Geometry and Energetics of Ocean Mesoscale Eddies and Their Rectified Impact on Climate (GEOMETRIC)

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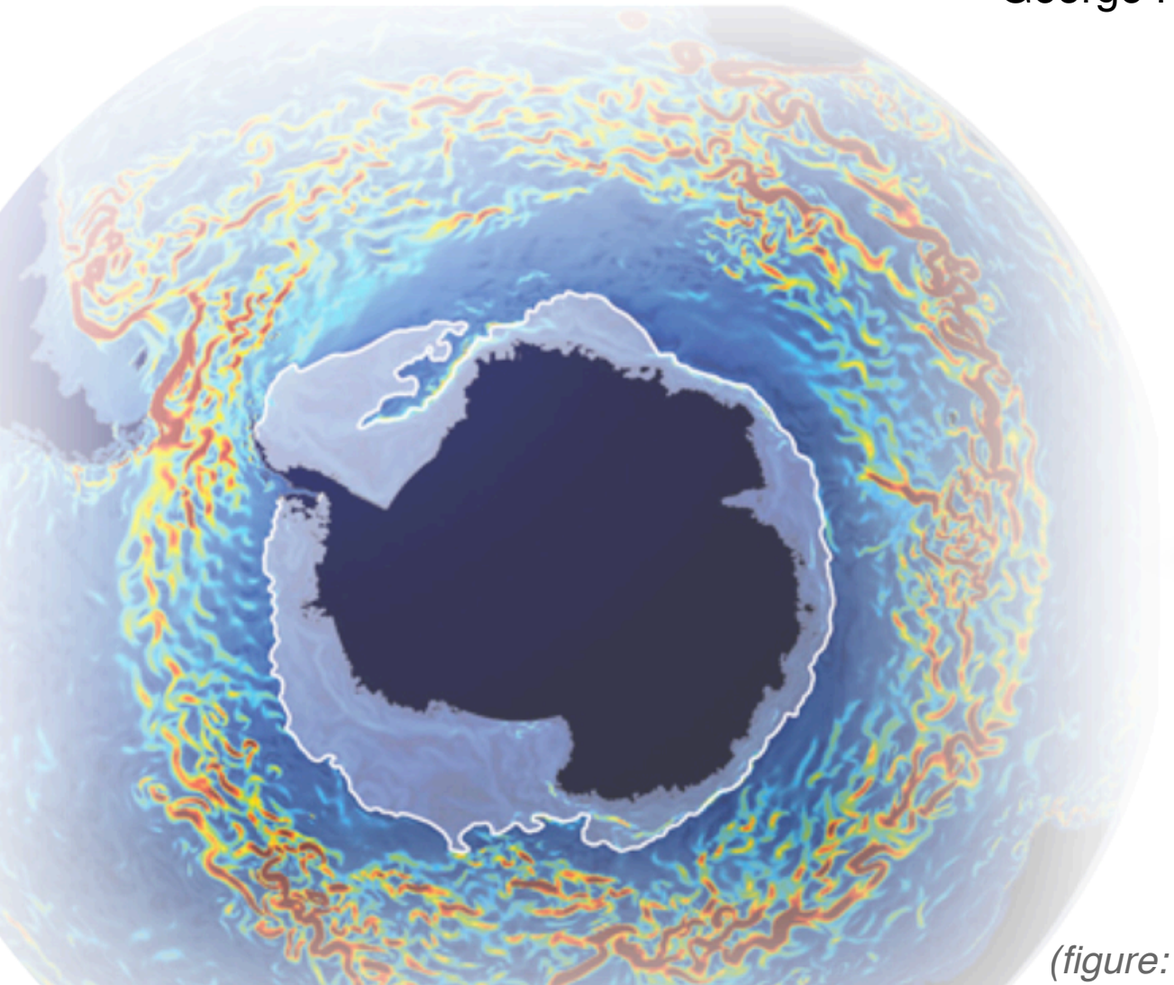


(figure: SOSE)

Structure:

- GEOMETRIC
- Proofs of concept
- Implementation in NEMO

(NEMO results removed from public copy)

NERC SCIENCE OF THE ENVIRONMENT

GEOMETRIC

(Marshall et al., 2012)

