbf{T}_h = N_{\rm e} \langle \mathbf{Q}_{\rm eh} \rangle,$$

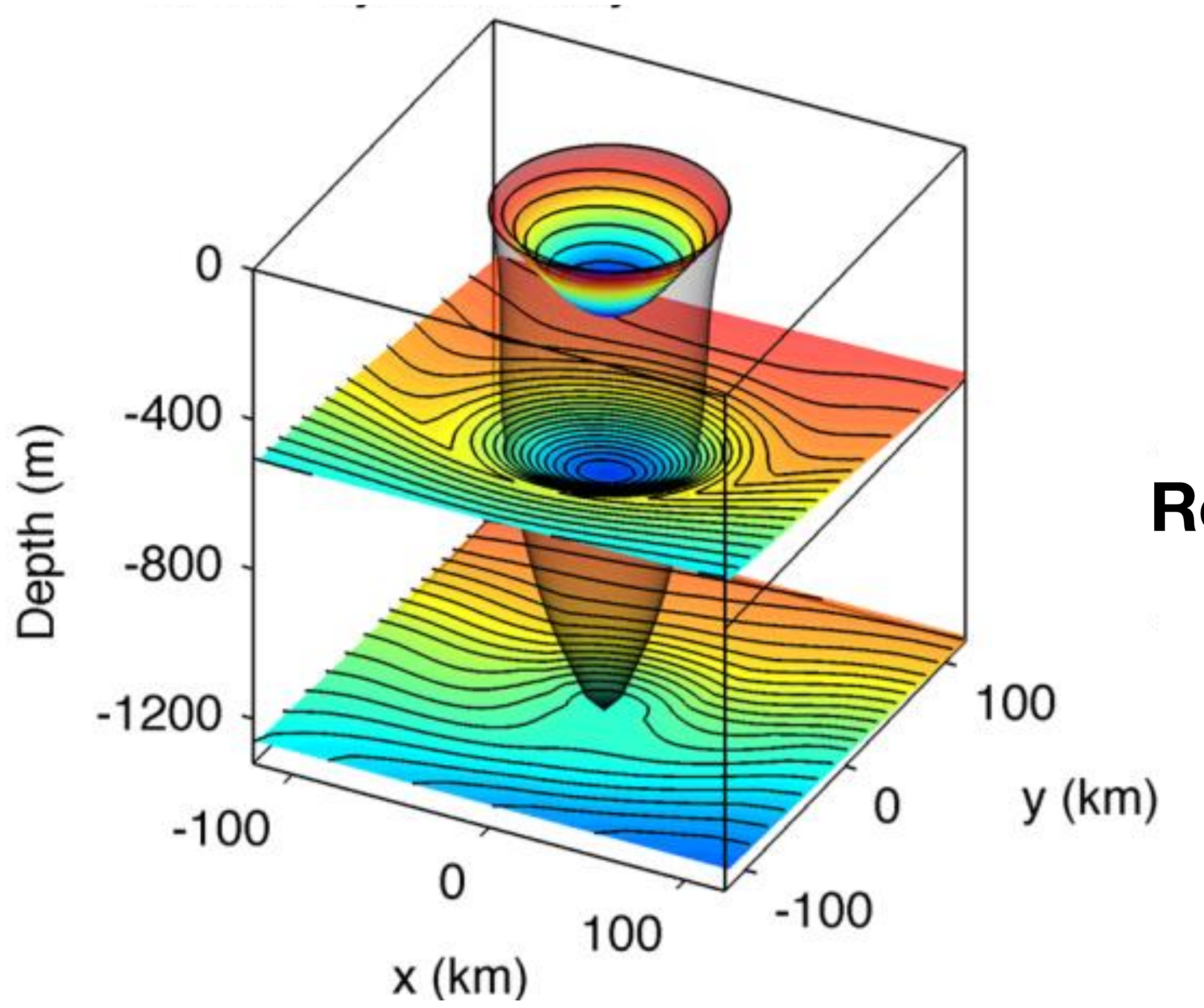


"…eddy heat and salt transports are mainly due to individual eddy movements."

Dong et al. (2014)

Are all the "eddies" seen in SSH actually materially coherent?

Material Volumes



Any scalar

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = \dot{\phi}$$

any volume *velocity of boundary*

Reynolds' Transport Theorem

$$\frac{\partial}{\partial t} \int_{\Omega} \phi dV = \int_{\Omega} \frac{\partial \phi}{\partial t} dV + \oint_{\partial \Omega} \phi \mathbf{u}_b \cdot \hat{\mathbf{n}} dA$$

Definition of material volume

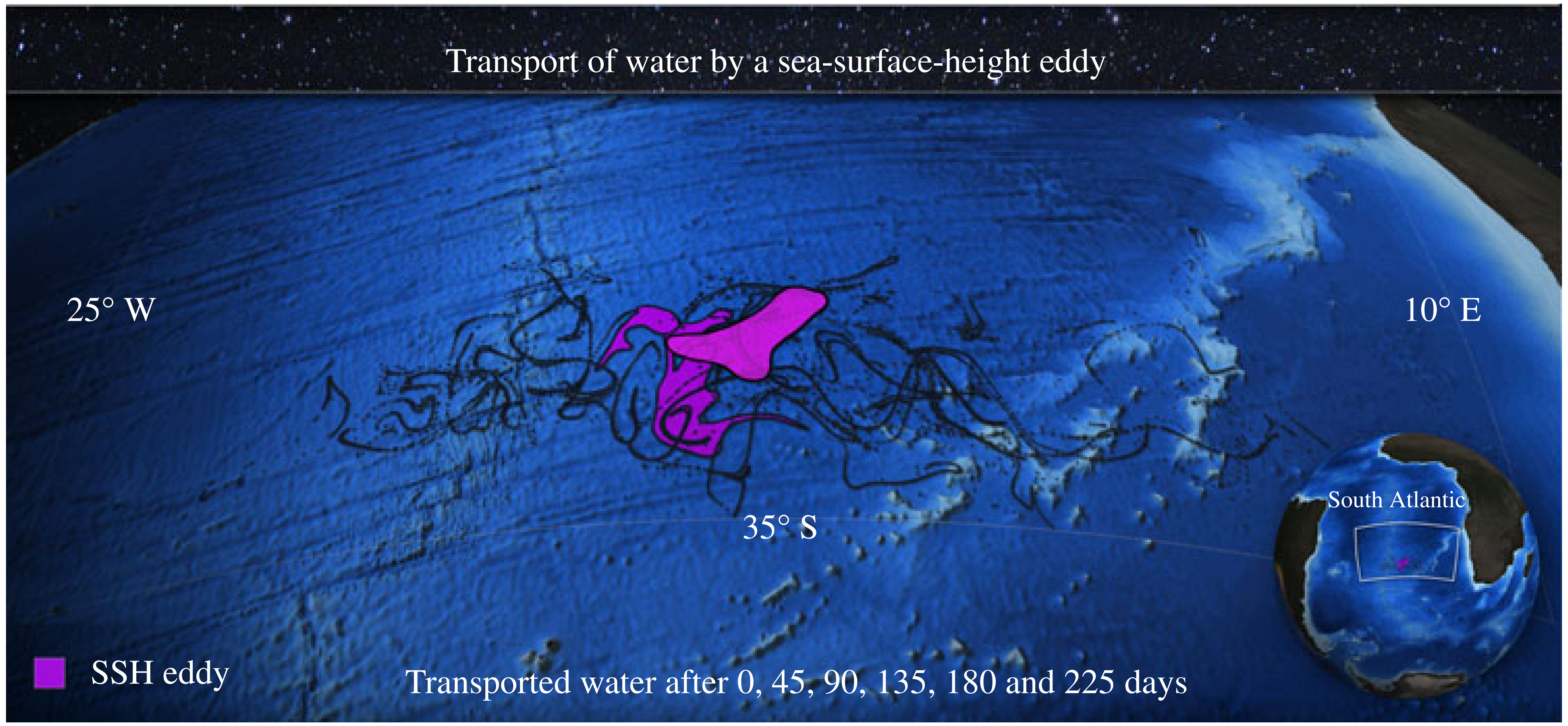
$$\mathbf{u}_b = \mathbf{u}$$

RTT for material volume

$$\frac{\partial}{\partial t} \int_{\Omega} \phi dV = \int_{\Omega} \dot{\phi} dV$$

No flux through boundary!

…SSH eddies are NOT material volumes!



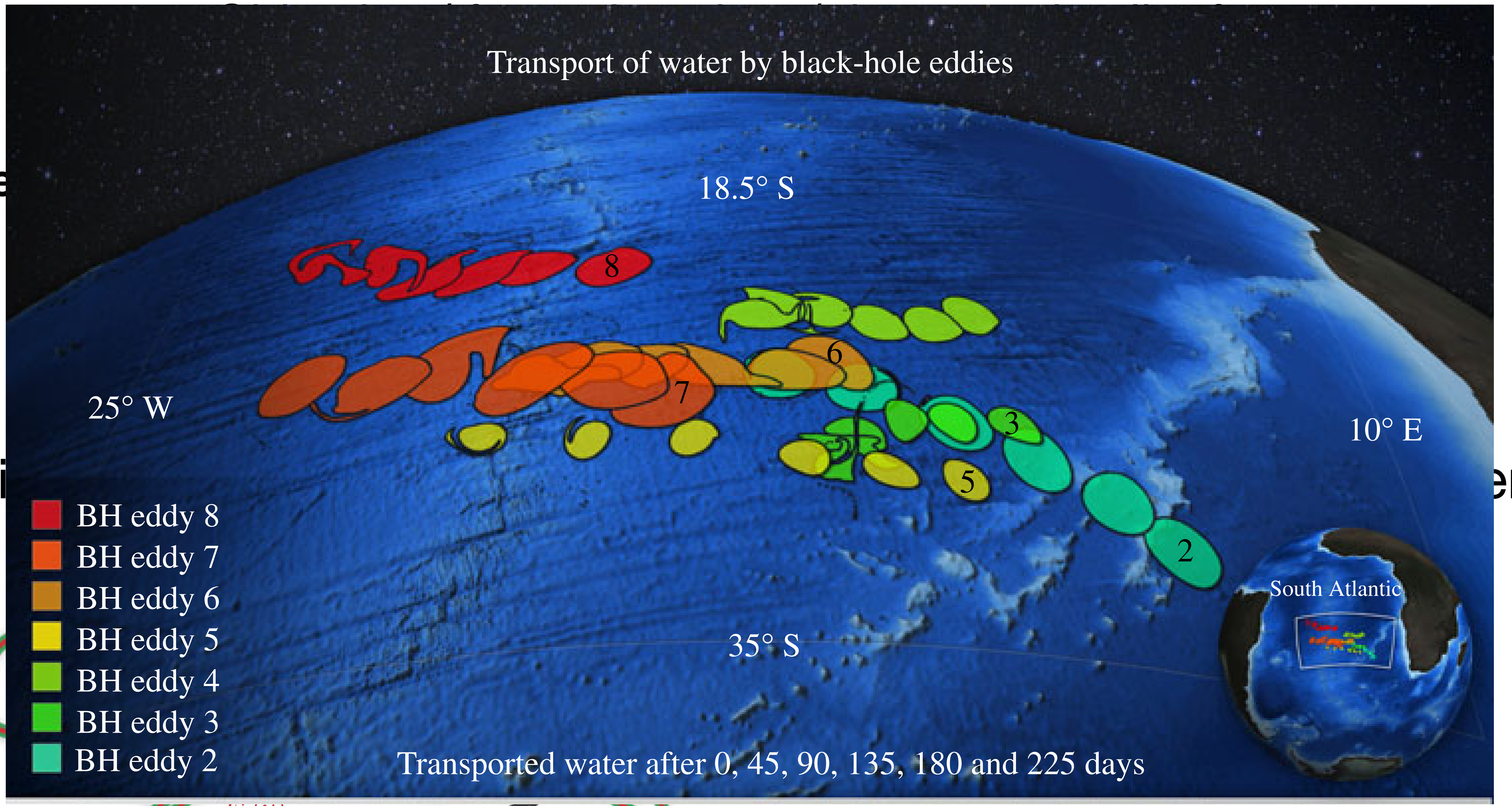
"Coherently evolving velocity features tend to differ substantially from coherently moving fluid parcels…in unsteady flows"

Haller & Beron Vera (2013)

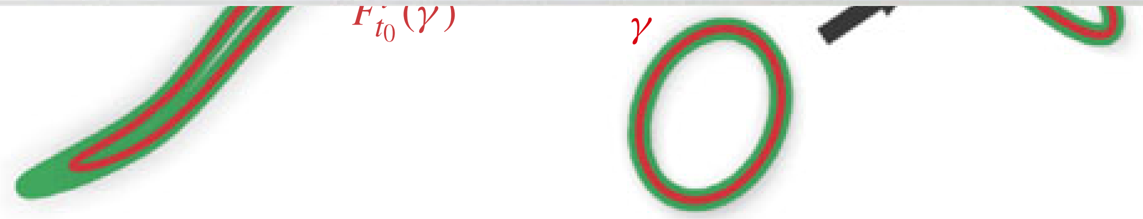
How do we find materially coherent eddies?

George Halle
ETH

Apply dynamical systems theory to the problem of eddy detection. Method must be:



Soluti ... erent



Cauchy-Green strain tensor:

$$C(x_0) = \left[\nabla F_{t_0}^{t_1}(x_0)\right]^T \nabla F_{t_0}^{t_1}(x_0).$$

Haller, G., 2015: Lagrangian Coherent Structures. Ann. Rev. of Fluid Mech., 47, 137–162.

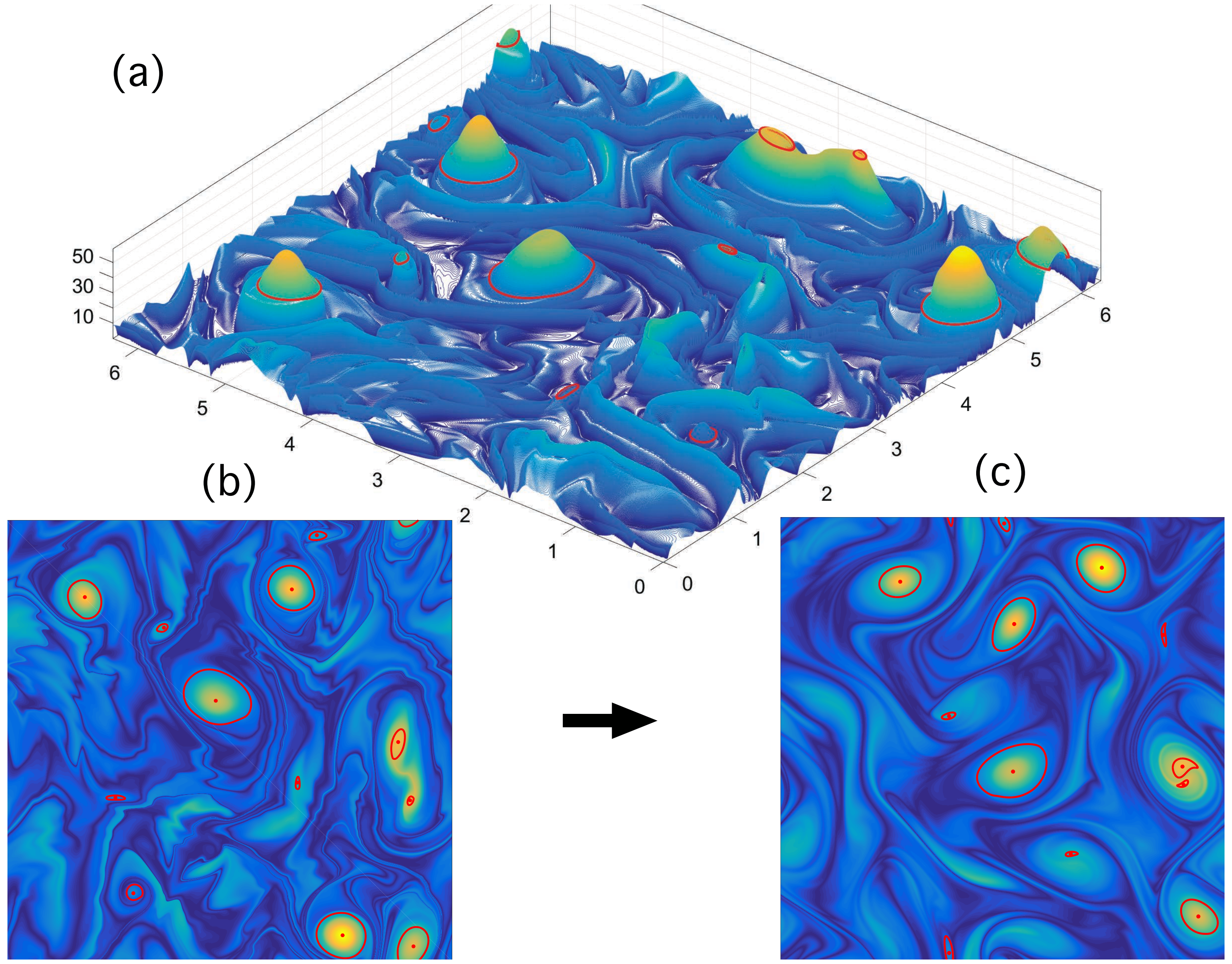
Rotationally Coherent Lagrangian Vortices

G. Haller, A. Hadjighasem, M. Farazamand & F. Huhn, 2016
Defining coherent vortices objectively from the vorticity. J. Fluid Mech.

