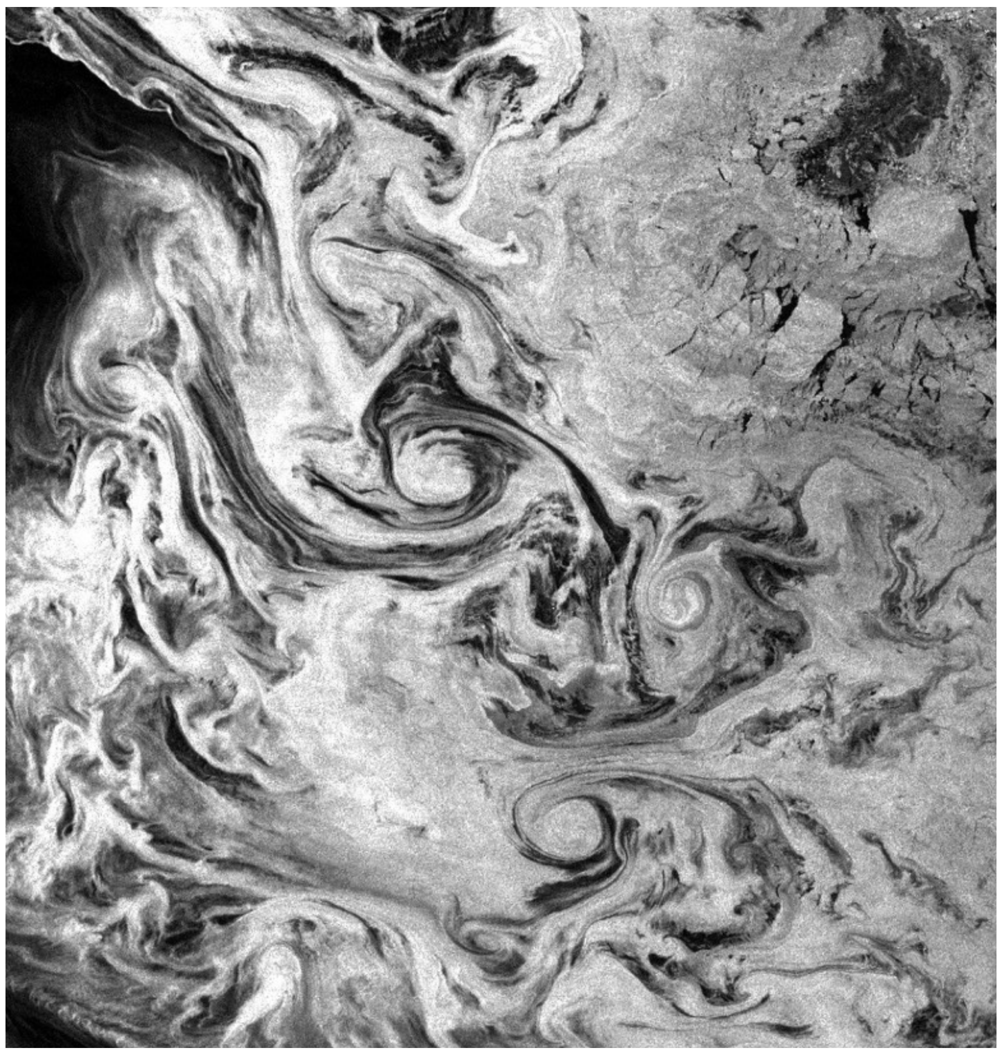
Characterising the chaotic nature of ocean ventilation

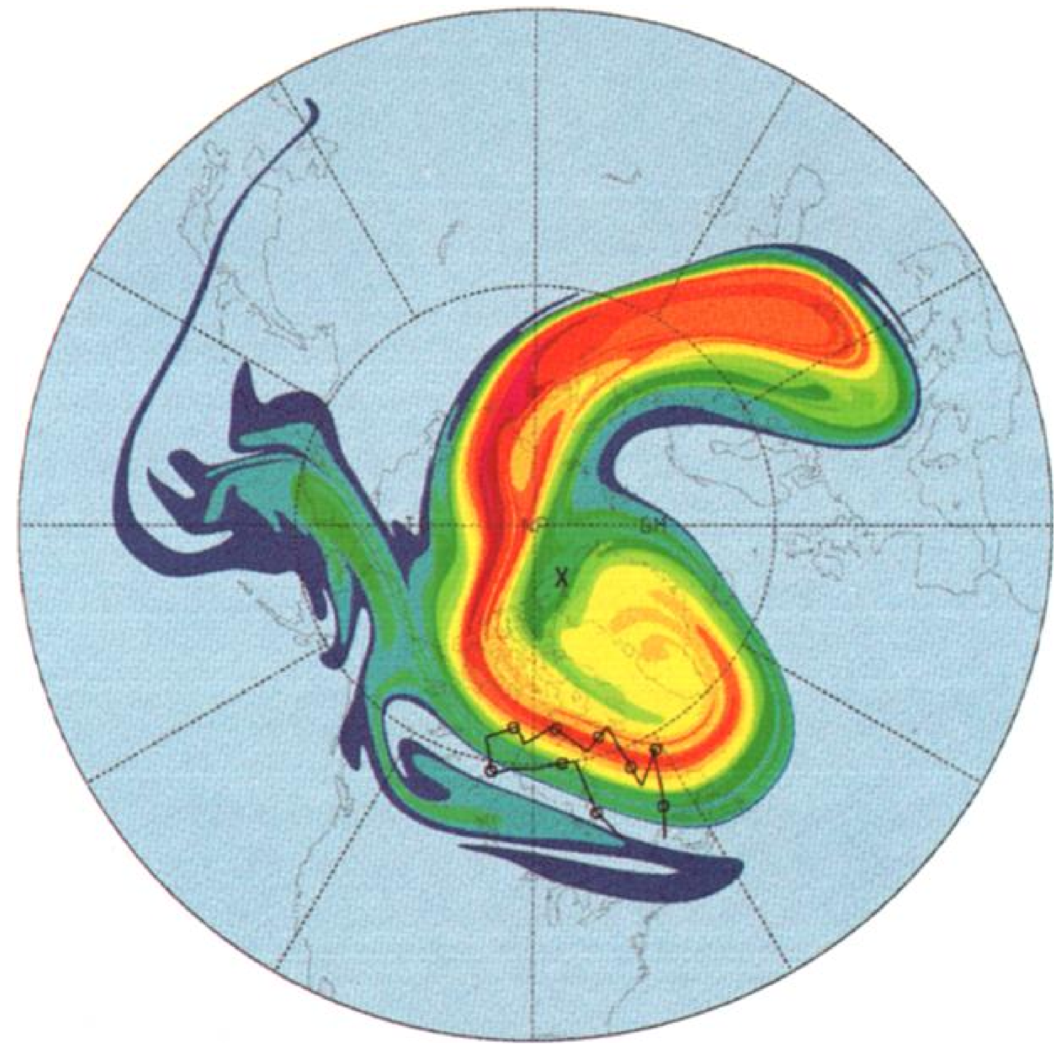
Graeme MacGilchrist, Helen Johnson, David Marshall, Camille Lique, Matthew Thomas

