

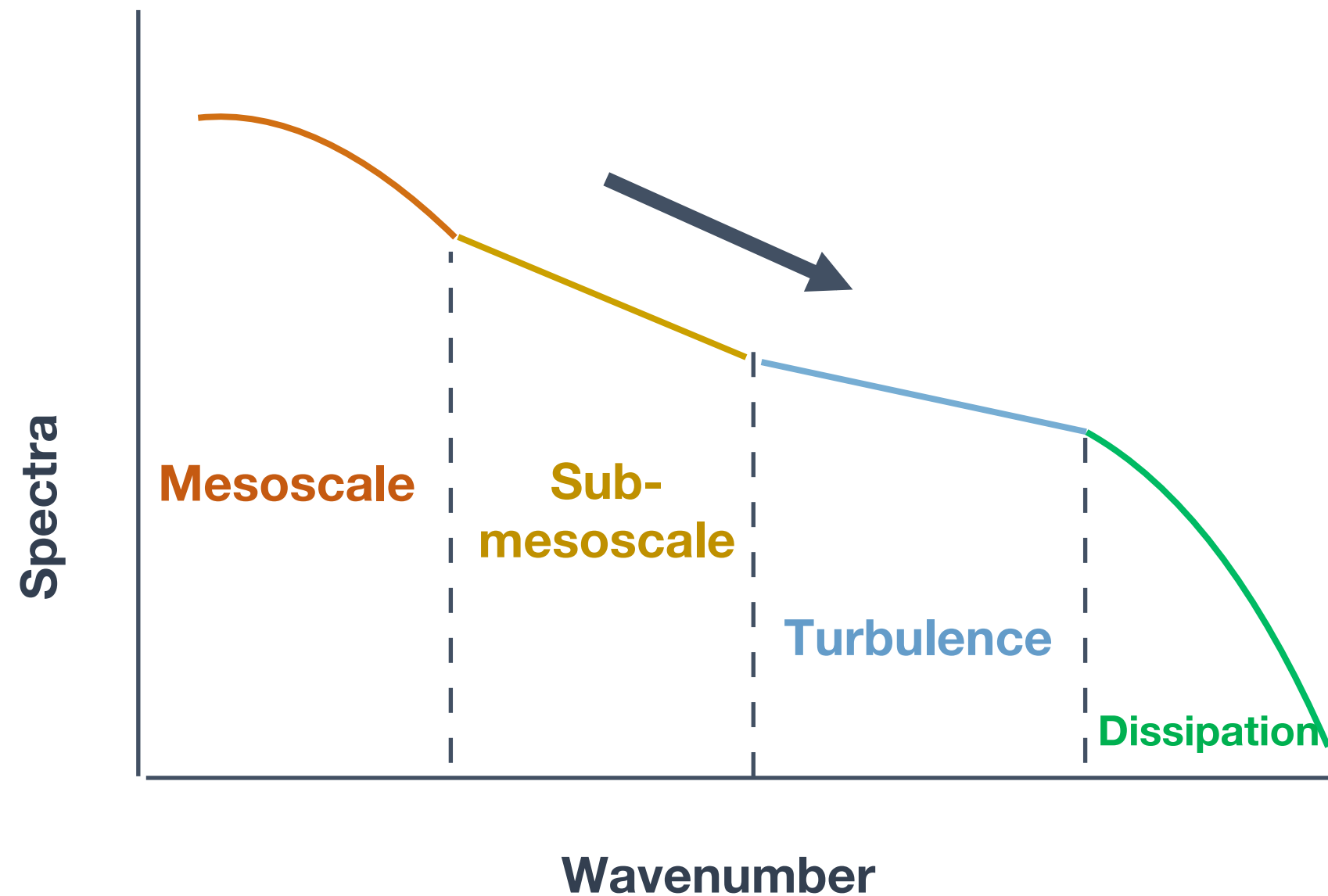


# Hidden Dangers in Potential Vorticity



Abigail Bodner

Baylor Fox-Kemper



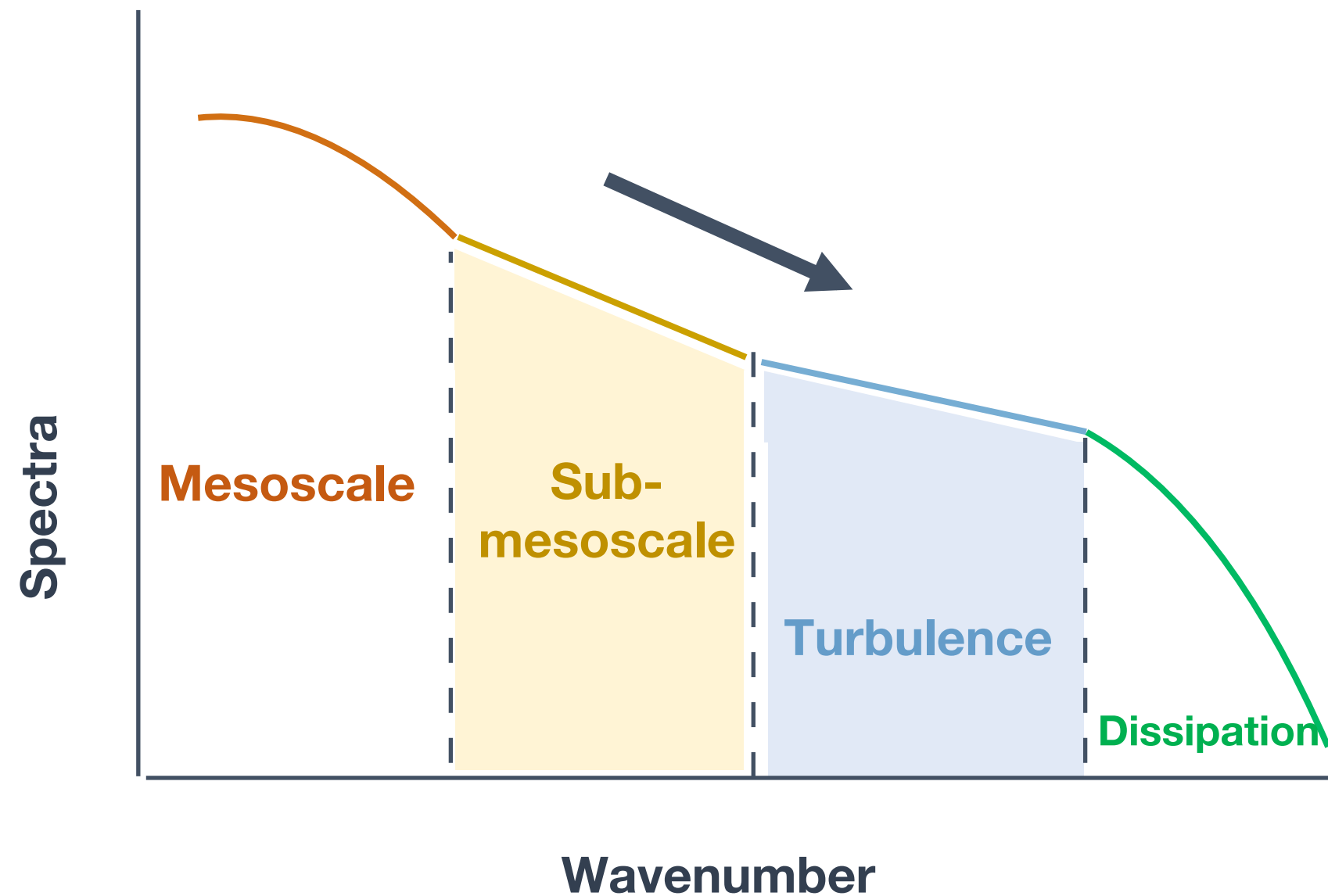


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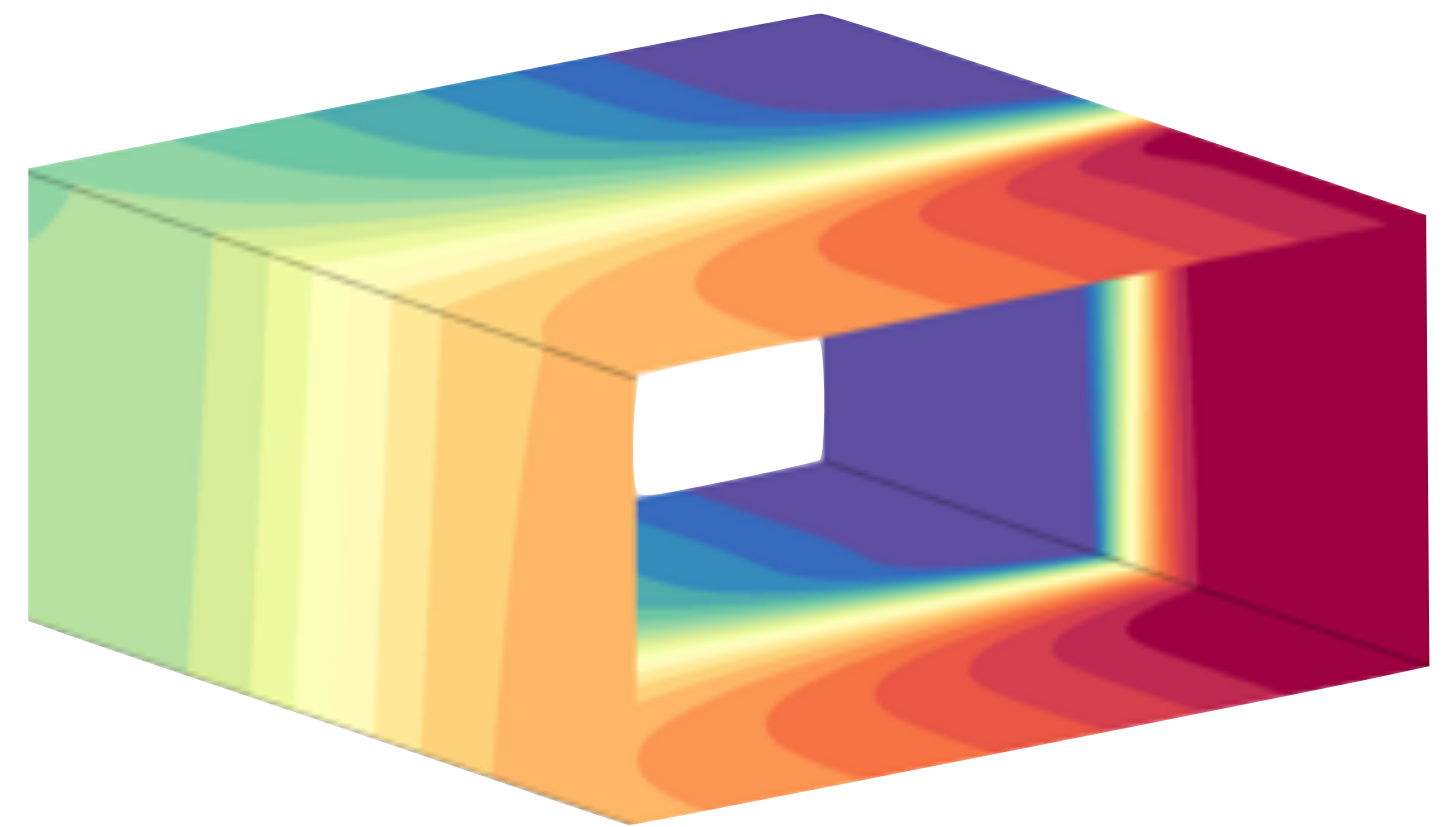


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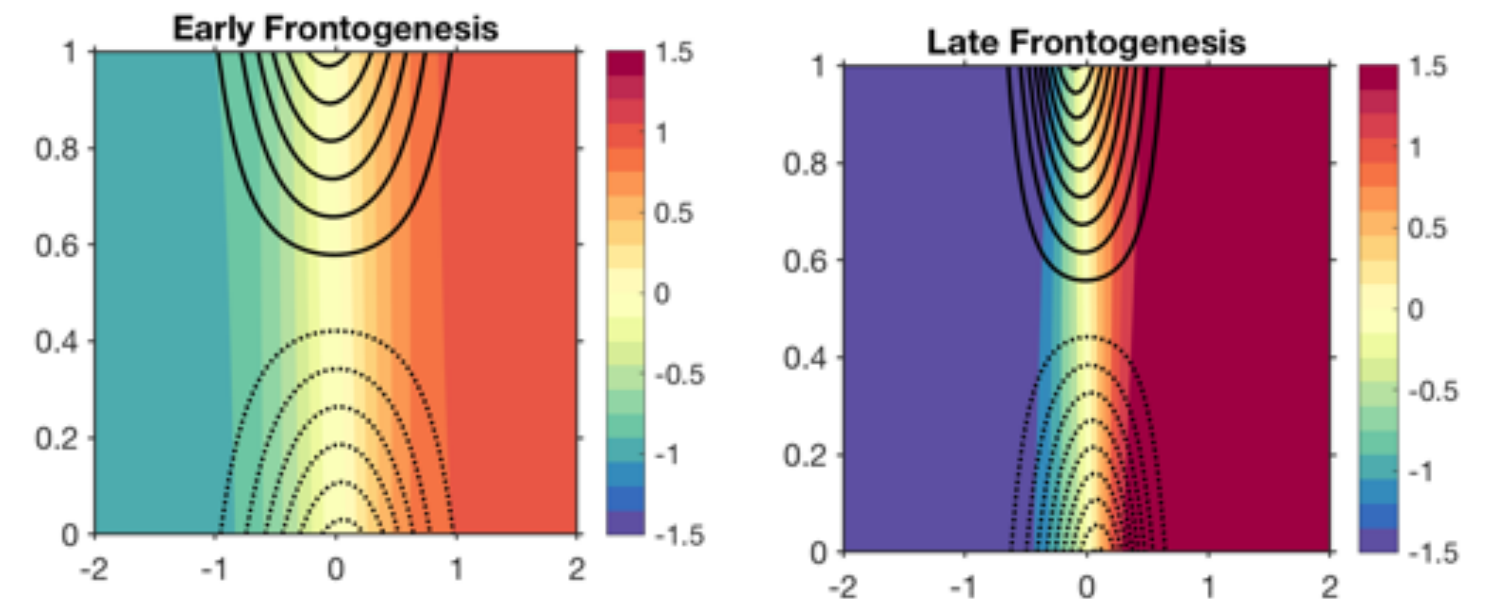