Lagrangian-Averaged Vorticity Deviation:

$$\mathrm{LAVD}_{t_0}^{t}(x_0) := \int_{t_0}^{t} |\omega(x(s;x_0),s) - \bar{\omega}(s)| \, ds$$

integral taken *along Lagrangian trajectory*



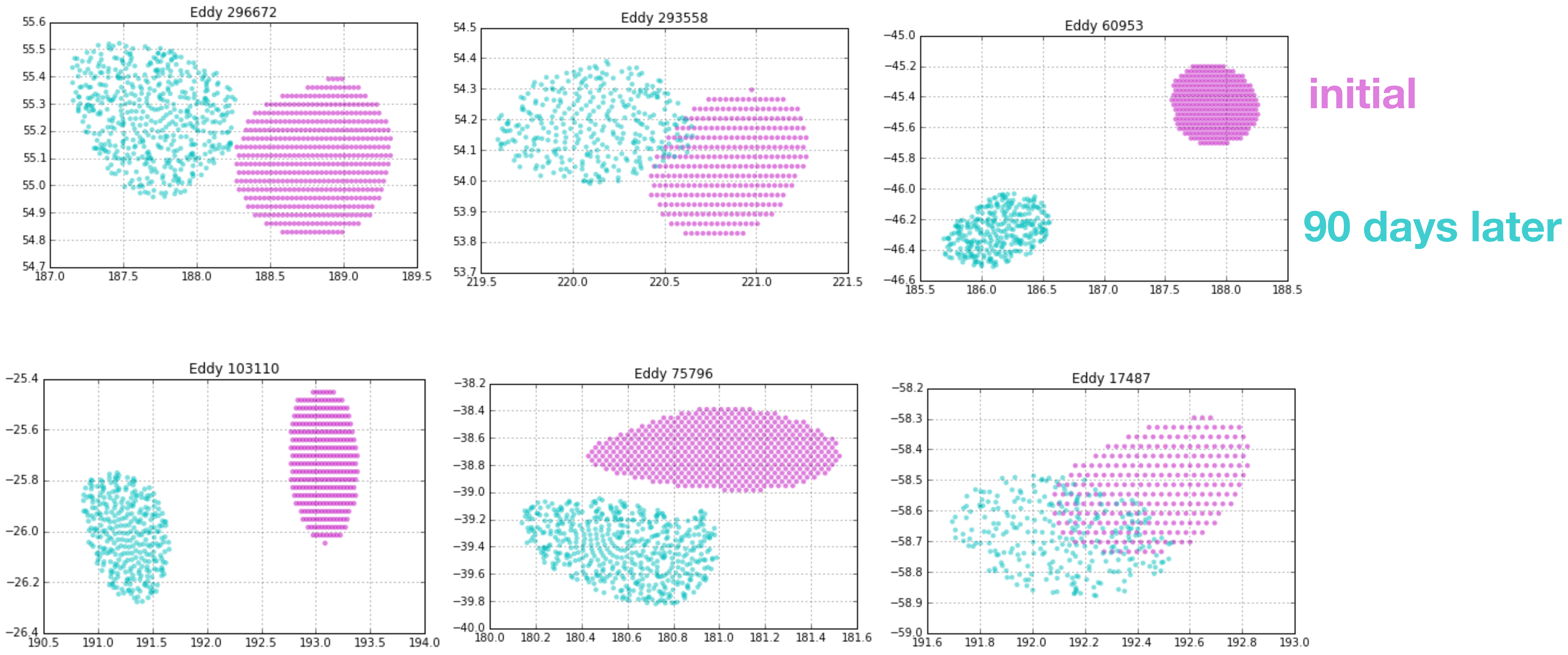
- Vortex centers: maxima of |LAVD|
- Vortex boundaries: outermost *convex* curves around maxima
- Coherence is defined over a *finite time interval*

Computing The Flow Map

initial position $\boldsymbol{x}_0 \to \boldsymbol{x}_1$ ***final position***

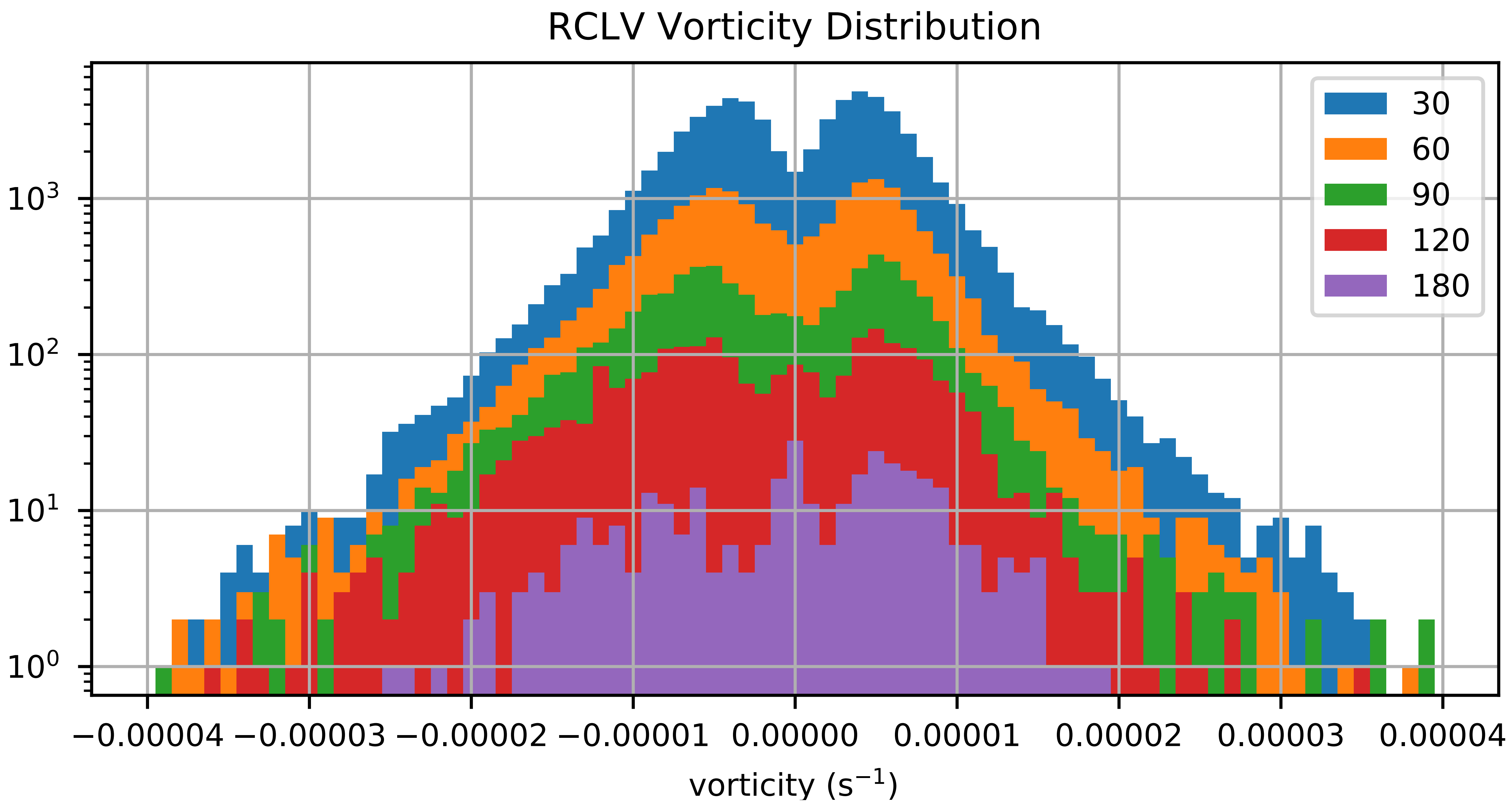
- Initialize a dense “mesh” of Lagrangian particles over the whole globe (50 million particles)
- Solve the equation $\frac{d}{dt}\boldsymbol{x} = \boldsymbol{v}_g$ for each particle using the AVSIO surface geostrophic velocities. (MITgcm offline)
- Run forward for 180 days.
- Output LAVD, position every 30 days
- Repeat for each segment of the ~23 year dataset (1993 - 2016)

90-day RCLVs


Eddy 296672
Eddy 293558
Eddy 60953
Eddy 103110
Eddy 75796
Eddy 17487
initial
90 days later

Global RCLV Census

Duration (days)	Count
30	64,978
60	19,457
90	6,540
120	2,467
180	331

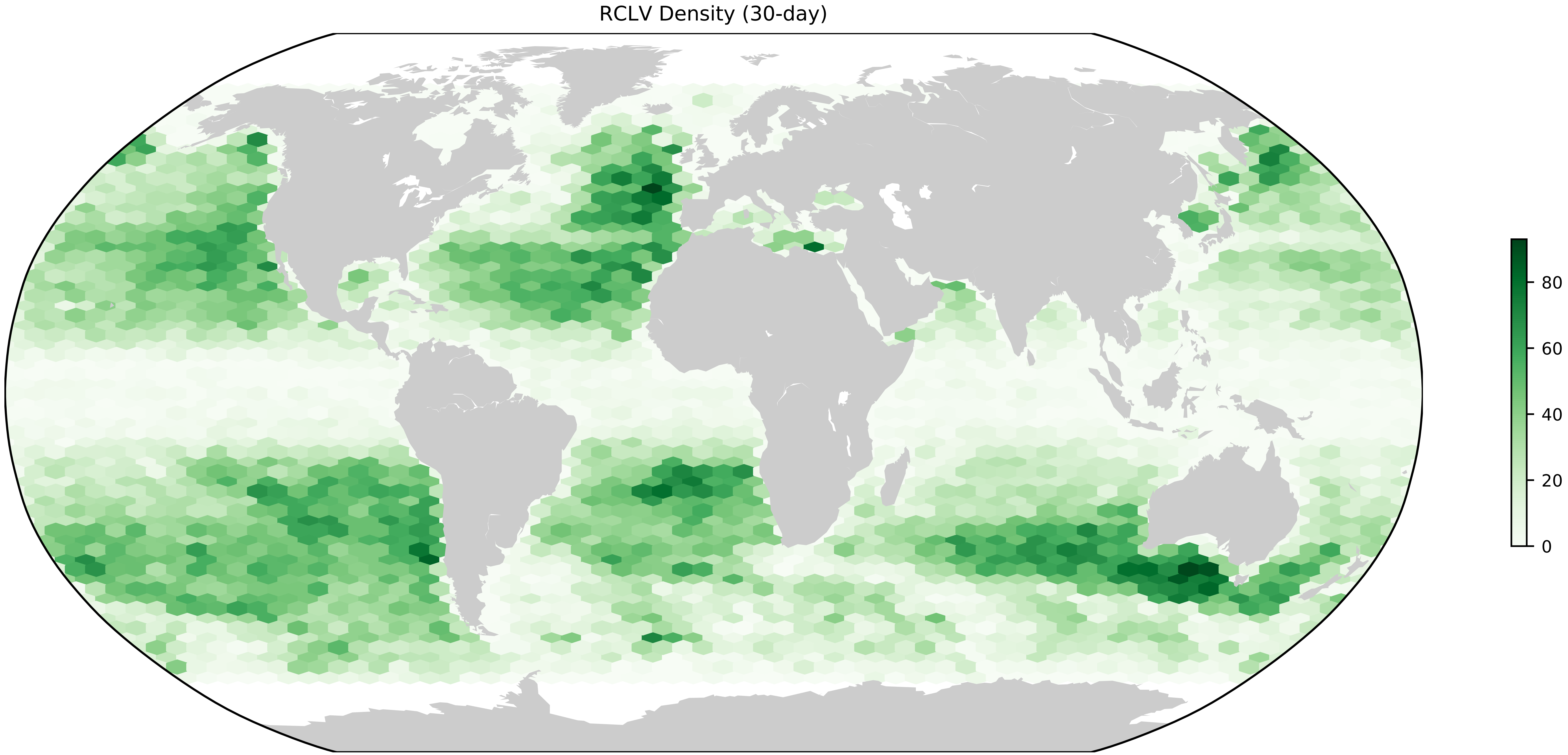


- Use AVISO altimetric velocities used to advect Lagrangian particles over finite time intervals (30, 60, 90, 120, 180 days):

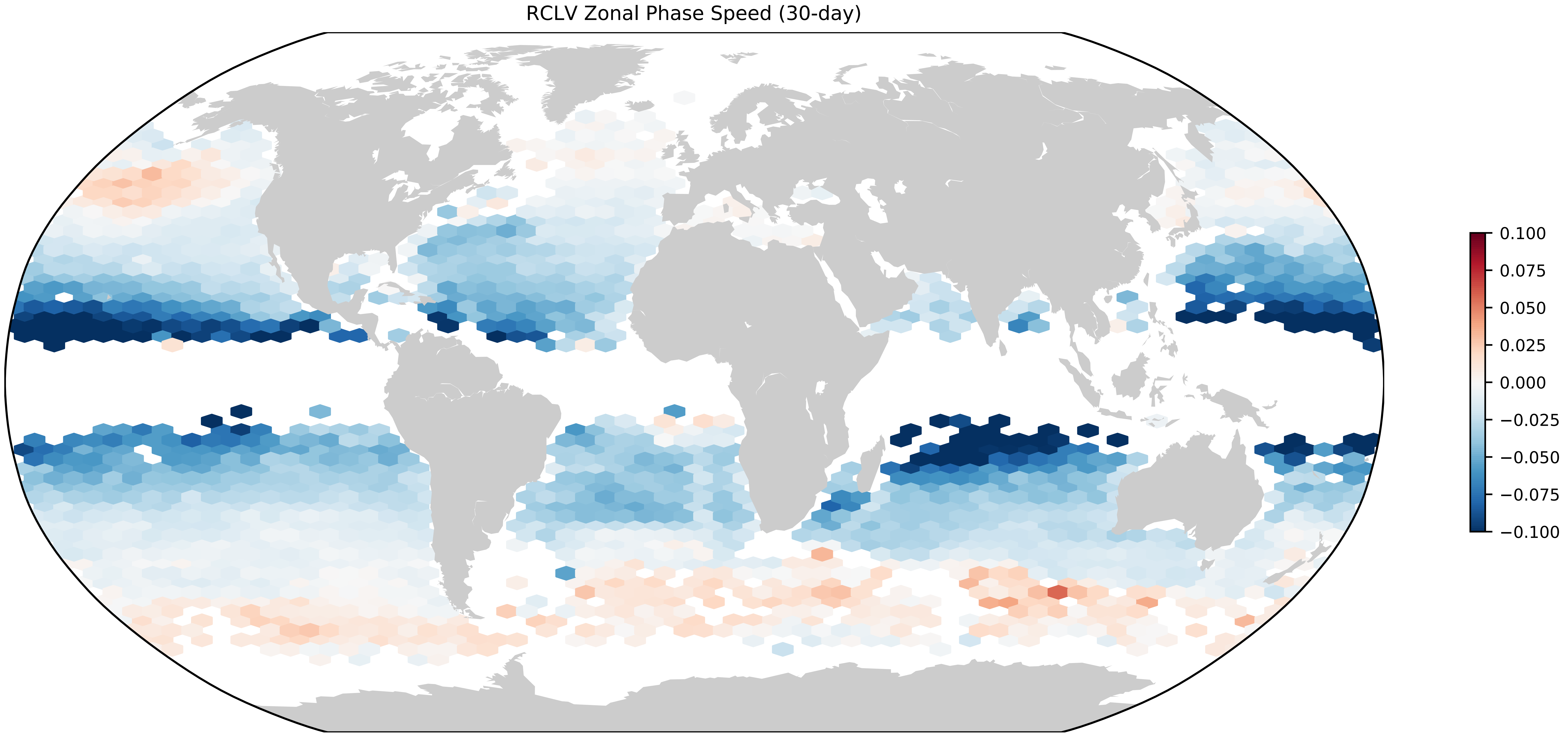
$$\frac{d\mathbf{X}}{dt} = \mathbf{v_g}$$