Eddy stirring and filamentation



Manucharyan and Thompson (2017)

Lagrangian tracing of filaments



Plumb *et al.* (1994)

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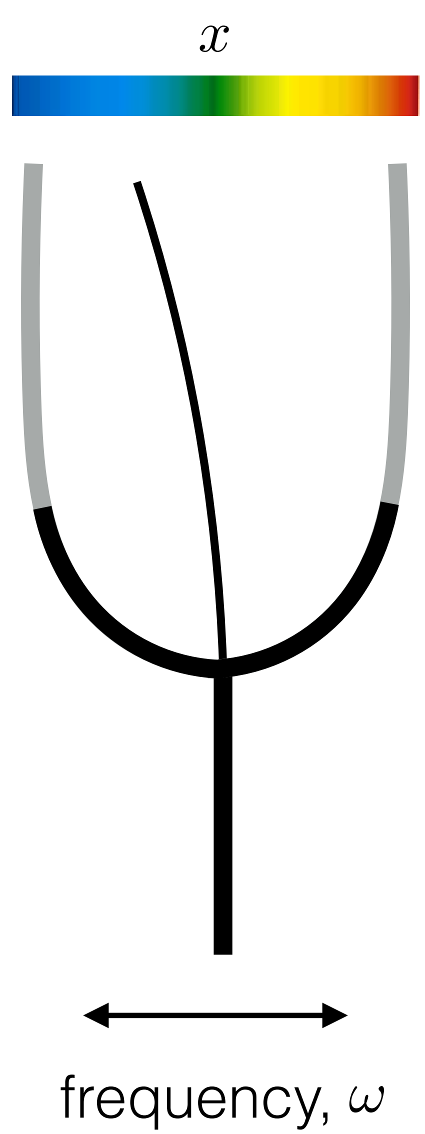
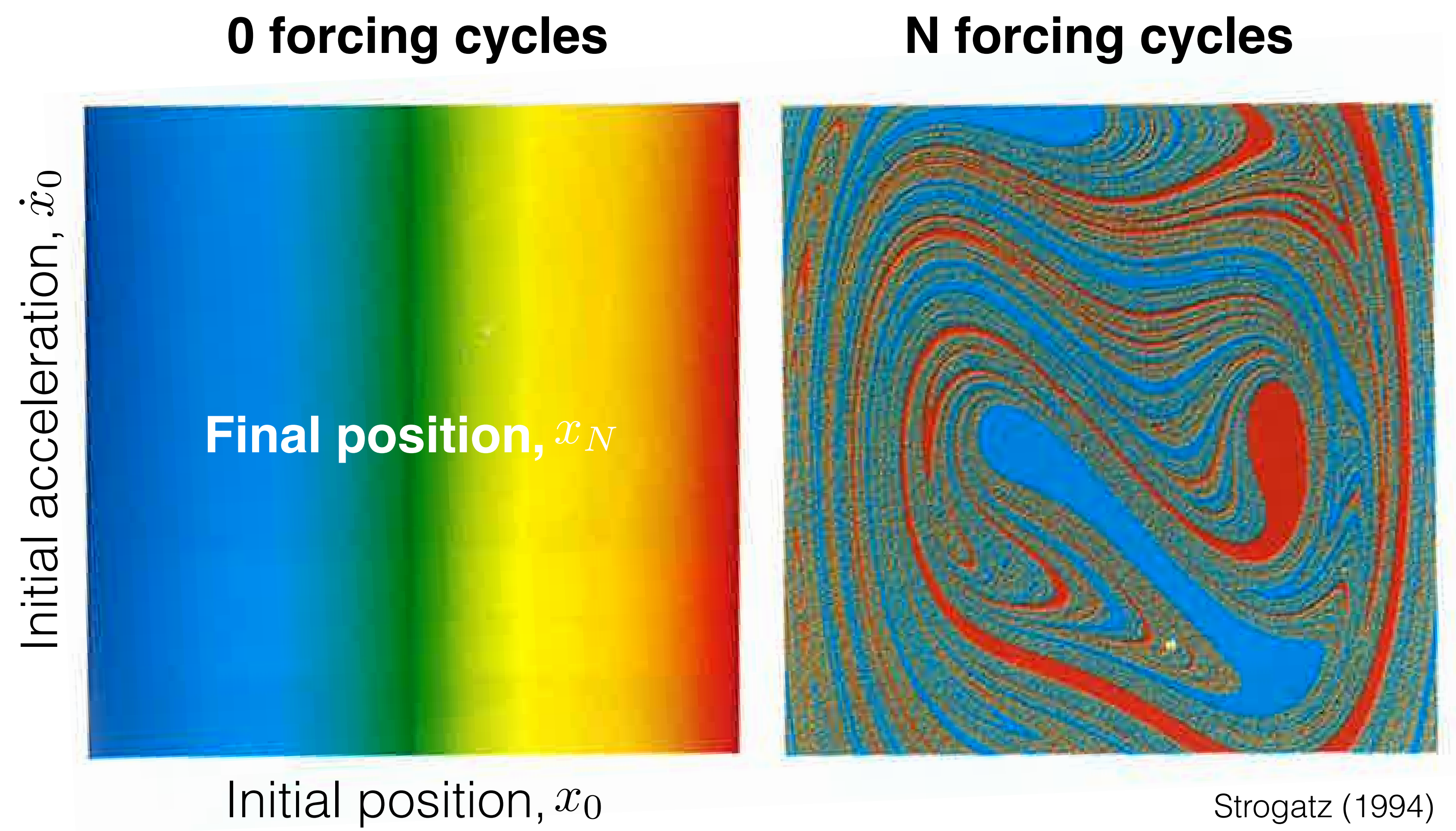
In nonlinear dynamical systems, *filament width* characterises the chaotic nature of trajectories by establishing *sensitivity to initial conditions*

Forced double-well oscillator



Lagrangian map from initial to final state



Strogatz (1994)

