Exploring the dynamical connections between GM and Redi Mixing Coefficients: Diagnosing Transport Tensor in Zonally Inhomogeneous Flows

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A Simple Recipe for Turbulent Transport

Tracer Equation $$\partial_t c + \nabla.(\mathbf{v}c) = \nabla.(\kappa_m \nabla c)$$

Reynold's Averaging

Tracer Equation For Mean/Coarse Model Variable $$\partial_t \overline{C} + \overline{\mathbf{V}}.\nabla\overline{C} = \nabla.(\kappa_m \nabla\overline{C}) - \nabla.\mathbf{F^c}$$

Mixing length + Scale Separation/ Small Amplitude+ Locality

$$\begin{aligned}
\mathbf{F^c} = \overline{\mathbf{v}'c'} &= -(\overline{\mathbf{v}'\mathbf{l}'})\nabla\overline{C} - (\overline{\mathbf{v}'\mathbf{l}'\mathbf{l}'})\nabla\nabla\overline{C} + O(l^4) \\
&\approx -(\overline{\mathbf{v}'\mathbf{l}'})\nabla\overline{C} \qquad \boxed{(l = l'/L_C << 1)} \\
&= -\begin{bmatrix} \overline{u'l'^x} & \overline{u'l'^y} & \overline{u'l'^z} \\ \overline{v'l'^x} & \overline{v'l'^y} & \overline{v'l'^z} \\ \overline{w'l'^x} & \overline{w'l'^y} & \overline{w'l'^z} \end{bmatrix} \begin{bmatrix} \partial_x\overline{C} \\ \partial_y\overline{C} \\ \partial_z\overline{C} \end{bmatrix} \\
&= -\mathbf{K}\nabla\overline{C}
\end{aligned}$$

(Taylor 1915 … Plumb and Mahlmann 1987… Bachmann et al 2015)

Ocean Meso-scale Parameterizations

Eddy Induced Advection

$$\mathbf{u}^* = -\partial_z(\kappa_{GM}\mathbf{s})$$
$$w^* = \nabla_H.(\kappa_{GM}\mathbf{s})$$

- Adiabatic isopycnal slumping.
- Sink of APE, models the conversion of APE to KE.

Gent and McWilliams 1990,
Gent et al 1995

$$\mathbf{K}^{\mathbf{GM}} = \begin{bmatrix} 0 & 0 & -\kappa_{GM}s^x \\ 0 & 0 & -\kappa_{GM}s^y \\ \kappa_{GM}s^x & \kappa_{GM}s^y & 0 \end{bmatrix}$$

Isopycnal Stirring

$$\tilde{\mathbf{F^c}}_{Redi} = -\kappa_{Redi}\tilde{\nabla}_{iso}\overline{C}$$

- Along isopycnal diffusion of tracers.
- Sink of variance - models the stirring enhanced variance dissipation in the tracer variance equation.

Solomon 1971,
Redi 1982

$$\mathbf{K}^{\mathbf{Redi}} = \kappa_{Redi}\begin{bmatrix} 1 & 0 & s^x \\ 0 & 1 & s^y \\ s^x & s^y & |\mathbf{s}|^2 \end{bmatrix}$$

$$\mathbf{K}^{\mathbf{OGCM}} = \begin{bmatrix} \kappa_{Redi} & 0 & (\kappa_{Redi} - \kappa_{GM})s^x \\ 0 & \kappa_{Redi} & (\kappa_{Redi} - \kappa_{GM})s^y \\ (\kappa_{Redi} + \kappa_{GM})s^x & (\kappa_{Redi} + \kappa_{GM})s^y & \kappa_{Redi}|\mathbf{s}|^2 \end{bmatrix}$$

Griffies 2004

Linking GM and Redi

For QG PV: $$\overline{\mathbf{u}q} = \overline{\mathbf{u}\zeta} + \partial_z \left(\frac{f\overline{\mathbf{u}b}}{N^2} \right) + \nabla \times \left(\frac{\overline{b^2}}{2N^2} \hat{z} \right)$$

$$\overline{vq} \approx \partial_z \left(\frac{f\overline{vb}}{N^2} \right)$$

In **Zonally Homogeneous** Simulations:

$$\kappa_q \partial_z \boldsymbol{s} \approx \partial_z \left(\kappa_{\mathrm{gm}} \boldsymbol{s} \right)$$

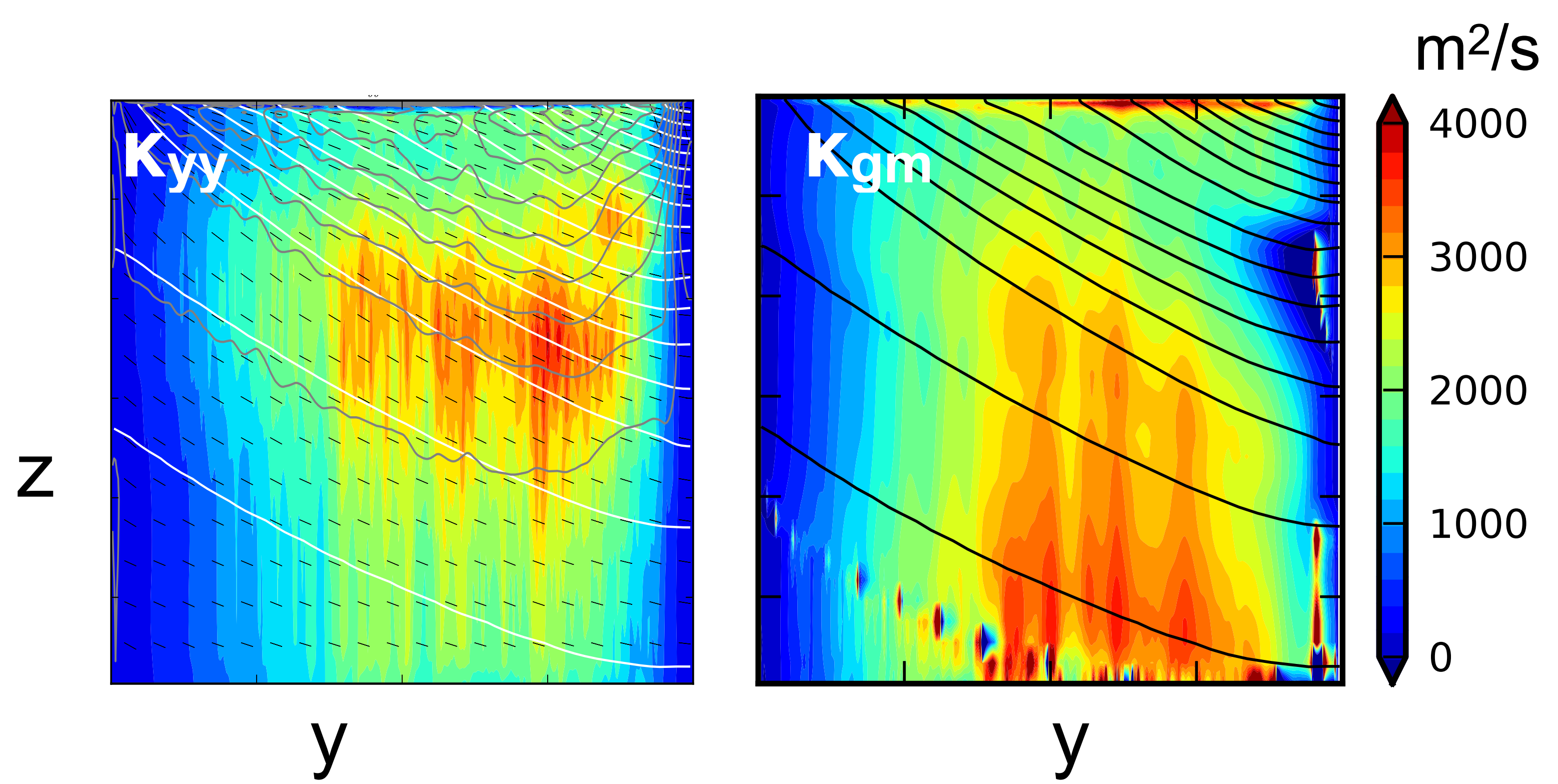
Treguier 1999, …
Smith & Marshall 2009,
Abernathey et al 2013

Empirically:

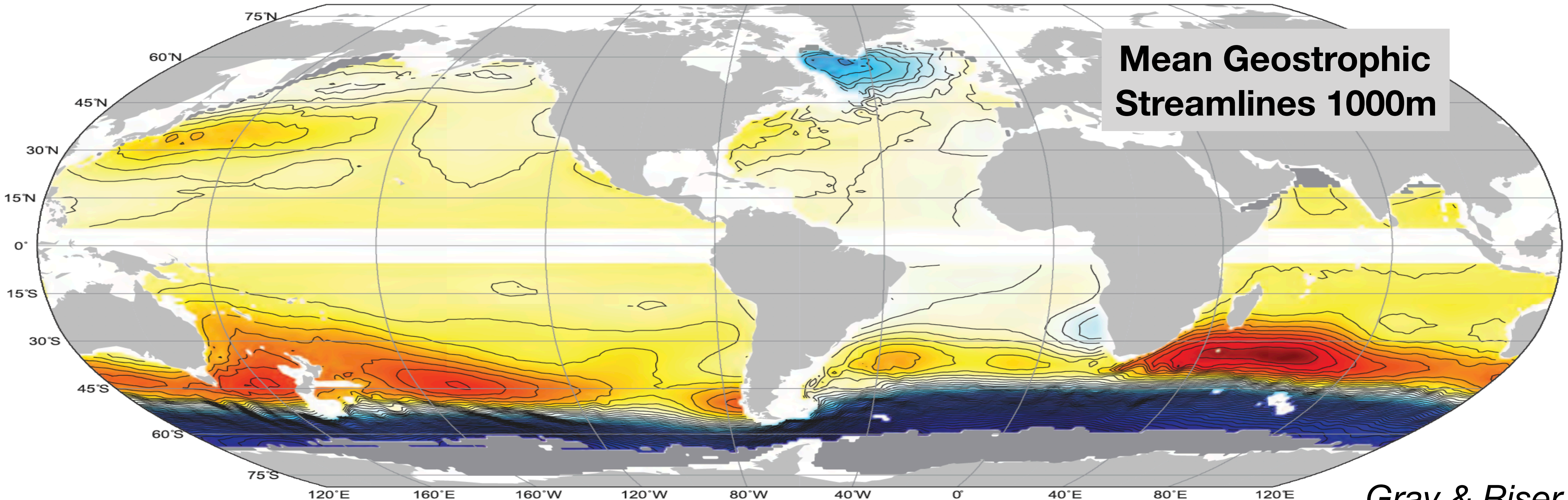
$K_y \sim K_q$,

K_y != K_{gm}

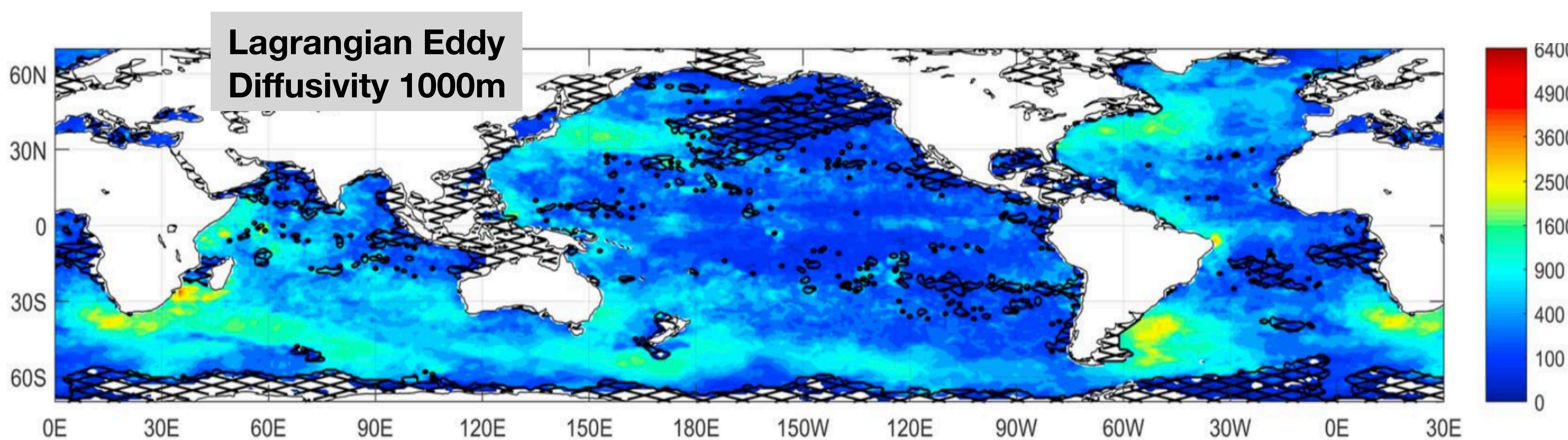
- Thus, a recipe for setting consistent parameters is available under the conditions of a zonally symmetric channel.