- 1/32-degree mesh is necessary to resolve LAVD field: ~50 million particles

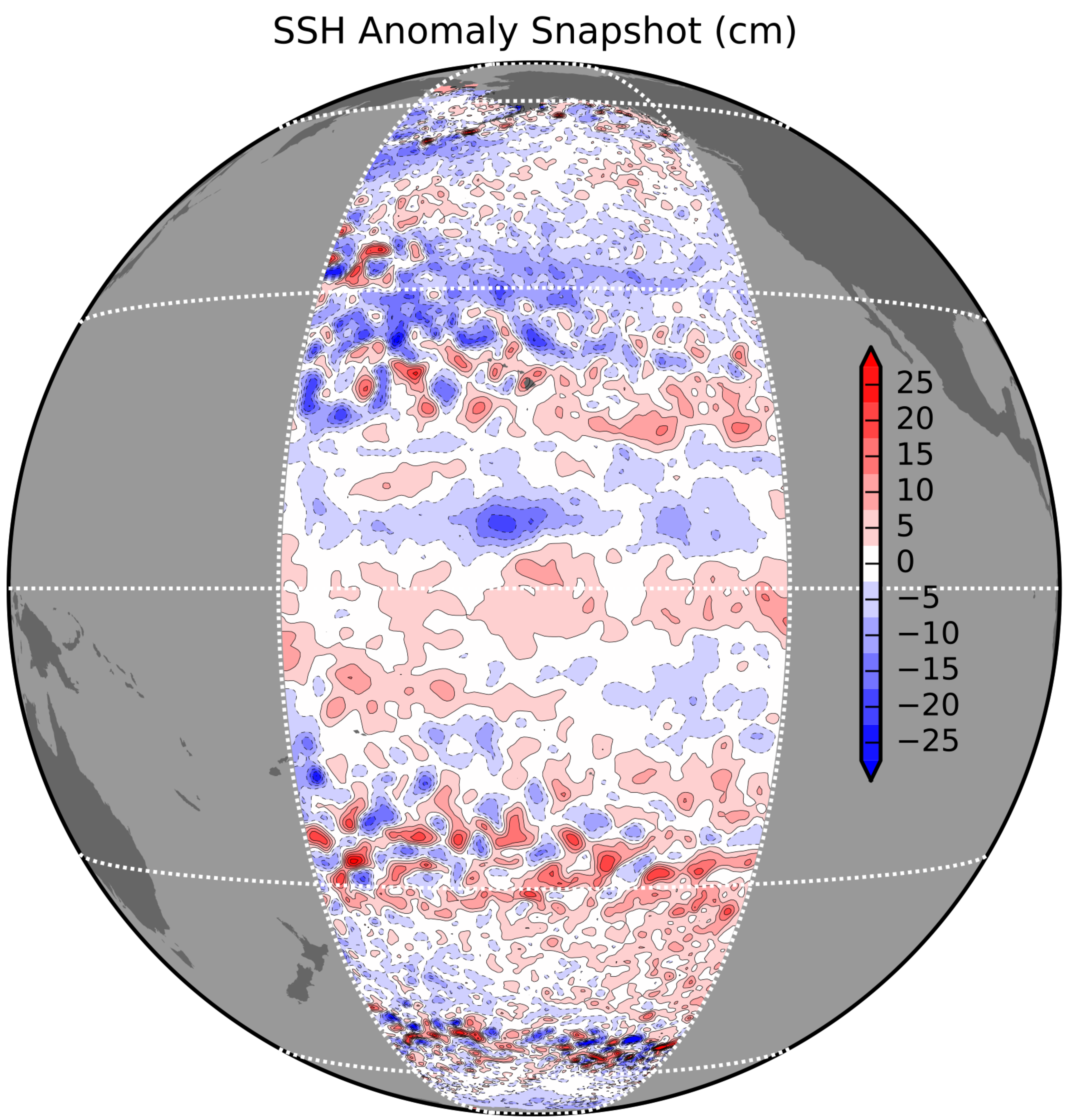
Global RCLV Distribution



Global RCLV Zonal Propagation



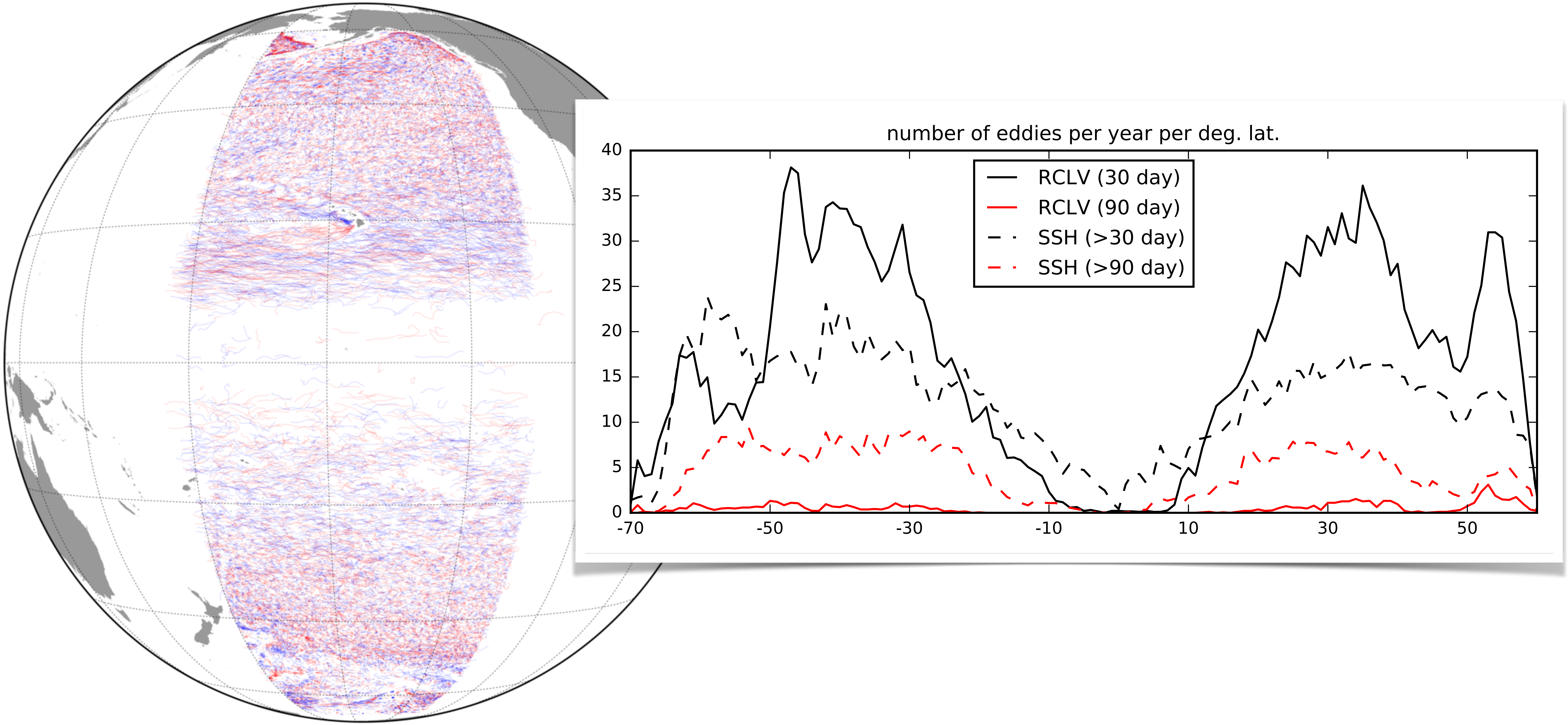
East Pacific Sector



$$\hat{\boldsymbol{k}} \times \boldsymbol{v}_g = -\frac{g}{f}\nabla\eta$$

- Very little land, mostly zonally symmetric statistics
- Straightforward comparison with Chelton et al. (2011)
- Eulerian eddy fluxes are very well studied (Abernathey & Marshall, 2013; Klocker & Abernathey, 2014; Abernathey & Wortham, 2015)

East Pacific Eddy Density



Lagrangian Dispersion

DIFFUSION BY CONTINUOUS MOVEMENTS

By G. I. TAYLOR.

[Received May 22nd, 1920.—Read June 10th, 1920.]

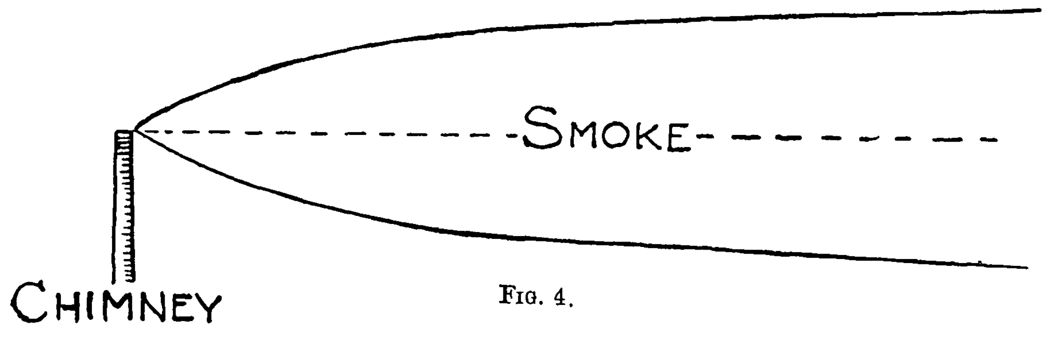


FIG. 4.

meridional heat flux $\overline{v'\theta'} = -K\frac{\partial\overline{\theta}}{\partial y}$

$K = \frac{1}{2}\frac{\partial}{\partial t}\overline{(\Delta Y)^2}$ **eddy diffusivity**

$\Delta Y = Y(x, y, t) - Y(x, y, 0)$ single-particle Lagrangian dispersion

Relative dispersion

$$K_{rel} = \frac{1}{2}\frac{\partial}{\partial t}\overline{(Y - \overline{Y})^2}$$

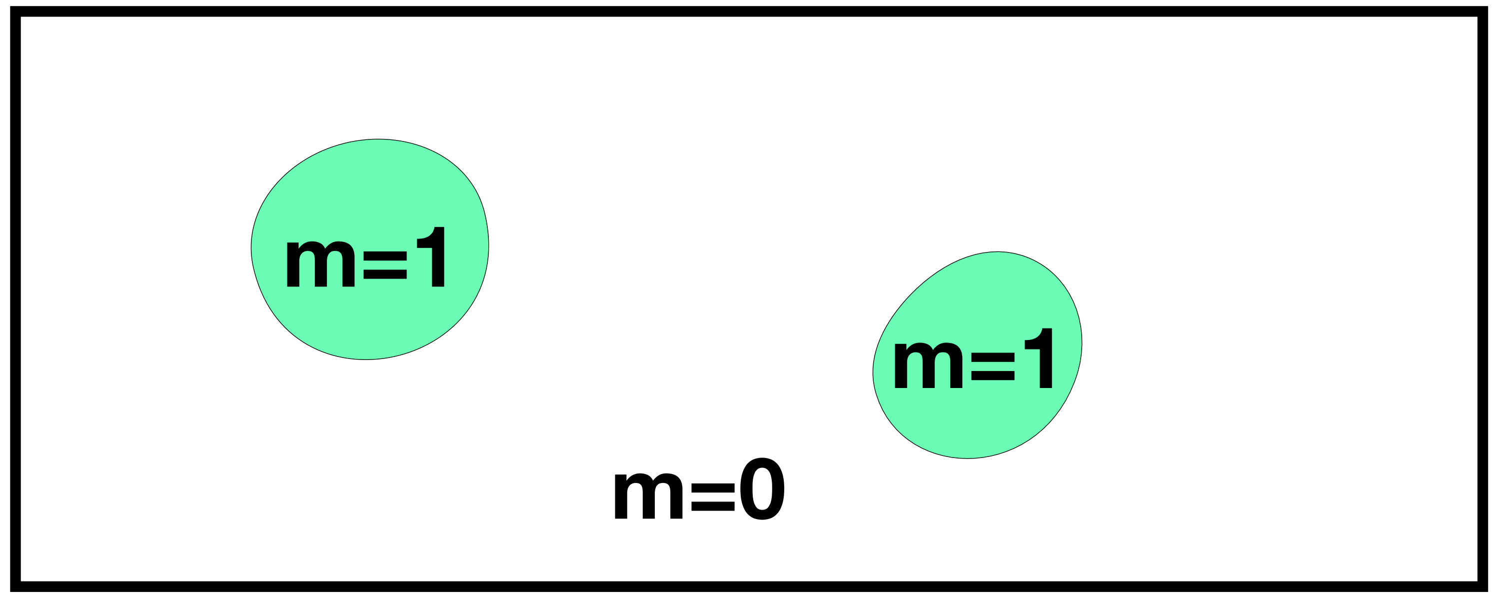
- Removes the dispersion due to “mean flow”
- Describes spreading of tracer patch
- Equivalent to K at large separation scales (i.e. long time scales)

Coherent Lagrangian Dispersion

Relative dispersion

$$K_{rel} = \frac{1}{2}\frac{\partial}{\partial t}\overline{(Y - \overline{Y})^2}$$

coherent structure mask



"Coherent" diffusivity

$$K_{rel}^{CS} = \frac{1}{2}\frac{\partial}{\partial t}\overline{m(Y - \overline{Y})^2}$$

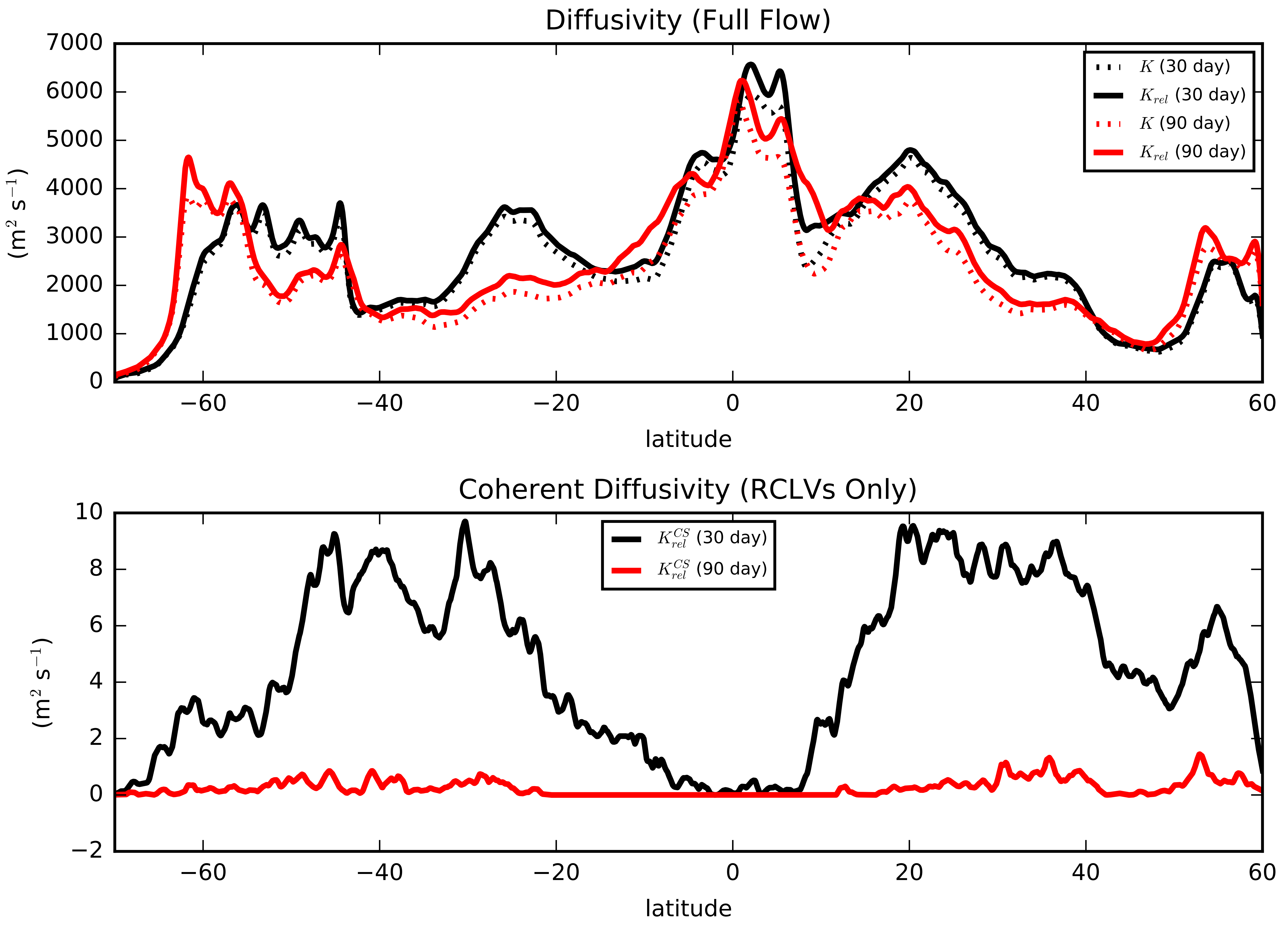
Assumes that the water outside the CSs is just moving with the mean flow