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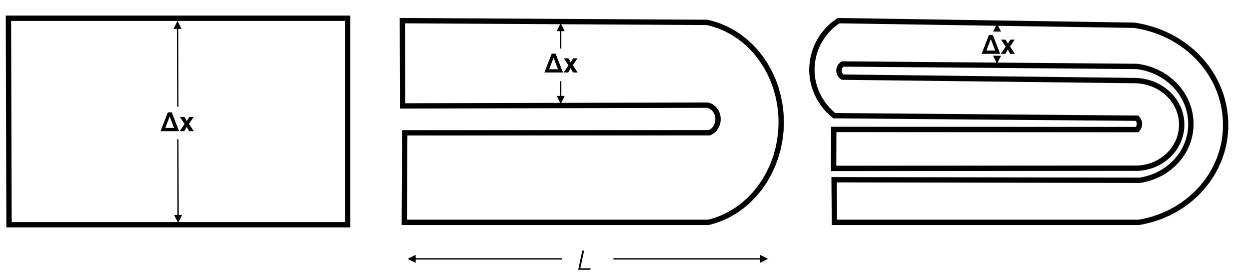
The thinning of filaments in dynamical systems is analogous to stretching and folding of puff pastry, at a rate defined by the *strain*



$$\frac{d\Delta x}{dt} = -\gamma \Delta x$$

$$\Delta x(t) = \Delta x(0) e^{-\int_0^t \gamma dt}$$
$$= \Delta x(0) e^{-\overline{\gamma}^t t}$$

$\overline{\gamma}^t$ *is the (average) vigour with which the baker rolls the pastry*

t *is the time they've been working for*

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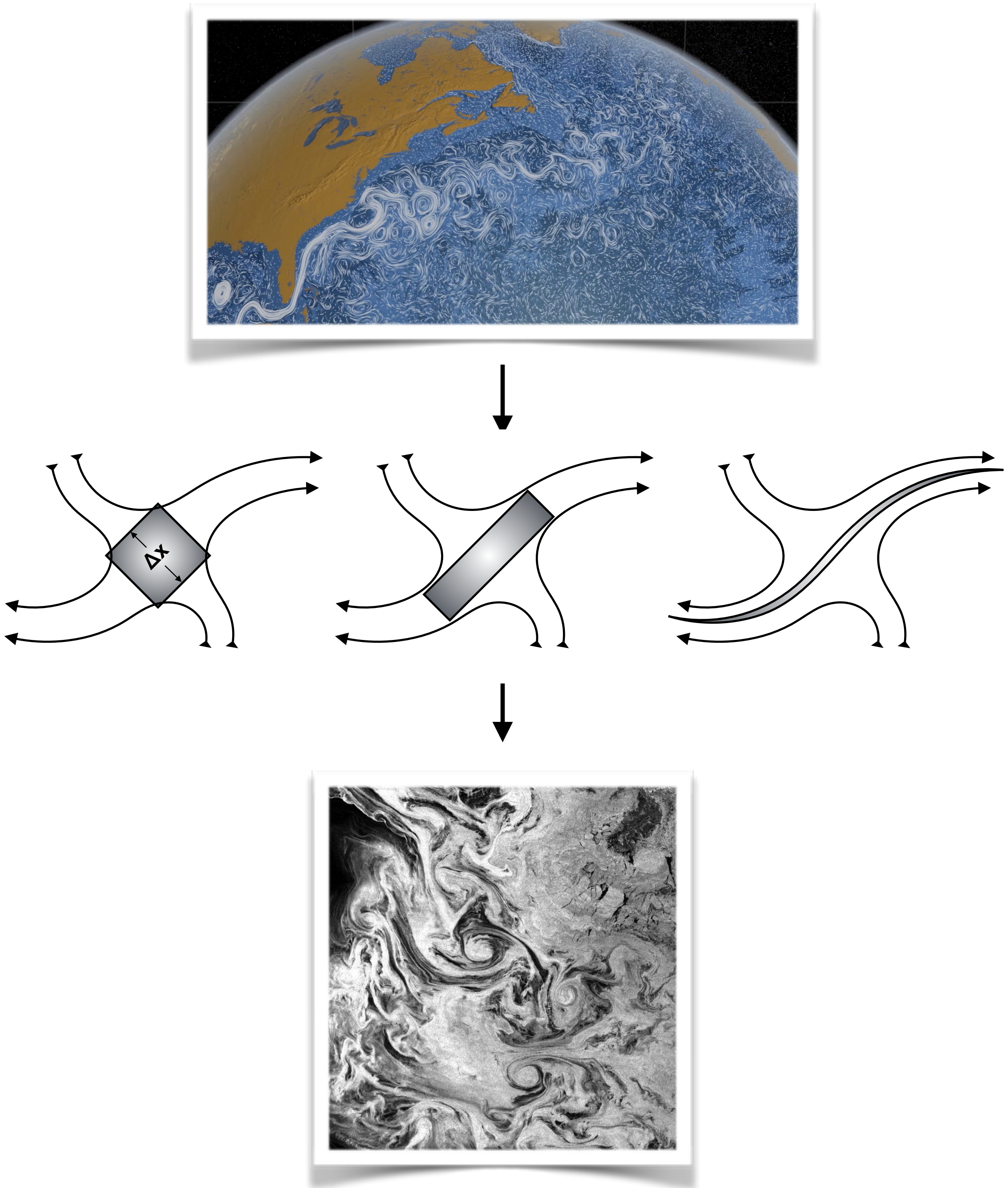
In the ocean, the role of the baker is played by the circulation, with the strain rate set by local velocity gradients



$$4\gamma^2 = \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2$$

$$\Delta x(t) = \Delta x(0)e^{-\overline{\gamma}^L t}$$

$\overline{\gamma}^L$ is the average strain rate following a Lagrangian trajectory