Non-Zonal & Inhomogeneous

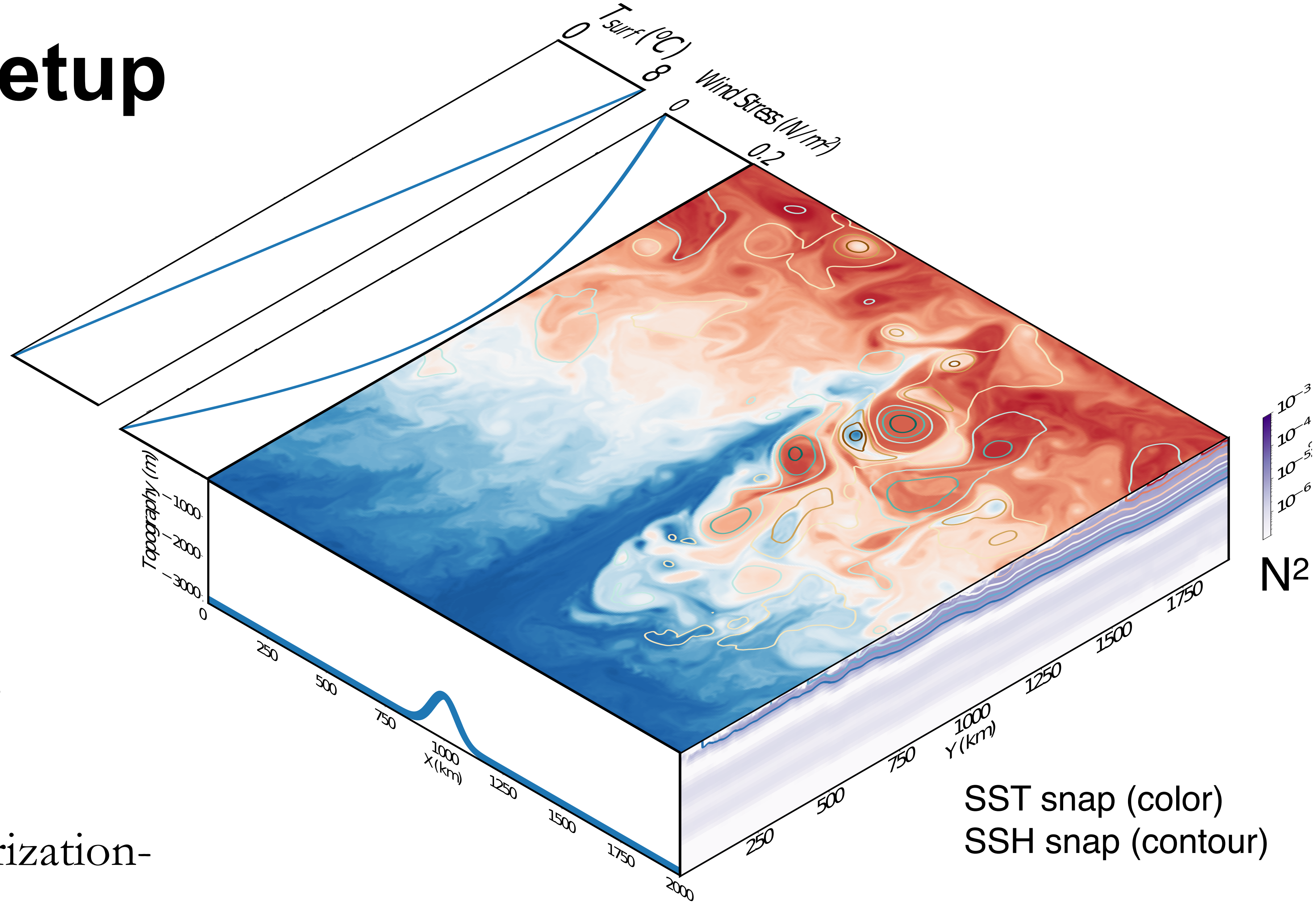


Gray & Riser 2014



Roach, Balwada & Speer 2018

Experimental Setup

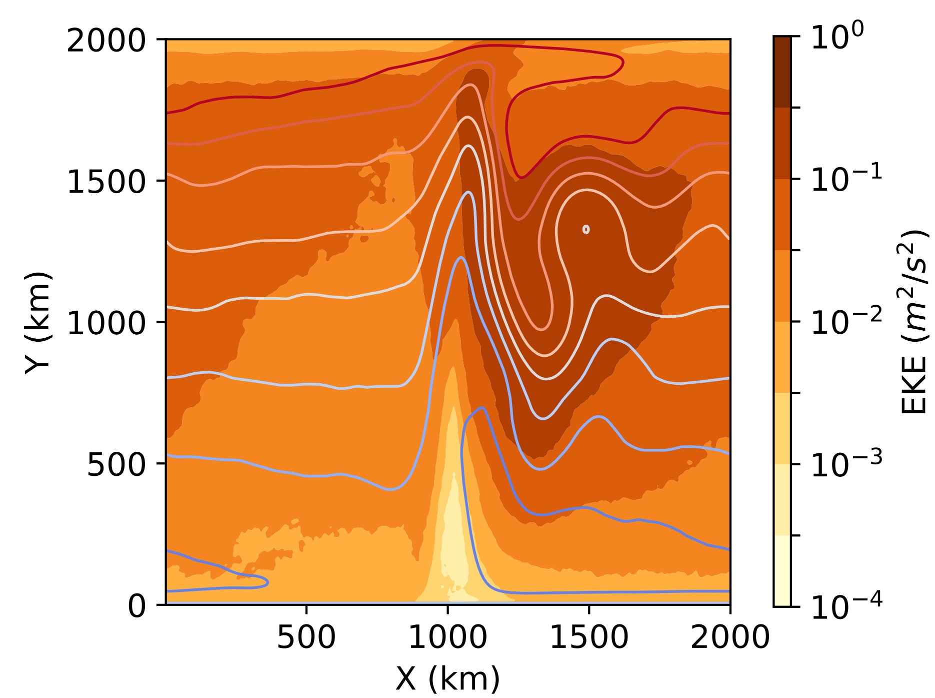


Physical Model

- MITgcm
- 2000²km x 3km
- Resolution: 5km x 40 levs
- Center: 50°S
- Channel with:
  - (A) No-slip sides
  - (B) Sponge at north wall
- SST restored to linear T(y)
- Steady sinusoidal windstress
- No salinity, linear EOS
- Quadratic drag
- LLC4320 subgrid parameterization- Leith dissipation
- 150 year spin-up

Passive Tracers

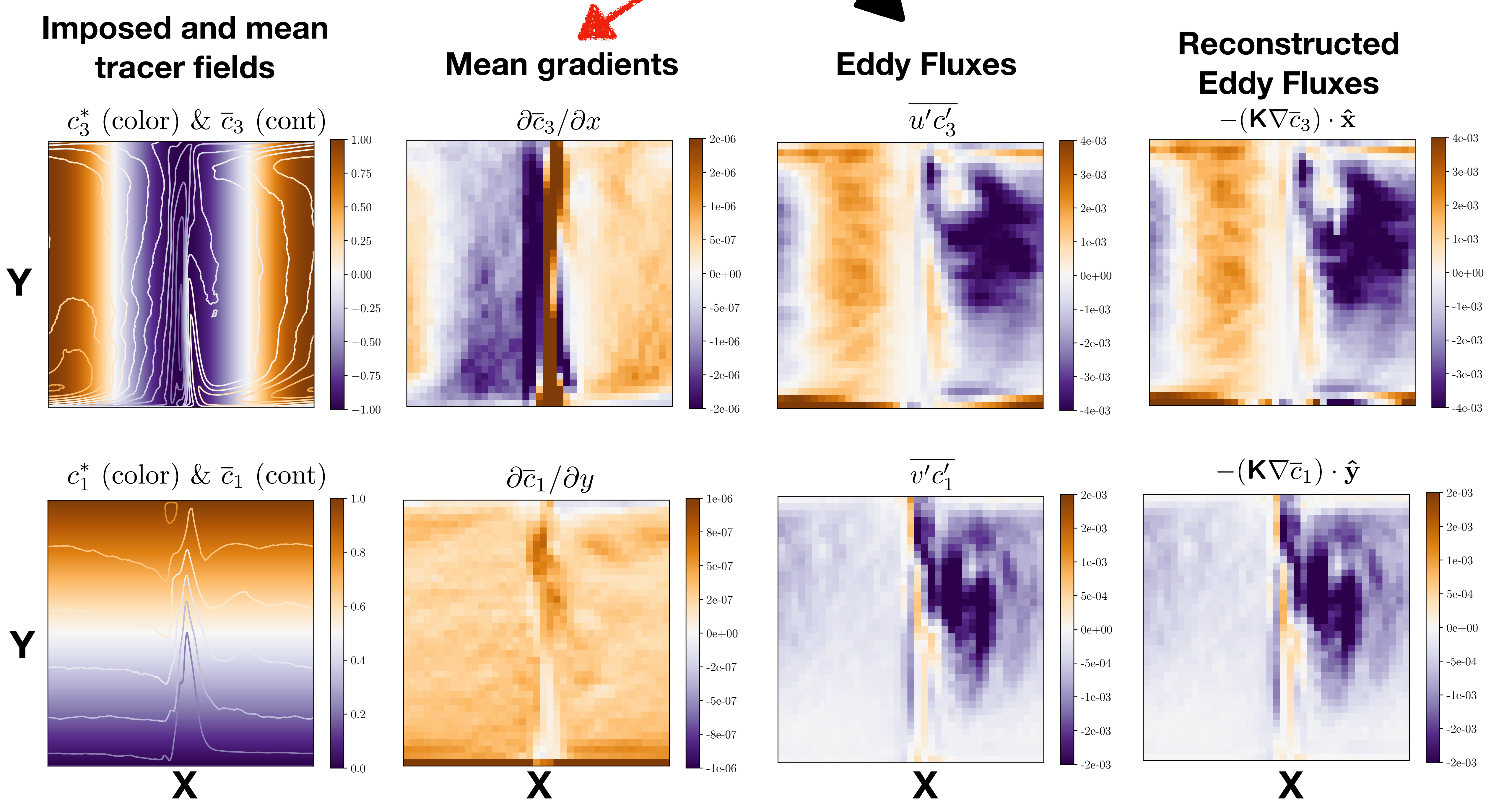
- 10 tracers, each varying in one direction
- 2 restoring time scales - 1& 6 years
- 35 years spin up
- 20 years of post-spinup statistics.



Diagnose Eddy Transport Tensor

$$\mathbf{K} = -\overline{\mathbf{v}'c'}(\nabla C)^{-1}$$

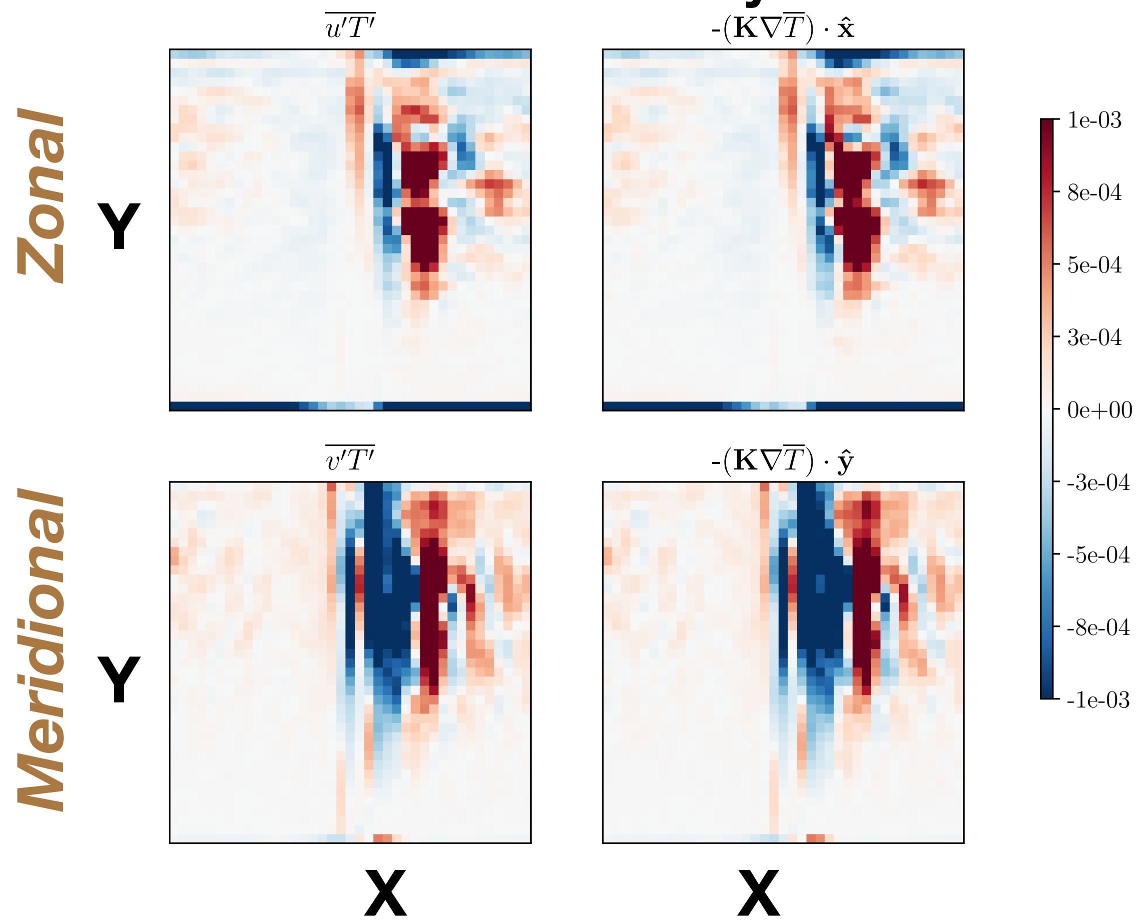
Plumb & Mahlmann 1987, …
Bachman & Fox-Kemper 2013,
Abernathey et al 2013,
Fox-Kemper et al 2013,
Bachman et al 2015