Goal: Develop a framework for **parameterising** and **interpreting** ocean eddy-mean flow interaction in which the relevant **symmetries** and **conservation laws** are preserved as far as possible

1. residual-mean momentum equation

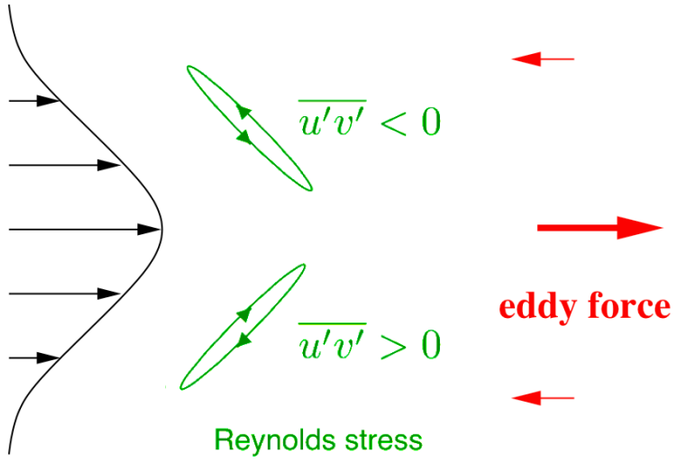
$$\frac{\partial \overline{\mathbf{u}}}{\partial t} = \ldots - \nabla \cdot \mathbf{E} + \text{forcing} - \text{dissipation}$$

eddy forcing

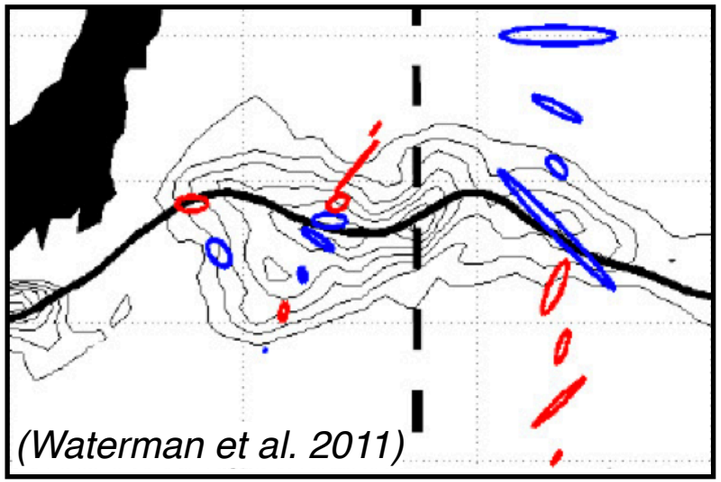
2. consistent eddy energy equation

$$\frac{\partial E}{\partial t} = \nabla \cdot (\ldots) + \overline{\mathbf{u}} \cdot \nabla \cdot \mathbf{E} + \text{forcing} - \text{dissipation}$$

(cf. Young, 2012; Maddison and Marshall, 2013)

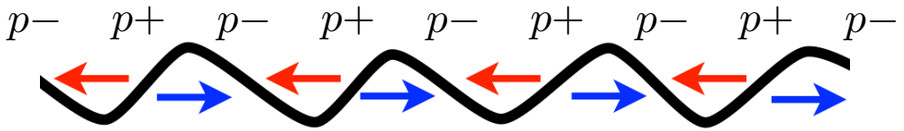


- eddy stress tensor $\mathbf{E}$ includes eddy Reynolds stresses and form stresses
- dimensional size of $\mathbf{E}$ set by eddy energy E
- **all** remaining unknowns are dimensionless and ≤ 1 in magnitude, measure anisotropy and direction of eddy fluxes


(Waterman et al. 2011)

Initial implementation: only vertical momentum flux - “eddy form stress”

vertical momentum transfer is equivalent to Gent and McWilliams *(Greatbatch and Lamb, 1990; Greatbatch, 1998)*



eddy form stress:

$$S = \frac{f}{N^2}\overline{b'v'} = \alpha\frac{f}{N}E \qquad (|\alpha| \leq 1)$$

$\Leftrightarrow$

Gent and McWilliams eddy diffusivity:

$$\kappa = \alpha\frac{N}{\partial\bar{b}/\partial y}E \qquad (|\alpha| \leq 1)$$