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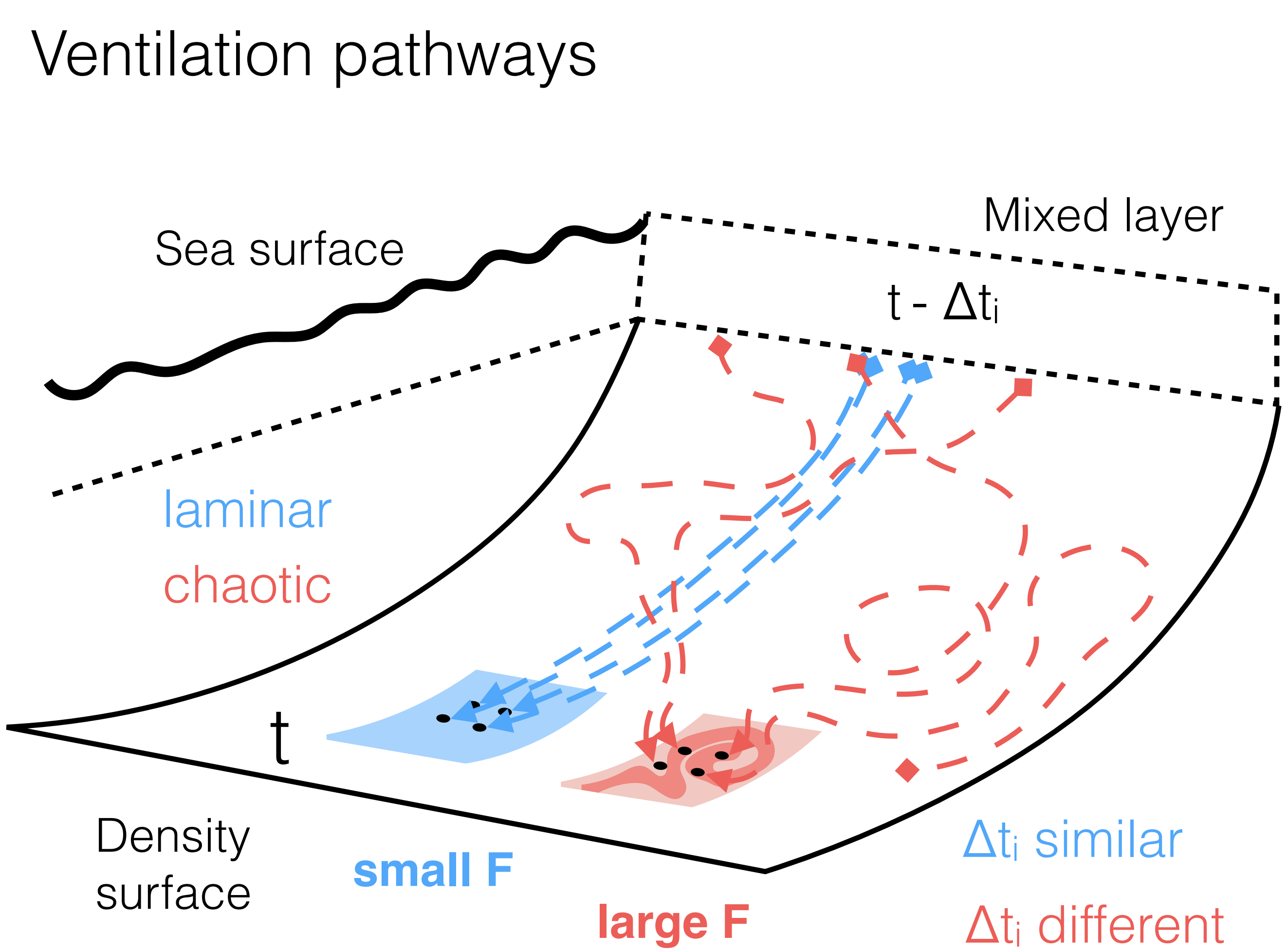
For a *ventilated* fluid parcel, the ‘time that the baker has been working for’ is the time since ventilation, allowing the definition of a *filamentation number,* **F**



$$\overline{\gamma}^L t = \frac{\tau_{\text{vent}}}{\overline{\tau}^L_{\text{strain}}} = F$$

$$\Delta x(t) = \Delta x(0) e^{-F}$$

In a region with F = 4, we would expect typically a 50-fold reduction in filament width since ventilation

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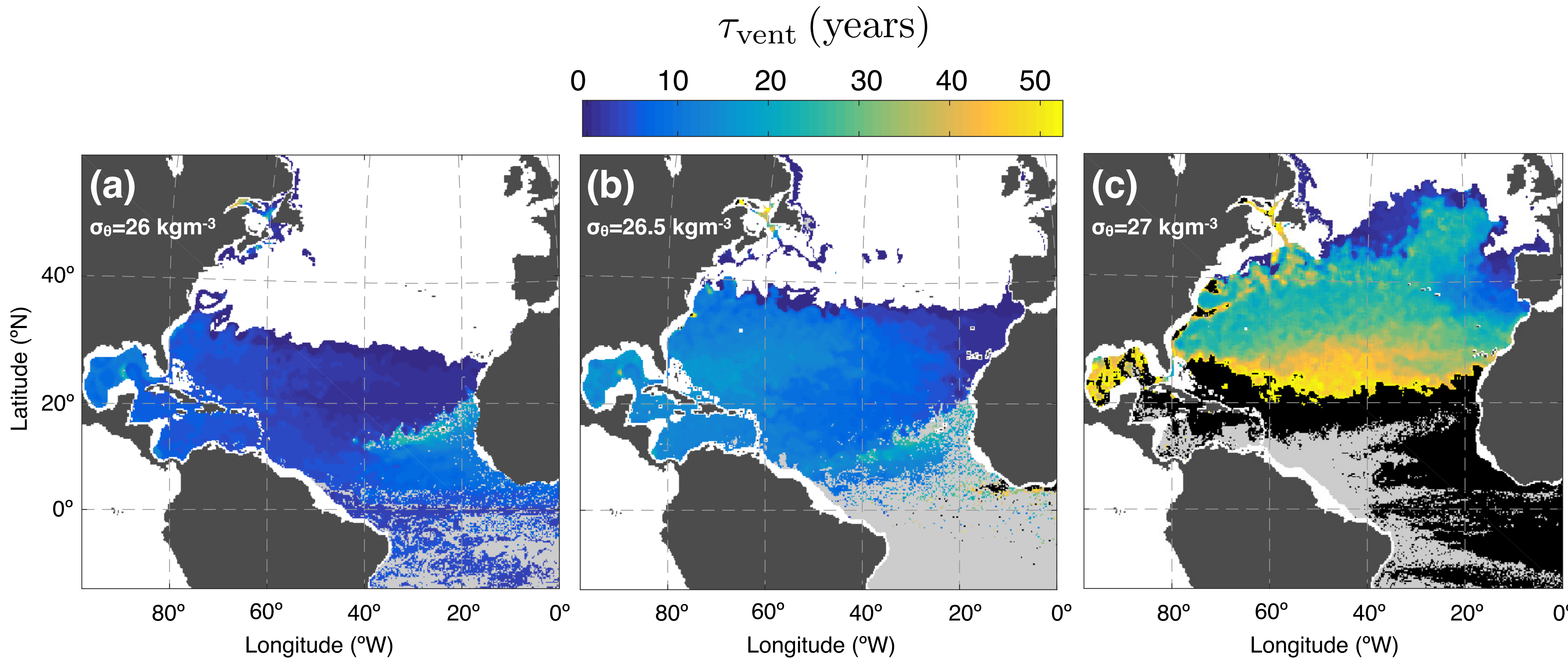
We calculated F in the subtropical thermocline of a 1/4° ocean model, using backwards-in-time Lagrangian trajectories



$\sigma_\theta = 26\ \text{kgm}^{-3}$	$\sigma_\theta = 26.5\ \text{kgm}^{-3}$	$\sigma_\theta = 27\ \text{kgm}^{-3}$
~100 - 300 m	~300 - 500 m	~400 - 700 m