Coherent Lagrangian Dispersion

256,000 particles

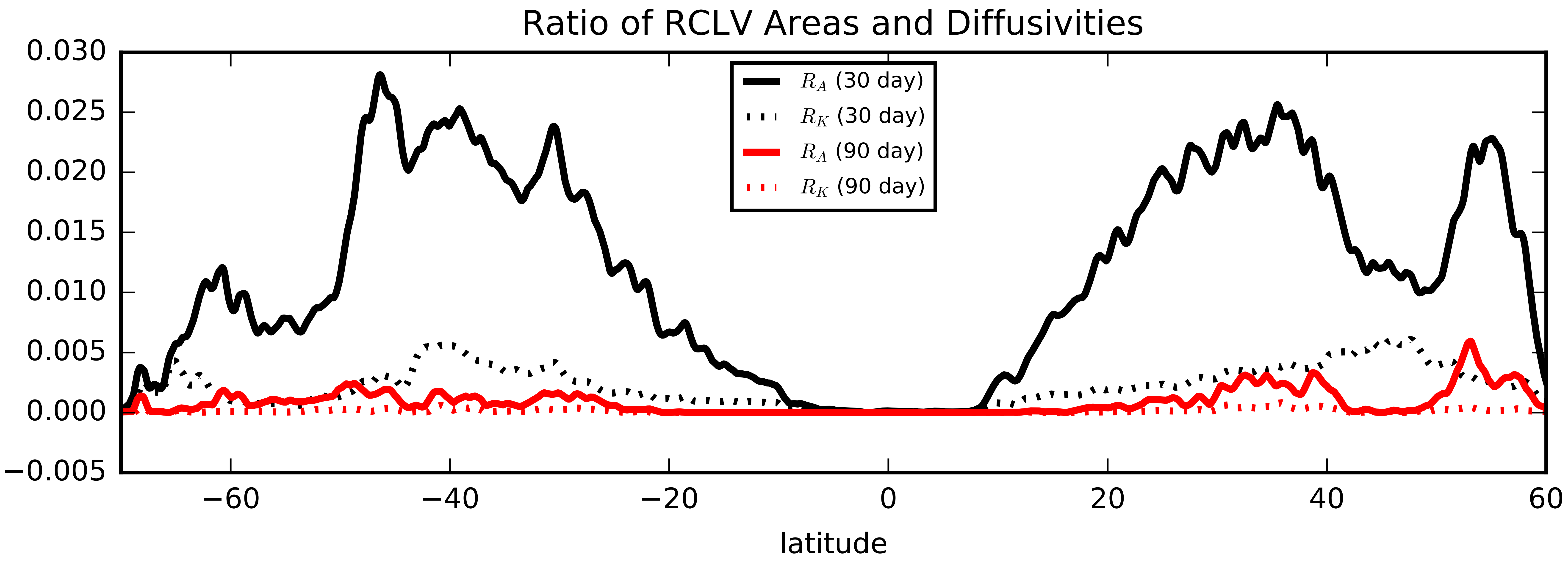
Coherent Lagrangian Dispersion

Diffusivity



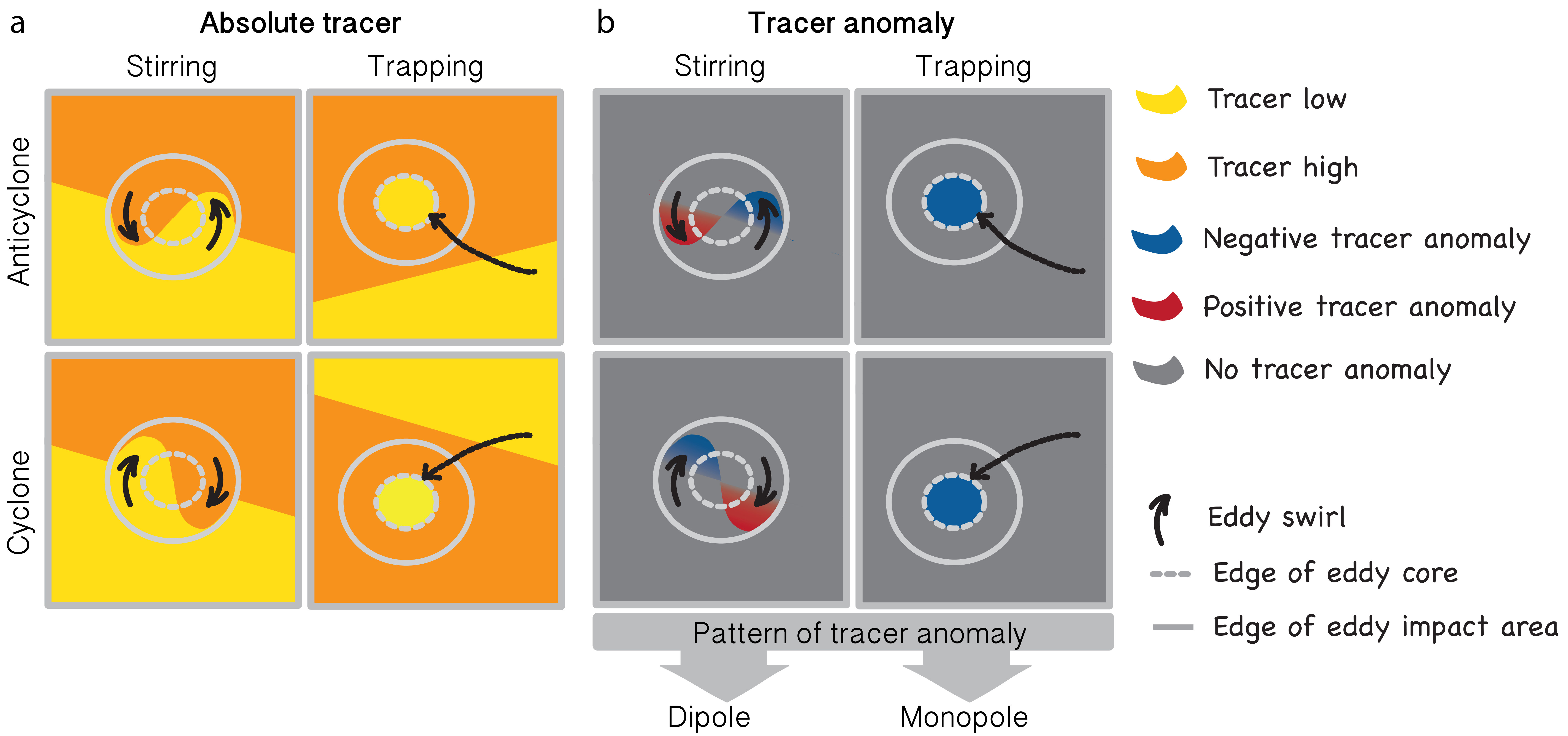
Area and Diffusivity Ratios

$$R_A = \frac{\text{area inside RCLVs}}{\text{total area}} \qquad R_K = \frac{\text{diffusivity from RCLVs}}{\text{total diffusivity}} \quad \text{(dashed)}$$



RCLVs are anomalously non-dispersive!

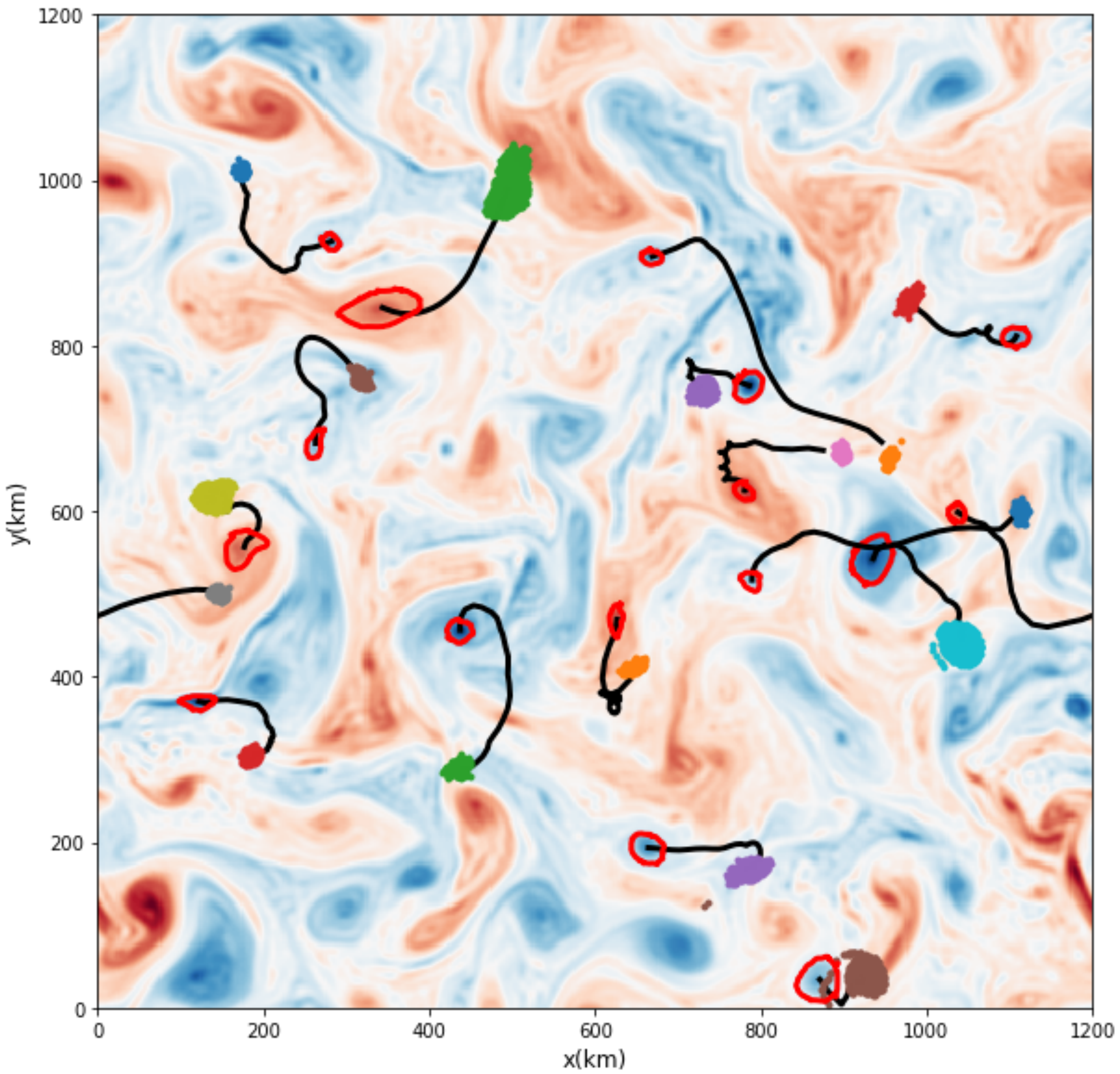
Phenomenology of eddy flux

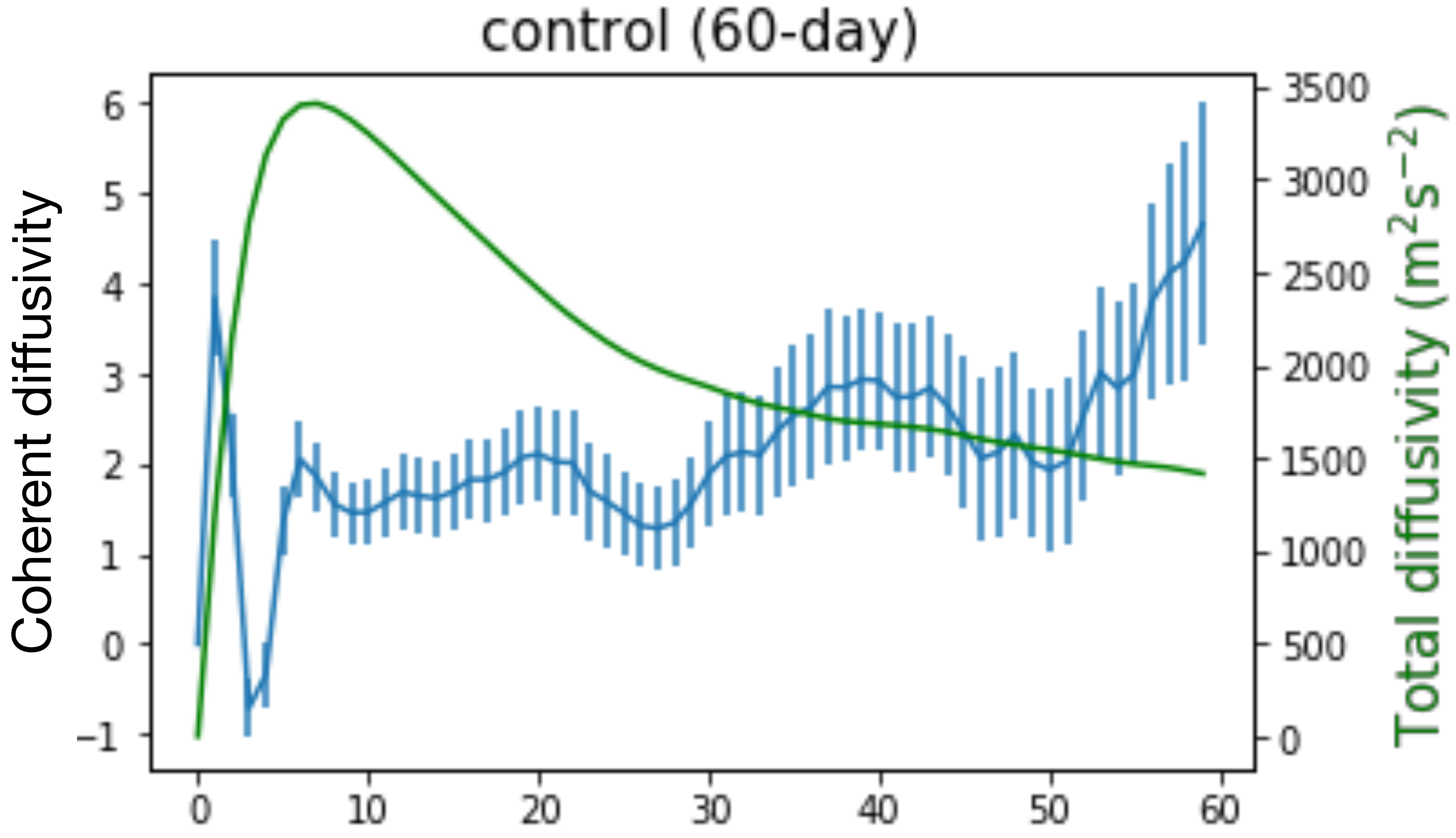


Frenger et al. (2015)

Coherent Diffusivity in Two Layer QG Model

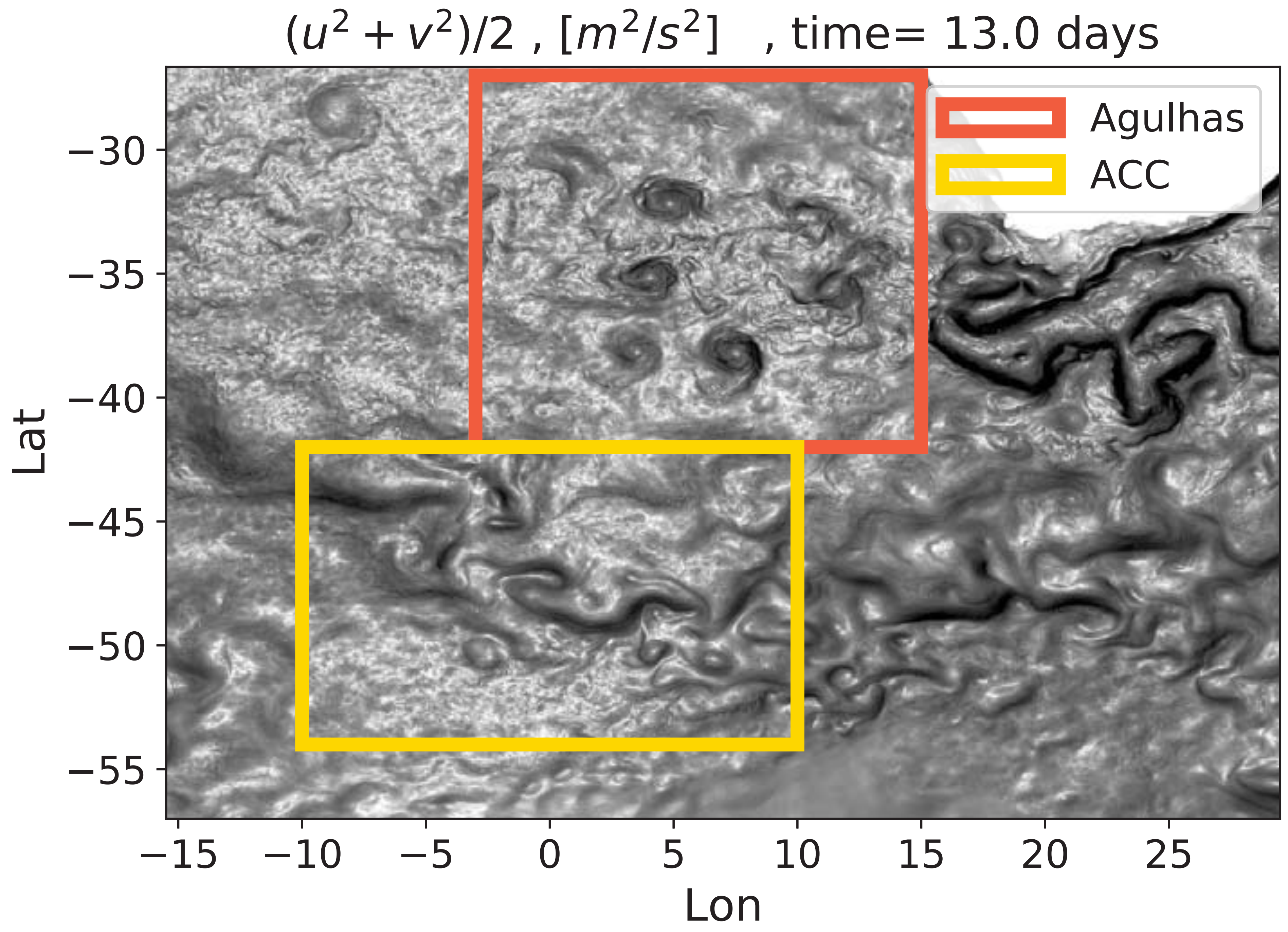
w/ Wenda Zhang and Christopher Wolfe (SUNY Stonybrook)





RCLVs in Ultra-High-Res Model

Ph.D. work by Dr. Anirhan Sinha (now @ Caltech)



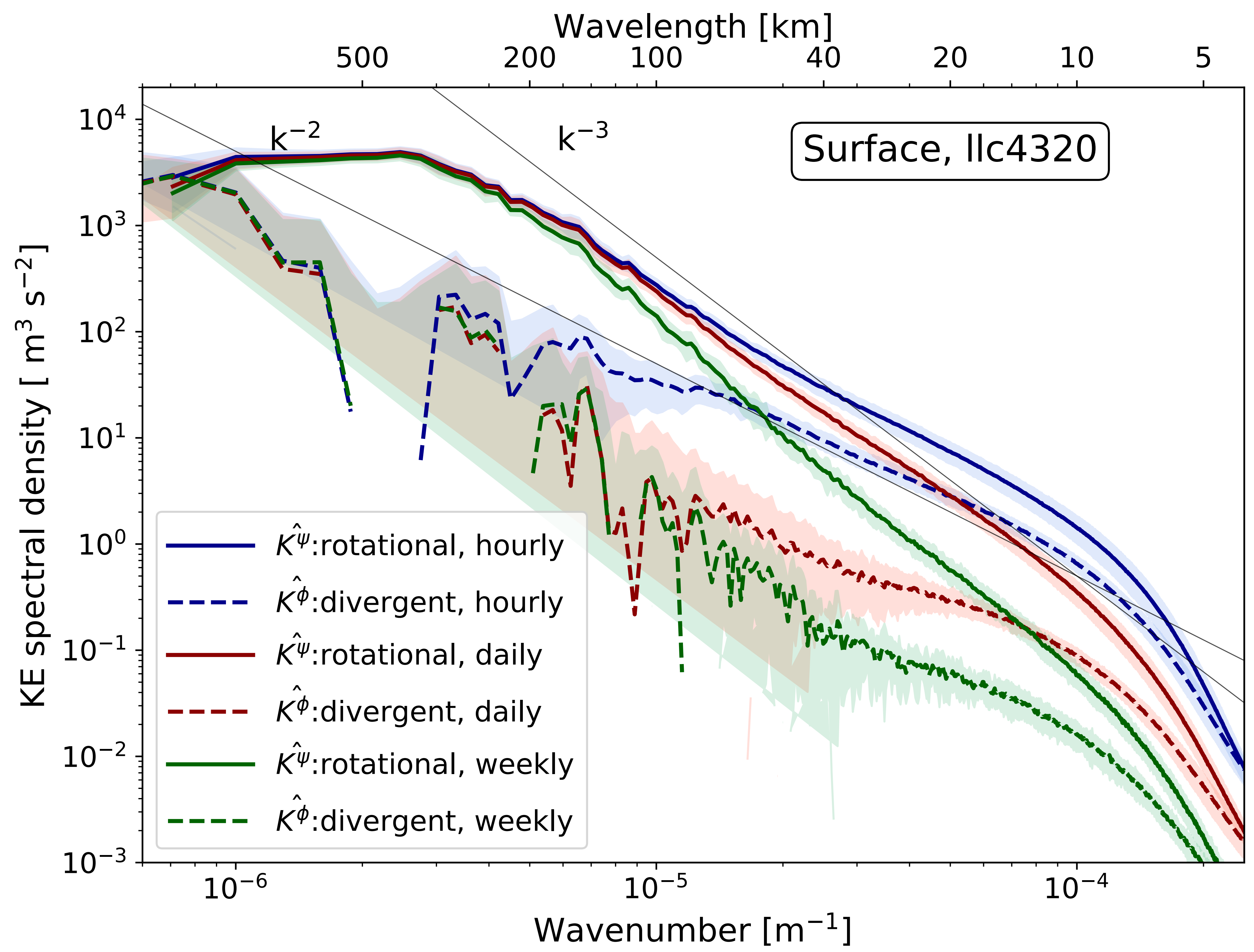
MITgcm LLC4320 (Hill & Menemenlis)