# Turbulence affects (submesoscale) frontal dynamics



⊙ Classic theory of fronts predicts that the cross-frontal scale becomes infinitely thin within a finite time

⊙ A modified theory including turbulence helps explain the scale observed



**A perturbation approach to understanding the effects of turbulence on frontogenesis.**

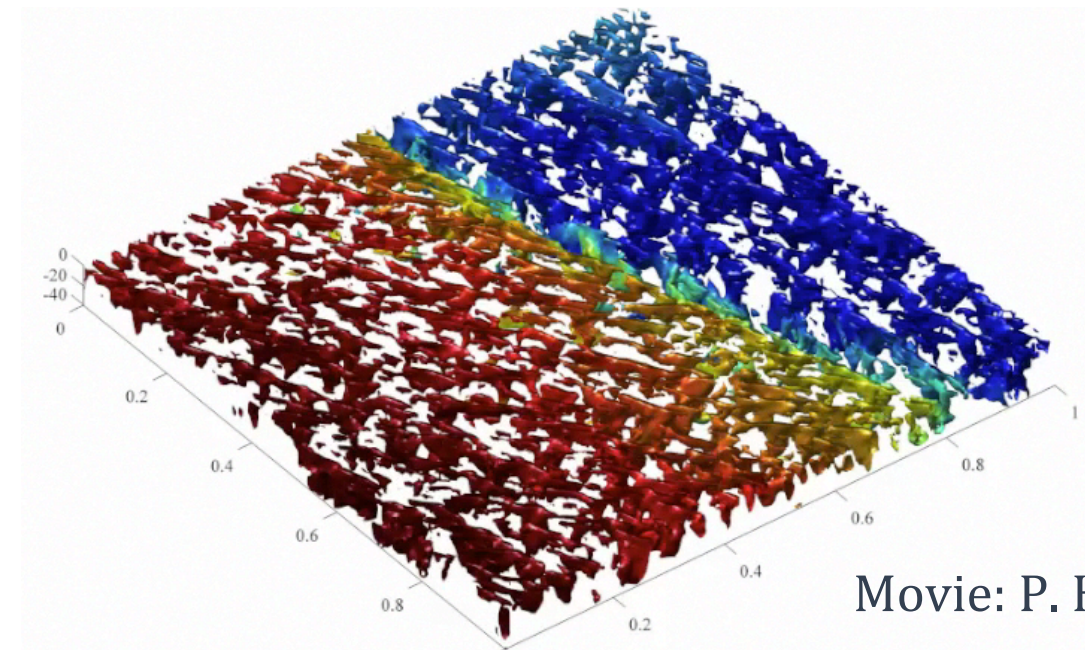
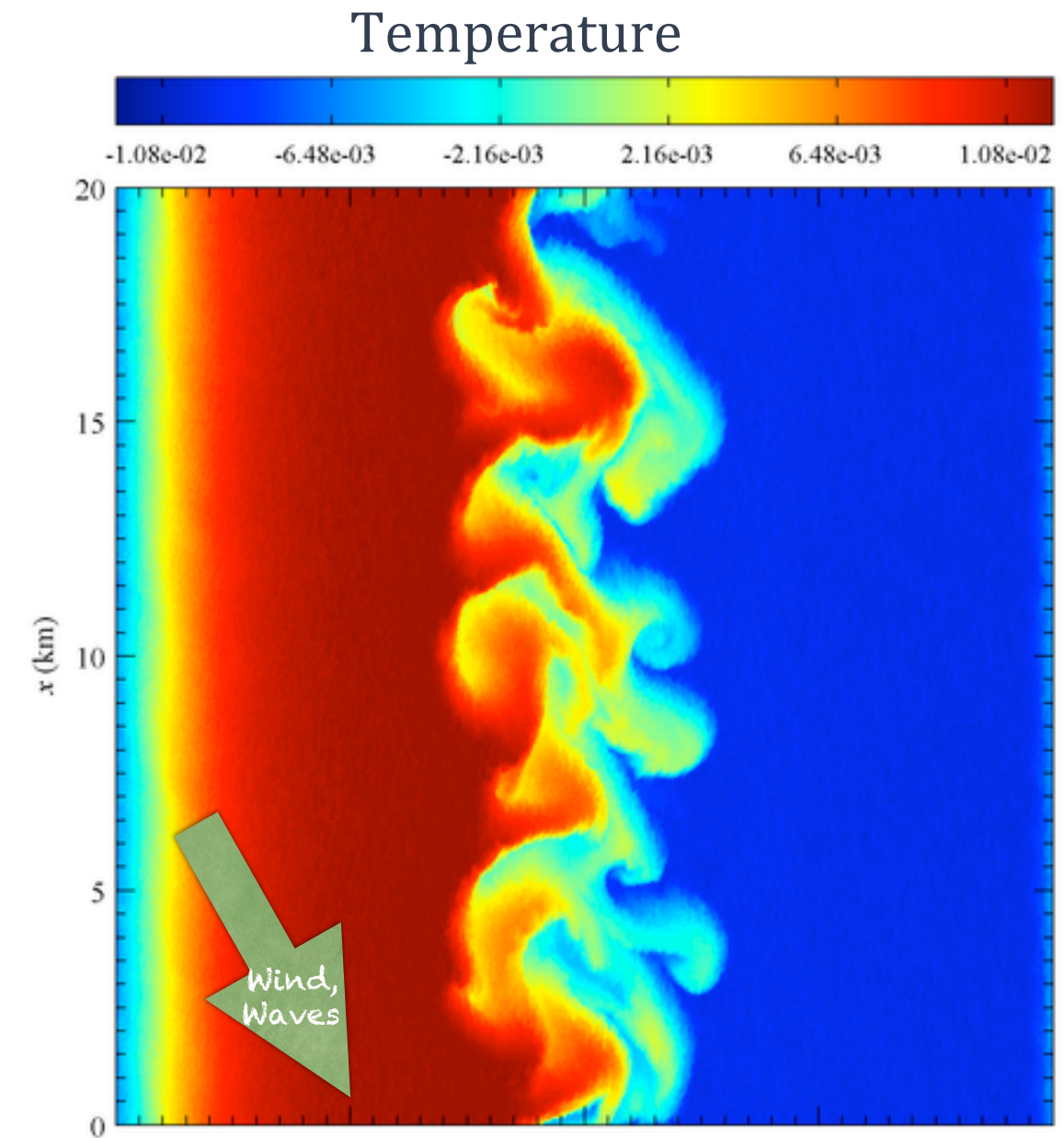
**A. Bodner, B. Fox-Kemper, L. Van Roekel, J. McWilliams, and P. Sullivan:**

**Journal of Fluid Mechanics (under review).**

# Turbulence affects (submesoscale) frontal dynamics

LES of submesoscale temperature front with Langmuir turbulence (waves & winds).

Domain size: 20km x 20km x -160m  
Resolution: 5m x 5m x -1.25m



Movie: P. Hamlington

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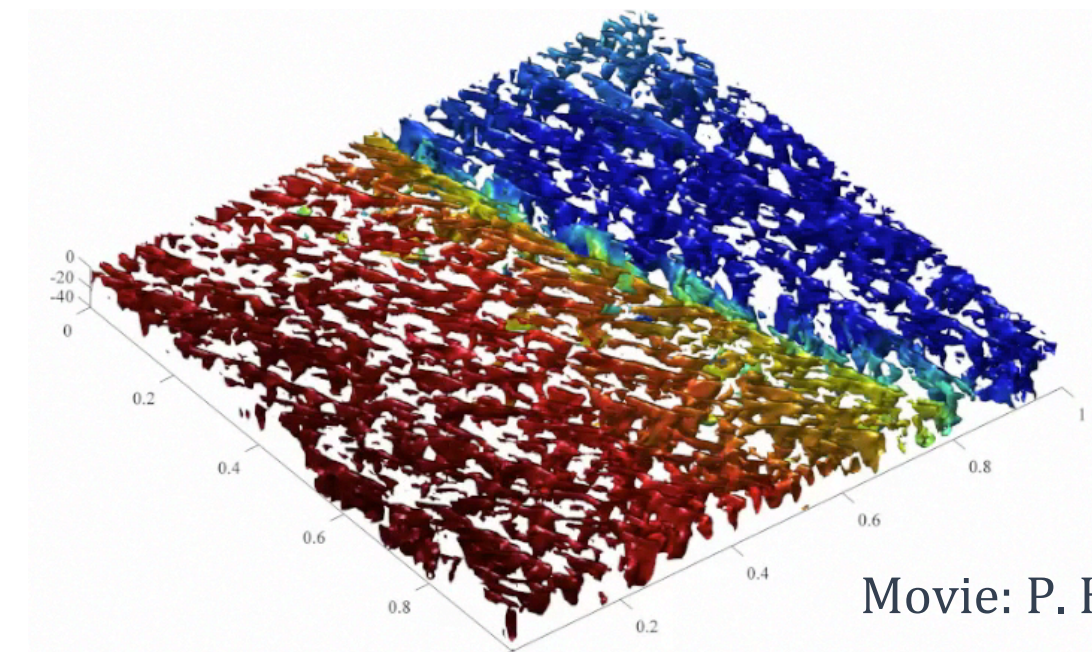
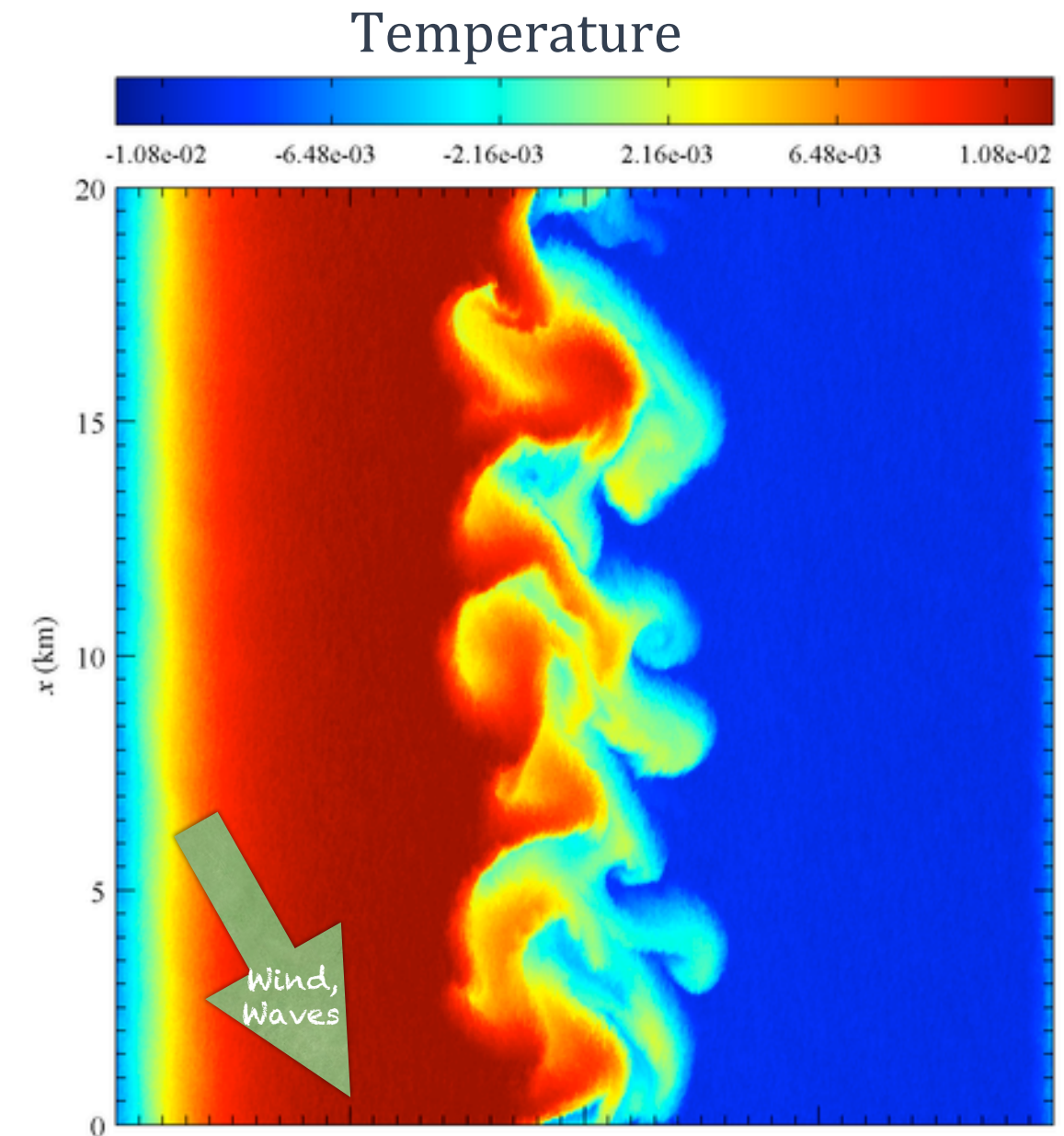
LES of submesoscale temperature front with Langmuir turbulence (waves & winds).