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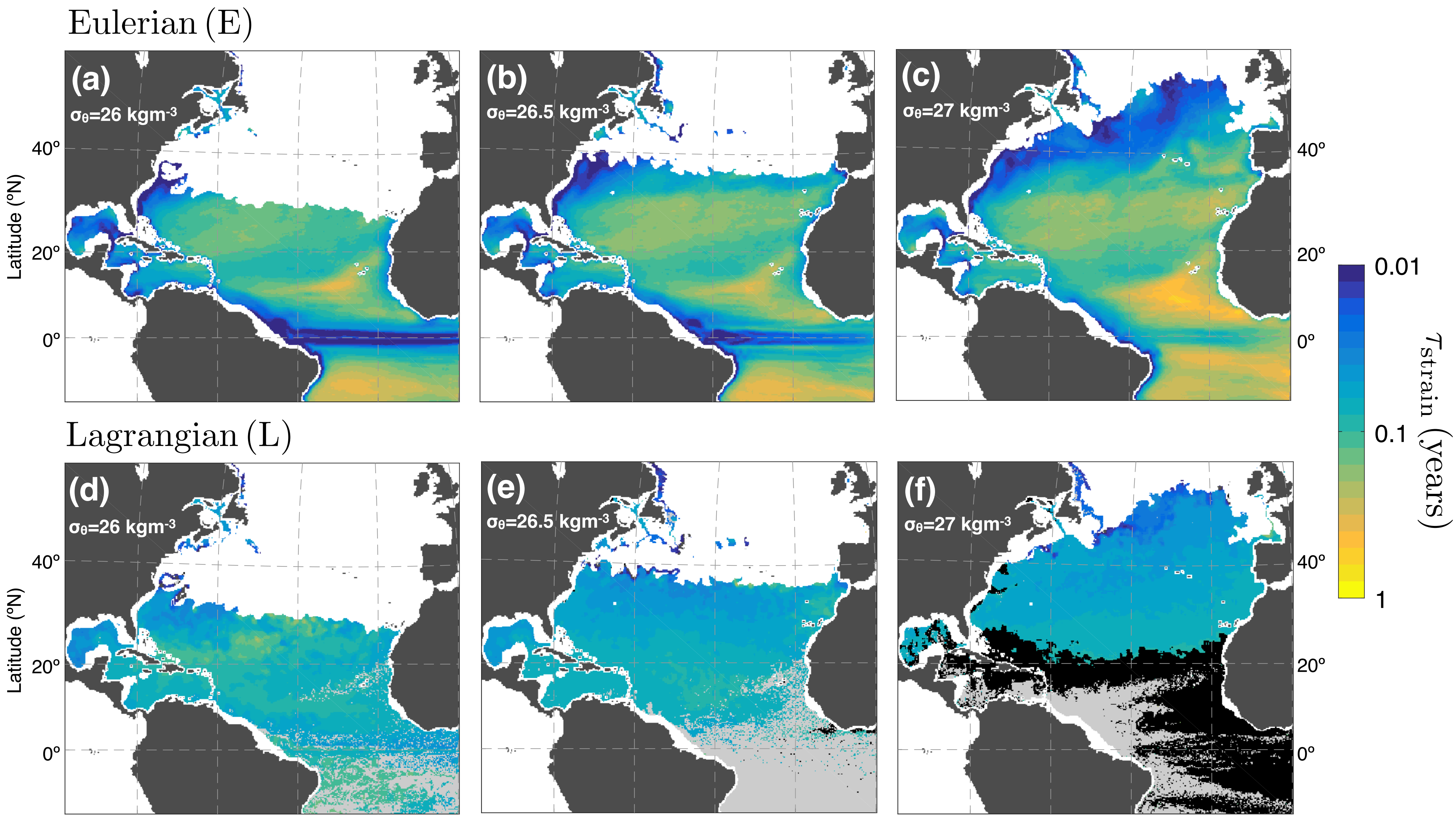

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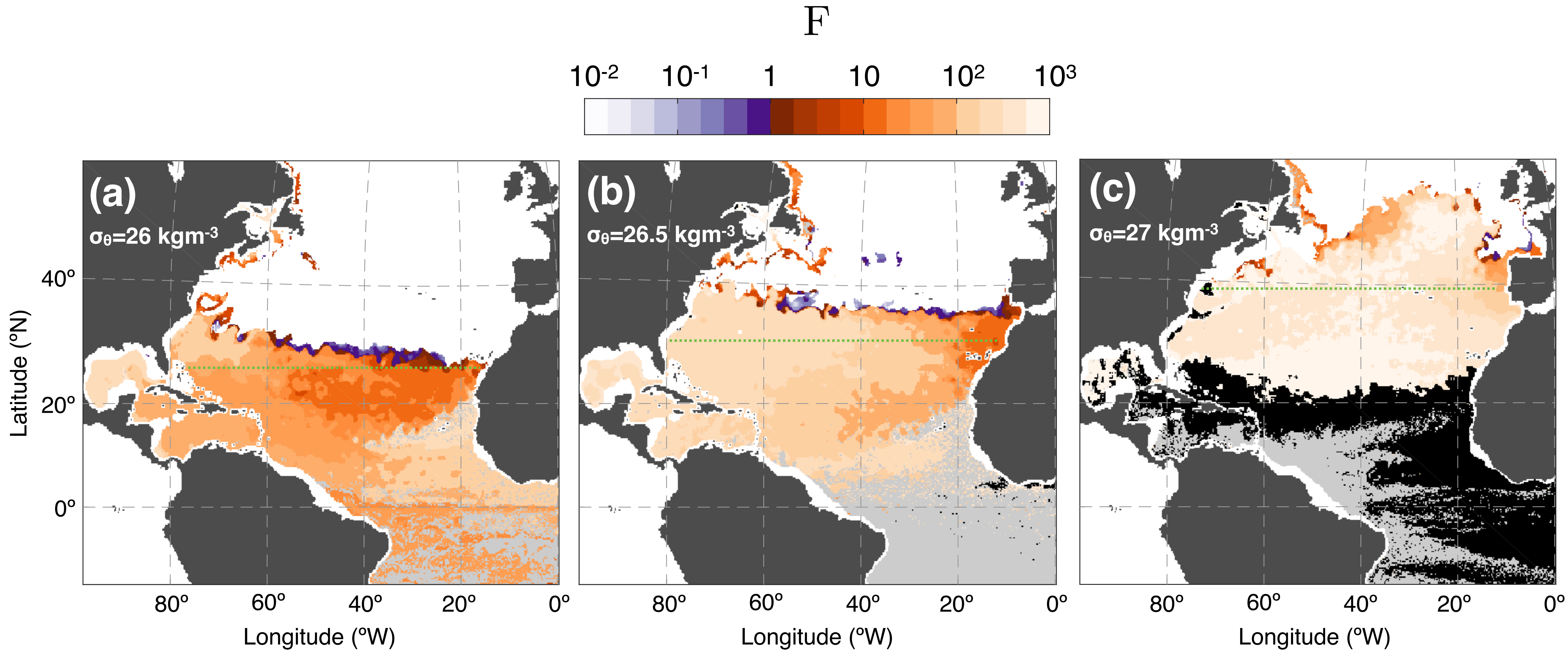

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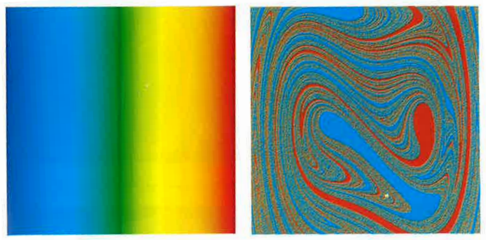
We resolve filaments directly, equivalent to a dynamical systems Lagrangian map, using year and longitude of ventilation as the 'final state'