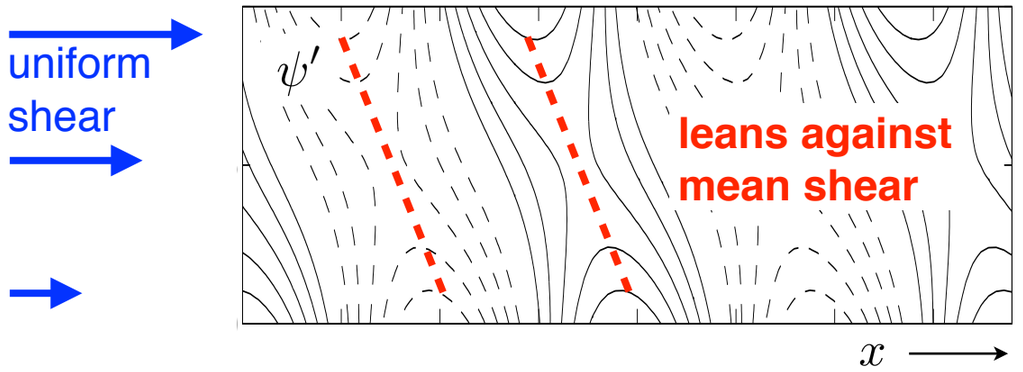
(Marshall et al., 2012; also see Jansen et al., 2015)

if eddy energy known, **only freedom is to specify the non-dimensional parameter** α

Proof of concept 1: Eady growth rate

(Marshall et al., 2012)

most **unstable** mode:



(figure: adapted from Vallis 2006)

Eddy energy budget:

eddy form stress

$$\frac{\partial}{\partial t}\iiint E\,dx\,dy\,dz = -\iiint \overline{u}\frac{\partial S}{\partial z}\,dx\,dy\,dz$$

substitute eddy stress tensor

$$= \iiint \frac{\partial \overline{u}}{\partial z} S\,dx\,dy\,dz$$

integrate by parts

$$= \boxed{\alpha \frac{f}{N}\frac{\partial \overline{u}}{\partial z}} \iiint E\,dx\,dy\,dz$$

