Energetically consistent, resolution aware, parameterization of ocean mesoscale eddies

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The Problem

- Model resolutions between ~ 1-1/8th degree occupy awkward “grey zone” where mesoscale “eddies” are (barely) permitted but not properly resolved.

- Challenging to leverage this resolution regime, arguably because existing parameterizations are overly dissipative for eddy permitting simulations.

- Current “state of the art” is to treat models as either “eddy-ing” or “non-eddy-ing” in a binary manner (potentially switching within domain - e.g. Hallberg 2013).

- Noting that there exists a continuum of (transient and standing) eddy scales, can we do better than that?
Energetics in Ocean models

Resolved PE

- GM
- diapycnal mixing
- Wind-driven overturning

Resolved KE

- Resolved geostrophic "eddies"
- Viscosity
- Bottom friction
- Wind stress
Energetics in Ocean models
“Backscatter” (e.g. Jansen et al. 2015)

Resolved PE

Wind-driven overturning

Resolved KE

Resolved geostrophic “eddies”

Unresolved KE

Diapycnal mixing

Wind stress

Bottom friction

Viscosity

Backscatter

Sub-grid dissipation
Energetics in Ocean models
“Energy budget-based GM” (e.g. Eden and Greatbatch 2008)
Energetics in Ocean models
Putting it all together

Resolved PE

Resolved KE

Resolved geostrophic “eddies”

Wind-driven overturning

Wind stress

GM

Unresolved KE

Unresolved KE informs GM

Diapycnal mixing

Bottom friction

Viscosity

Backscattering

Sub-grid dissipation
Energetics in Ocean models
Putting it all together

Resolved PE

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Wind-driven overturning

Resolved geostrophic "eddies"

GM

Unresolved KE informs GM

Unresolved KE

Bottom friction

Viscosity

Backscatter

Sub-grid dissipation

see also: Bachman 2019, Ocean modelling
Energetics in Ocean models
Equations

\[
\partial_t e = \dot{e}_{GM} + \dot{e}_{\nu_4} - \dot{e}_{\nu_2} - \dot{e}_{\text{diss}} + \frac{1}{H} \nabla \cdot (HF_e)
\]
Energetics in Ocean models

Equations

\[ \dot{e} = \dot{e}_{GM} + \dot{e}_{\nu_4} - \dot{e}_{\nu_2} - \dot{e}_{diss} + \frac{1}{H} \nabla \cdot (H \mathbf{F}_e) \]

\[ \nu_4 = (c_{smag} |D| + \tau_{\nu_4}^{-1}) \Delta^4 \]
Energetics in Ocean models

Equations

\[
\partial_t e = \dot{e}_{GM} + \dot{e}_\nu - \dot{e}_{\nu_2} - \dot{e}_{diss} + \frac{1}{H} \nabla \cdot (HF_e)
\]

\[
\nu_4 = (c_{smag} |D| + \tau_{\nu_4}^{-1}) \Delta^4
\]

\[
K_{GM} = -\nu_2 = \alpha \sqrt{2e} L_{mix} R(\Delta k_d)
\]

\[
L_{mix} = \min(\Delta, L_{\beta*})
\]

\[
R(\Delta k_d) = \left[ 1 + \left( \frac{\Delta k_d}{\pi} \right)^{-2} \right]^{-1}
\]
Energetics in Ocean models

Equations

\[
\begin{align*}
\partial_t e &= \dot{e}_{GM} + \dot{e}_{\nu_4} - \dot{e}_{\nu_2} - \dot{e}_{\text{diss}} + \frac{1}{H} \nabla \cdot (H \mathbf{F}_e) \\
\dot{e}_{\text{diss}} &= -c_d \sqrt{(e + u_{bot}^2 + U_{bg}^2)} \, e
\end{align*}
\]

\[
\nu_4 = (c_{\text{smag}} |D| + \tau_{\nu_4}^{-1}) \Delta_4
\]

\[
K_{GM} = -\nu_2 = \alpha \sqrt{2e} \, L_{\text{mix}} \, R(\Delta k_d)
\]

\[
L_{\text{mix}} = \min(\Delta, L_{\beta^*})
\]

\[
R(\Delta k_d) = \left[ 1 + \left( \frac{\Delta k_d}{\pi} \right)^{-2} \right]^{-1}
\]
Neverland