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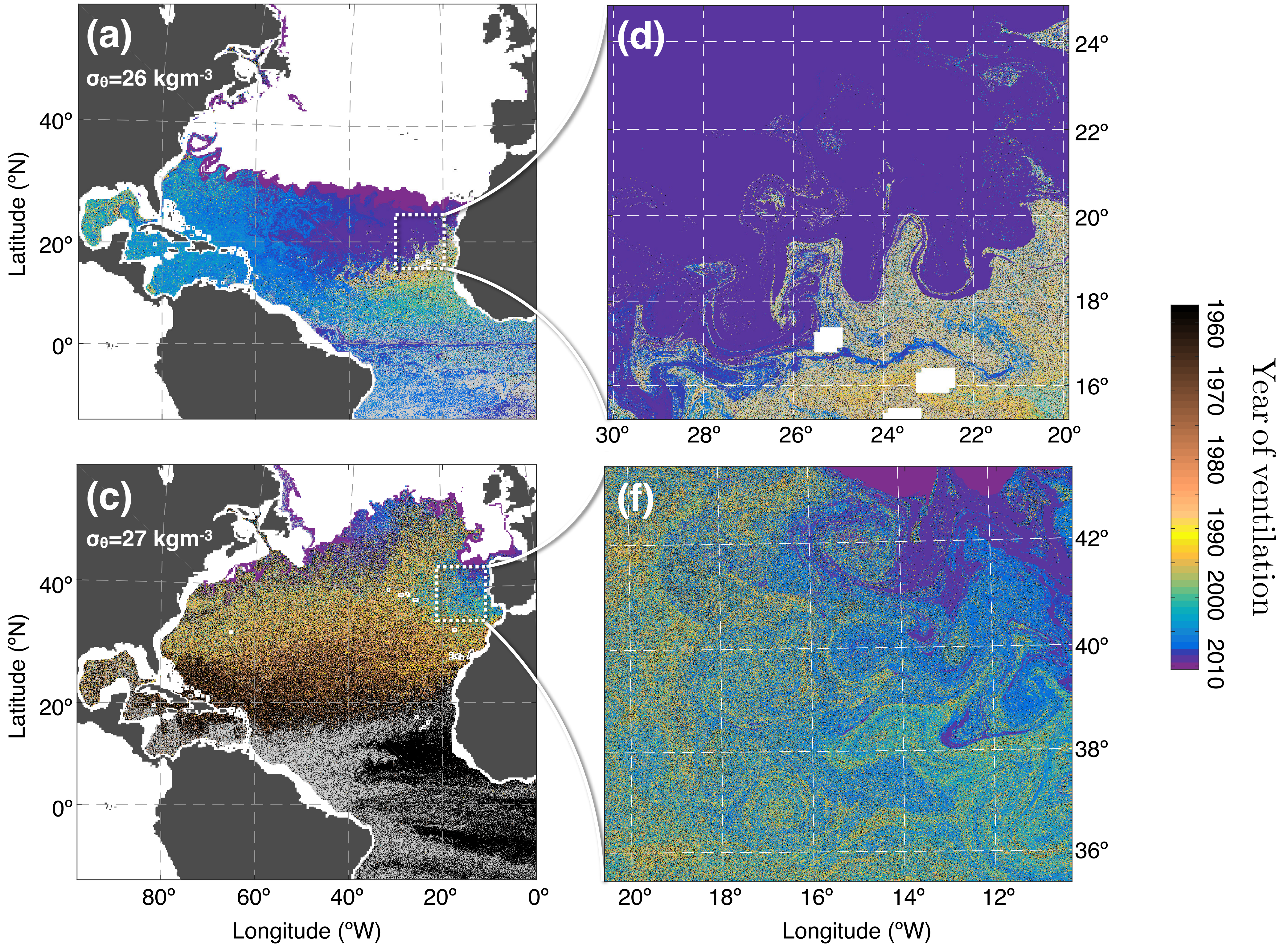

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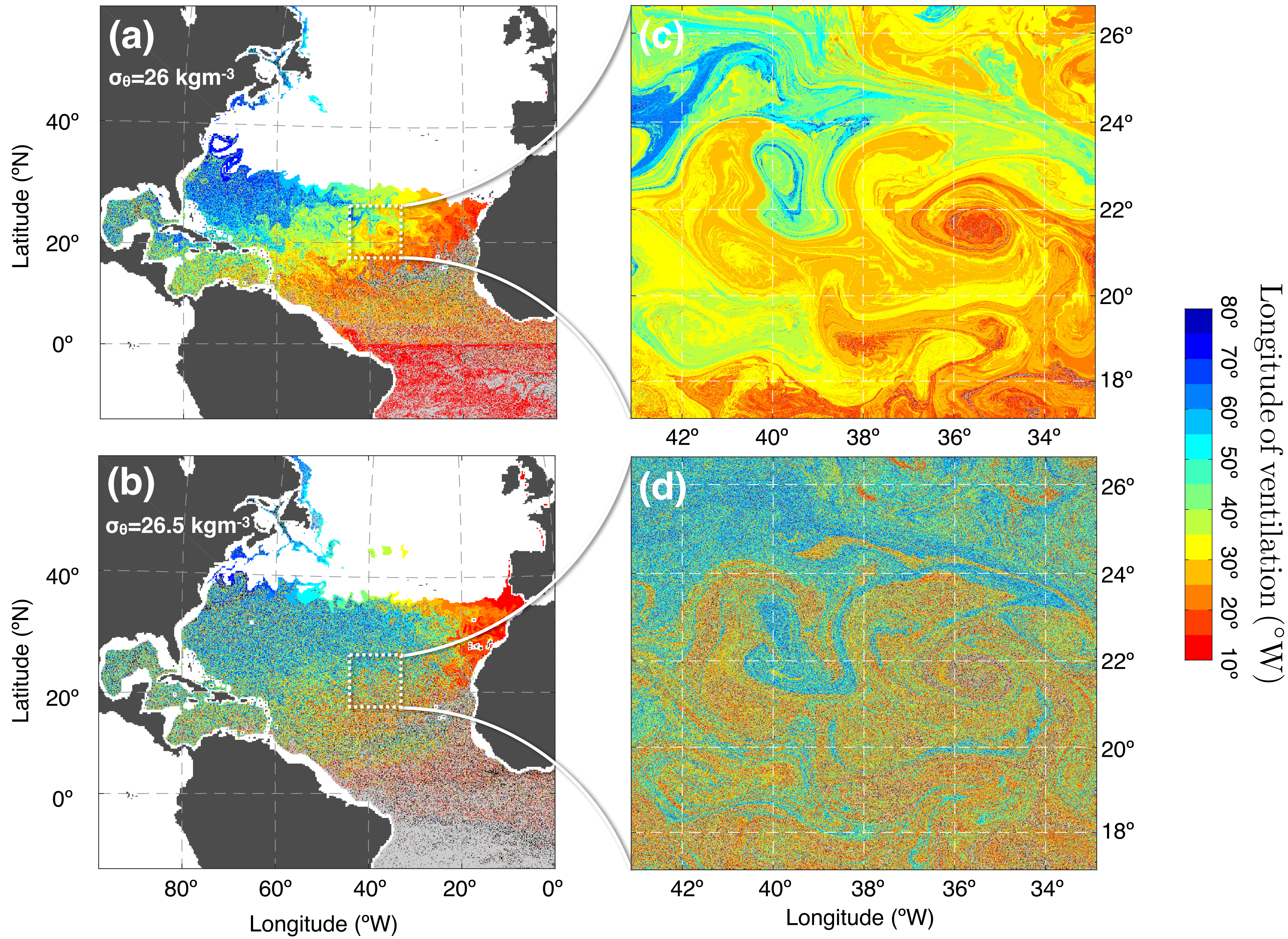