1/48-degree global simulation
Complete tidal forcing, rich IGW spectrum
Hourly output, > 1PB of data for 1yr of simulation

- Satellite observations are relatively coarse and have poor temporal sampling
- What happens to RCLV detection in the presence of energized submesoscale and IGWs?
- Let’s use a model to answer these questions…

Time-filtering of velocities

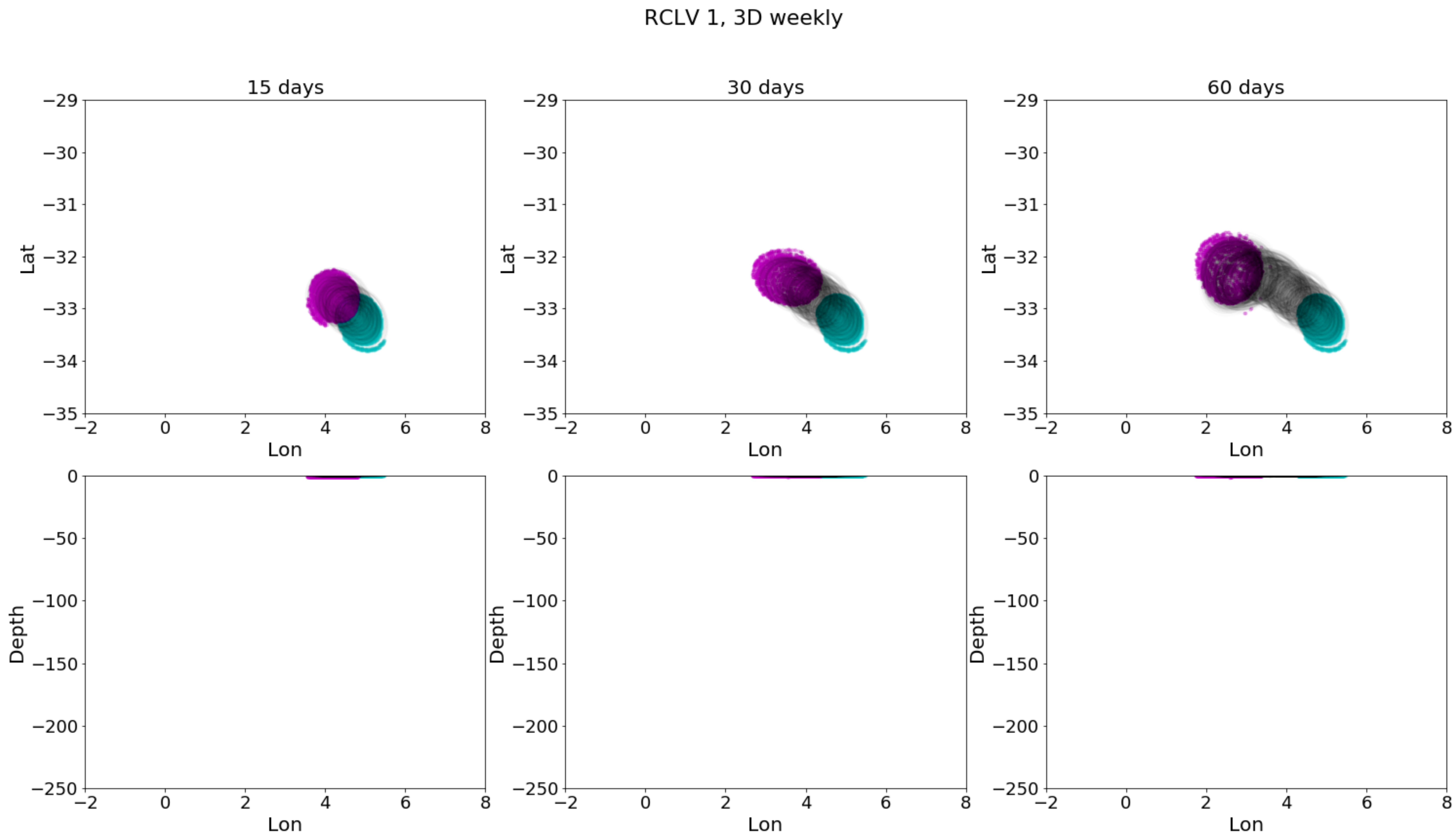
Ph.D. work by Dr. Anirhan Sinha (now @ Caltech)



- Resample hourly velocity fields to daily & weekly resolution
- Calculate rotational / divergent wavenumber energy spectra
- Use these filtered velocities to drive particle advection experiments

RCLVs in LLC4320

Ph.D. work by Dr. Anirhan Sinha (now @ Caltech)



RCLVs in LLC4320

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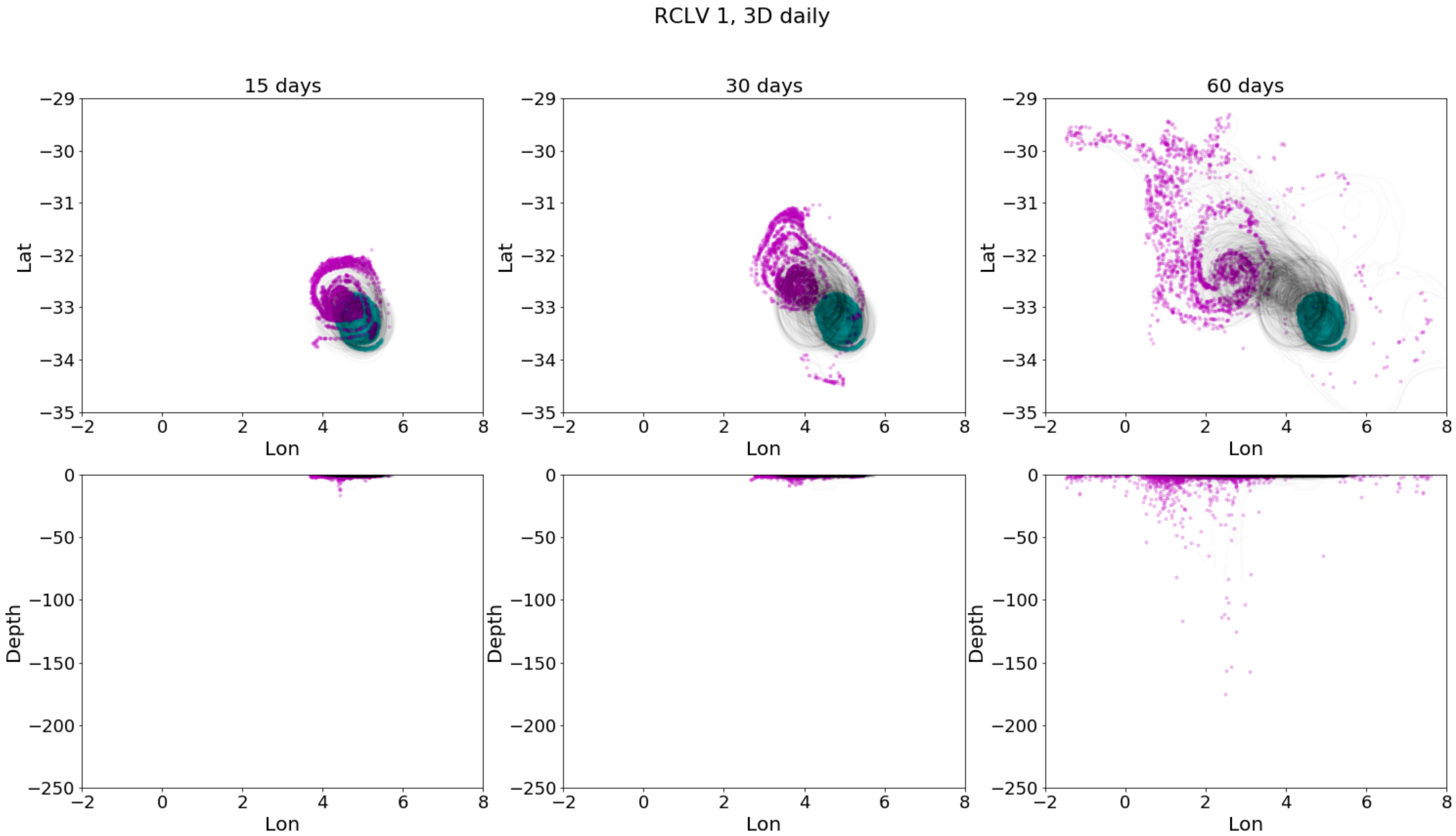

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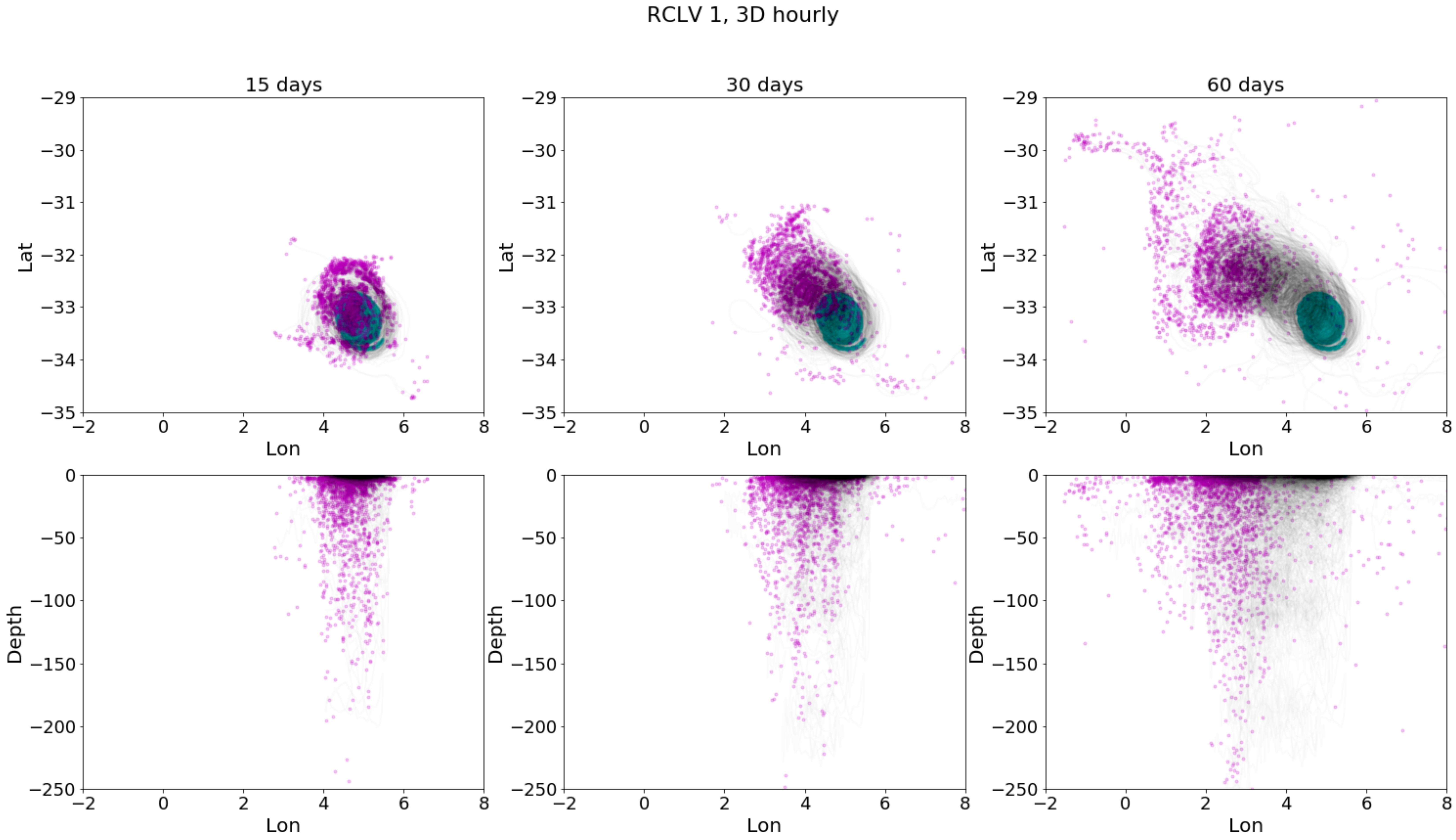

RCLVs in LLC4320

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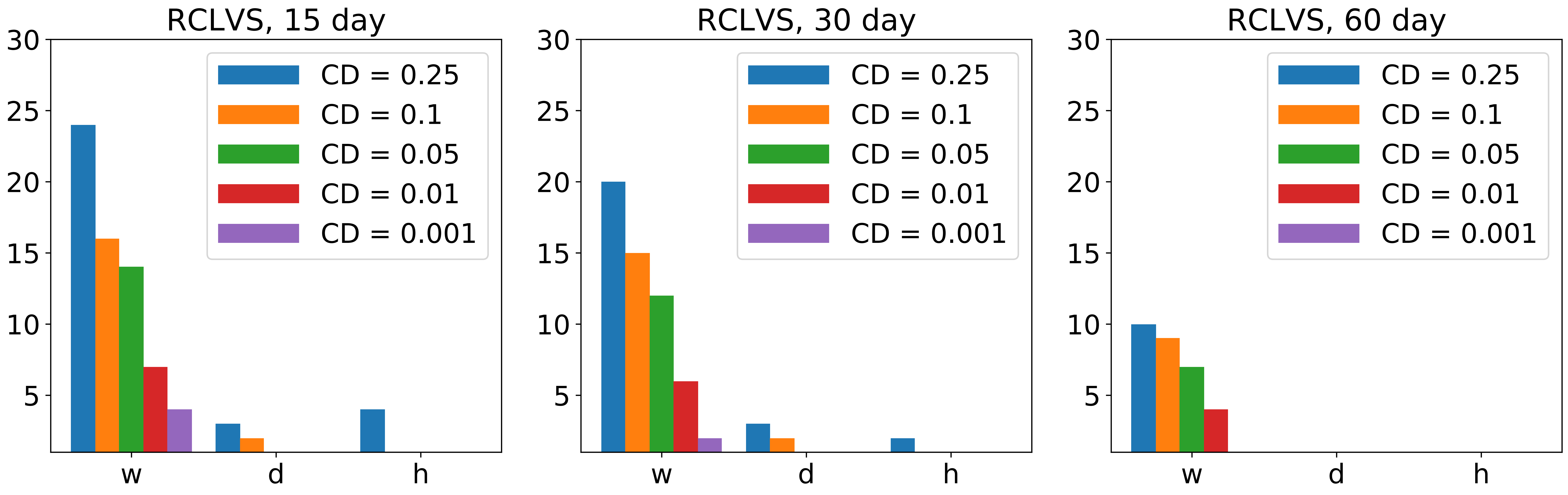
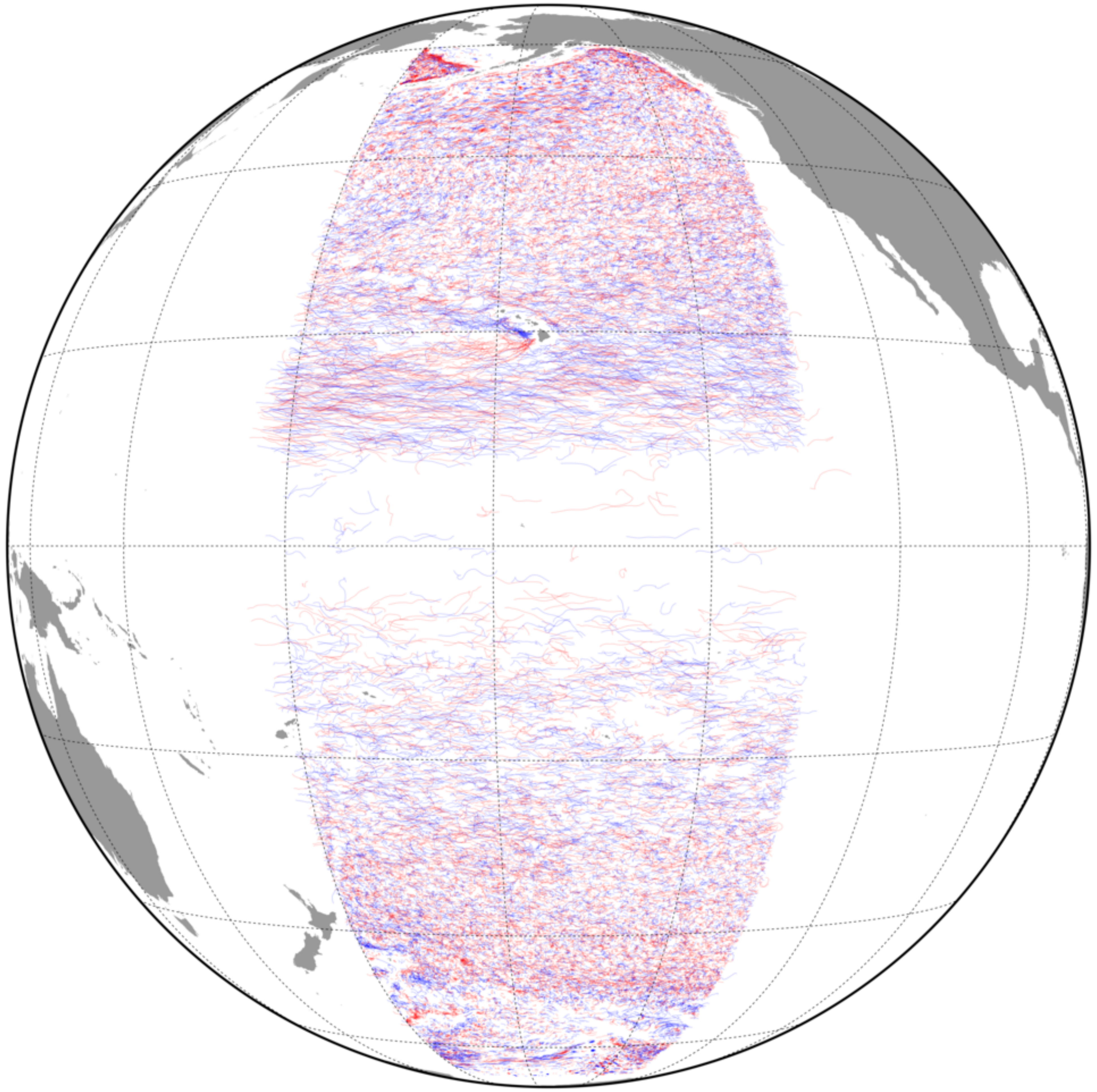