How well does K reconstruct buoyancy/temperature flux?

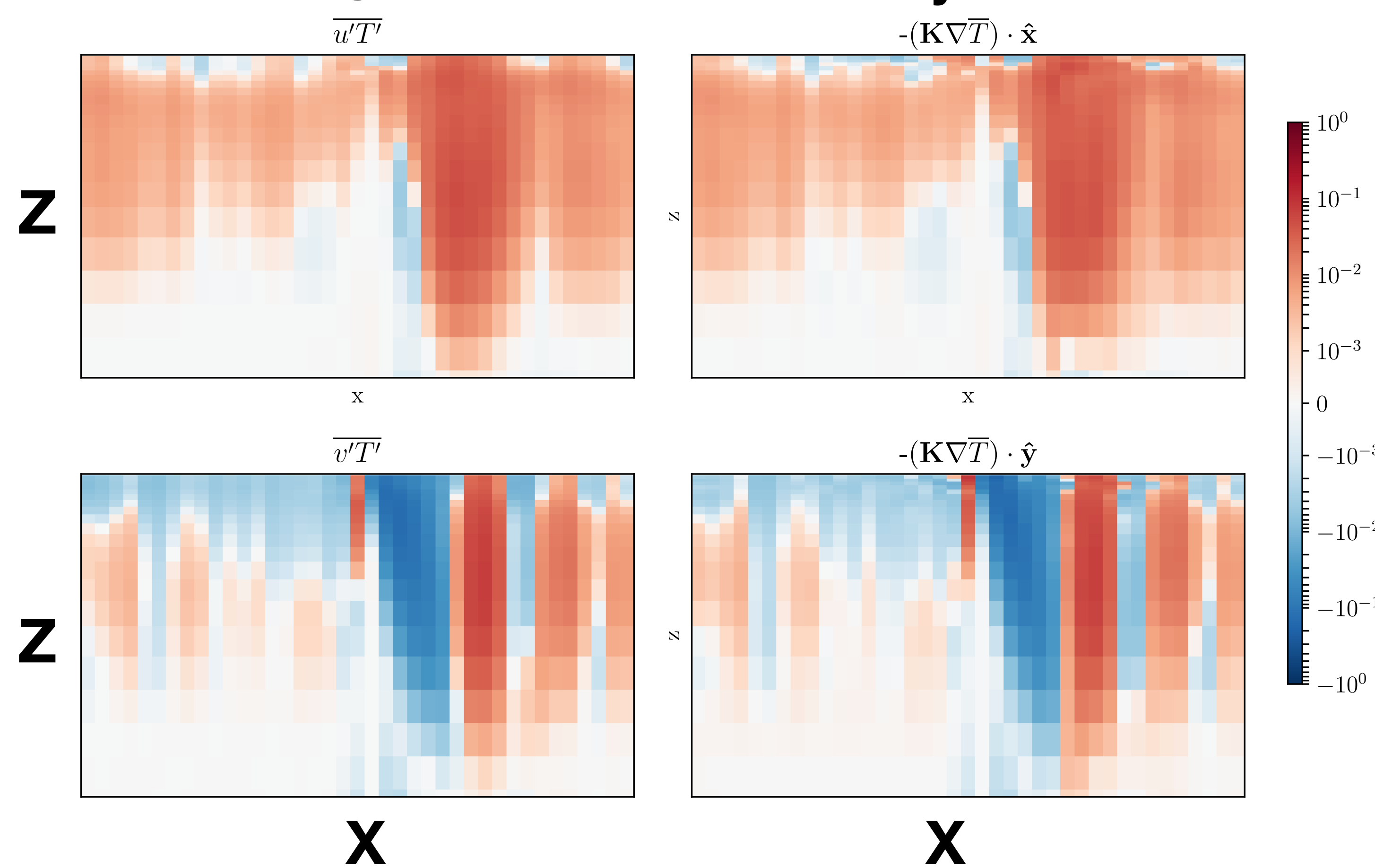
@Z=1500m

Eddy Fluxes | **Reconstructed Eddy Fluxes**

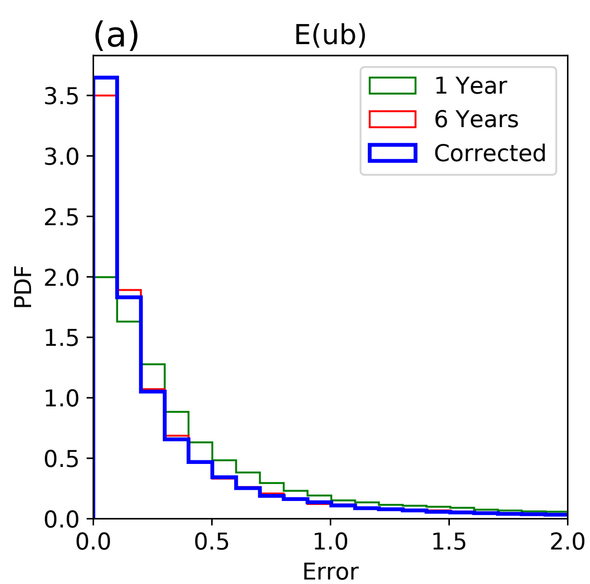


@Y=1000km

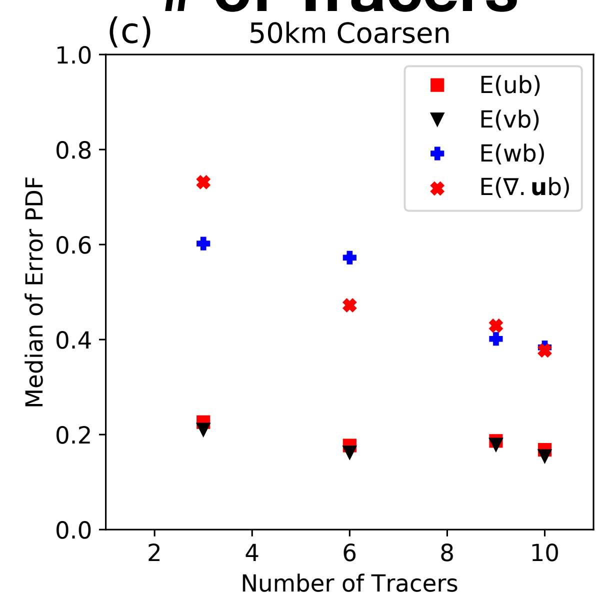
Eddy Fluxes | **Reconstructed Eddy Fluxes**