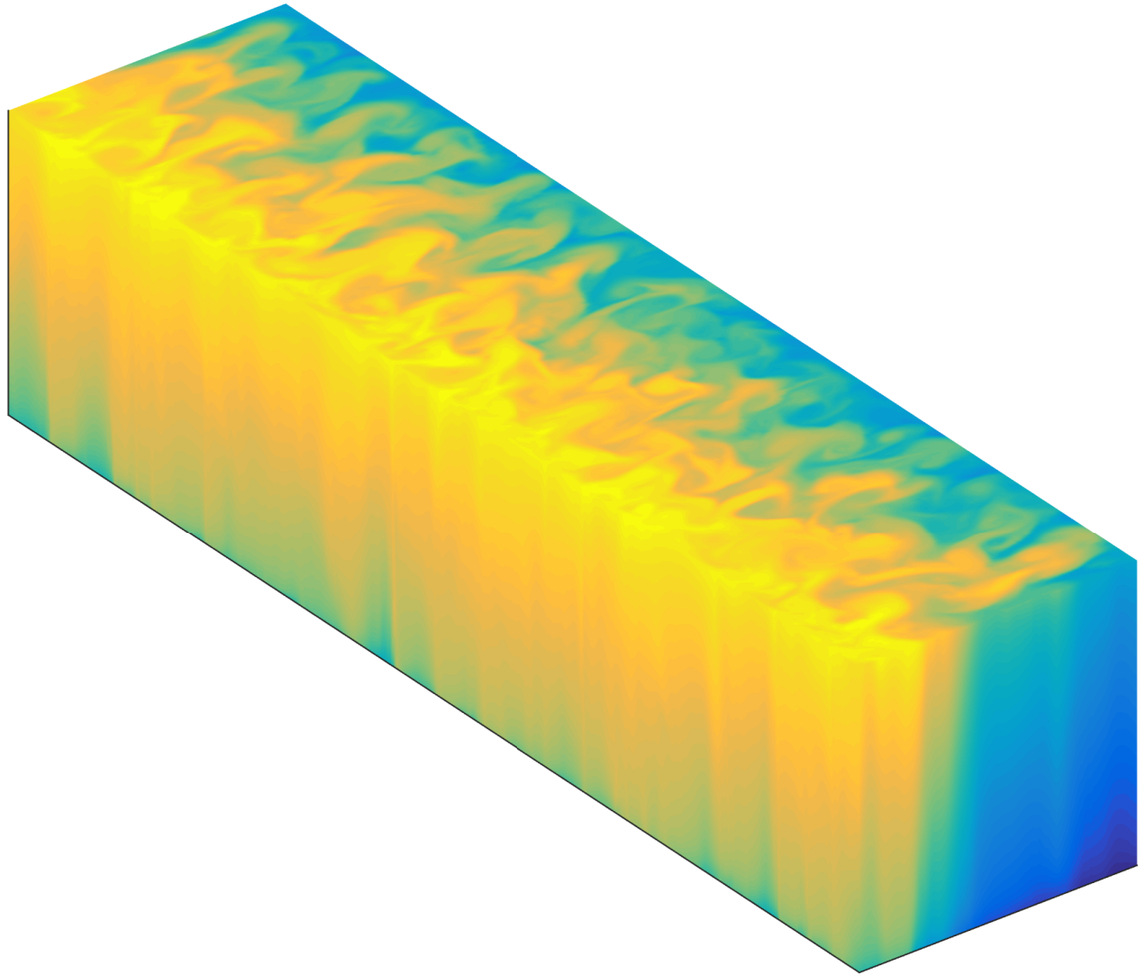
apply energy bound, use Cauchy-Schwarz

Eady growth rate
if $\alpha = 0.61$

$$(|\alpha| \leq 1)$$

Proof of concept 2: implied eddy diffusivity

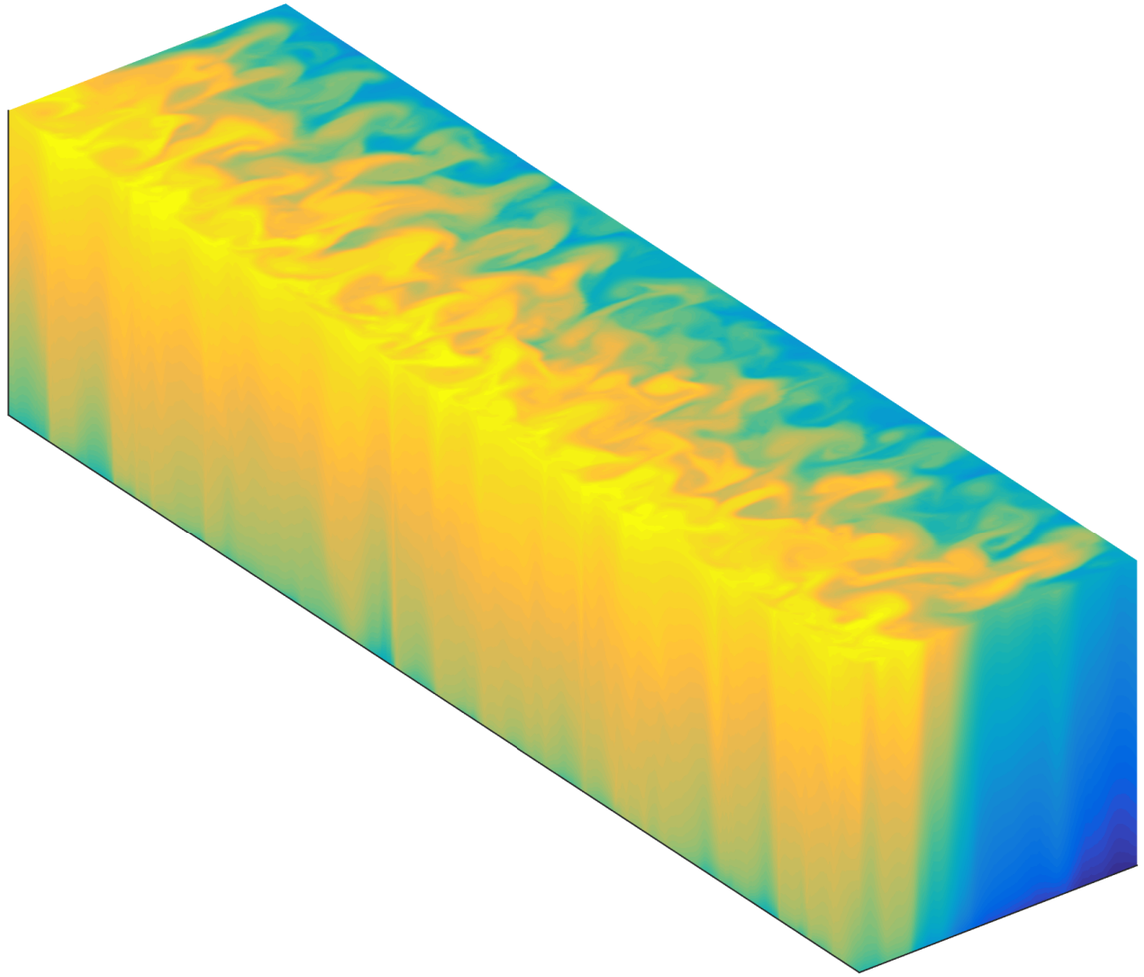
(Bachman et al., 2017)