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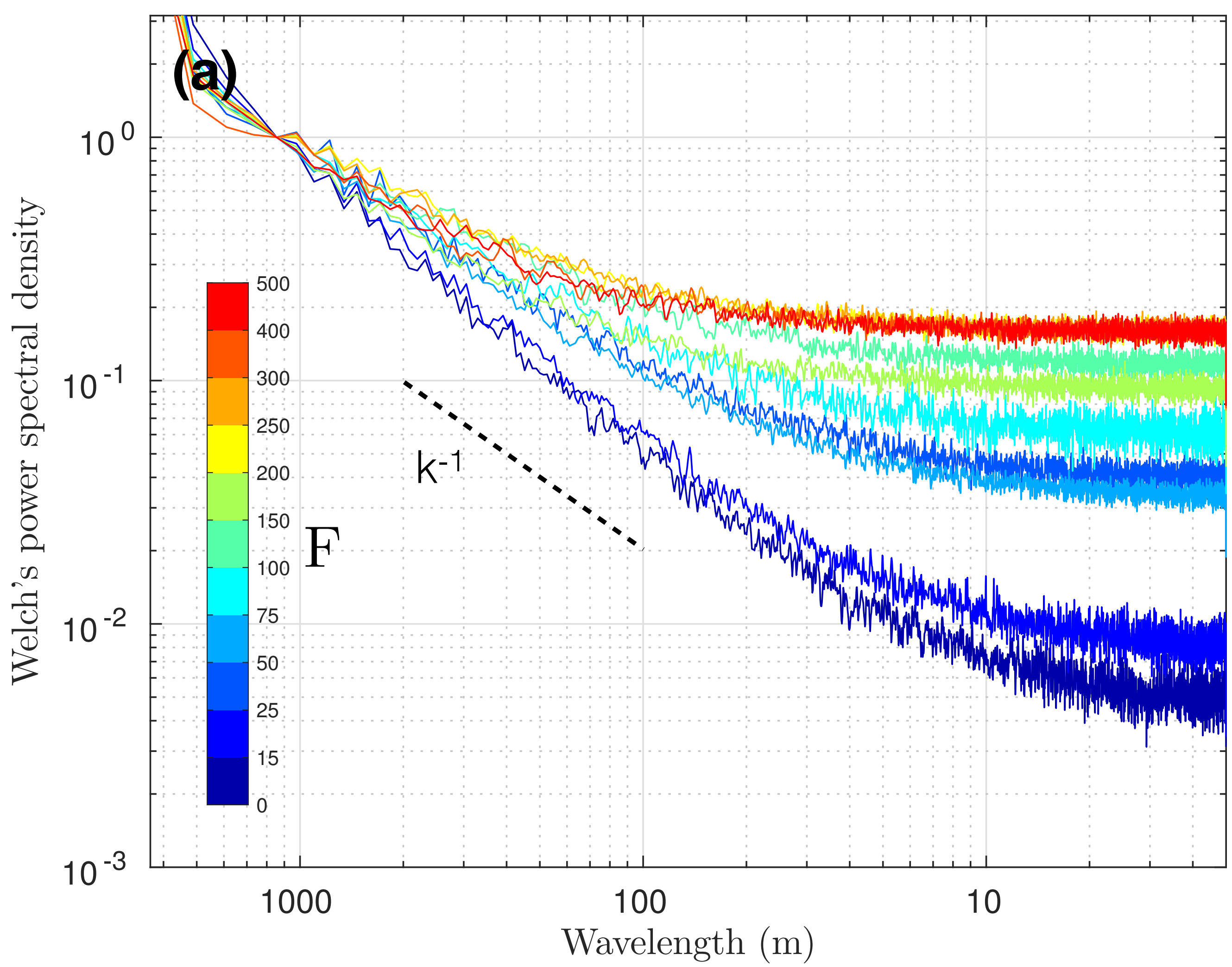
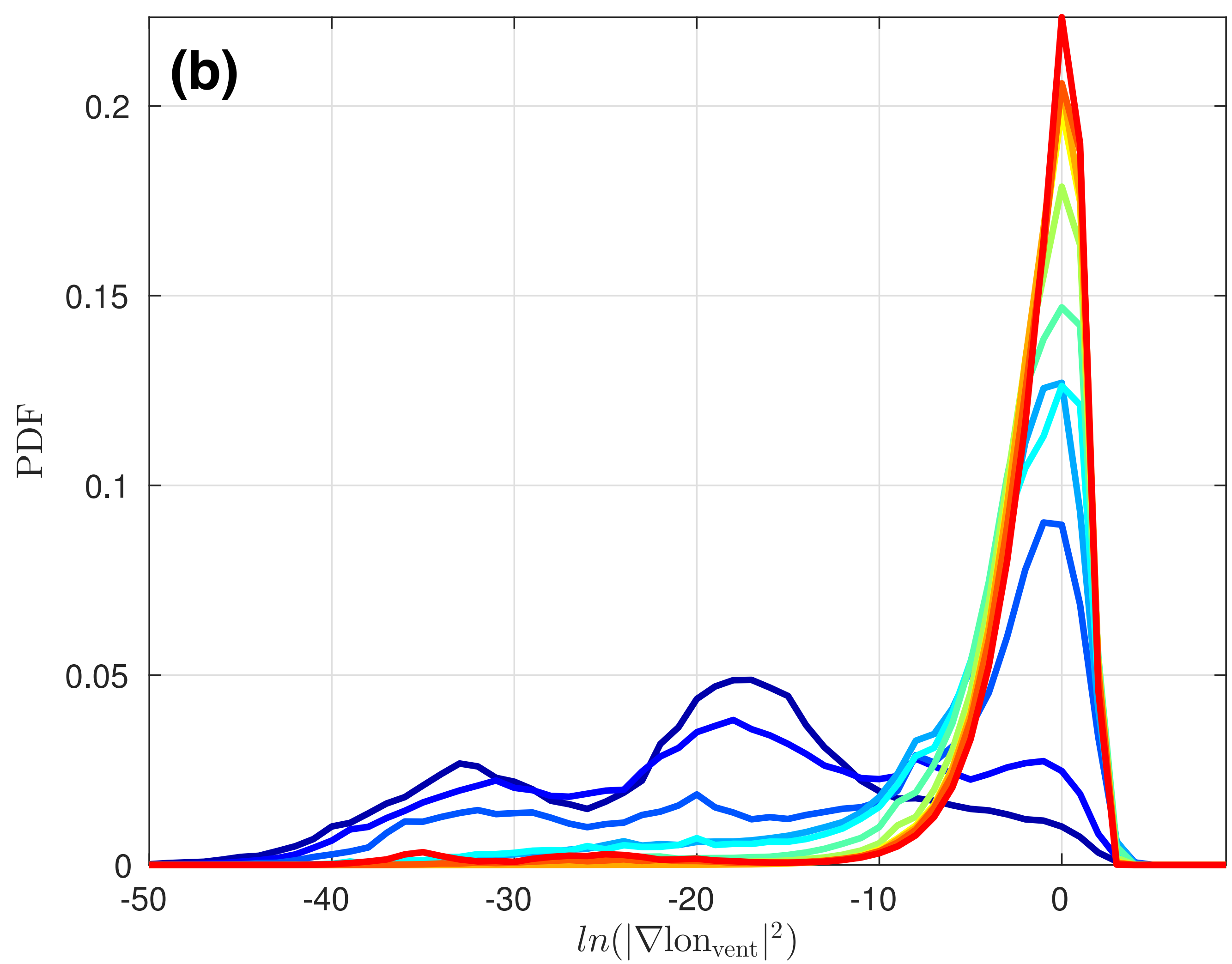
The filament width of the Lagrangian maps exhibits the expected behaviour: smaller filaments for larger F