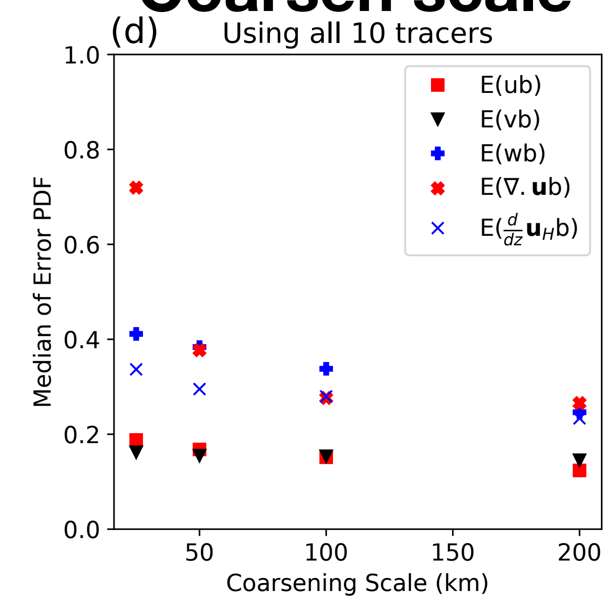
$$E(\overline{ub}) = \frac{|\overline{ub} - (-\mathbf{K}\nabla B)|}{|\overline{ub}|}$$



# of Tracers



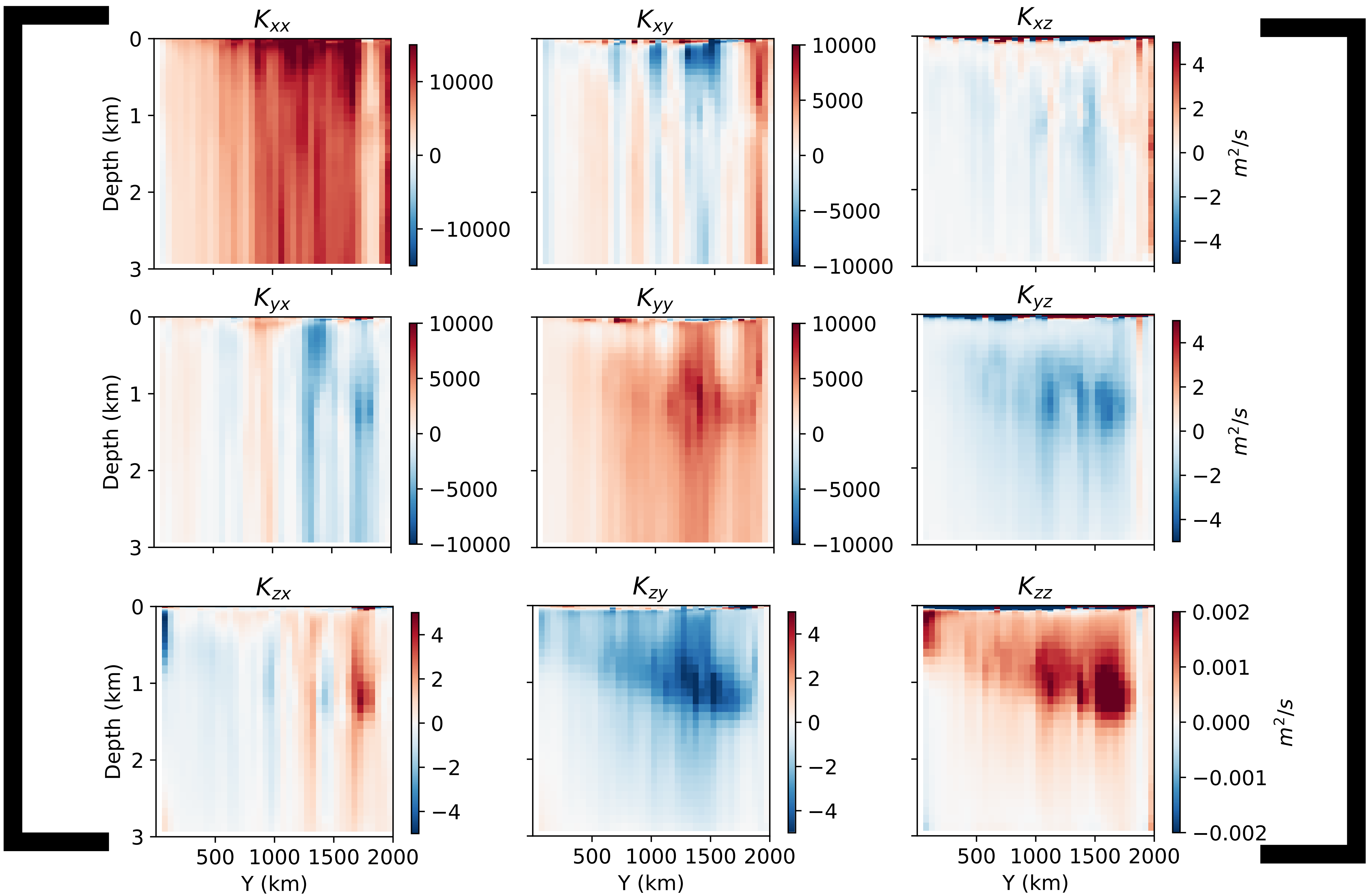
Coarsen scale



Diagnosed Tensor

@X = 0km

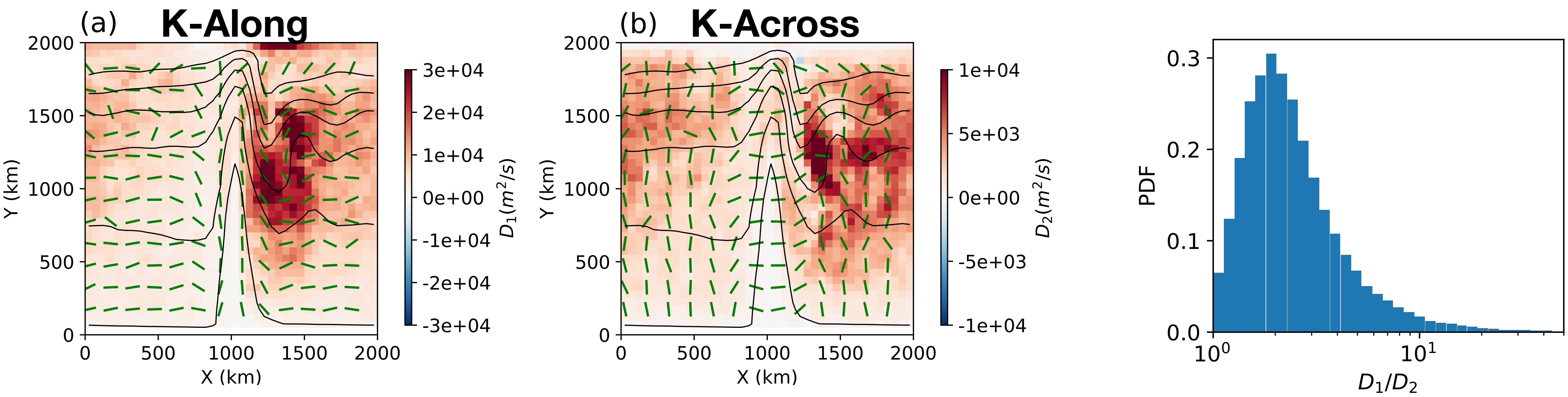
K =



Compare to:

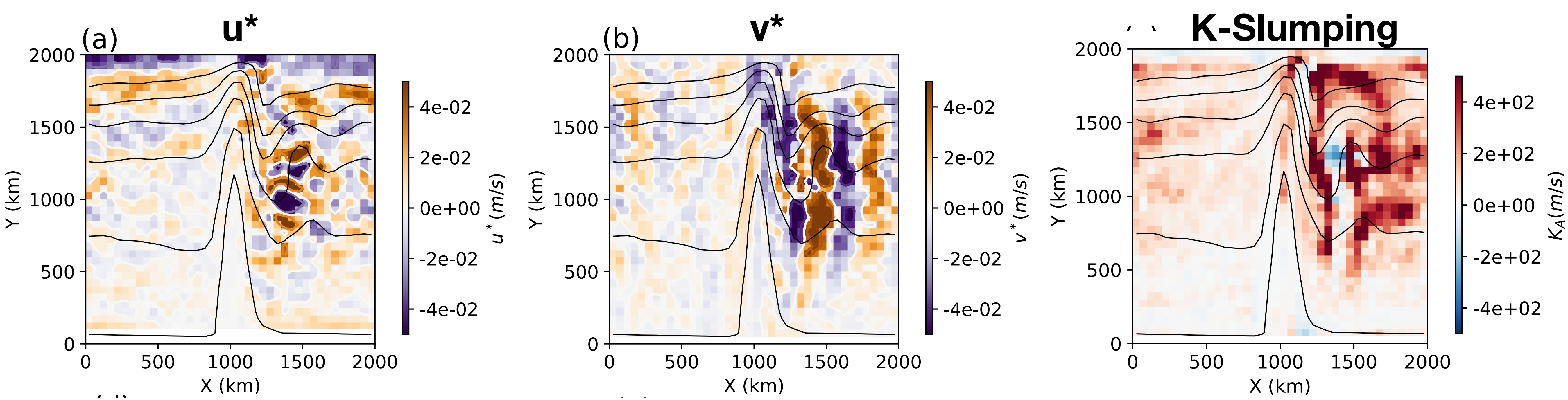
$$\mathbf{K}^{\mathbf{OGCM}} = \begin{bmatrix} \kappa_{Redi} & 0 & (\kappa_{Redi} - \kappa_{GM})s^x \\ 0 & \kappa_{Redi} & (\kappa_{Redi} - \kappa_{GM})s^y \\ (\kappa_{Redi} + \kappa_{GM})s^x & (\kappa_{Redi} + \kappa_{GM})s^y & \kappa_{Redi}|\mathbf{s}|^2 \end{bmatrix}$$