nonlinear (fully turbulent) Eady spin-down

Proof of concept 2: implied eddy diffusivity

(Bachman et al., 2017)



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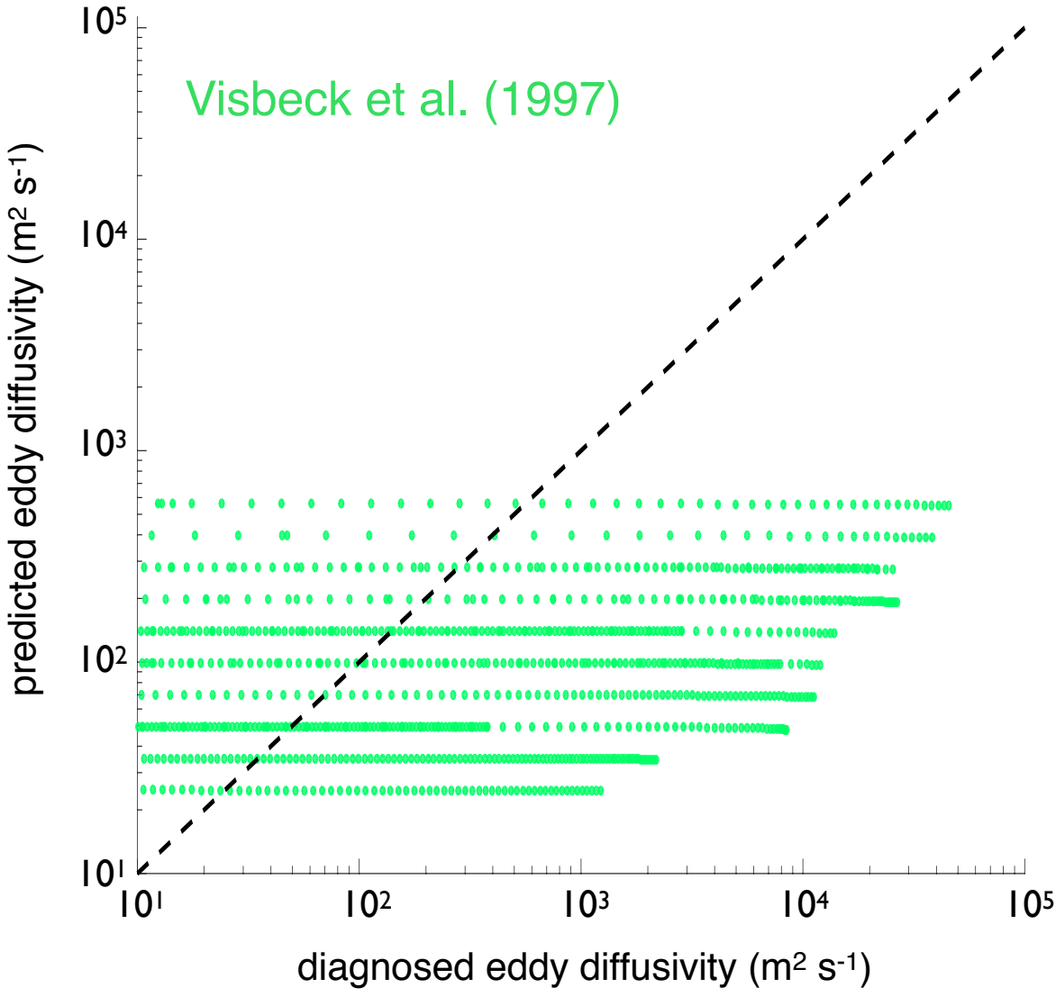

