Statistical Simulation of Climate Time Series for Hypothesis Testing

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Introduction

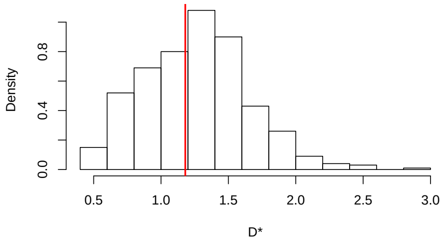
Main idea: decompose a time series into a signal series and a noise series, then resample the noise.

For instance: comparing two time series, a “benchmark", **Z**, and a “target", **X**,

- decompose each into a signal series and a noise series
- compute D, the distance between the benchmark and target signals’ series
- transform D into a probability...

Introduction

- consider D to be the value of a test statistic for testing H_0 : the benchmark and the target share the same signal
- the p-value of this test is a measure of the agreement of the data with the hypothesis, H_0 (Wasserstein and Lazar, 2016)

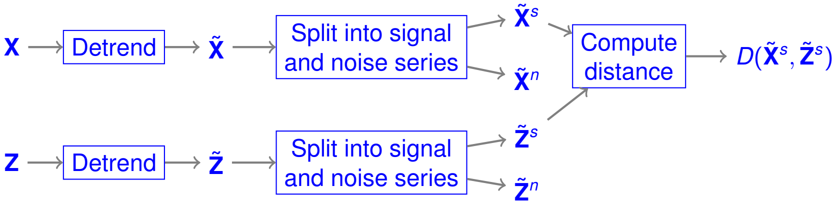


Introduction

- *This p-value is the probability of getting a value of D at least as extreme as the one obtained from the benchmark and the target, if they really did share the same signal.*

- The main technical challenge is obtaining the samplling distribution of D under H_0.

Statistical model and test statistic

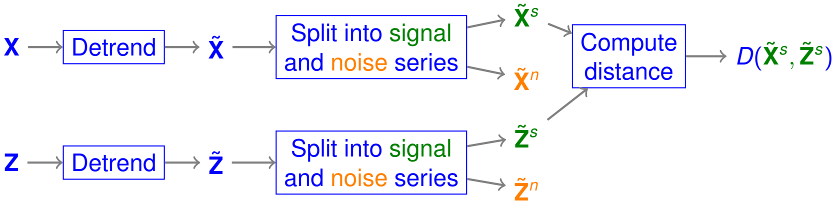


$$\mathbf{X} = (X(1), X(2), \ldots, X(T))', \quad X(t) = \hat{\alpha}_X + \hat{\beta}_X t + \tilde{X}(t), \quad t = 1, \ldots, T.$$

$$\mathbf{Z} = (Z(1), Z(2), \ldots, Z(T))', \quad Z(t) = \hat{\alpha}_Z + \hat{\beta}_Z t + \tilde{Z}(t), \quad t = 1, \ldots, T.$$

Convenient assumption: T is an exact power of 2; $T = 2^J$.

Statistical model and test statistic

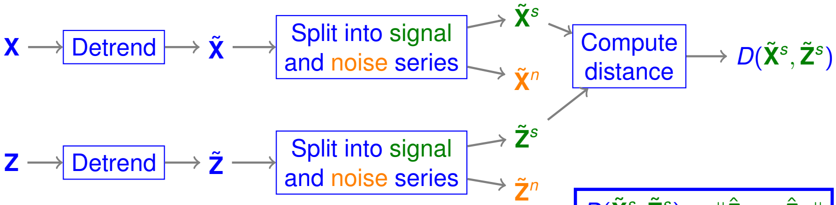


Wavelet decomposition:

$$\tilde{X}(t) = \tilde{X}^s(t) + \tilde{X}^n(t) = \sum_{j=0}^{J_0-1} \sum_{k=1}^{2^j-1} \hat{\gamma}_{Xjk} W_{jk}(t/T) + \sum_{j=J_0}^{J-1} \sum_{k=1}^{2^j-1} \hat{\gamma}_{Xjk} W_{jk}(t/T),$$

$$\tilde{Z}(t) = \tilde{Z}^s(t) + \tilde{Z}^n(t) = \sum_{j=0}^{J_0-1} \sum_{k=1}^{2^j-1} \hat{\gamma}_{Zjk} W_{jk}(t/T) + \sum_{j=J_0}^{J-1} \sum_{k=1}^{2^j-1} \hat{\gamma}_{Zjk} W_{jk}(t/T),$$

Statistical model and test statistic



$$D(\tilde{\mathbf{X}}^s, \tilde{\mathbf{Z}}^s) = \|\hat{\boldsymbol{\Gamma}}_X - \hat{\boldsymbol{\Gamma}}_Z\|$$