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## Potential Vorticity

- ⊙ Fundamental in mesoscale / submesoscale dynamics
- ⊙ Used to study frictional fluxes

$$q = (f + \nabla \times \vec{u}) \cdot \nabla \theta$$



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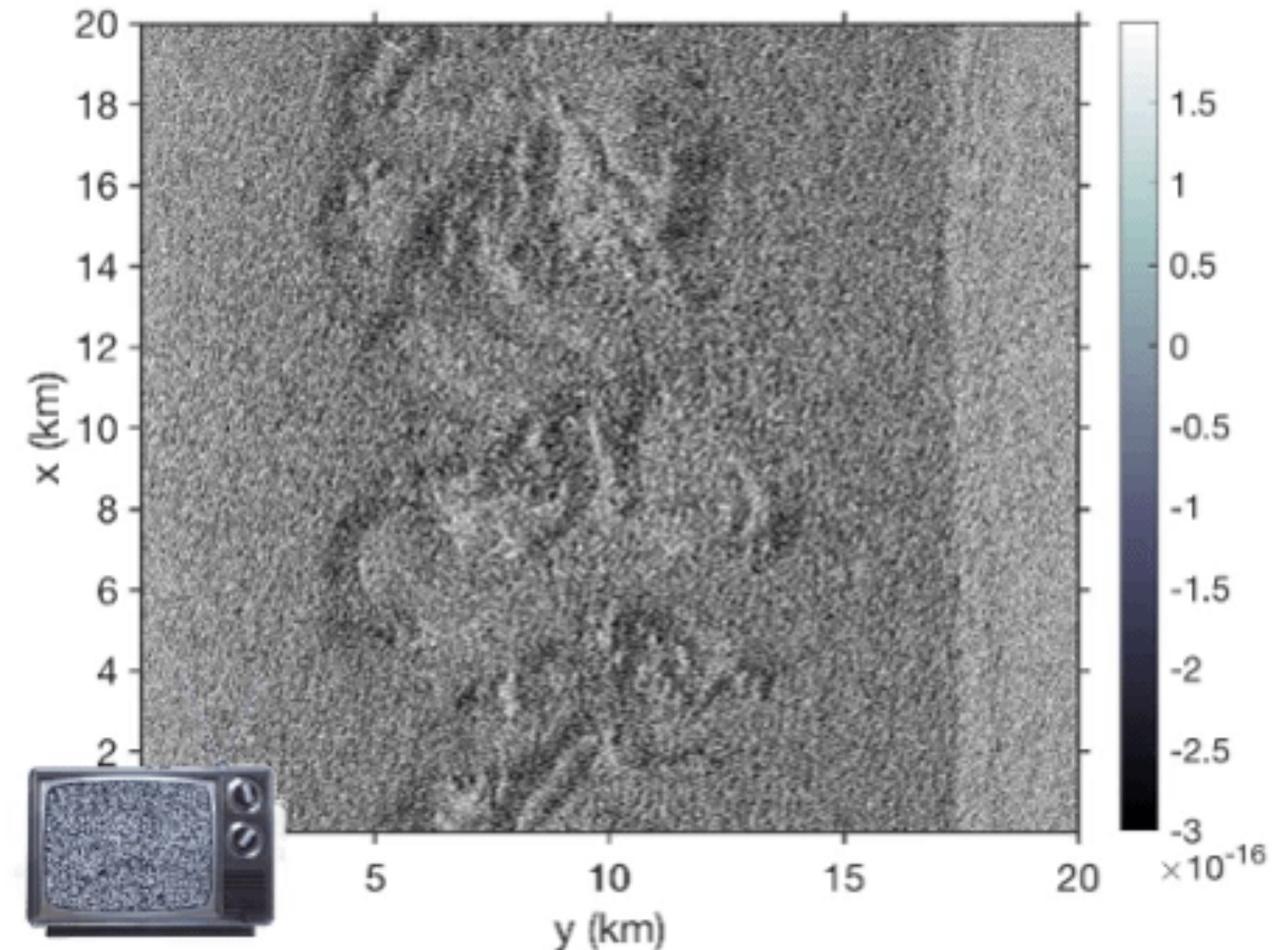
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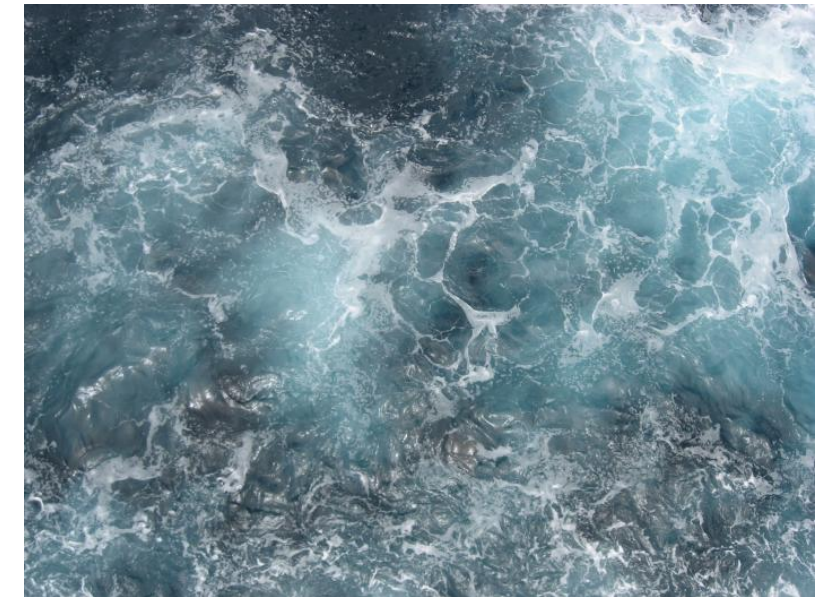
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## Potential Vorticity



# Potential Vorticity in Isotropic Turbulence

$$q = (f + \nabla \times \vec{u}) \cdot \nabla \theta$$



No statistically meaningful directionally

$$\overline{\nabla \times \vec{u}} = 0$$

$$\overline{\nabla \theta} = 0$$

# Potential Vorticity in Isotropic Turbulence

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Nonlinearity of PV retains gradients

$$\overline{(f + \nabla \times \vec{u}) \cdot \nabla \theta} \neq 0$$



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Nonlinearity of PV retains gradients

$$\overline{(f + \nabla \times \vec{u}) \cdot \nabla \theta} \neq 0$$

$$\overline{A'} = 0, \overline{B'} = 0 \quad \rightarrow \quad \overline{A'B'} \neq 0$$

Similar to Reynolds average

## Spectral Analysis

$$E(k) = \hat{u}(k) \cdot \hat{u}^*(k)$$

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### Submesoscale

$$E(k) \propto k^{-2}$$

### 3D Turbulence

$$E(k) \propto k^{-\frac{5}{3}}$$

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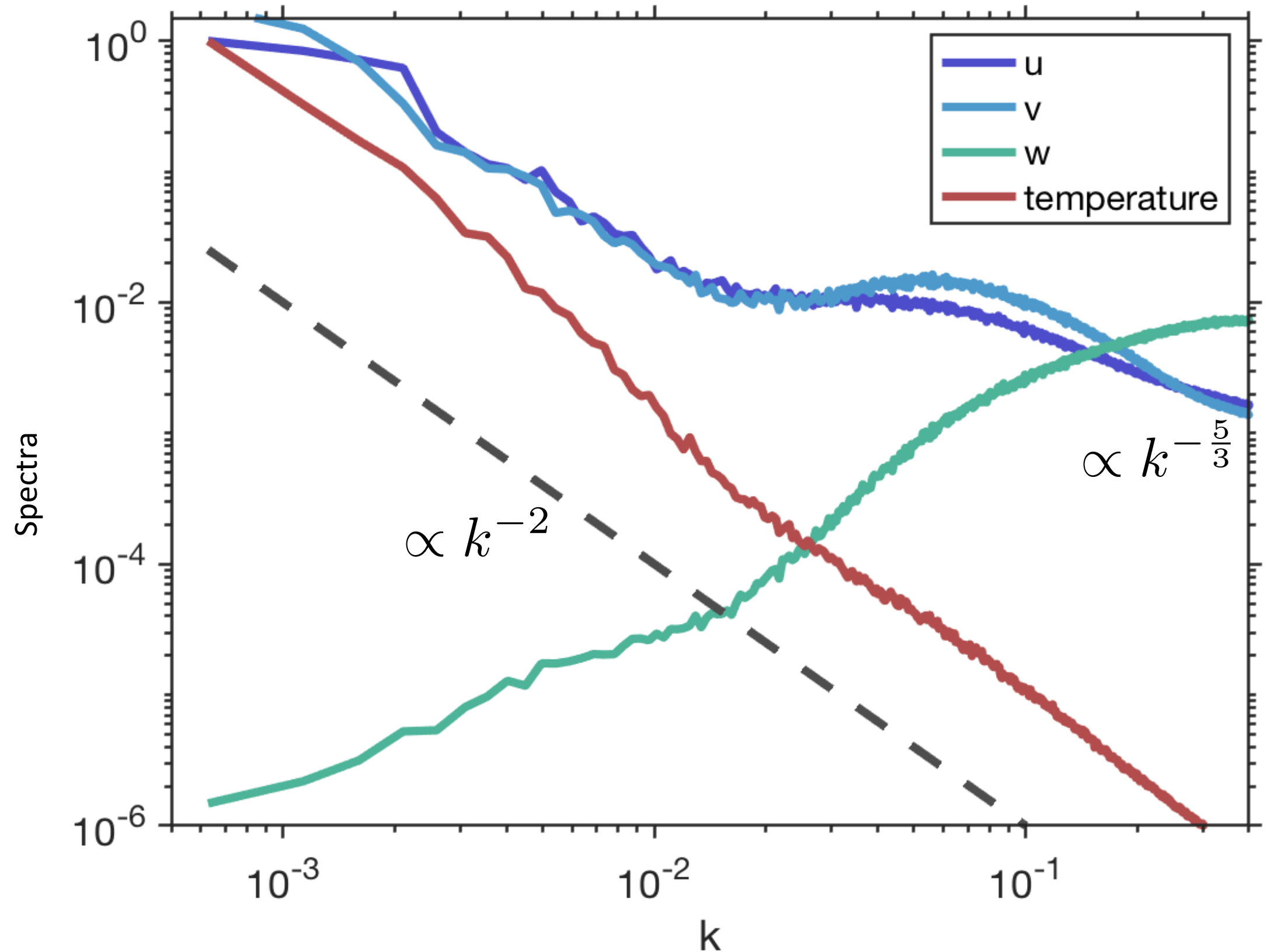
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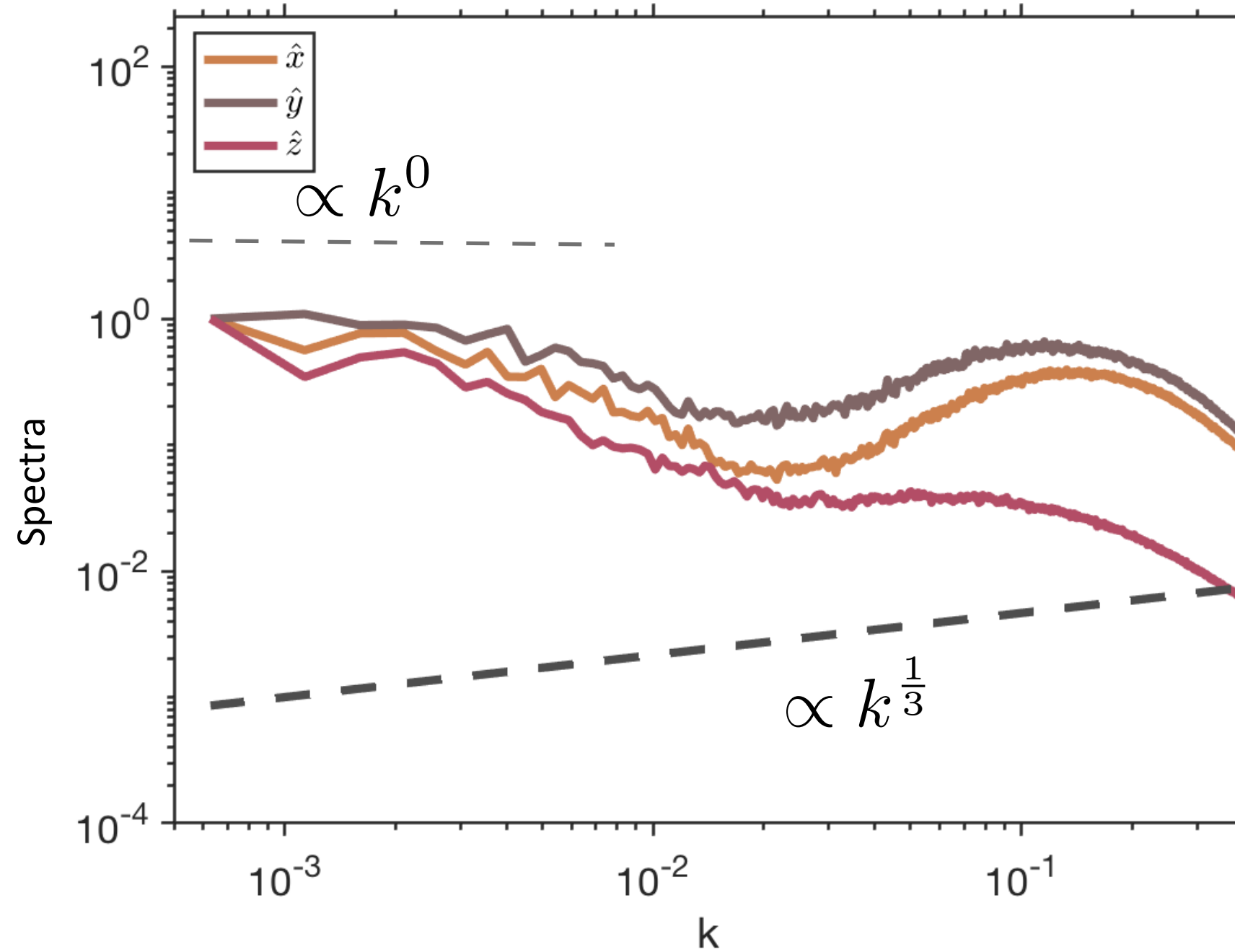
## Velocity and Temperature