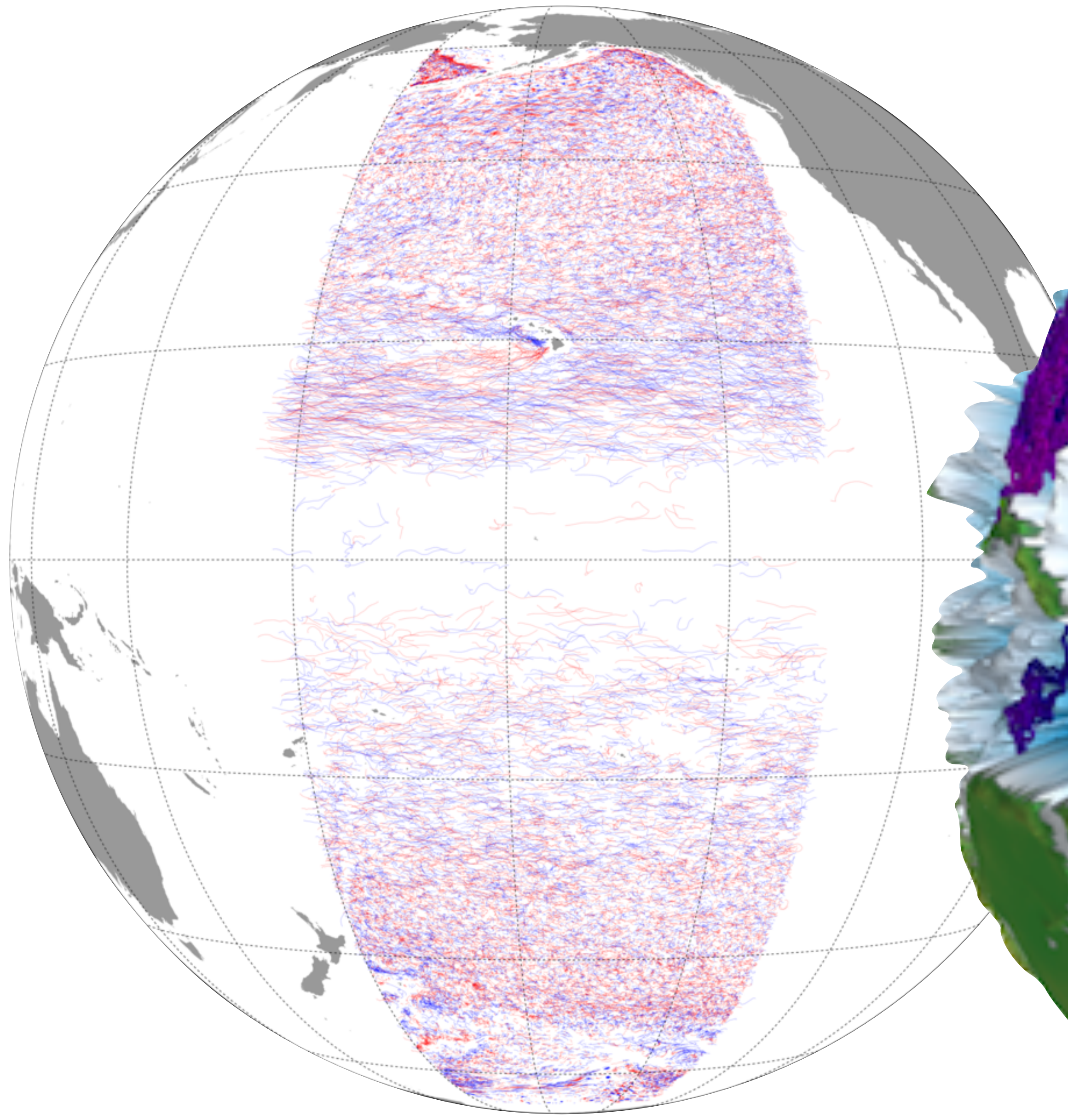
This method can't really detect ***any*** RCLVs in highest resolution data!

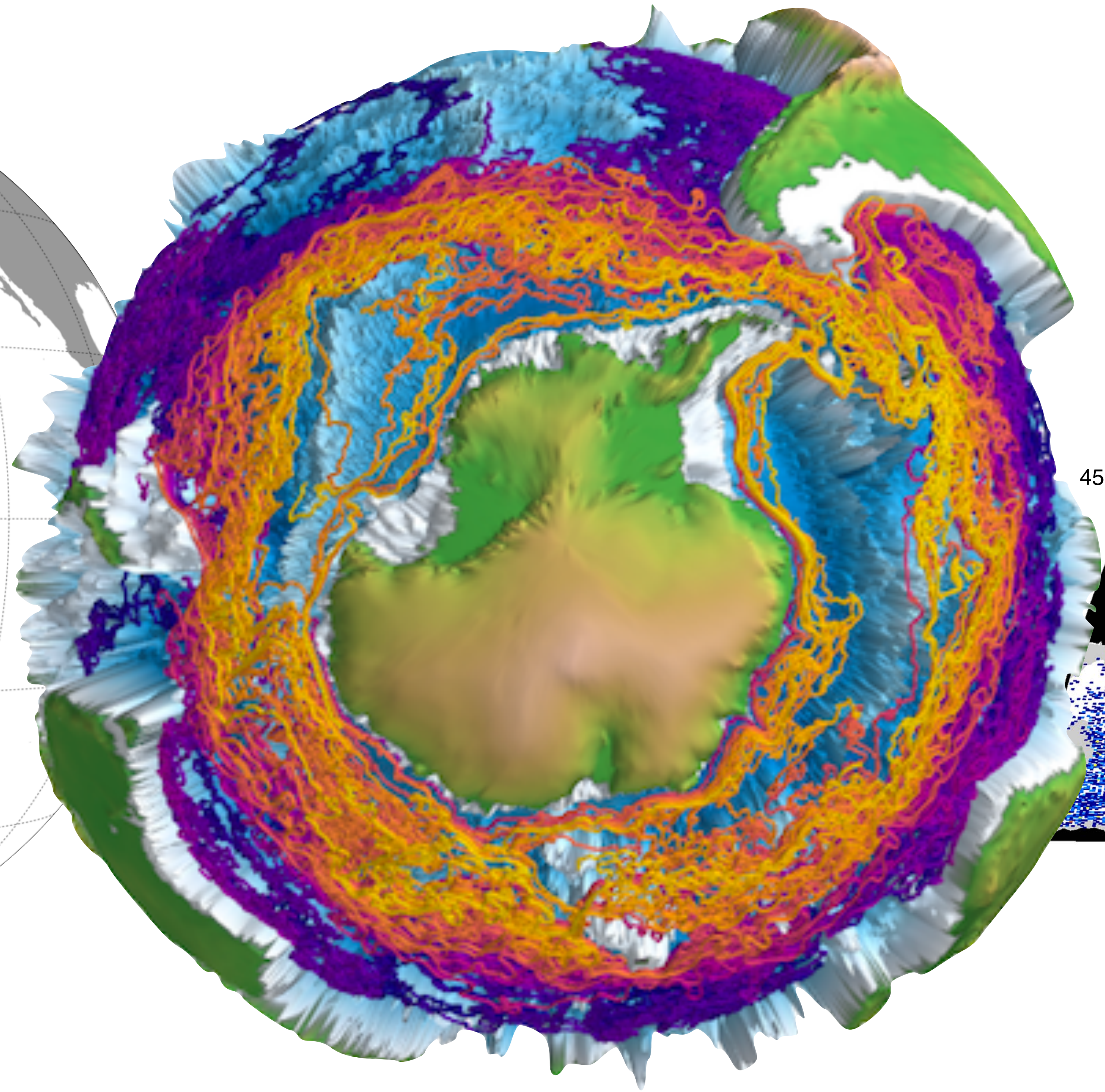
Conclusions



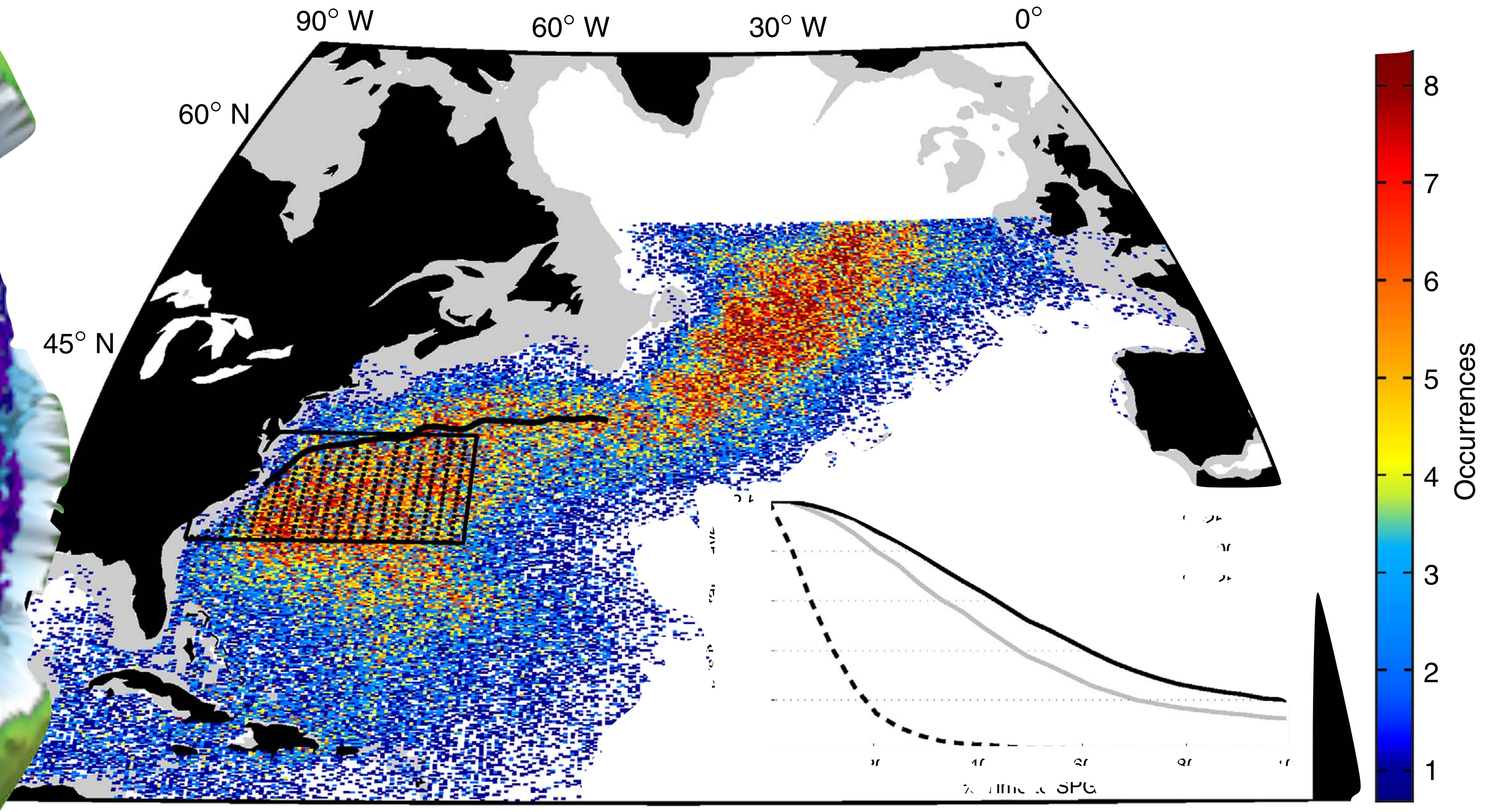
- It is feasible to perform a comprehensive "census" of RCLVs on a global scale using the entire altimetric dataset
- Certain statistics of RCLVs are broadly similar to SSH-eddies (e.g. propagation speed)…
- …But the RCLVs are much shorter lived
- Calculated the contributions RCLVs to net meridional dispersion (relative diffusivity): **totally negligible!**
- Same result in QG model
- RCLVs are very rare in submesoscale resolving model

Future Directions





Tamsitt et al. (2016)
Image by Henry Drake (MIT)



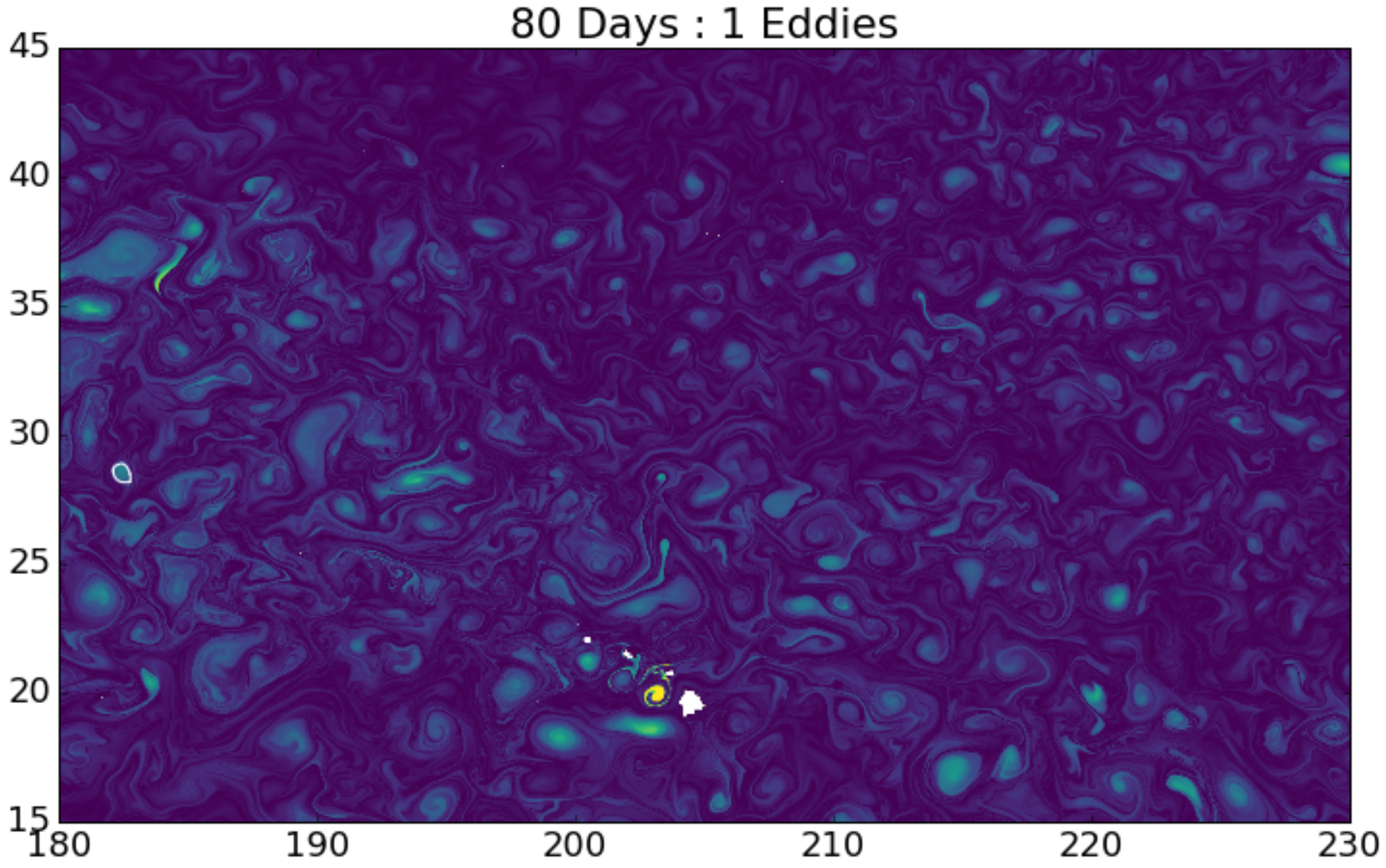
Foukal & Lozier (2017)

“Dense” Lagrangian trajectory datasets have enormous scientific potential…

…but they bring huge computational challenges.