- MOM6
- 6 isopycnal layers
- fully adiabatic
Neverland
A high resolution reference solution
Testing parameterizations

• We will here consider three different parameterizations:

  - MEKE GM + Backscatter:
    \[ K_{GM} = -\nu_2 = \alpha \sqrt{2e} L_{mix} R(\Delta k_d) \quad \text{with } \alpha = 0.16 \]

  - Constant GM:
    \[ K_{GM} = \text{const.} = 1000 \text{ m}^2\text{s}^{-1} \]

  - A “resolution aware” version of Visbeck (1997):
    \[ K_{GM} = \gamma sN\Delta^2 \quad \text{with } \gamma = 1.1 \]

• In all cases one parameter is tuned to match mean-state APE from high-res simulation at 1/2° resolution!
Results

Constant GM at 1/2 degree resolution

![Image of a contour plot showing m^2/s values over a map with latitude and longitude axes.]
Results
“Visbeck” at 1/2 degree resolution
Results
Energy-budget based GM + backscatter at 1/2 degree resolution
Resolved and sub-grid KE as a function of resolution
Mean flow as a function of resolution

1) Metrics

Three metrics are here considered to quantitatively test mean state representation:

1) Mean-state APE: 
\[ APE = \sum_n \int \int g_n' \bar{\eta}_n^2 dxdy \]  
(used to “tune” parameterizations)

2) Weighted interface height error: 
\[ e^2 = \frac{1}{A} \sum_n \int \int g_n' (\bar{\eta}_n - \bar{\eta}_{n-1/16})^2 dxdy \]

3) ACC transport: 
\[ T_{ACC} = \sum_n \int (\bar{v}h)_n dx \]
Mean flow as a function of resolution

2) Results

Used for tuning
Conclusions

• Energy budget based mesoscale eddy closures can be useful if we account for energy “backscatter” to the resolved flow.

• Including backscatter can lead to significant improvements, particularly in barely eddy permitting “grey zone”:
  a) More energetic flow with improved internal variability
  b) Smooth connection between the non-eddying and eddy-resolving regimes, with
  c) Mean flow properties as good or better than GM (at relatively coarse res) or no GM (at higher res)

• Important questions (and room for improvement) remain, including:
  a) Vertical structure of eddy energy and fluxes
  b) Transport of (sub-grid) eddy energy
  c) Sub-grid dissipation?!
Variance spectra
Kinetic energy spectra

The graph shows the kinetic energy spectra as a function of wave number ($k$) for different cases. The y-axis represents the energy density ($E(k)$) in units of $m^2/s^2$, and the x-axis represents the wave number in units of $k/2\pi$.

Different cases are indicated by distinct line styles and colors:
- **1/16°**: Black line
- **1/2° MEKE GM + BS**: Blue line
- **1/2° $K_{GM}=1000$**: Red line
- **1/2° $K_{GM}=\alpha sN \Delta^2$**: Yellow line

The energy spectra are compared with the theoretical $k^{-3}$ behavior, which is indicated by the dashed line in the graph.
Mean Flow
SSH

1/16° $\nu_4$ only

1/2° $\nu_4$ only

1/2° MEKE GM + BS

1/2° $K_{GM} = 1000$

1/2° MEKE GM

1/2° $K_{GM} = \alpha sN \Delta^2$
Mean Flow
Isopycnal Depth (1027 kg/m³)
Mean Flow
Isopycnal Depth (1027.5 kg/m$^3$)
Energy Budgets