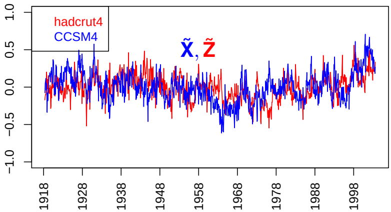
Wavelet decomposition:

$$\tilde{X}^s(t) = \sum_{j=0}^{J_0-1} \sum_{k=1}^{2^j-1} \hat{\gamma}_{Xjk} W_{jk}(t/T) \longrightarrow \hat{\boldsymbol{\Gamma}}_X = \left(\hat{\gamma}_{X00}, \ldots, \hat{\gamma}_{X(J_0-1)(2^{(J_0-1)}-1)}\right)',$$

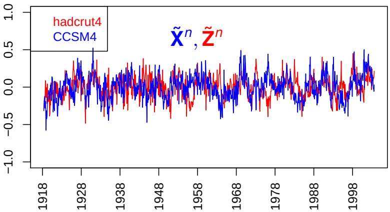
$$\tilde{Z}^s(t) = \sum_{j=0}^{J_0-1} \sum_{k=1}^{2^j-1} \hat{\gamma}_{Zjk} W_{jk}(t/T) \longrightarrow \hat{\boldsymbol{\Gamma}}_Z = \left(\hat{\gamma}_{Z00}, \ldots, \hat{\gamma}_{Z(J_0-1)(2^{(J_0-1)}-1)}\right)'.$$

Sampling distribution under H_0

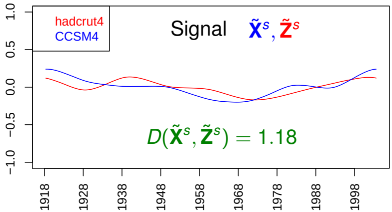
Detrended global surface temperature anomaly



Noise



Is 1.18 large or small?



Sampling distribution under H_0

$\{\mathbf{Z}_b^*, \mathbf{X}_b^*\}_{b=1}^B$

$\mathbf{Z}_1^*$	$\mathbf{X}_1^*$
$\mathbf{Z}_2^*$	$\mathbf{X}_2^*$
$\vdots$	$\vdots$
$\mathbf{Z}_B^*$	$\mathbf{X}_B^*$

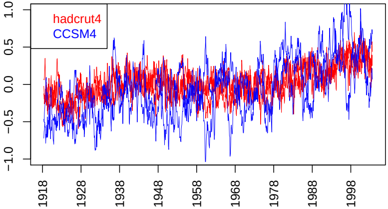
→ Detrend, split, compute distance →

$\{D_b^*\}_{b=1}^B$

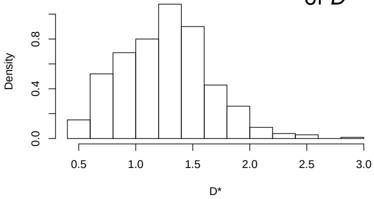
D_1^*
D_2^*
$\vdots$
D_B^*

Alternative plausible realizations of **Z** *and* **X** *give rise to sampling distribution of* D^*

Linked ensemble members



Sampling Distribution of D^*



Sampling distribution under H_0

$\mathbf{X} \rightarrow$ Detrend $\rightarrow \tilde{\mathbf{X}} \rightarrow$ Split into signal and noise series $\rightarrow \tilde{\mathbf{X}}^s$, $\tilde{\mathbf{X}}^n$

Detrend $\rightarrow (\hat{\alpha}_X, \hat{\beta}_X)$

$\tilde{\mathbf{X}}^n \rightarrow$ Fit time series model $\rightarrow$ ARIMA(p_X, d_X, q_X) $\rightarrow$ (bth draw) $\rightarrow \tilde{\mathbf{X}}_b^{n*}$

$\mathbf{Z} \rightarrow$ Detrend $\rightarrow \tilde{\mathbf{Z}} \rightarrow$ Split into signal and noise series $\rightarrow \tilde{\mathbf{Z}}^s$, $\tilde{\mathbf{Z}}^n$

Detrend $\rightarrow (\hat{\alpha}_Z, \hat{\beta}_Z)$

$\tilde{\mathbf{Z}}^n \rightarrow$ Fit time series model $\rightarrow$ ARIMA(p_Z, d_Z, q_Z) $\rightarrow$ (bth draw) $\rightarrow \tilde{\mathbf{Z}}_b^{n*}$

bth linked ensemble member:

$$\mathbf{Z}_b^* = \hat{\alpha}_Z \mathbf{1} + \hat{\beta}_Z \mathbf{t} + \tilde{\mathbf{Z}}^s + \tilde{\mathbf{Z}}_b^{n*},$$

$$\mathbf{X}_b^* = \hat{\alpha}_X \mathbf{1} + \hat{\beta}_X \mathbf{t} + \tilde{\mathbf{Z}}^s + \tilde{\mathbf{X}}_b^{n*}.$$

$\mathbf{1}$ = length T vector of one's; $\mathbf{t} = (1, 2, \ldots, T)'$.