Symmetric Tensor



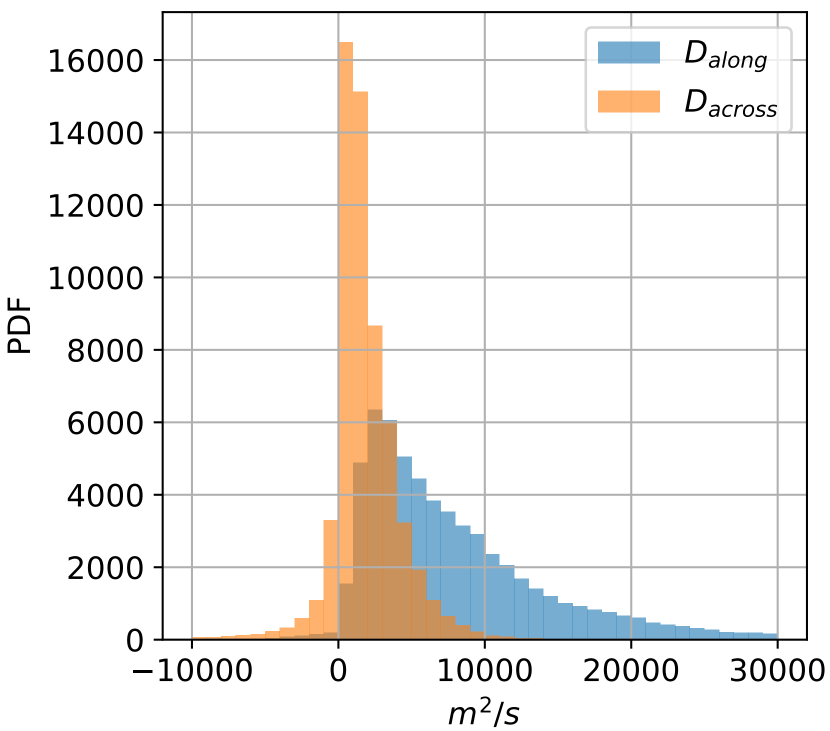
@Z=1500m

AntiSymmetric Tensor



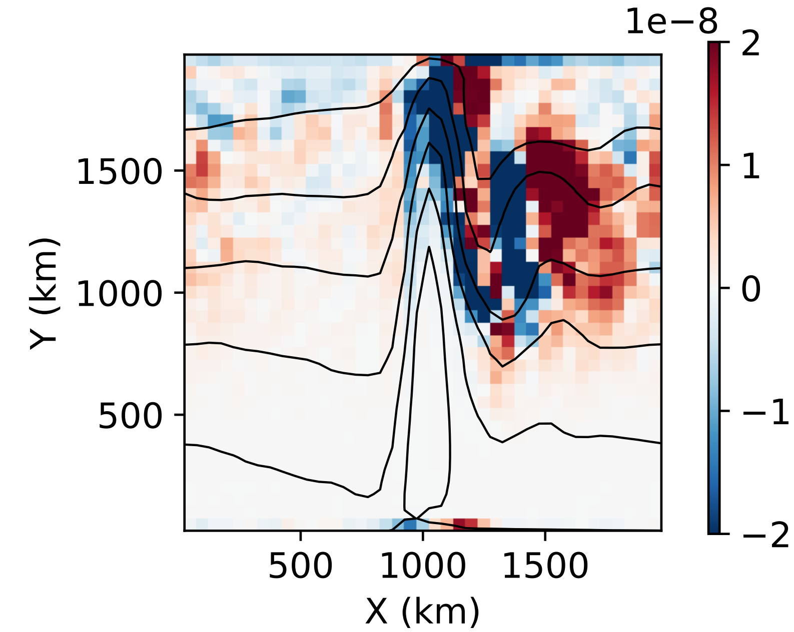
Does the diagnosed tensor have the expected properties?

- Does **S** dissipate variance? Are eigenvalues of S positive?

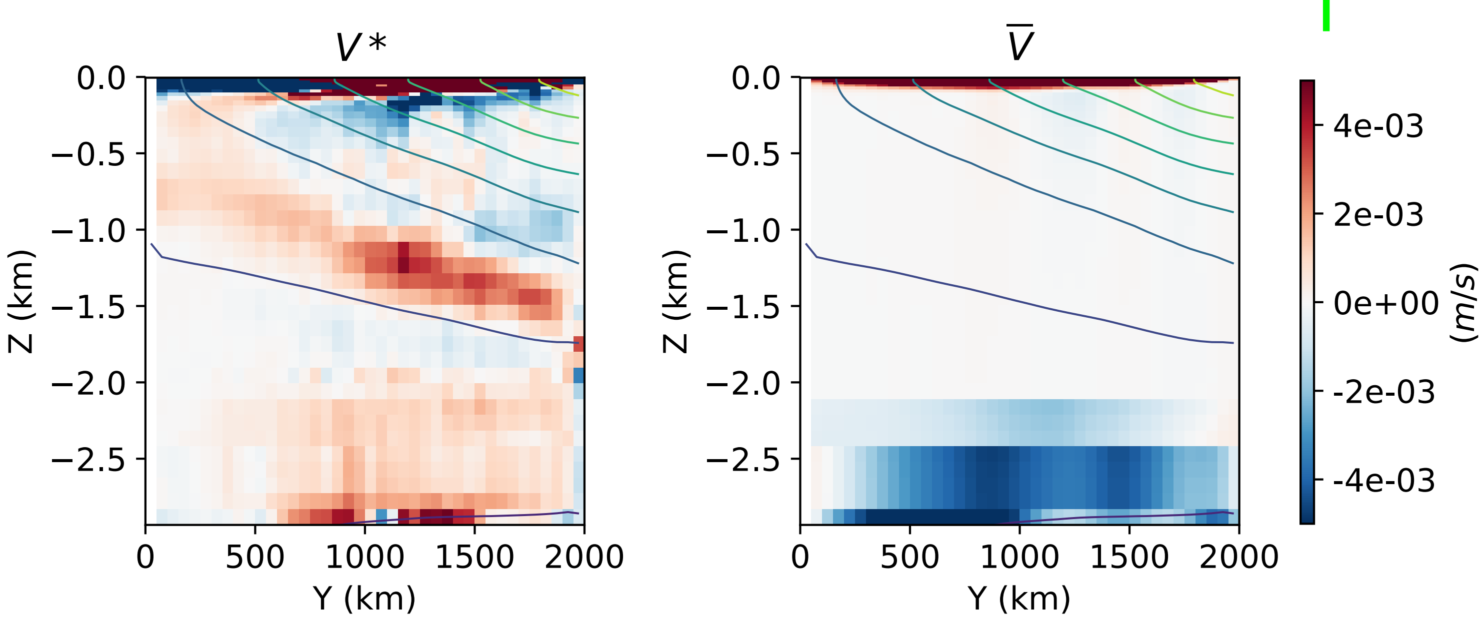


- Is buoyancy transported along mean isopycnals?