Power spectra of ventilation longitude



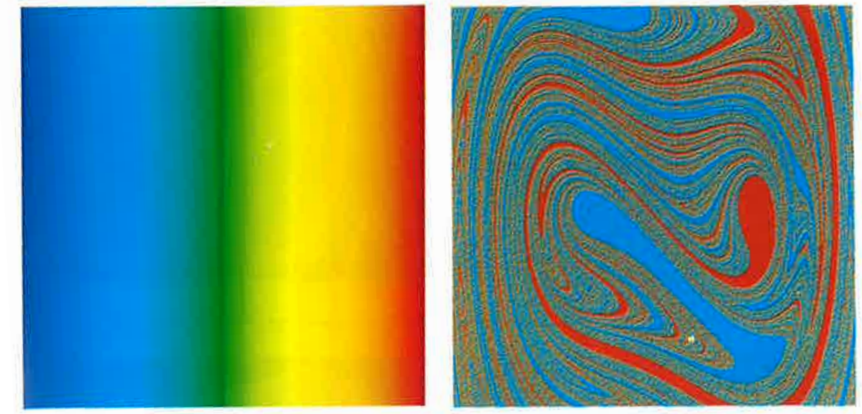
PDFs of ventilation longitude gradients

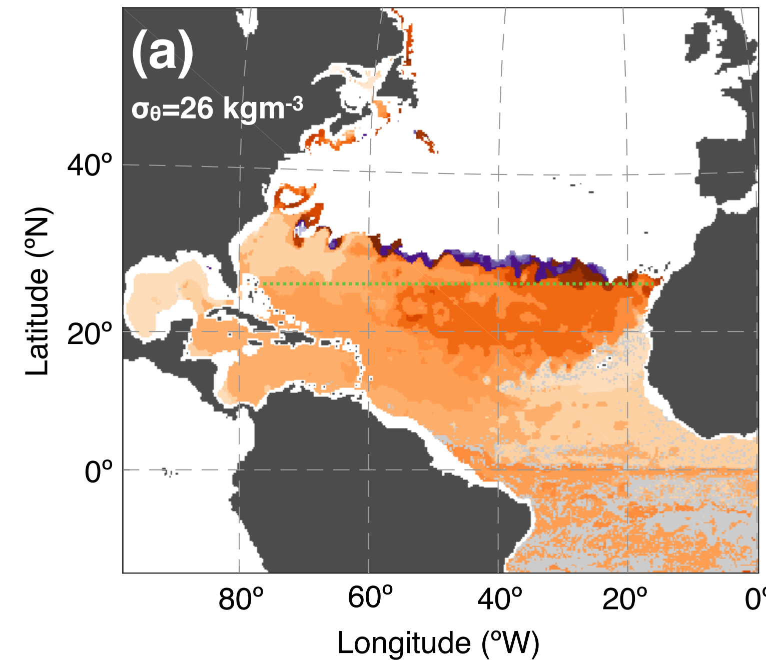


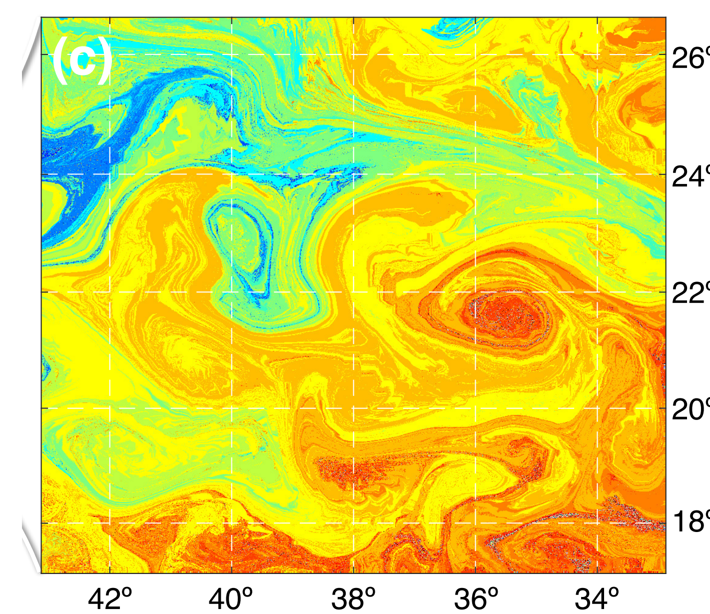
Summary

- By analogy to dynamical systems, the chaotic nature of ocean ventilation can be characterised by a reduction in filament width since subduction.
- This is quantified by the non-dimensional number *F*, a ratio of *ventilation* and *strain* timescales.
- *F* is large across three density surfaces in the subtropical North Atlantic thermocline.
- Resolving filament width directly (through backwards-in-time Lagrangian maps) shows the expected relationship with *F*.

MacGilchrist *et al.* (2017) Characterizing the chaotic nature of ocean ventilation, *JGR Oceans*, 122.







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