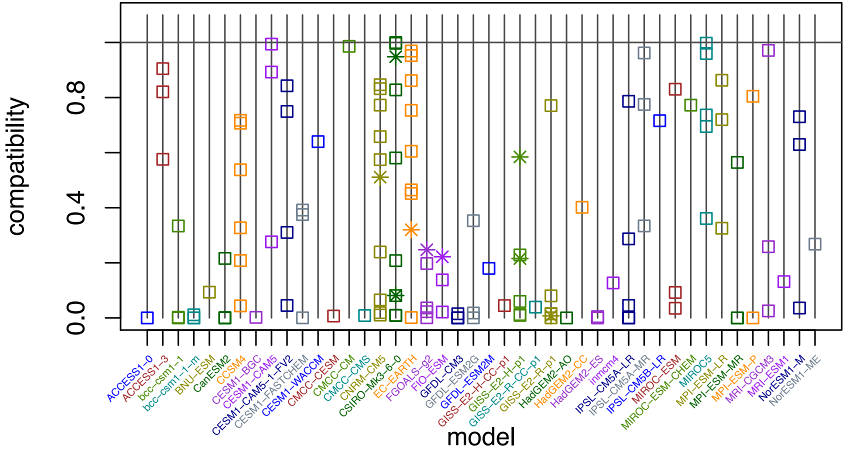
Application to CMIP5 historical runs

- Observational benchmark: HadCRUT4 (Morice et al., 2012) monthly global average surface temperature anomaly, May 1918 through August 2003 (1024 months).
- Model targets: 139 model runs using 44 different models from the CMIP5 historical experiment, May 1918 through August 2003.
- All time series computed relative to their 1961–1990 averages.
- “Signal" defined as the coarsest three wavelet decomposition levels ($J_0 = 3$), which corresponds to approximately ten-year cycles.
- Resampling performed with $B = 5000$ trials.

Application to CMIP5 historical runs

Model	*Center*	*Members*	*Model*	*Center*	*Members*
ACCESS1-0	CSIRO-BOM (Australia)	1	GFDL-ESM2M	GFDL (USA)	1
ACCESS1-3	CSIRO-BOM (Australia)	3	GISS-E2-H p1	NASA GISS (USA)	1
BCC-CSM-1	Beijing Climate Center (PRC)	3	GISS-E2-H-CC p1	NASA GISS (USA)	6
BCC-CSM-1-M	Beijing Climate Center (PRC)	3	GISS-E2-R p1	NASA GISS (USA)	1
BNU-ESM	Beijing Normal Univ. (PRC)	1	GISS-E2-R-CC p1	NASA GISS (USA)	6
CanSM2	CCCMA (Canada)	5	HadGEM2-AO	NIMR/KMA (UK/Korea)	1
CCSM4	NCAR (USA)	6	HadGEM2-CC	MOHC/INPE (UK/Brazil)	1
CESM1-BGC	NCAR/DOE/NSF (USA)	1	HadGEM2-ES	MOHC/INPE (UK/Brazil)	4
CESM1-CAM5	NCAR/DOE/NSF (USA)	3	INMCM4	INM (Russia)	1
CESM1-CAM5-1-FV2	NCAR/DOE/NSF (USA)	4	IPSL-CM5A-LR	IPSL (France)	6
CESM1-FASTCHEM	NCAR/DOE/NSF (USA)	3	IPSL-CM5A-MR	IPSL (France)	3
CESM1-WACCM	NCAR/DOE/NSF (USA)	1	IPSL-CM5B-LR	IPSL (France)	1
CMCC-CESM	CMCC (Italy)	1	MIROC-ESM	MIROC (Japan)	3
CMCC-CM	CMCC (Italy)	1	MIROC-ESM-CHEM	MIROC (Japan)	1
CMCC-CMS	CMCC (Italy)	1	MIROC5	MIROC (Japan)	5
CNRM-CM5	CNRM (France)	10	MPI-ESM-LR	MPI (Germany)	3
CSIRO-Mk3-6-0	CSIRO (Australia)	10	MPI-ESM-MR	MPI (Germany)	3
EC-EARTH	EC-EARTH Consortium (Europe)	9	MPI-ESM-P	MPI (Germany)	2
FGOALS-g2	LASG (PRC)	5	MRI-CGM3	MRI (Japan)	3
FIO-ESM	FIO (PRC)	3	MRI-ESM1	MRI (Japan)	1
GFDL-CM3	GFDL (USA)	5	NorESM1-M	NCC (Norway)	3
GFDL-ESM2G	GFDL (USA)	3	NorESM1-ME	NCC (Norway)	1

Results



∗ indicates runs for which ARIMA assumptions failed.

Discussion

- Nearly all models' outputs with more than one ensemble member show considerable variation.
- This is a property of their time series, particularly their noise series.
- We exploit their structures to mimic "internal variability" through resampling.
- Is this the right definition of internal variability?
- See Braverman, A., Chatterjee, S., Heyman, M., and Cressie, N.: Probabilistic evaluation of competing climate models, Adv. Stat. Clim. Meteorol. Oceanogr., 3, 93-105, https://doi.org/10.5194/ascmo-3-93-2017, 2017.

Question or comments? Contact Amy.Braverman@jpl.nasa.gov.