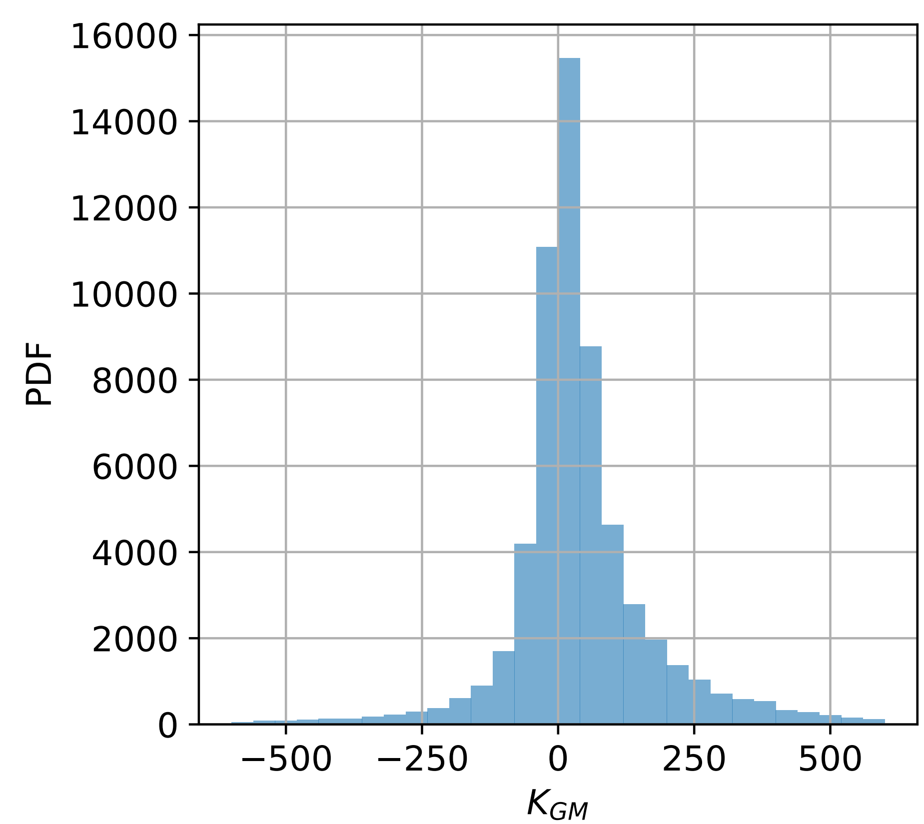
$$\mathbf{F}^b.\nabla B = 0?$$



- Does the zonally averaged eddy driven velocity help in closing overturning circ?



- Does **A** dissipate APE?



Why are mesoscale eddy fluxes seemingly diabatic?

$$\partial_t \overline{b'^2}/2 + \nabla.(\overline{\mathbf{u}}\overline{b'^2}/2 + \overline{\mathbf{u}'b'^2}) = \mathbf{F}^b.\nabla B = -(\mathbf{S}\nabla B).\nabla B$$

For Zonal Steady Flows:

$$(\mathbf{S}\nabla B).\nabla B \approx 0$$

However, for Non-Zonal InHomogeneous Flows:

$$(\mathbf{S}\nabla B).\nabla B \neq 0$$

Hence,

$$\mathbf{K} \approx \mathbf{K}_{GM} + \mathbf{K}_{Redi} + \mathbf{K}_{Var-Transport} + ??$$

Summary

- GM and Redi diffusivity coefficients are not the same, for flows with vertical variations and non-constant interior PV.
- A recipe for setting consistent GM and Redi diffusivity coefficients is available in the literature, based on zonal homogeneous flows.
- We are attempting to extend this idea to non-zonal and inhomogeneous flows, which are more apt for the real ocean.
- A multiple tracer method helps diagnose a diffusivity tensor that can skillfully reconstruct the buoyancy and PV fluxes.
- Is this the tensor we want?
  - Yes, it can correctly capture tracer fluxes.
  - No, because it does not have some of the favorable properties we would like the parameterized tensor to have.
- Main issue - variance transport, which is not included in 0-D theories.
- In regions that are ~zonal and ~homogeneous the QG link between symmetric and antisymmetric tensor holds.

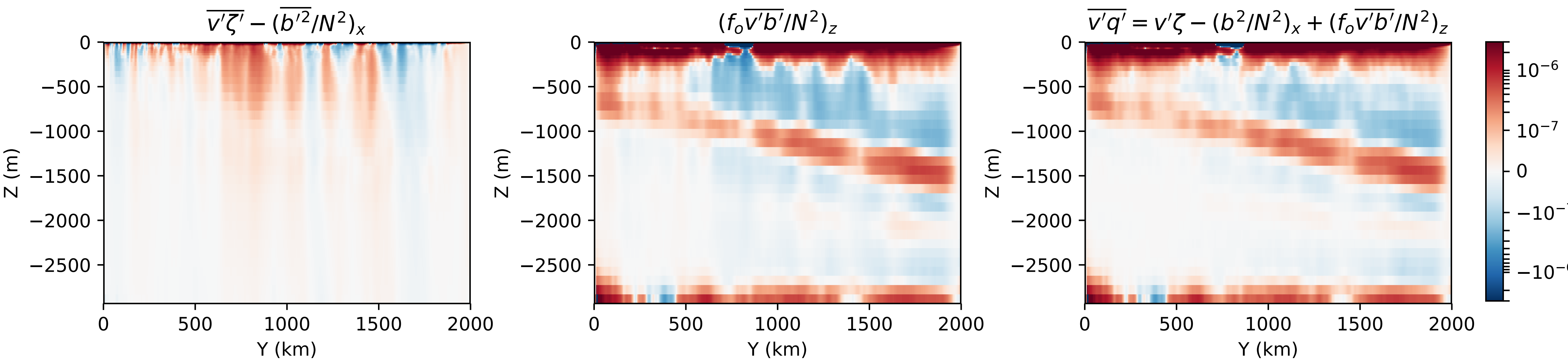
Future Work

- Some clever way to remove the contribution of variance transport before diagnosing the tensor might be helpful (numerous papers by Eden and Greatbatch 200X).
- Strategies for including variance transport in OGCM parameterizations.