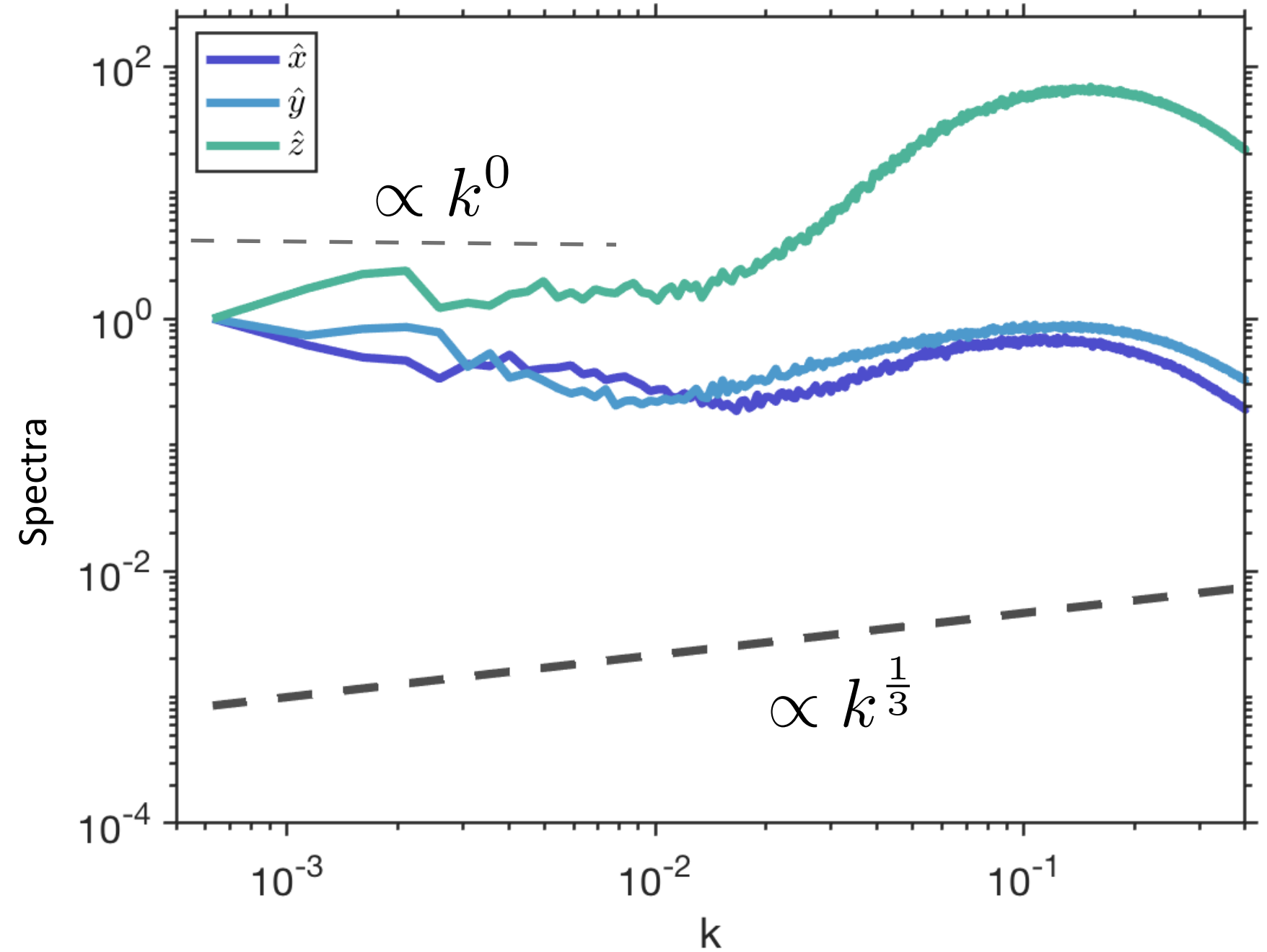
Data from Hamlington et. al. (2014), Suzuki et. al. (2016)