Identifying RLCVs

dependence on time interval



color = |LAVD|

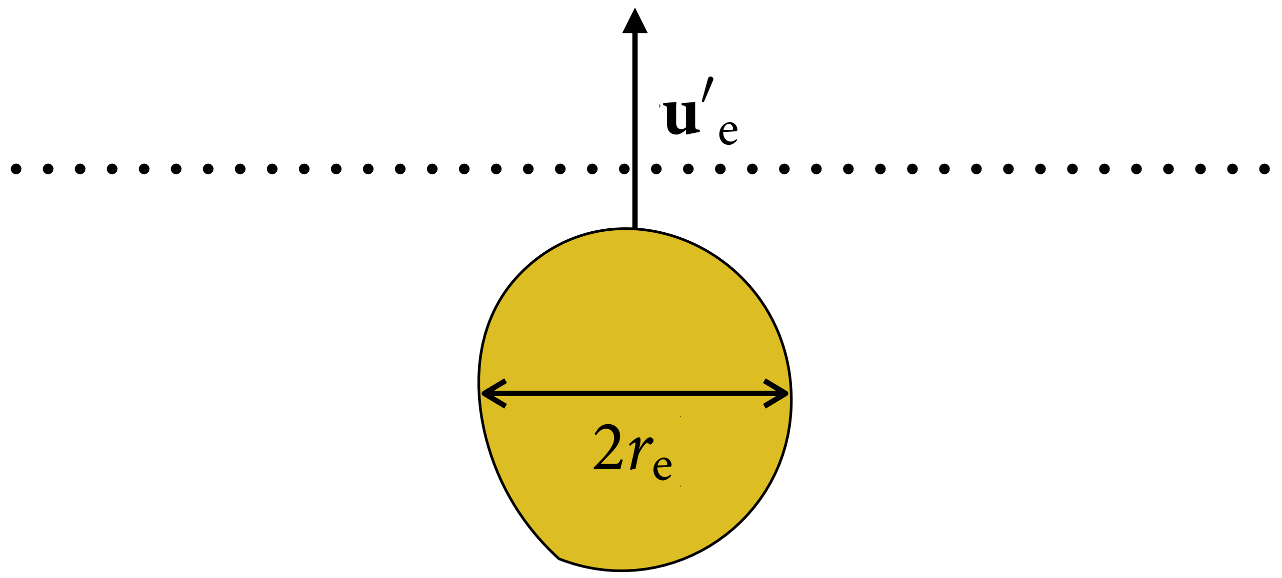
“Transport” by a coherent eddy

$$\mathbf{Q}_{\mathrm{eh}} = s\mathbf{u}'_{\mathrm{e}} \int dz \rho_{\mathrm{o}} C_{\mathrm{po}} (2r_{\mathrm{e}}) T'_{\mathrm{e}},$$

$$\mathbf{T}_h = N_{\mathrm{e}} \langle \mathbf{Q}_{\mathrm{eh}} \rangle,$$

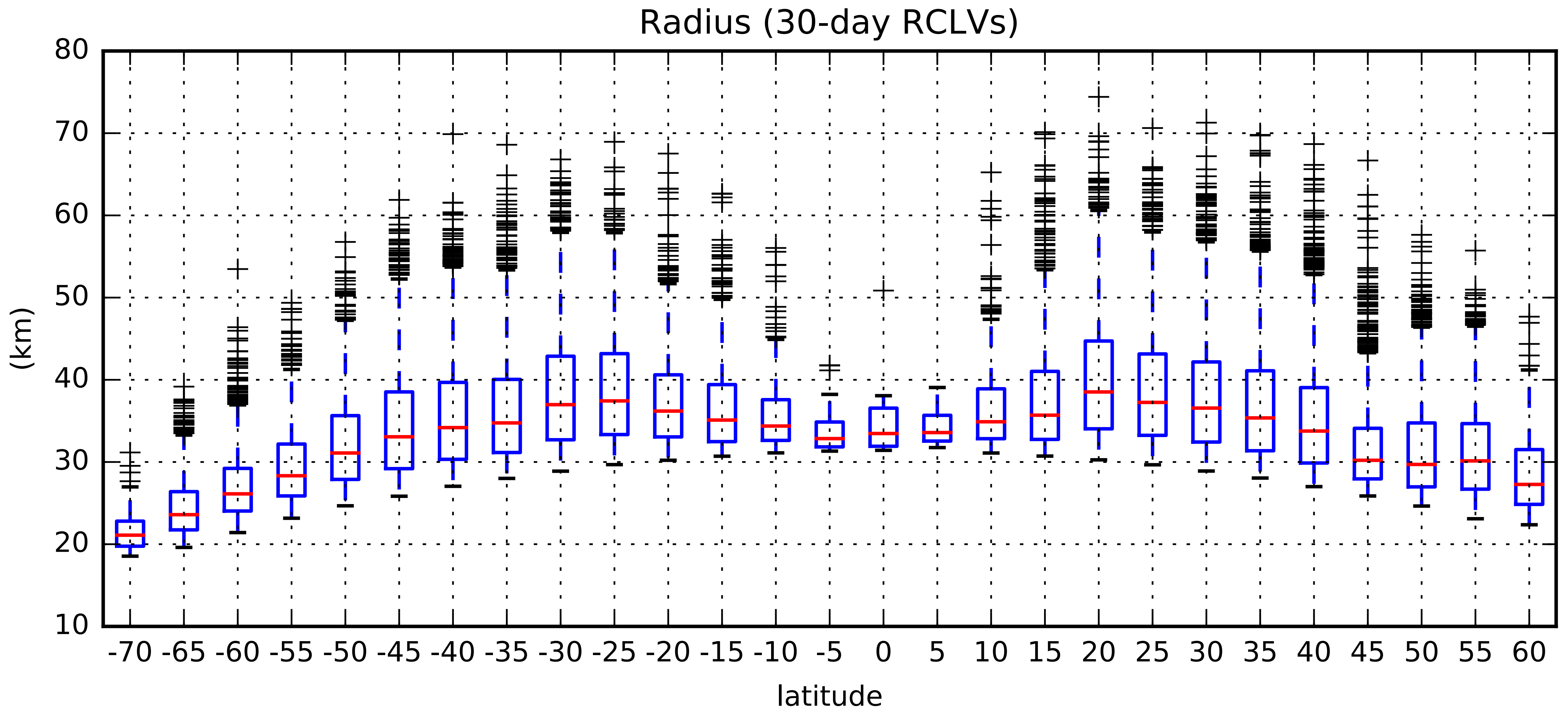
“…eddy heat and salt transports are mainly due to individual eddy movements.”

Dong et al. (2014)



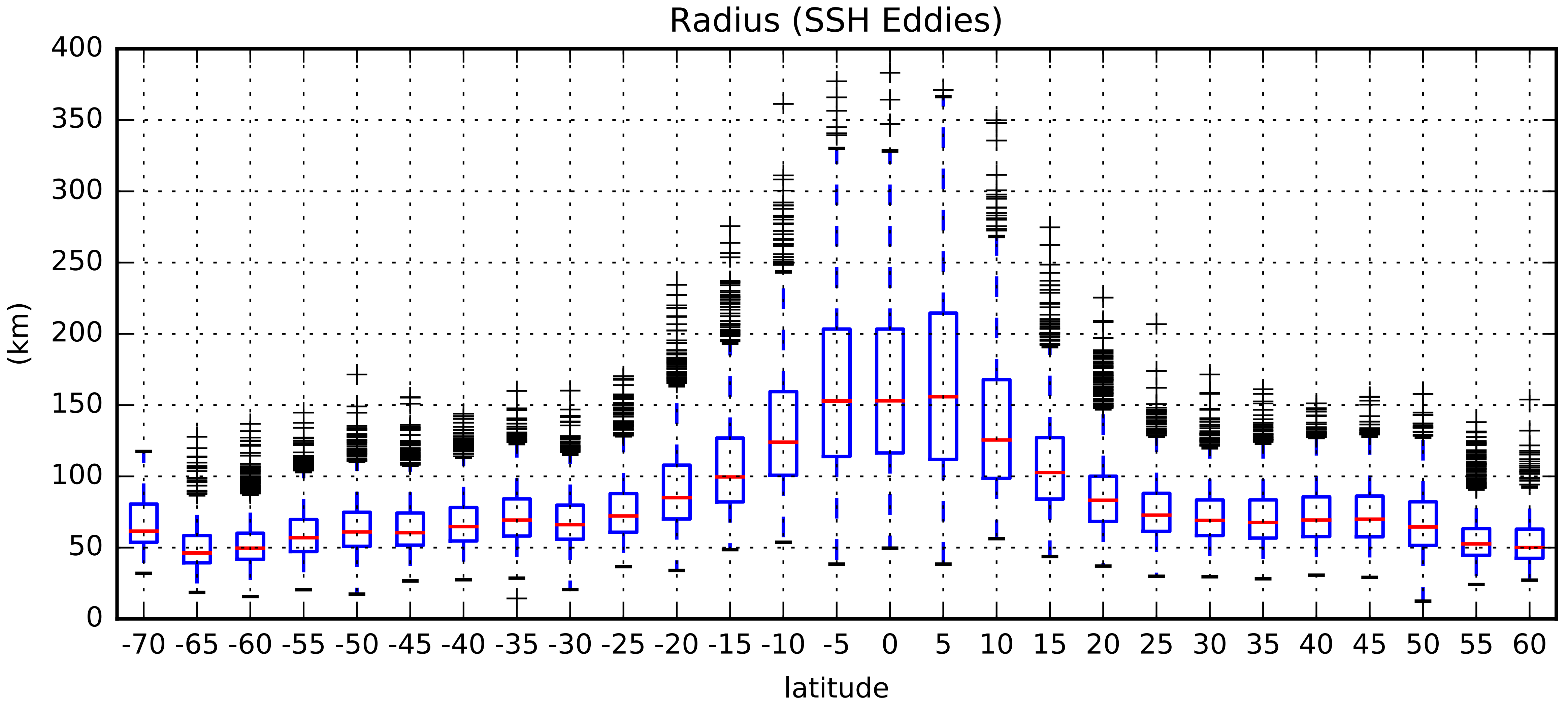
Eddy Statistics: Radius

RCLV
(30 day)



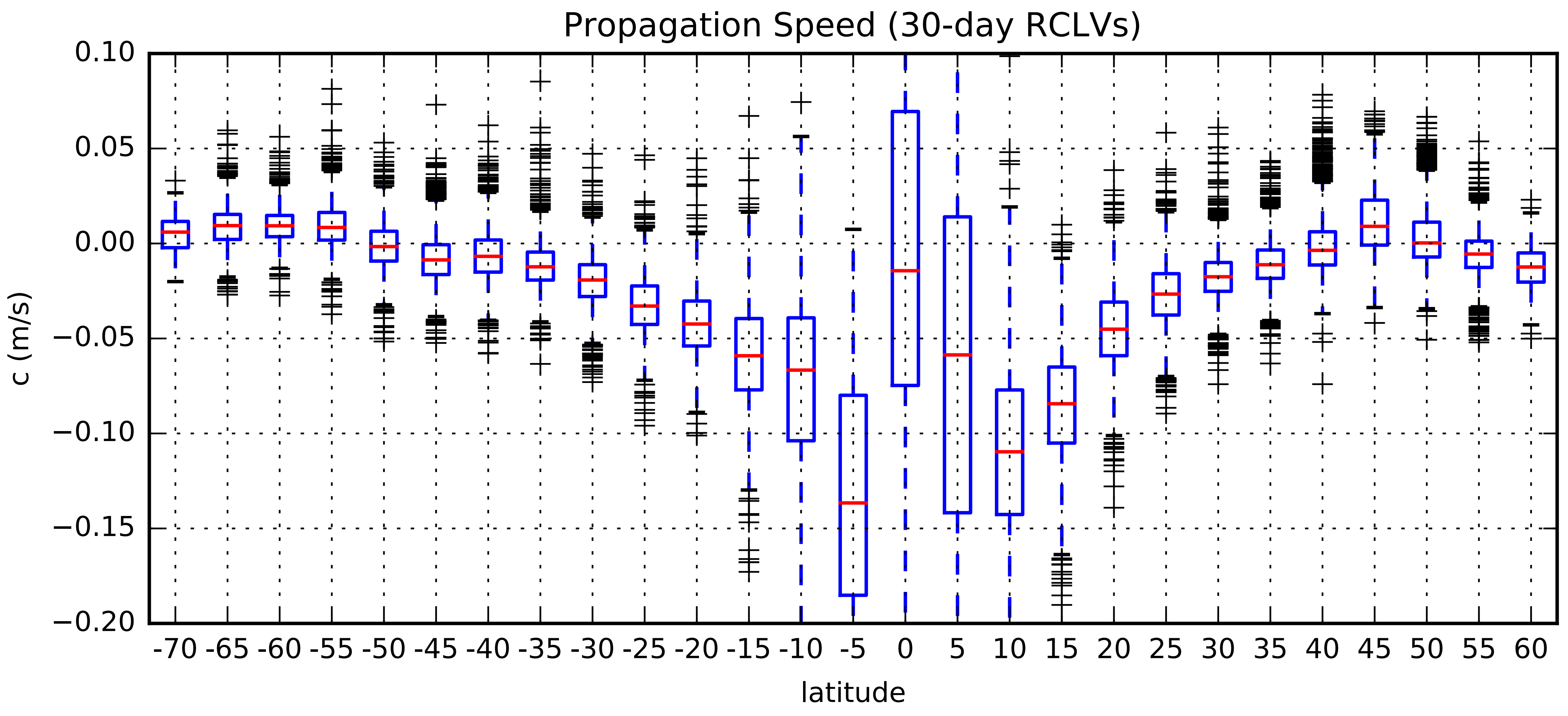
"mesoscale"

SSH
(Chelton)

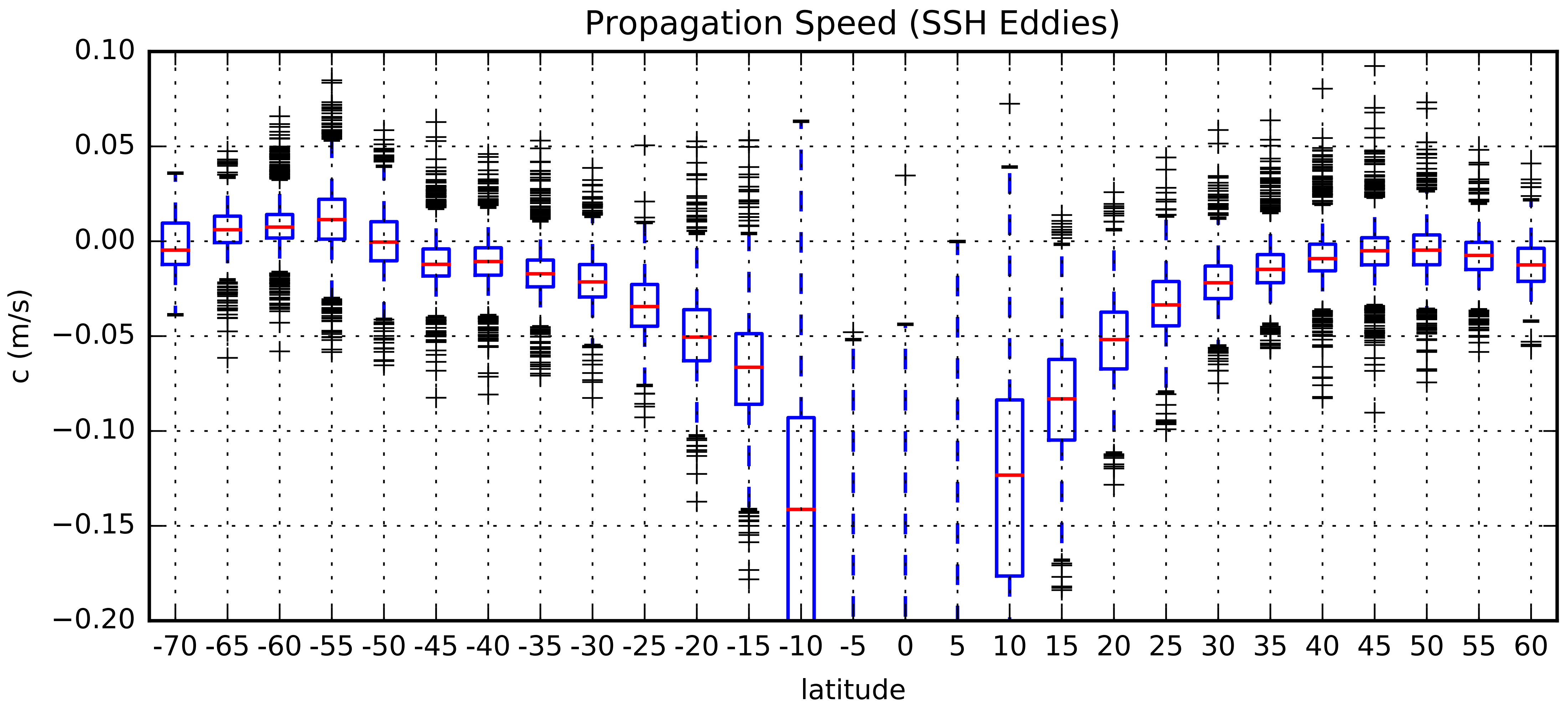


Eddy Statistics: Zonal Propagation

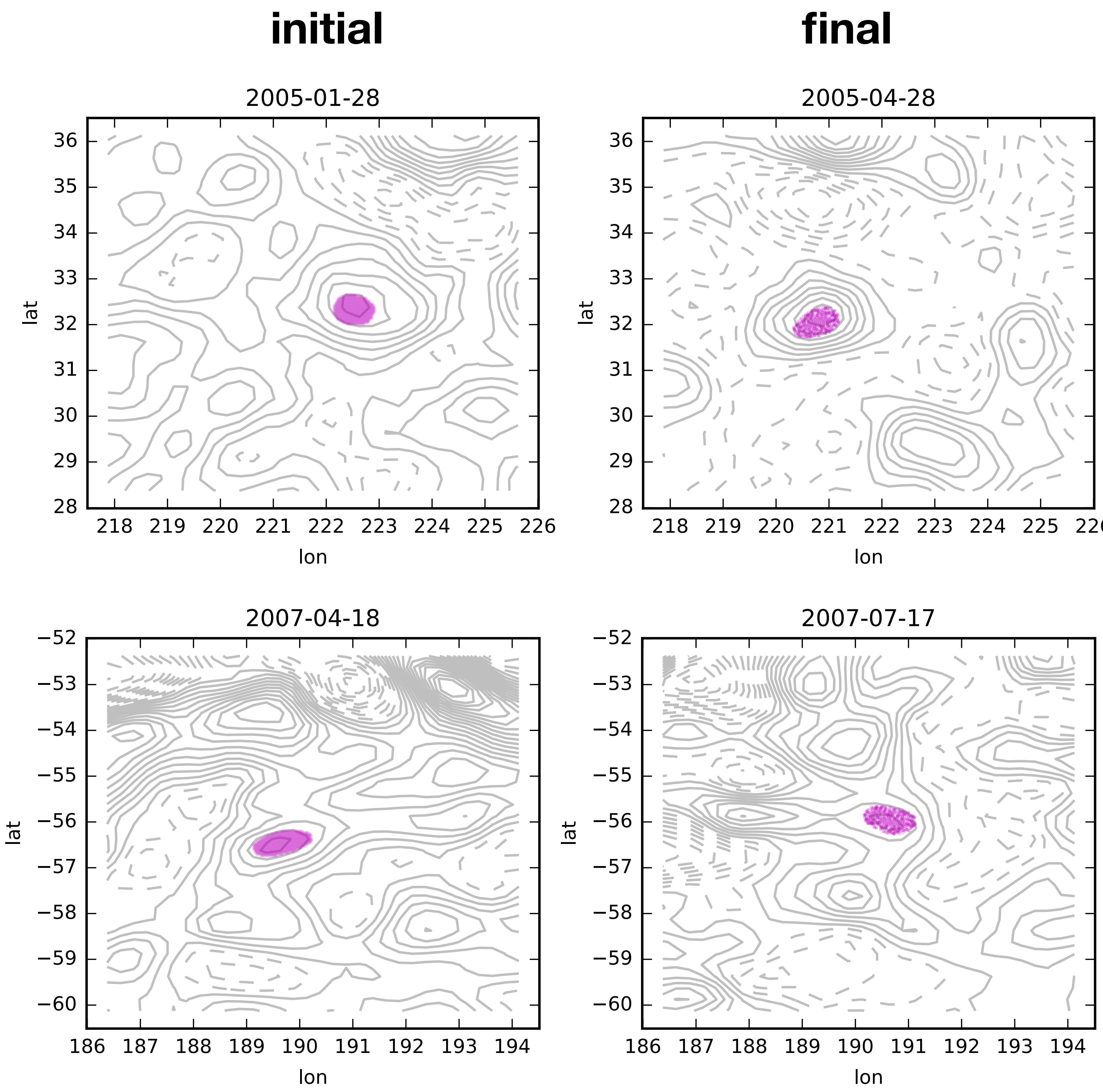
RCS (30 day)