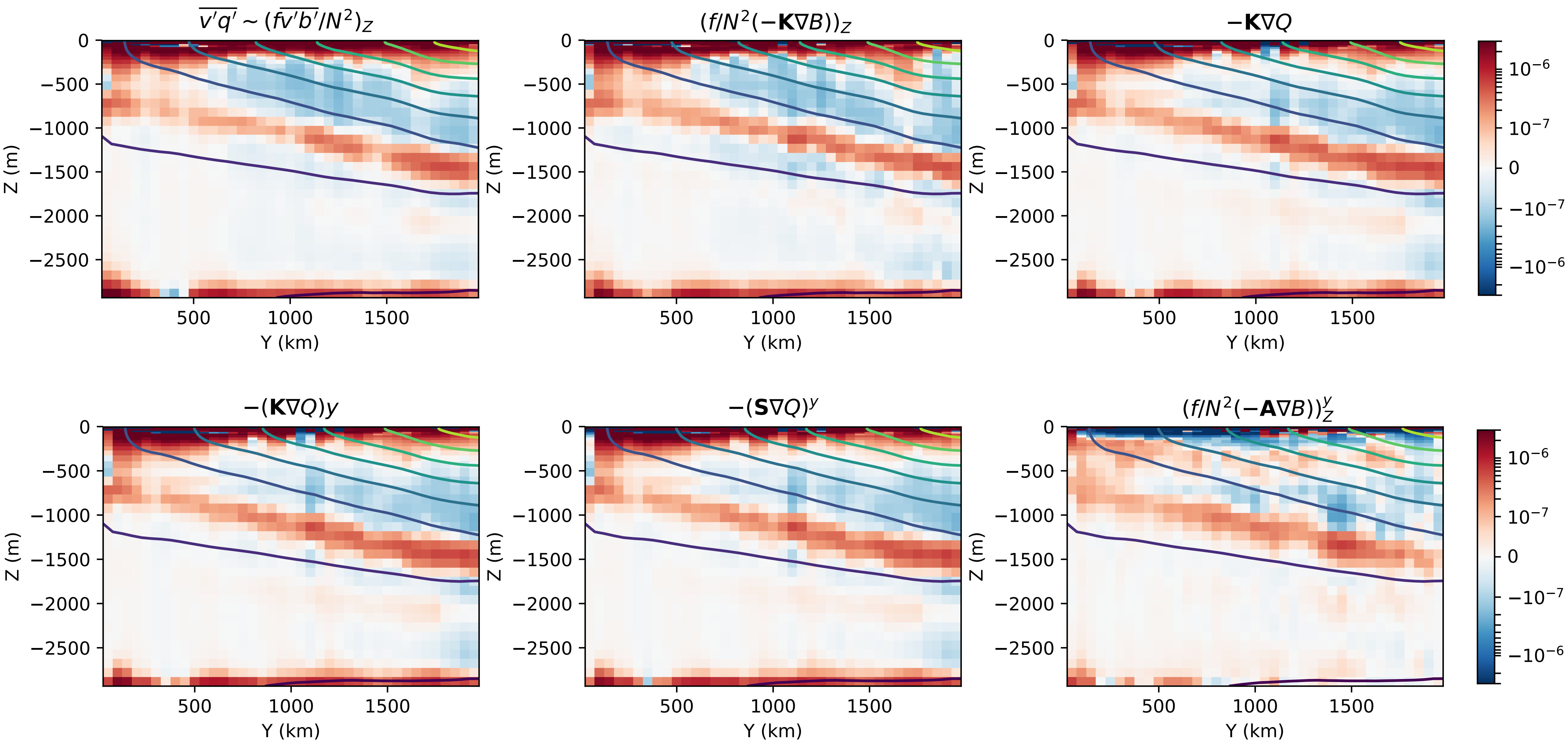
To be continued …

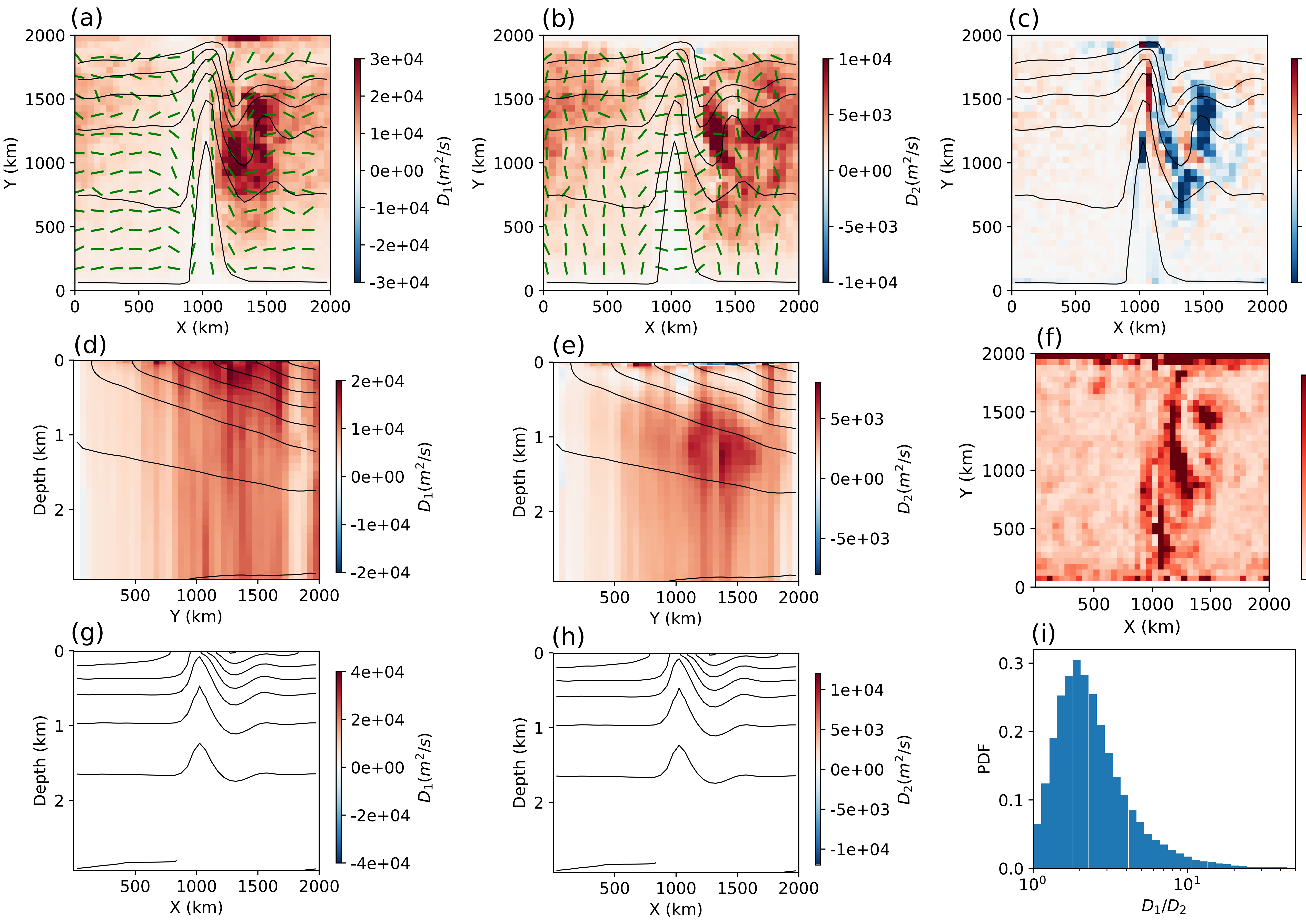
Extras

Check whether PV flux mostly due to buoyancy flux…

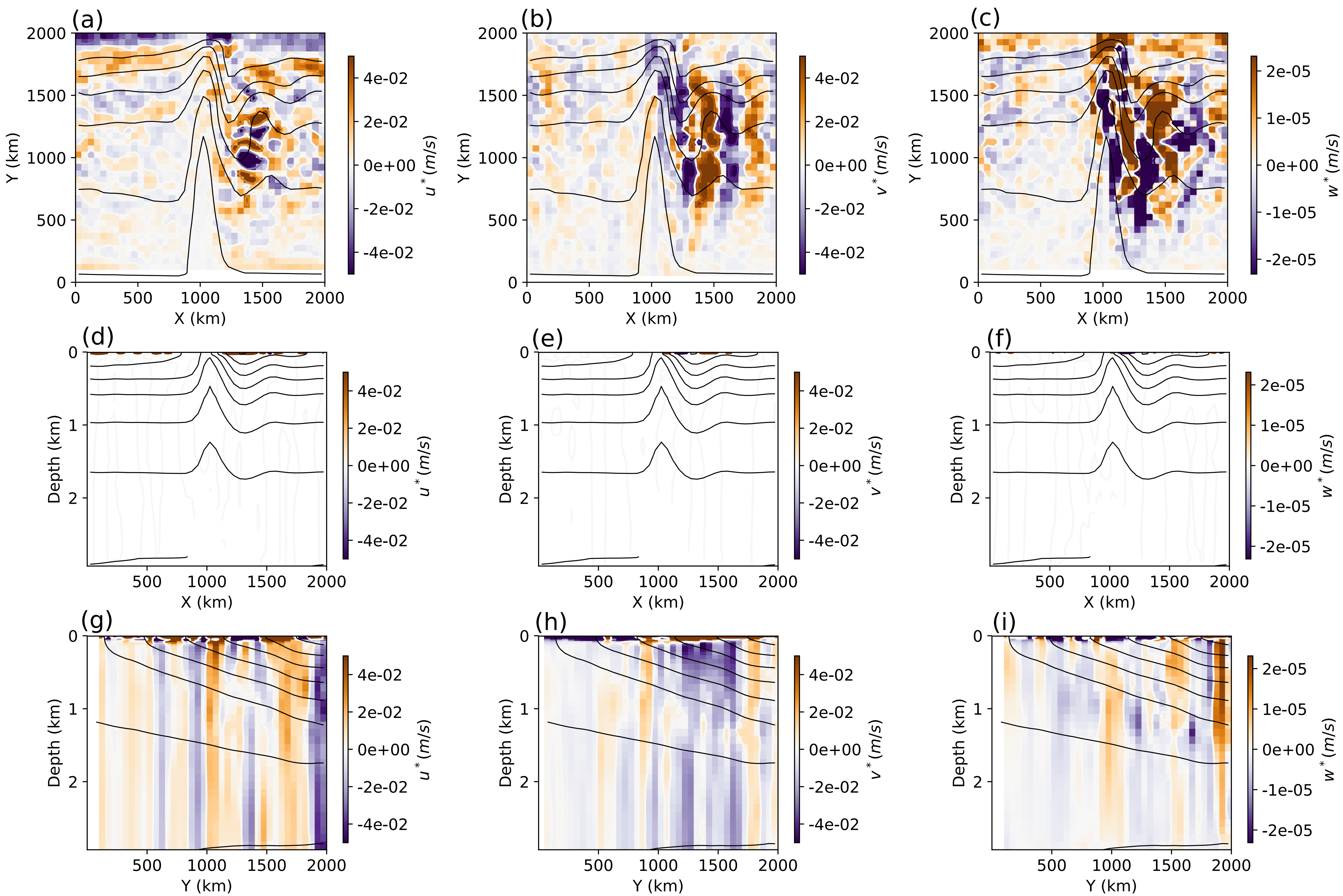


Check whether predicted dynamical connection holds




(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
Y (km)
X (km)
Depth (km)
$D_1(m^2/s)$
$D_2(m^2/s)$
$D_3(m^2/s)$
D_1/D_2
PDF

AntiSymmetric Tensor



Role of Redi diffusivity in Ocean/Climate Models

- Heat and salt transport (Griffies 2015)
- Oxygen
- Changes in SST and SSS, resulting in coupled responses.