$$E_{\nabla}(k) \propto k^2 E(k) \propto k^0$$

Temperature gradient  $\nabla\theta$

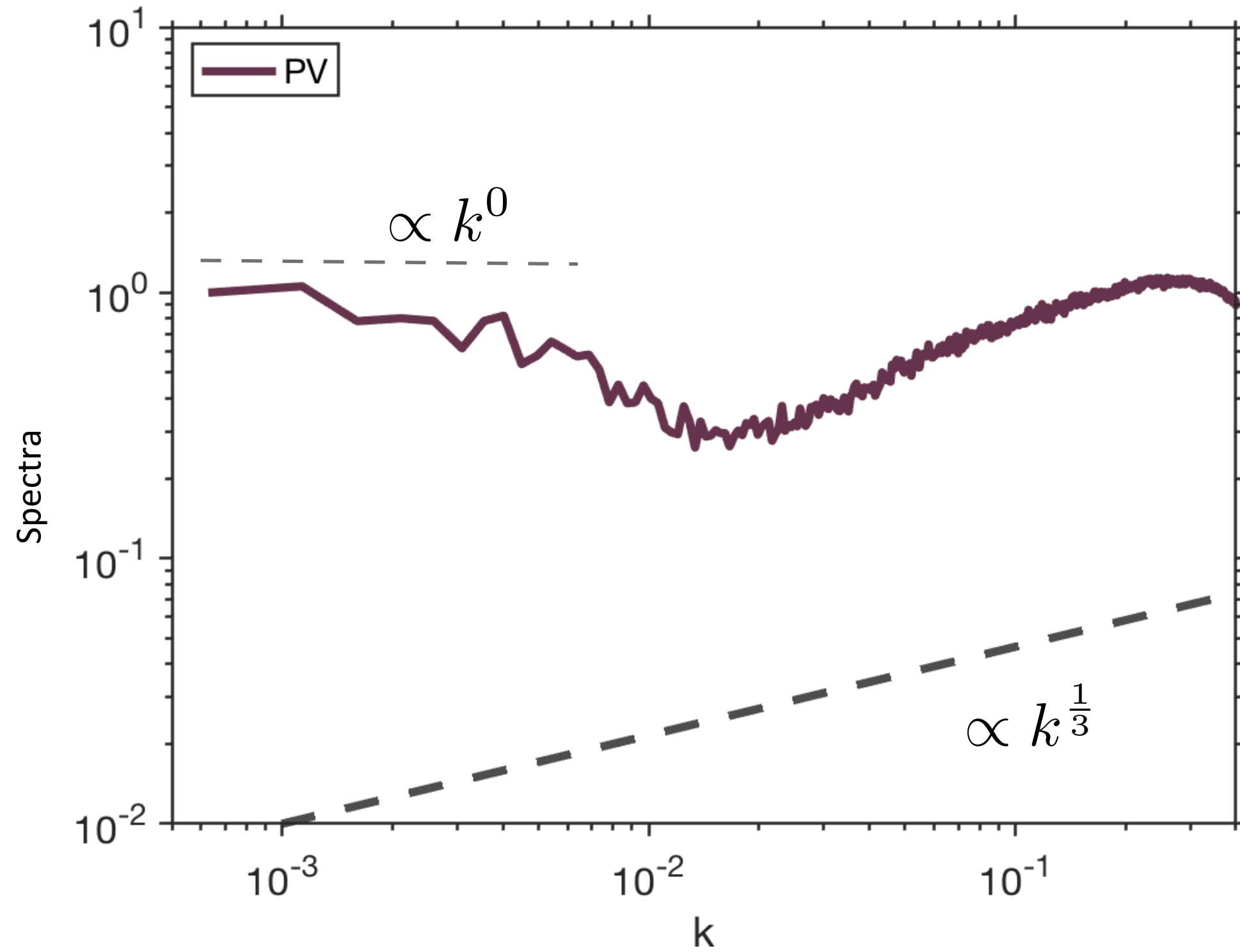


Vorticity  $\nabla \times \vec{u}$

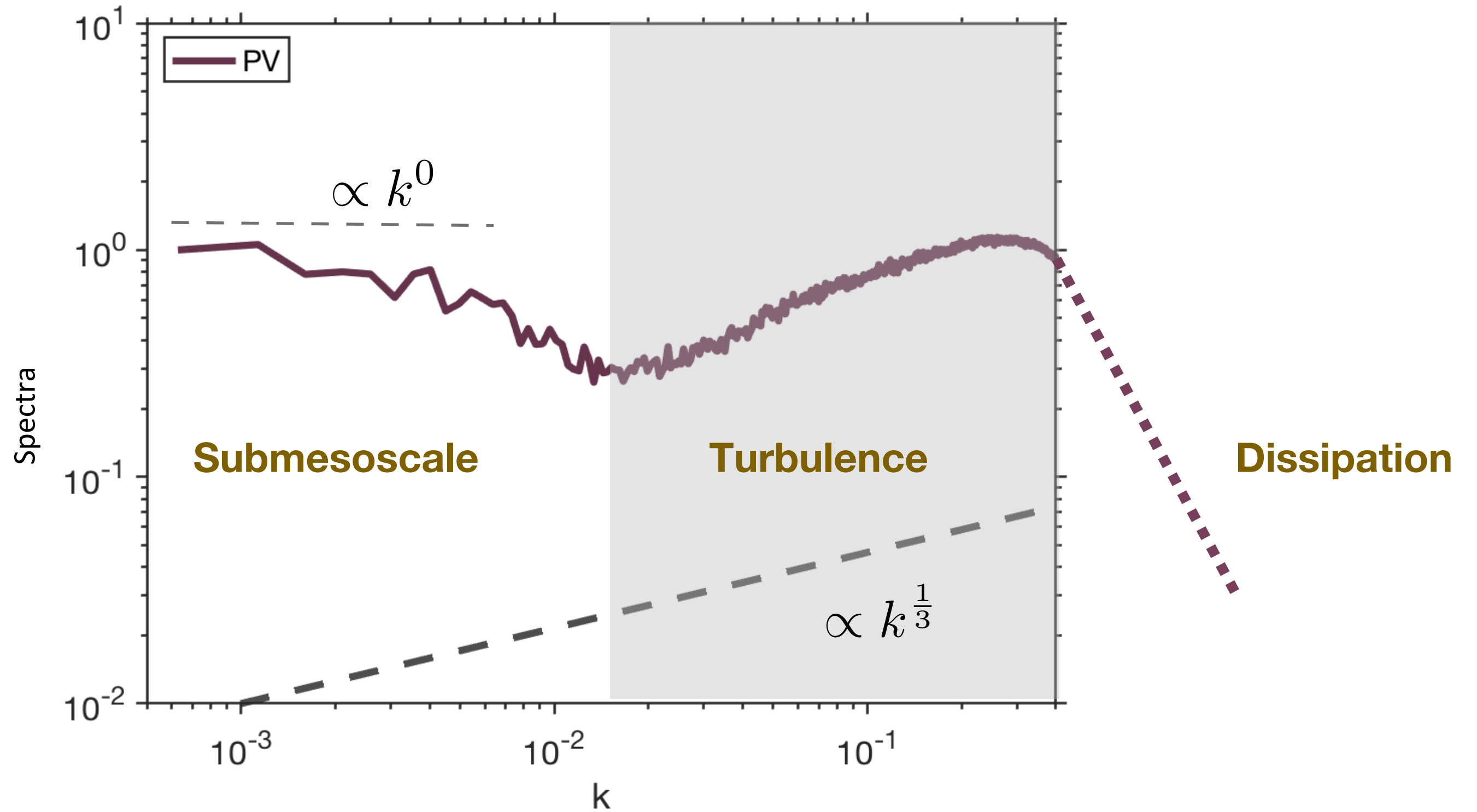


**Smallest scales dominate !!**  
**Ultraviolet catastrophe !!**

# Potential Vorticity $q = (f + \nabla \times \vec{u}) \cdot \nabla \theta$



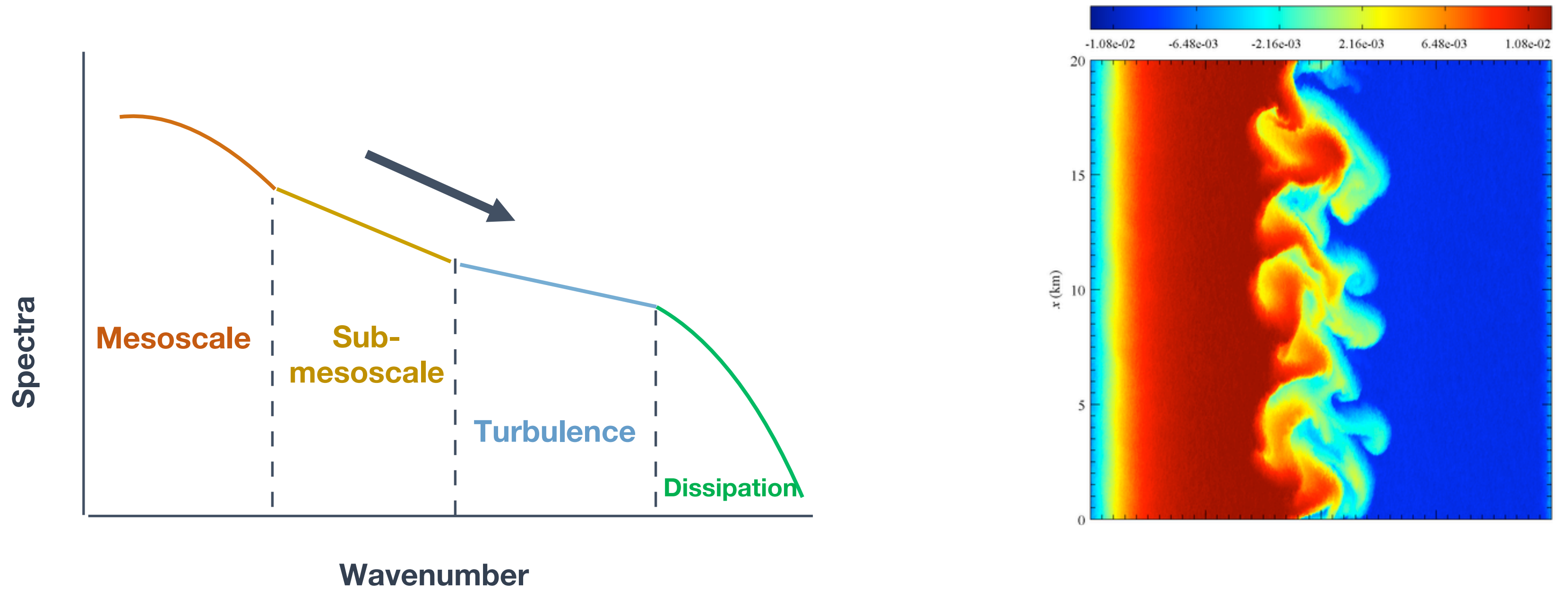
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The failure of PV signals the bottom limit of the submesoscales

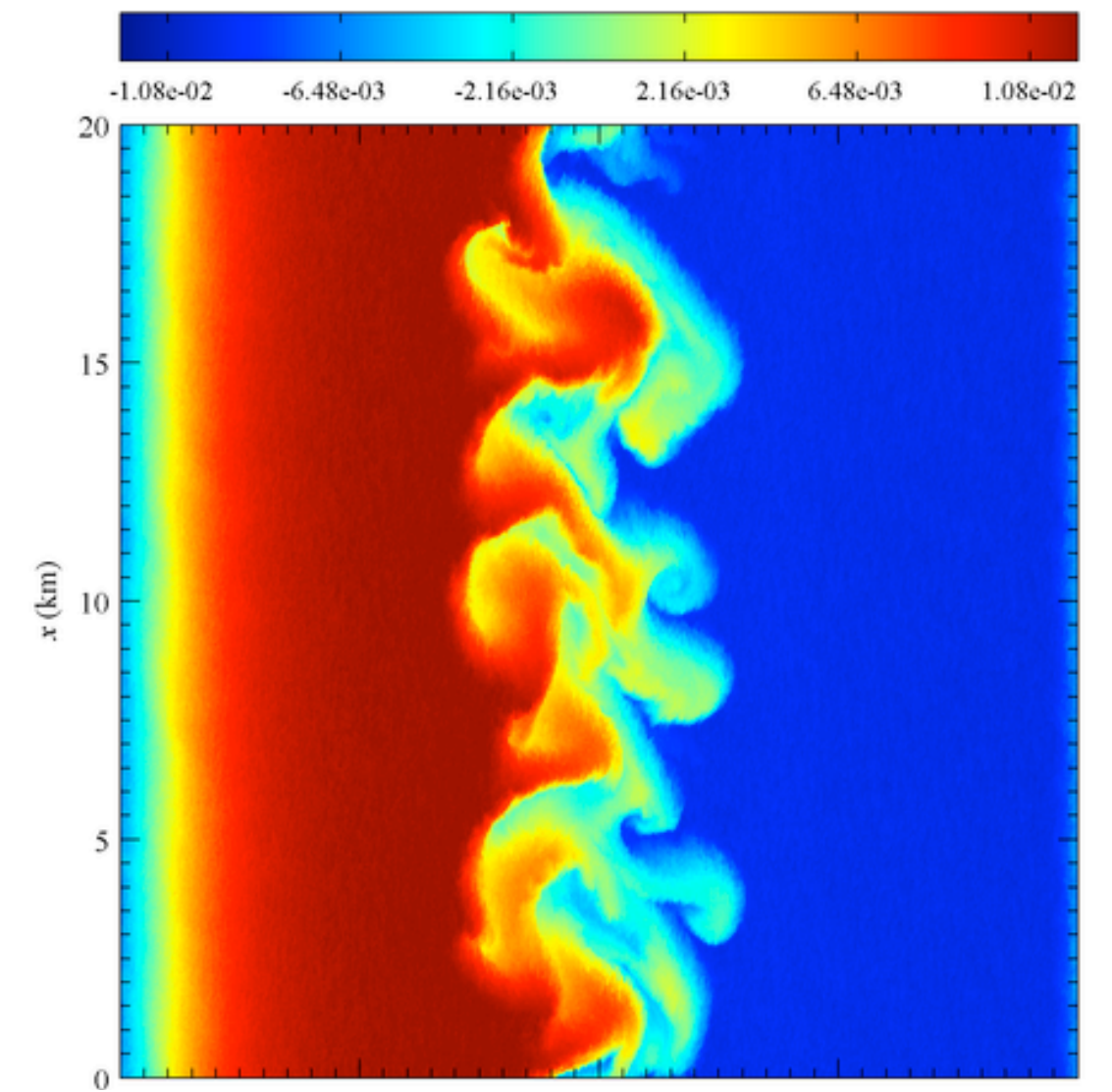
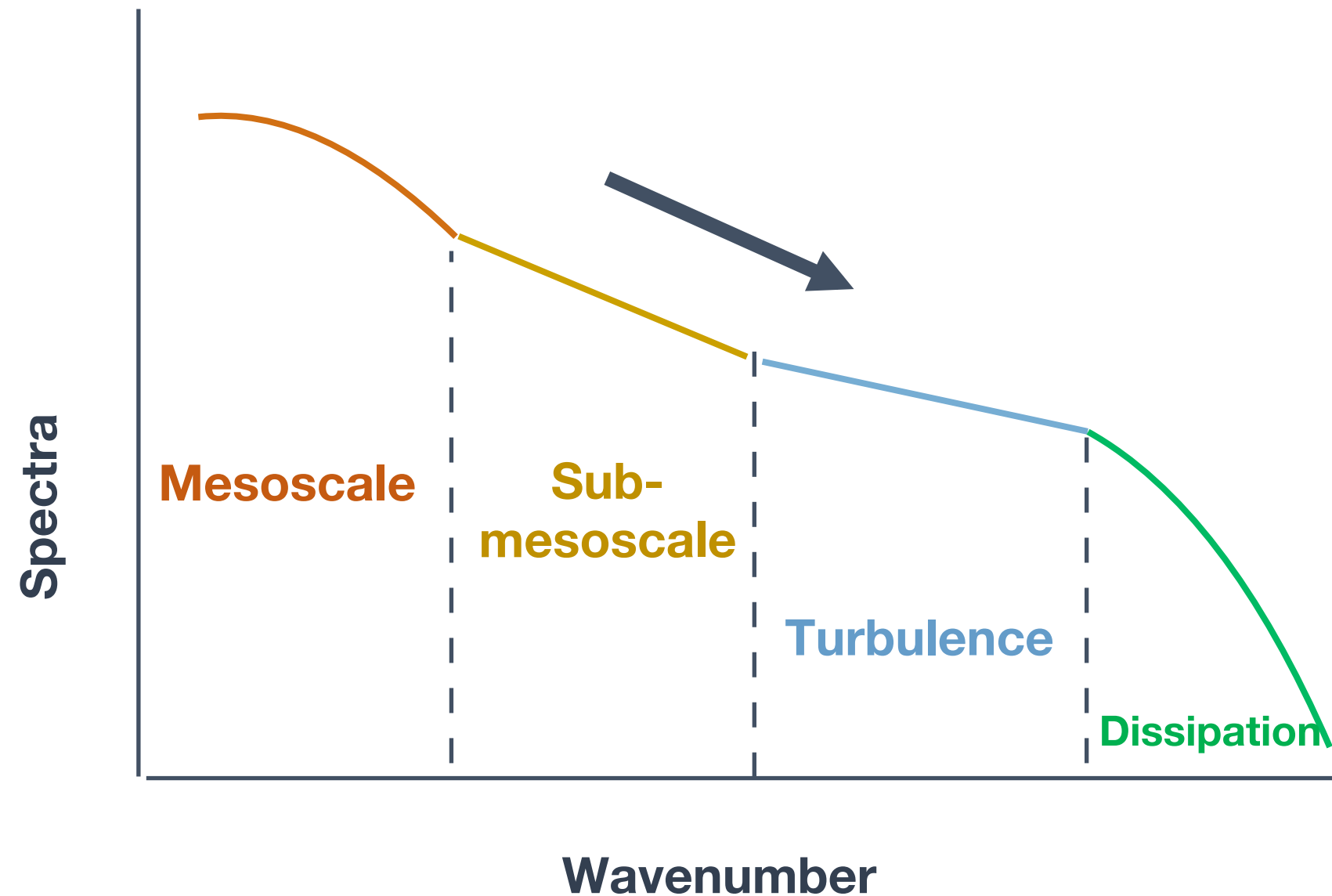