SSH (Chelton)

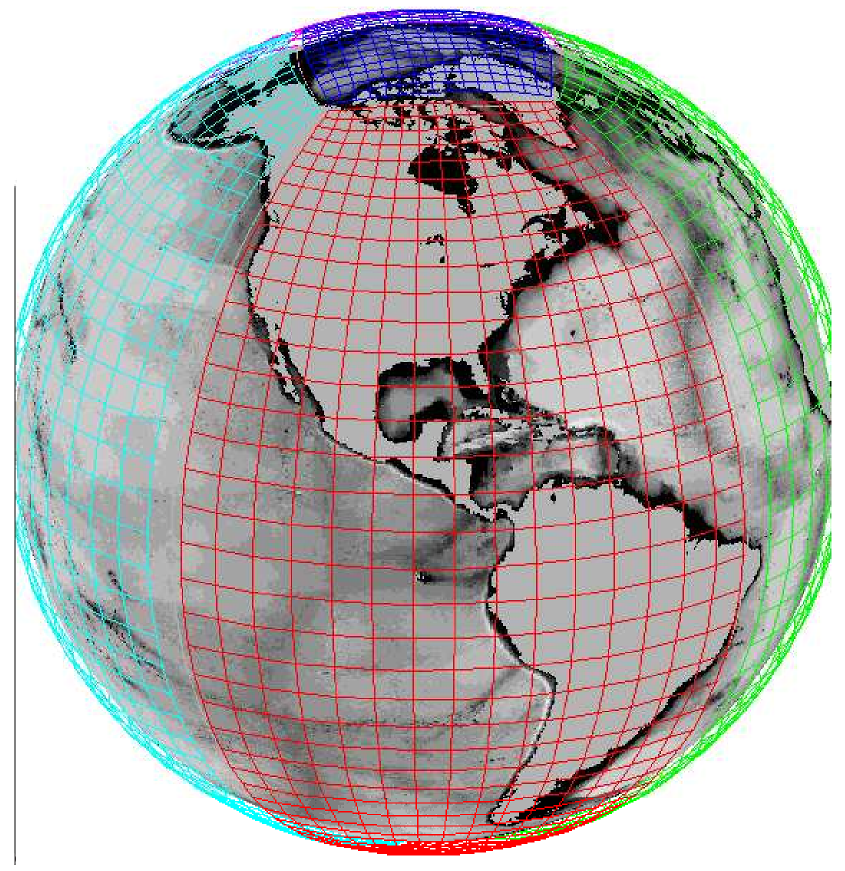


both “feel” Rossby wave dispersion relation



initial
final
2005-01-28
2005-04-28
2007-04-18
2007-07-17
lat
lon

The Tools: MITgcm for Lagrangian Advection



- Has a built in Lagrangian “particle tracking” via FLT package
- Originally designed to simulate ARGO floats
- Full 3D RK4 with trilinear interpolation in space
- “Turbulence” options, but I didn’t use them
- Combined with “offline mode,” which reads velocity fields from files
- Fully MPI parallelized, scales to 100s of processors
- IO bound

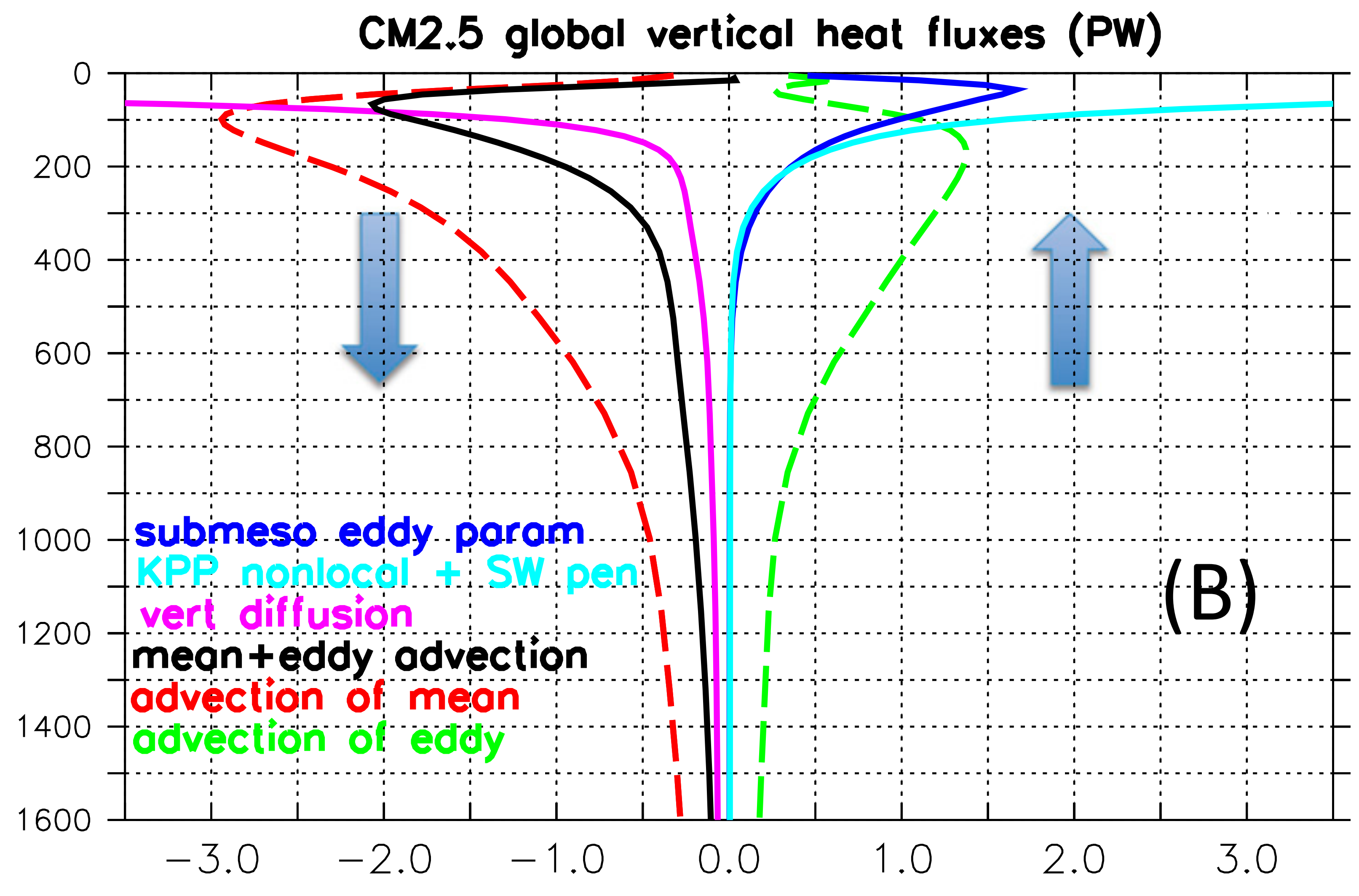
Eddy Parameterization

- Eddy fluxes must be "parameterized" in terms of resolved model fields, i.e.

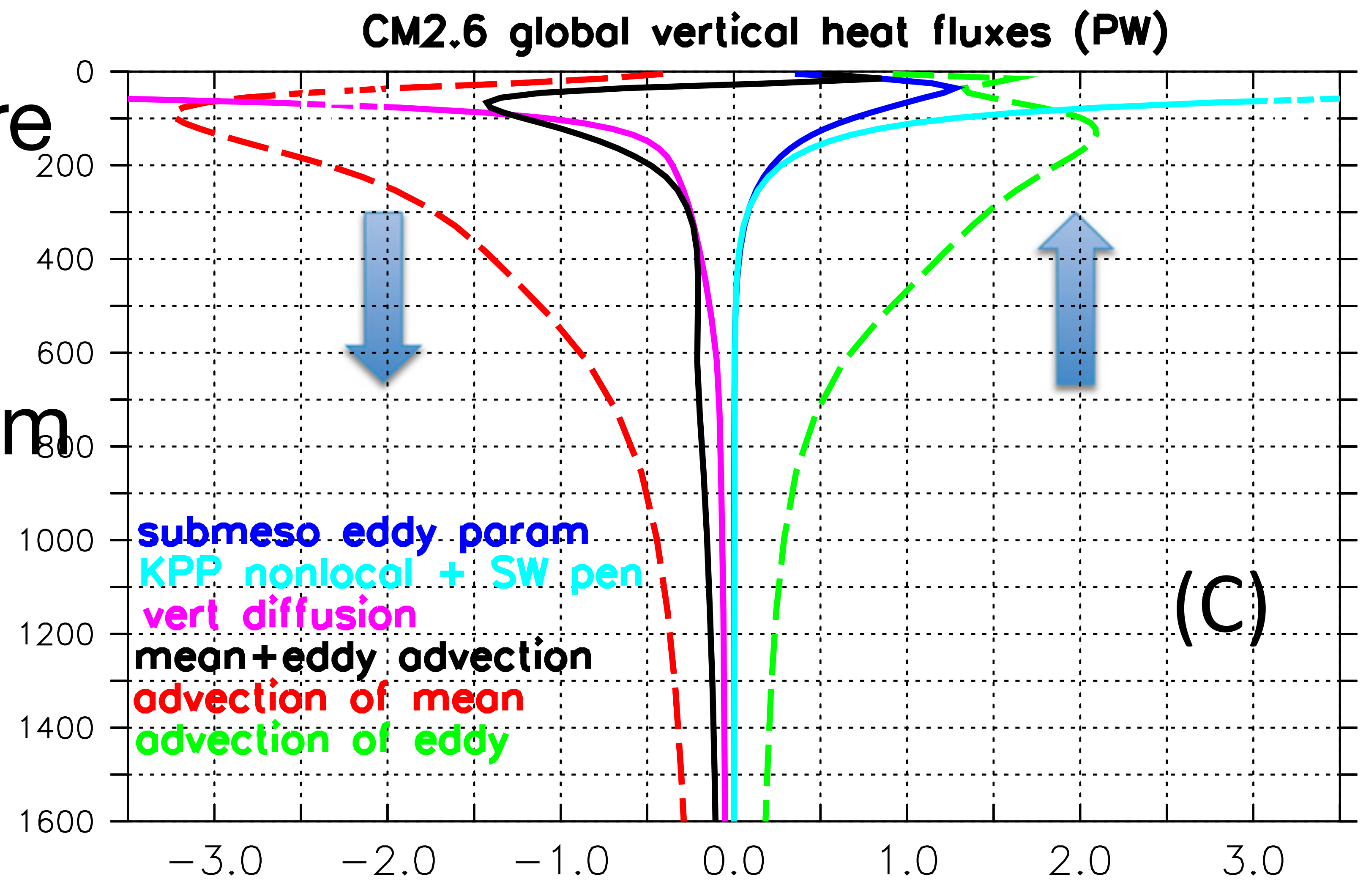
$$\overline{\mathbf{v}'\theta'} = -K\nabla\overline{\theta}$$

- We have decent eddy parameterizations, but they are still far from perfect

- High resolution models perform better than low resolution in many ways

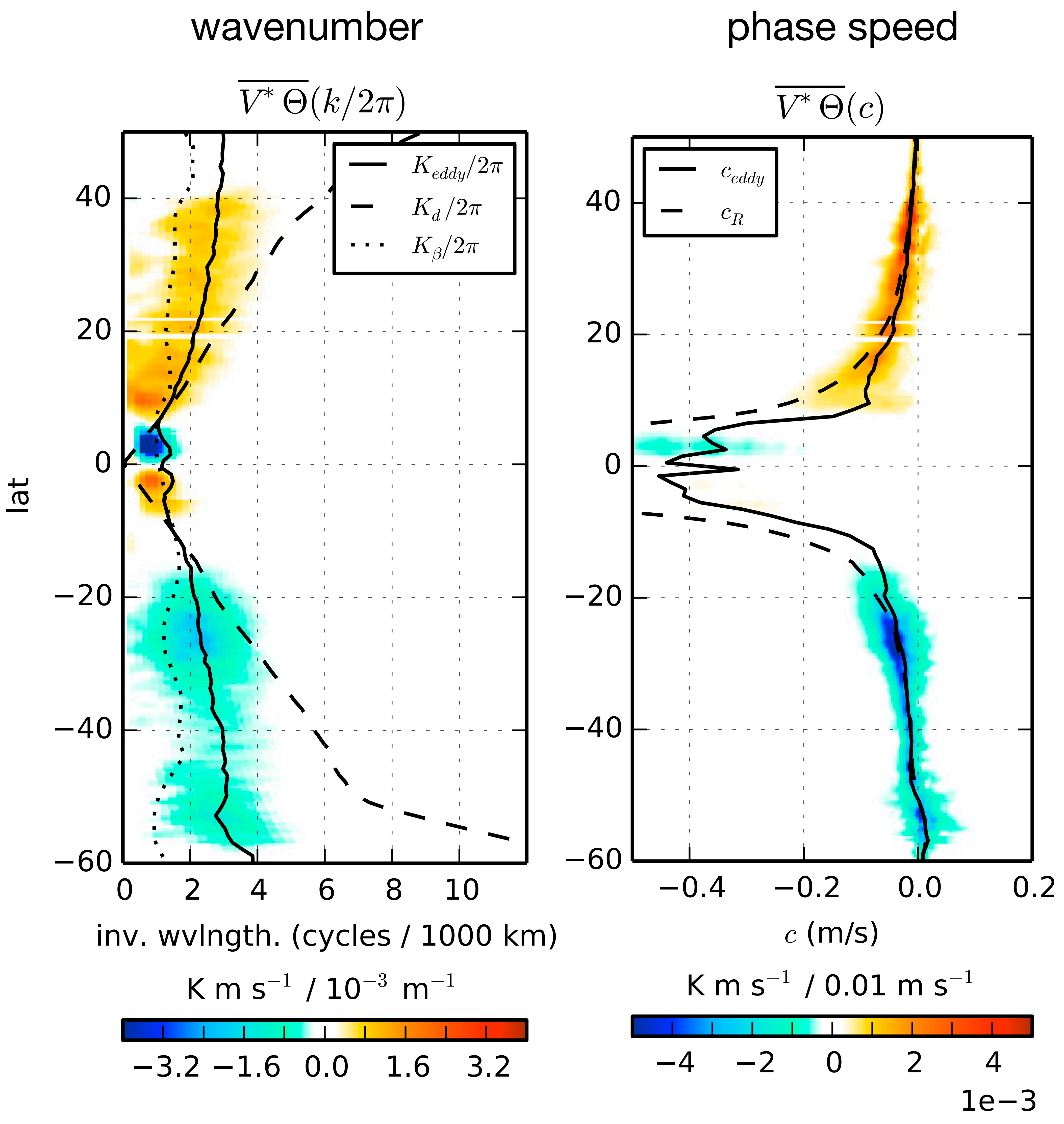


lo-res
(1 deg.)



hi-res
(0.1 deg.)

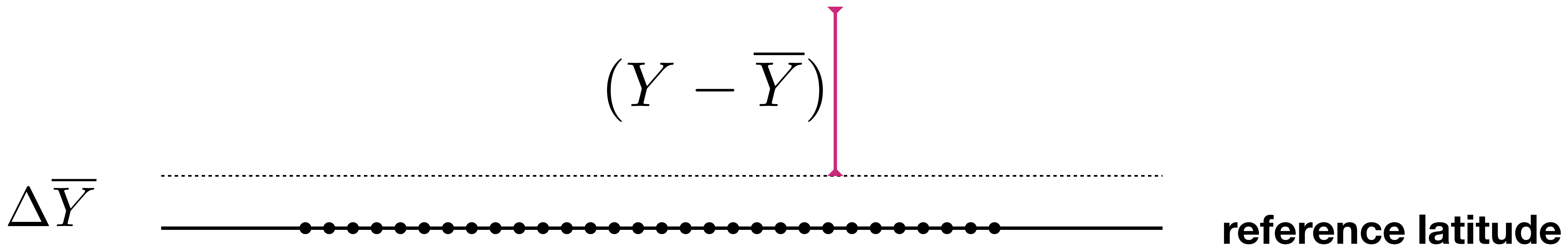
East Pacific Sector - Heat Flux from Satellite Obs.



Abernathey & Wortham (2015)

- Cross-spectral analysis of SST and SSH-derived velocity
- Spectral peaks are aligned with the properties of tracked SSH eddies (Chelton)
- Implies an important role for coherent SSH “features” in eddy heat transport

Lagrangian Dispersion



Relative dispersion

$$K_{rel} = \frac{1}{2}\frac{\partial}{\partial t}\overline{(Y - \overline{Y})^2}$$