Models such as LES and ROMS rely on smallest scales having the smallest variance



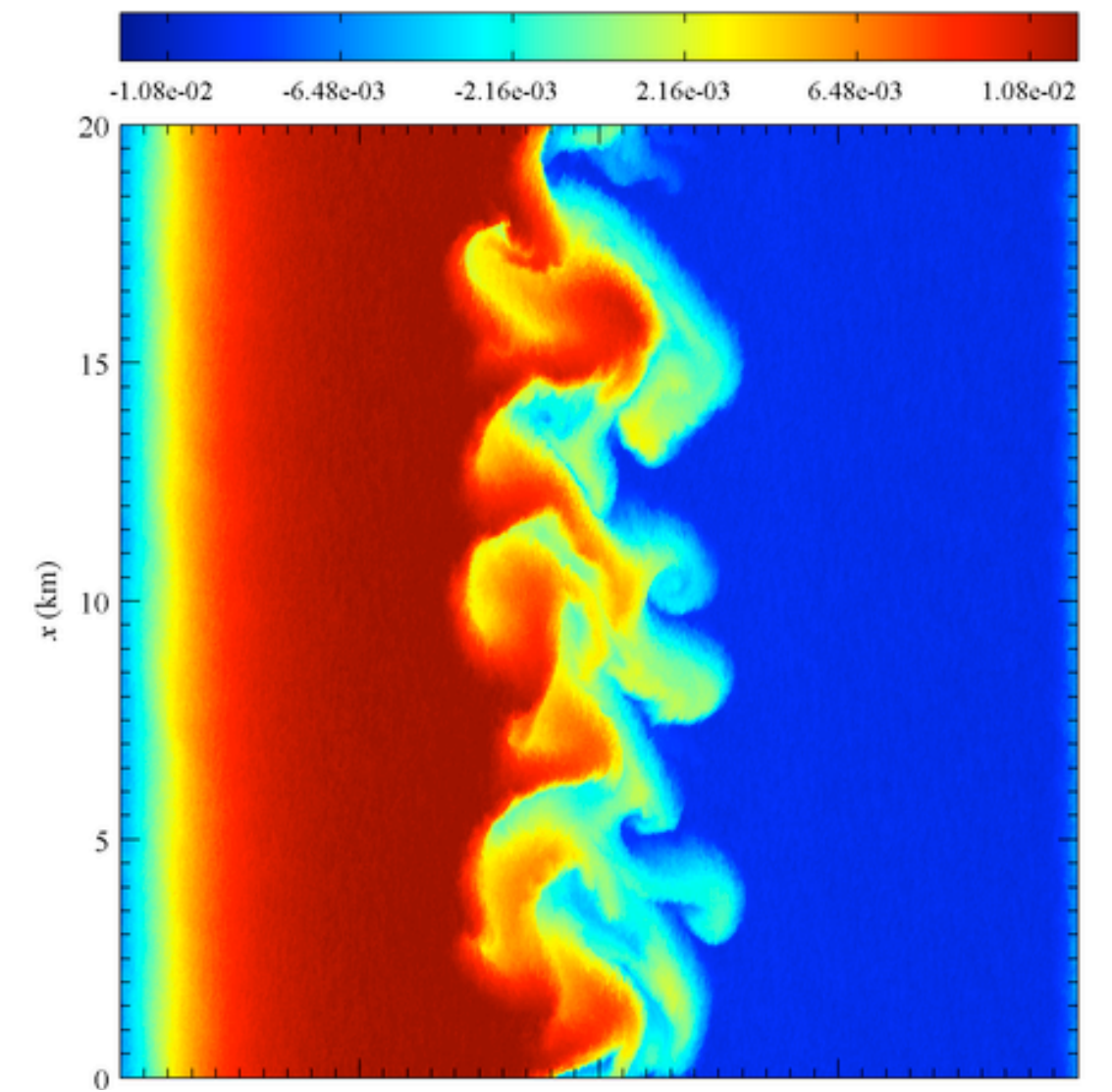
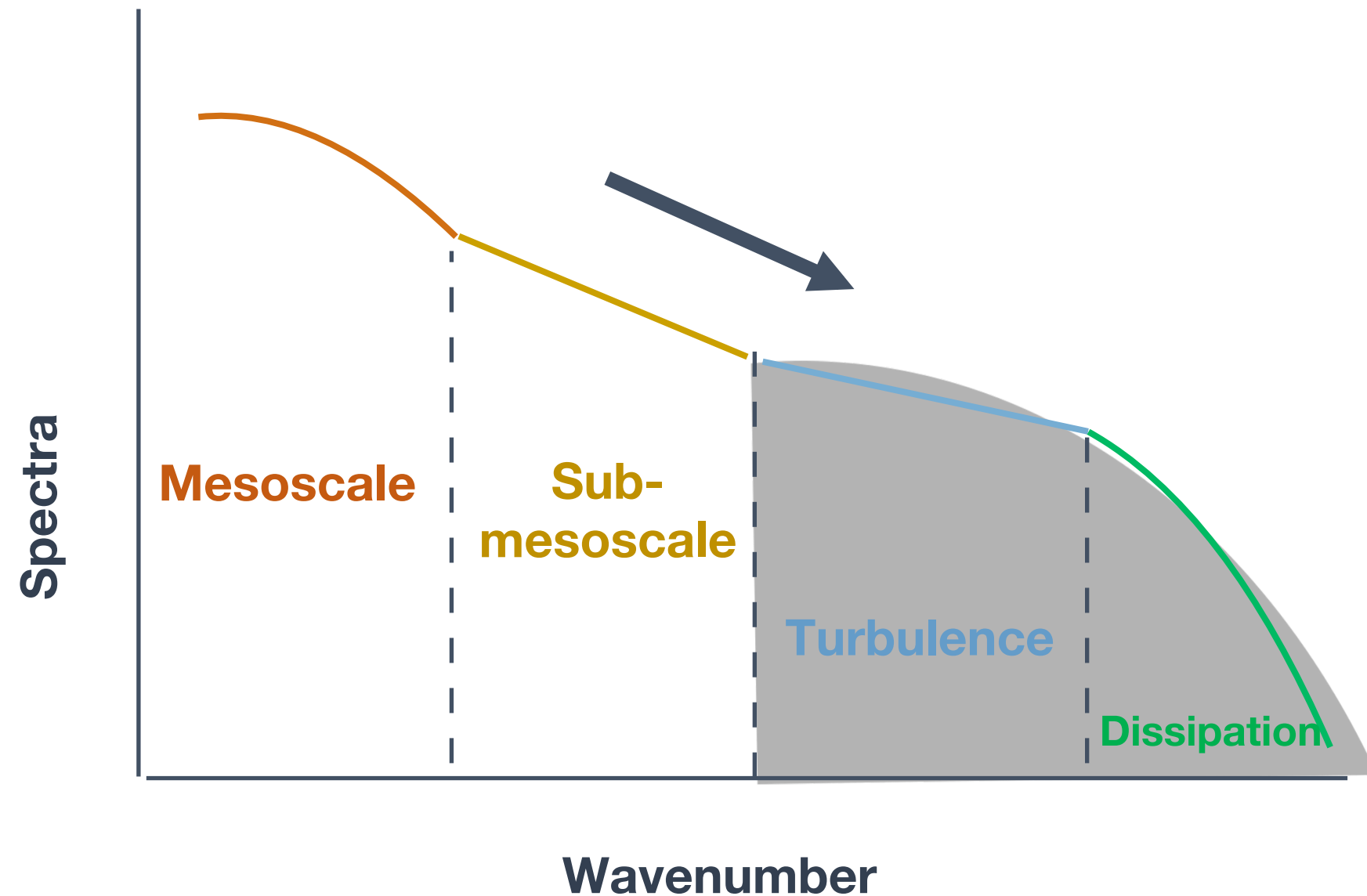
Models such as LES and ROMS rely on smallest scales having the smallest variance

→ Only fully resolved (DNS) potential vorticity is meaningful below the submesoscale

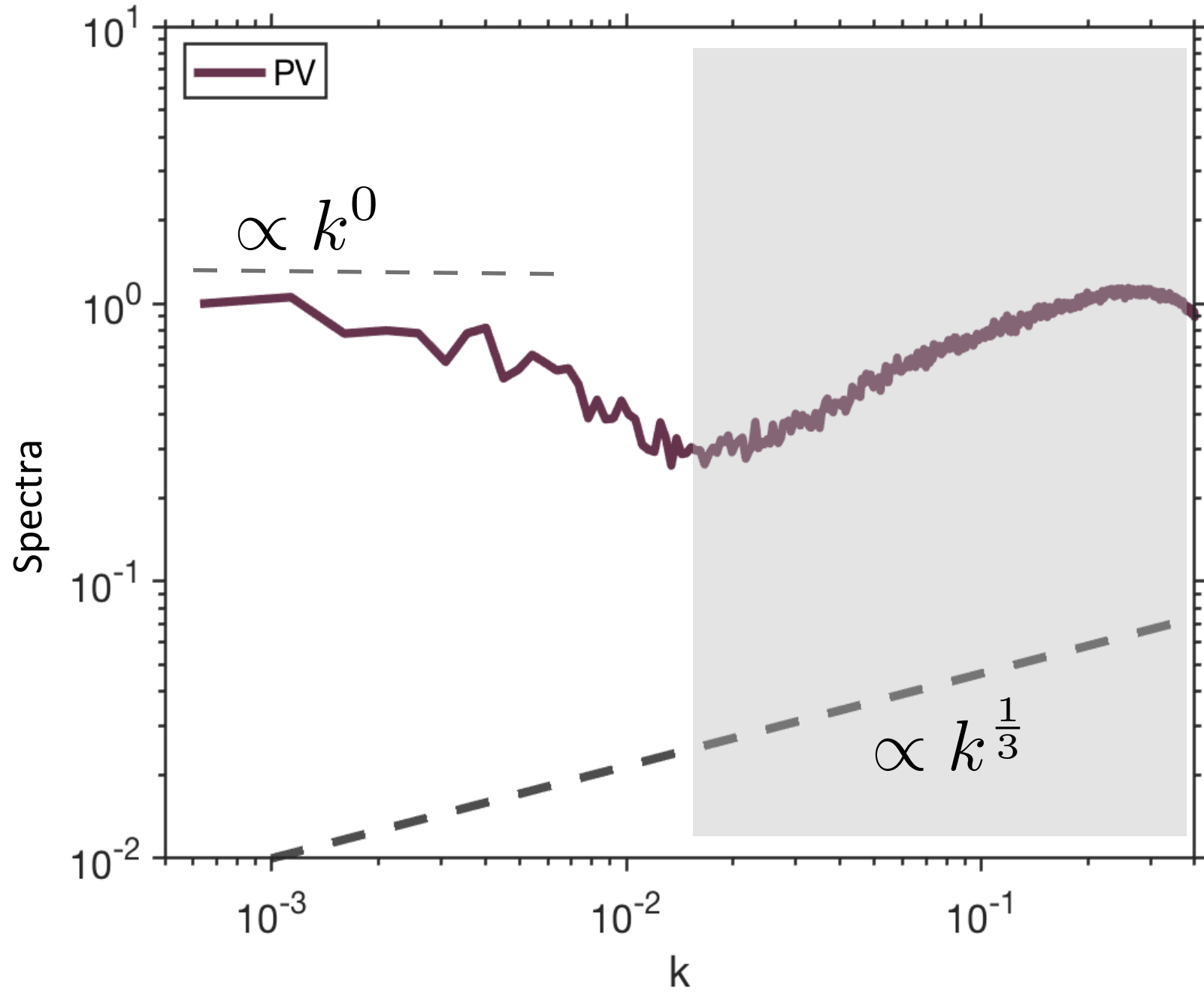


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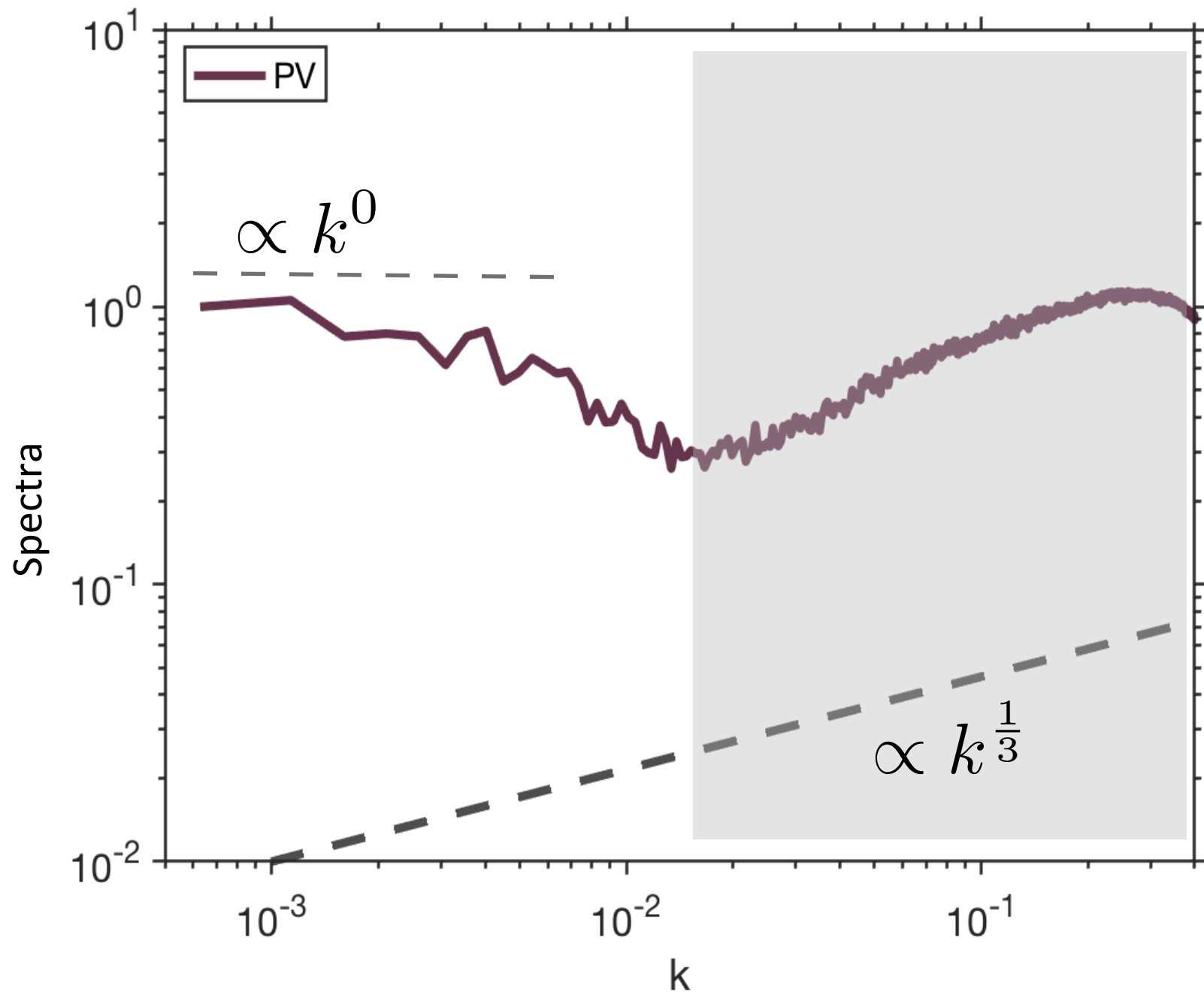
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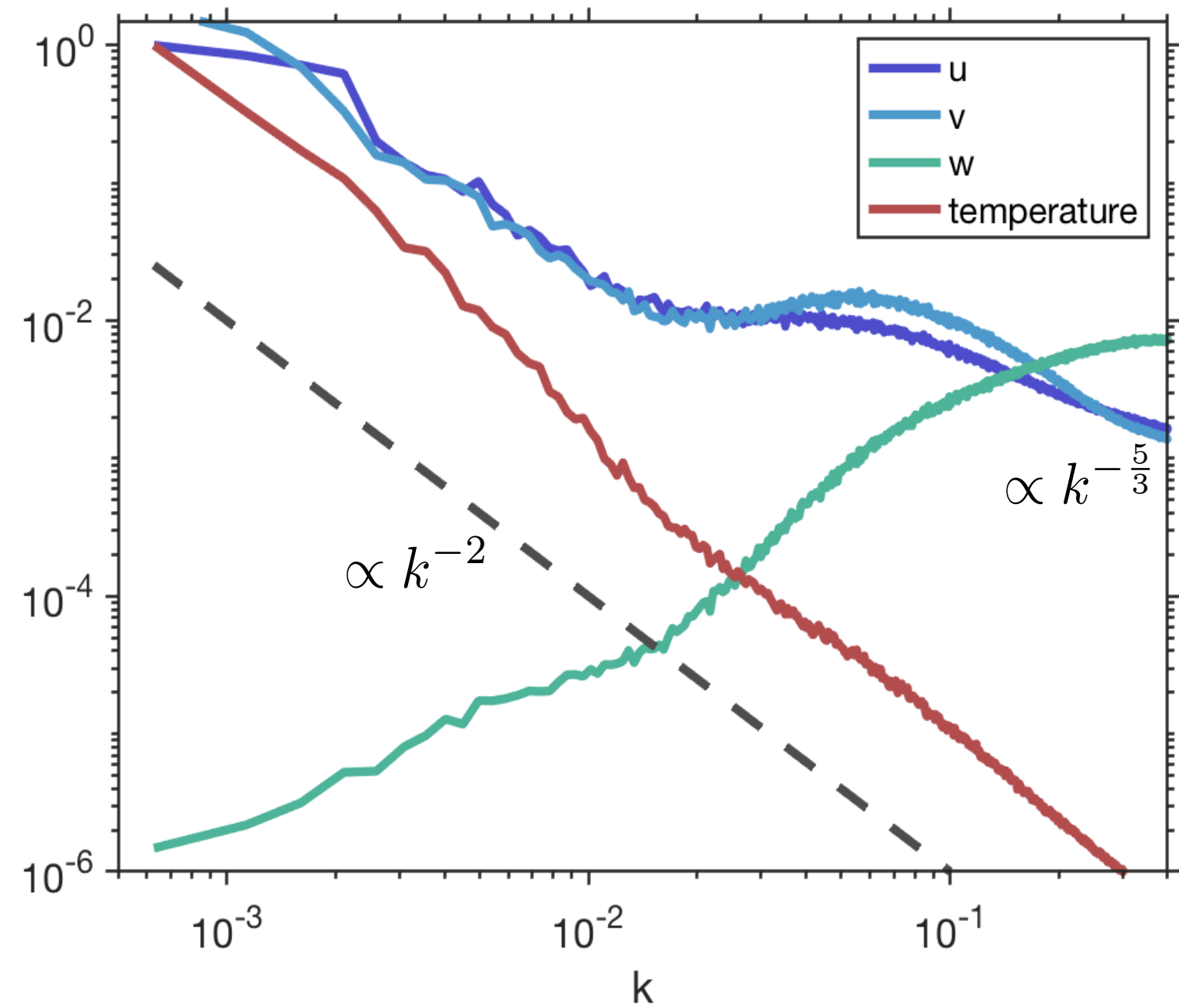
# Submesoscale limit defined by potential vorticity



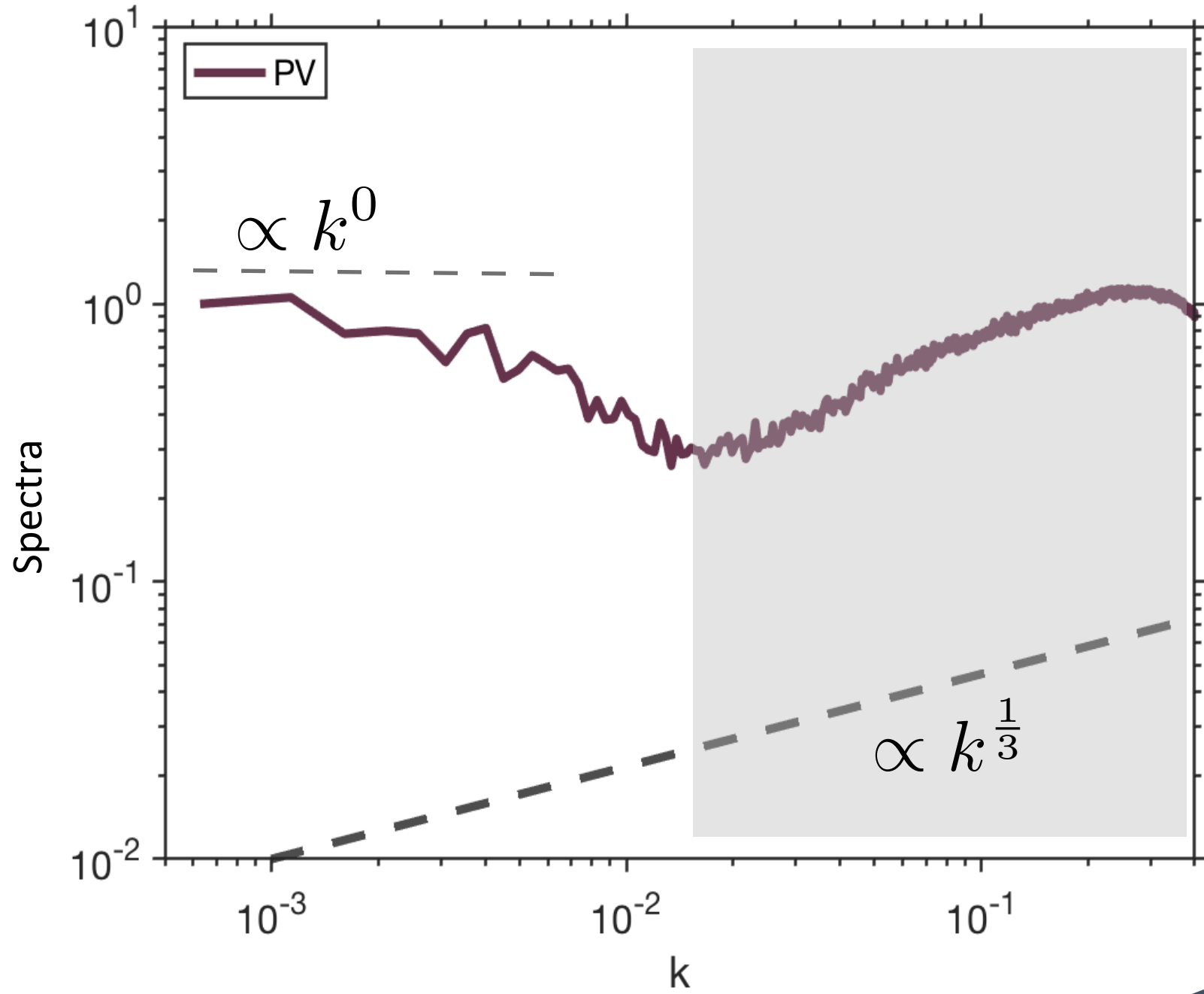
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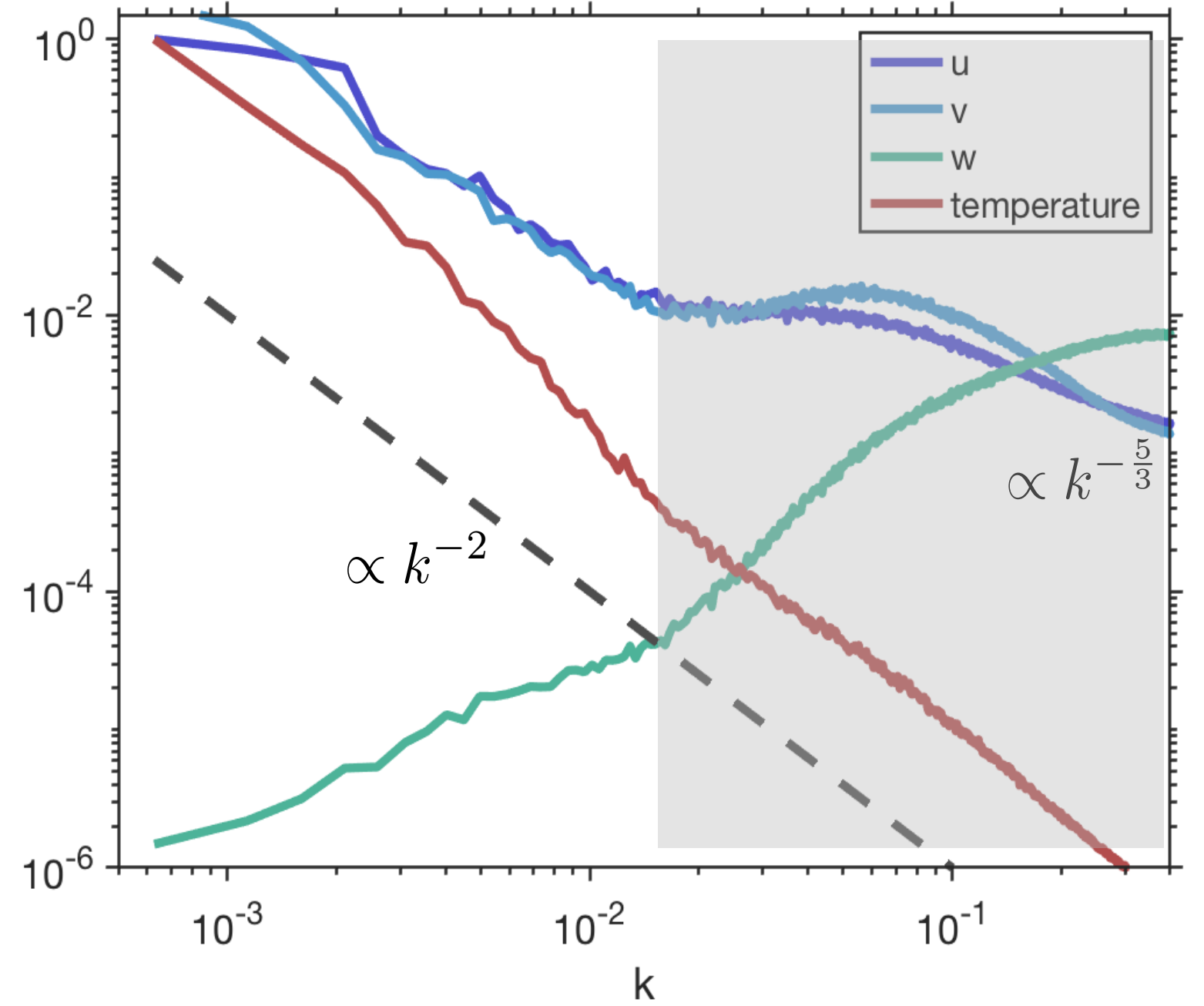
# filter velocity and temperature



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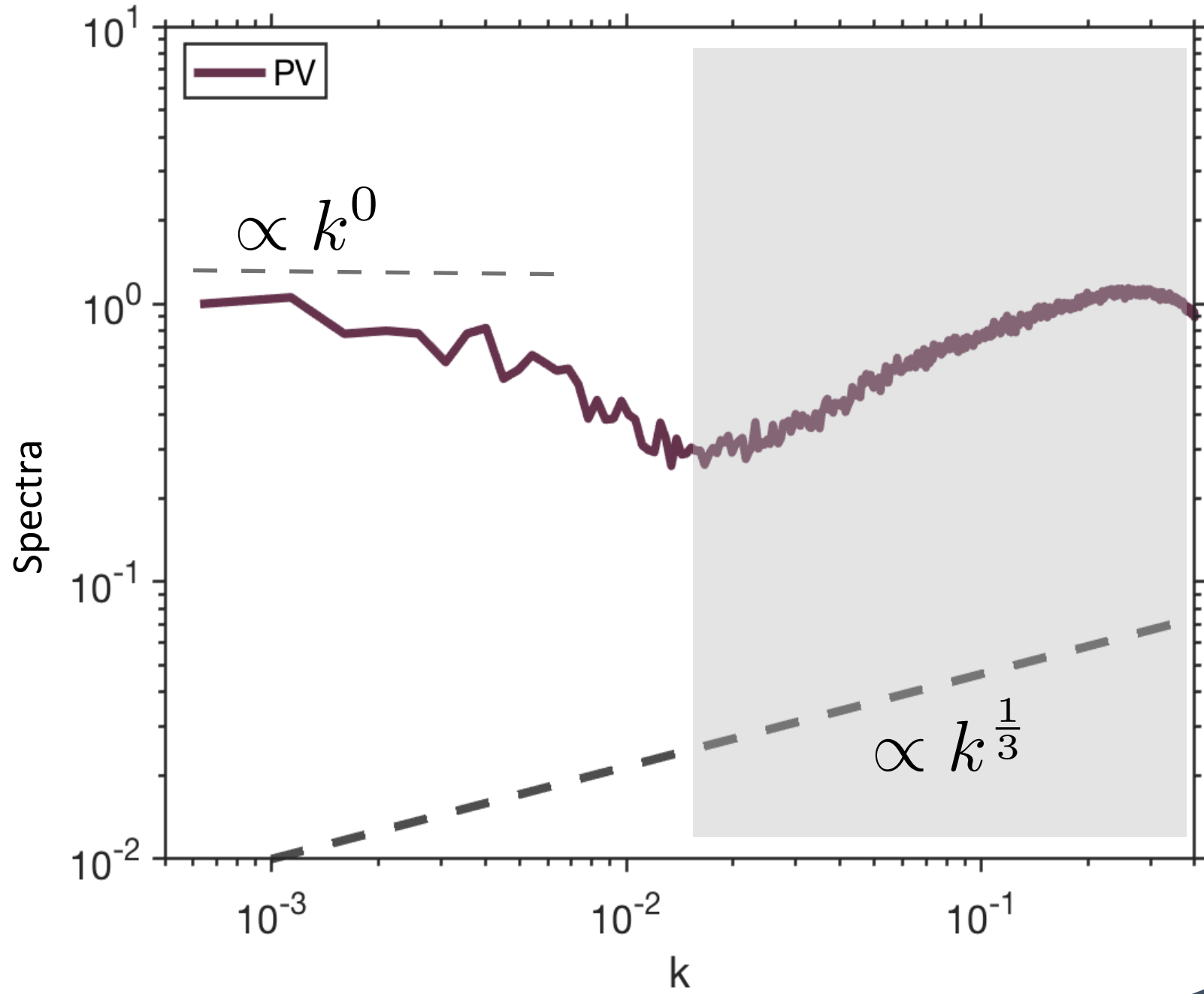


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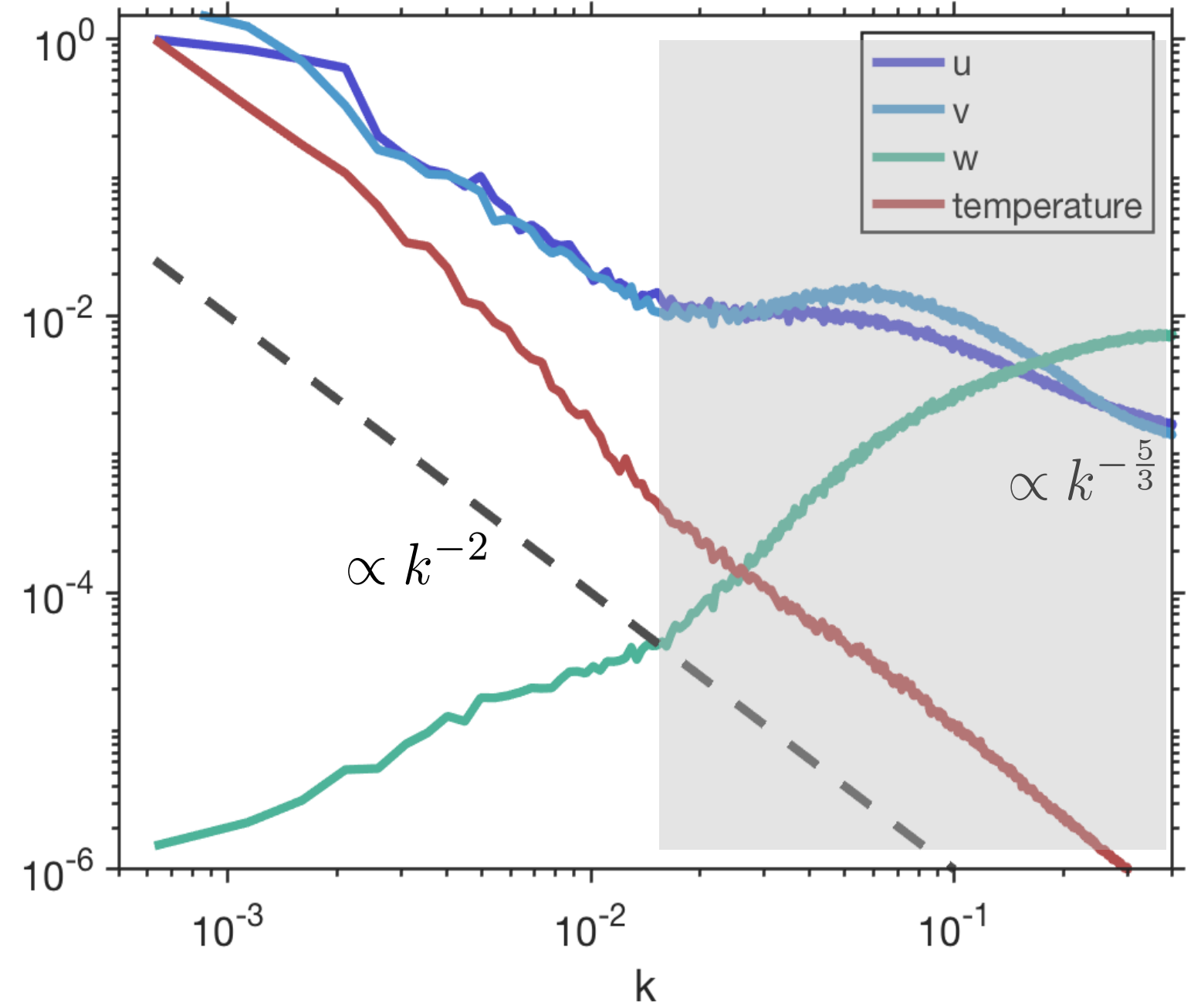


$\tilde{u}, \tilde{v}, \tilde{w}, \tilde{\theta}$

### Submesoscale limit defined by potential vorticity



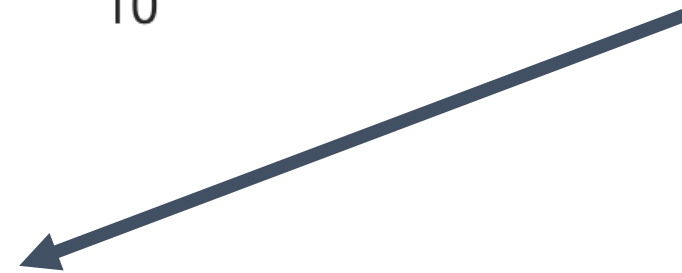
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$$\tilde{u}, \tilde{v}, \tilde{w}, \tilde{\theta}$$

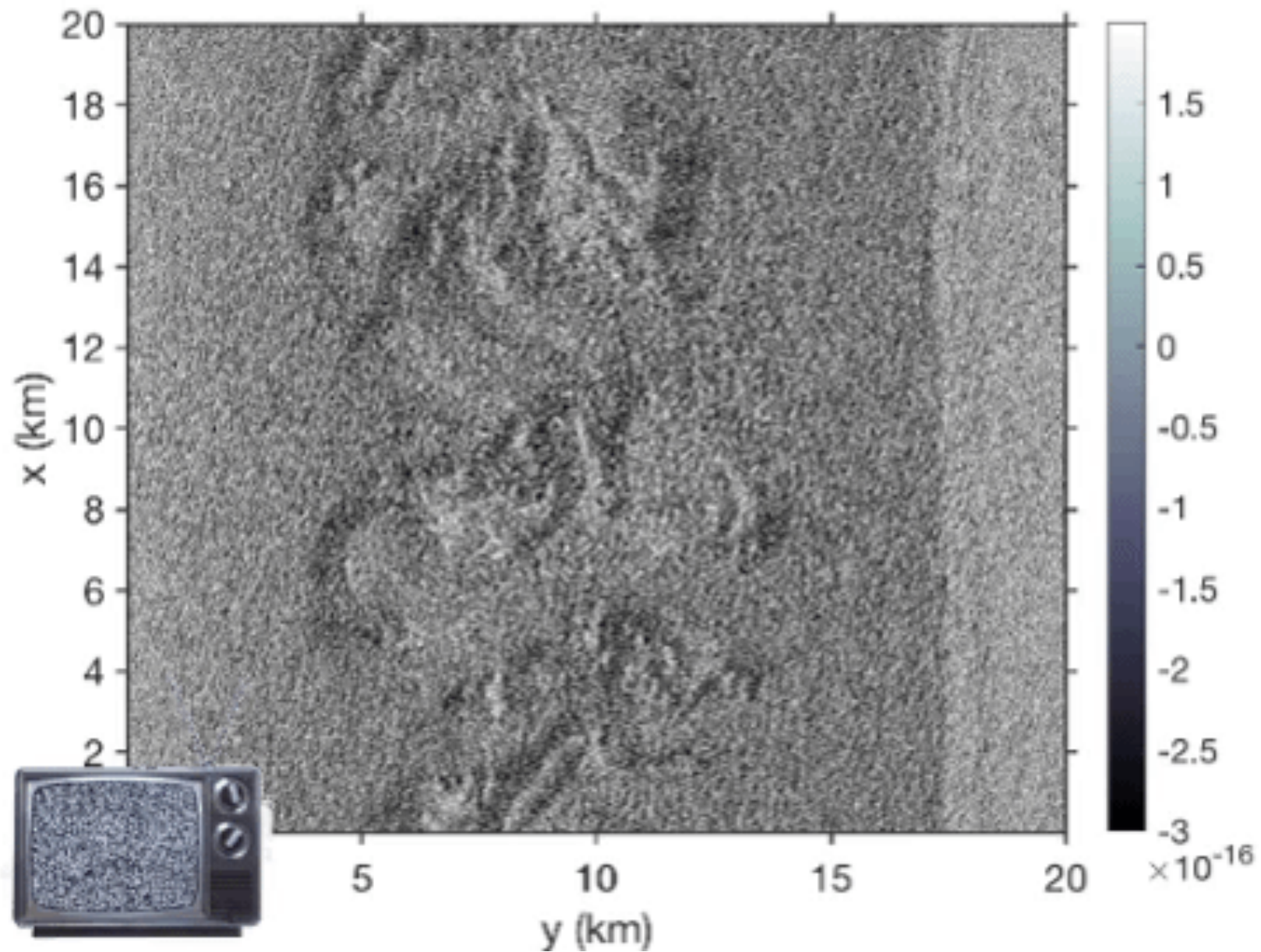


$$\tilde{q} = (f + \nabla \times \tilde{u}) \cdot \nabla \tilde{\theta}$$



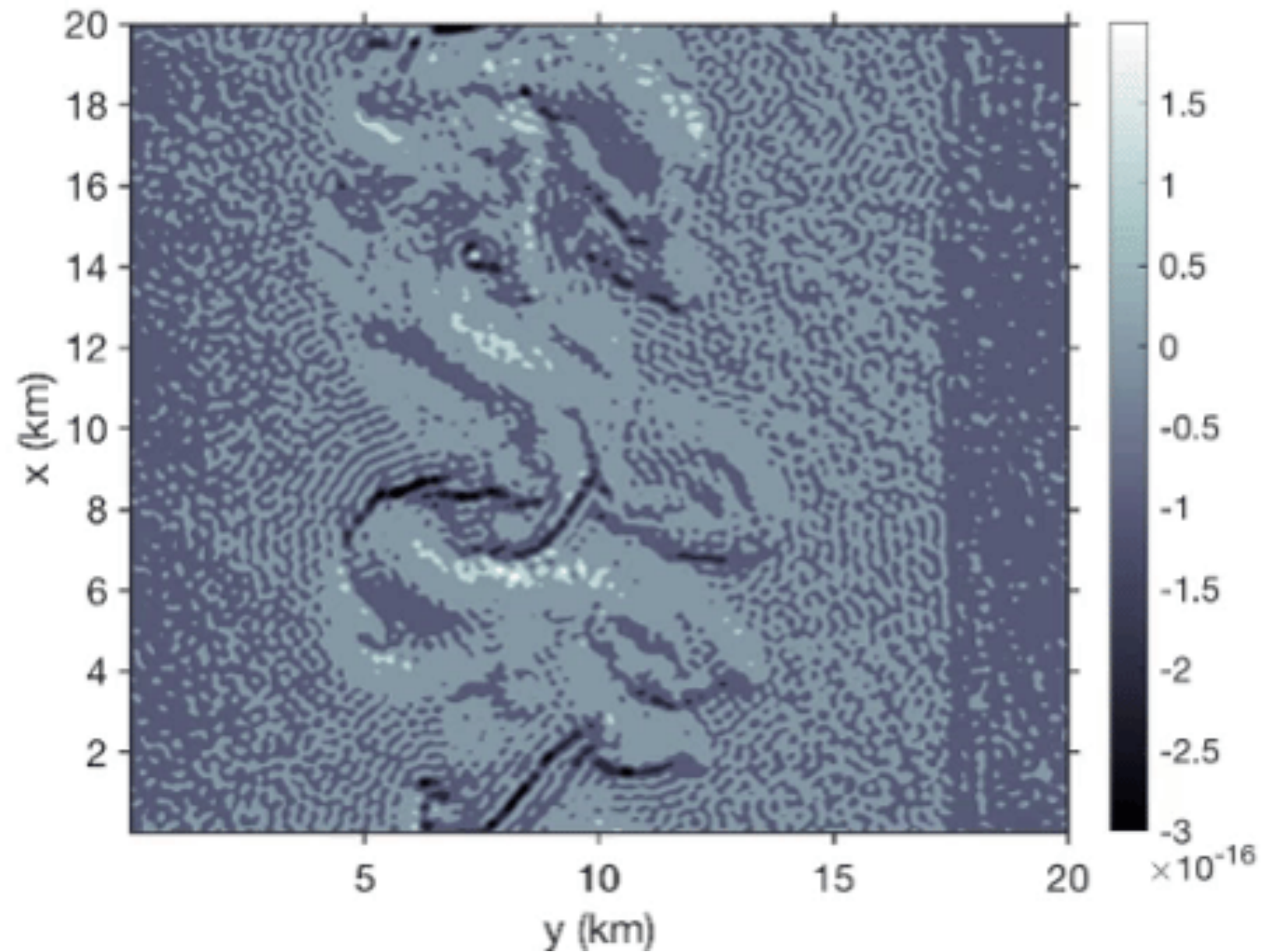
## “noisy” potential vorticity

$$q = (f + \nabla \times \vec{u}) \cdot \nabla \theta$$



## pre-filtered potential vorticity

$$\tilde{q} = (f + \nabla \times \tilde{u}) \cdot \nabla \tilde{\theta}$$





# Summary

- 1. The nonlinearity of PV results in an ultraviolet catastrophe**
- 2. The failure of PV signals the bottom limit of the submesoscales**
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