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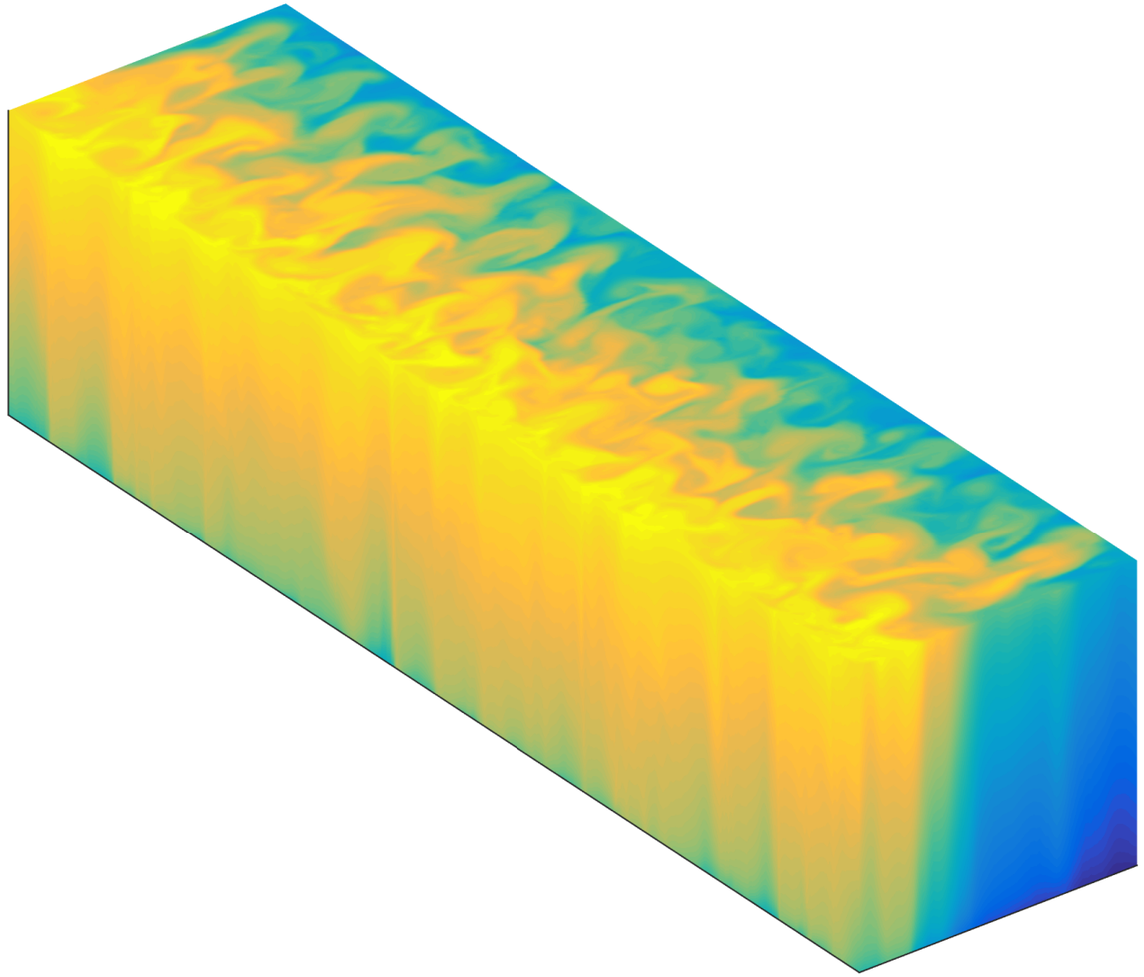

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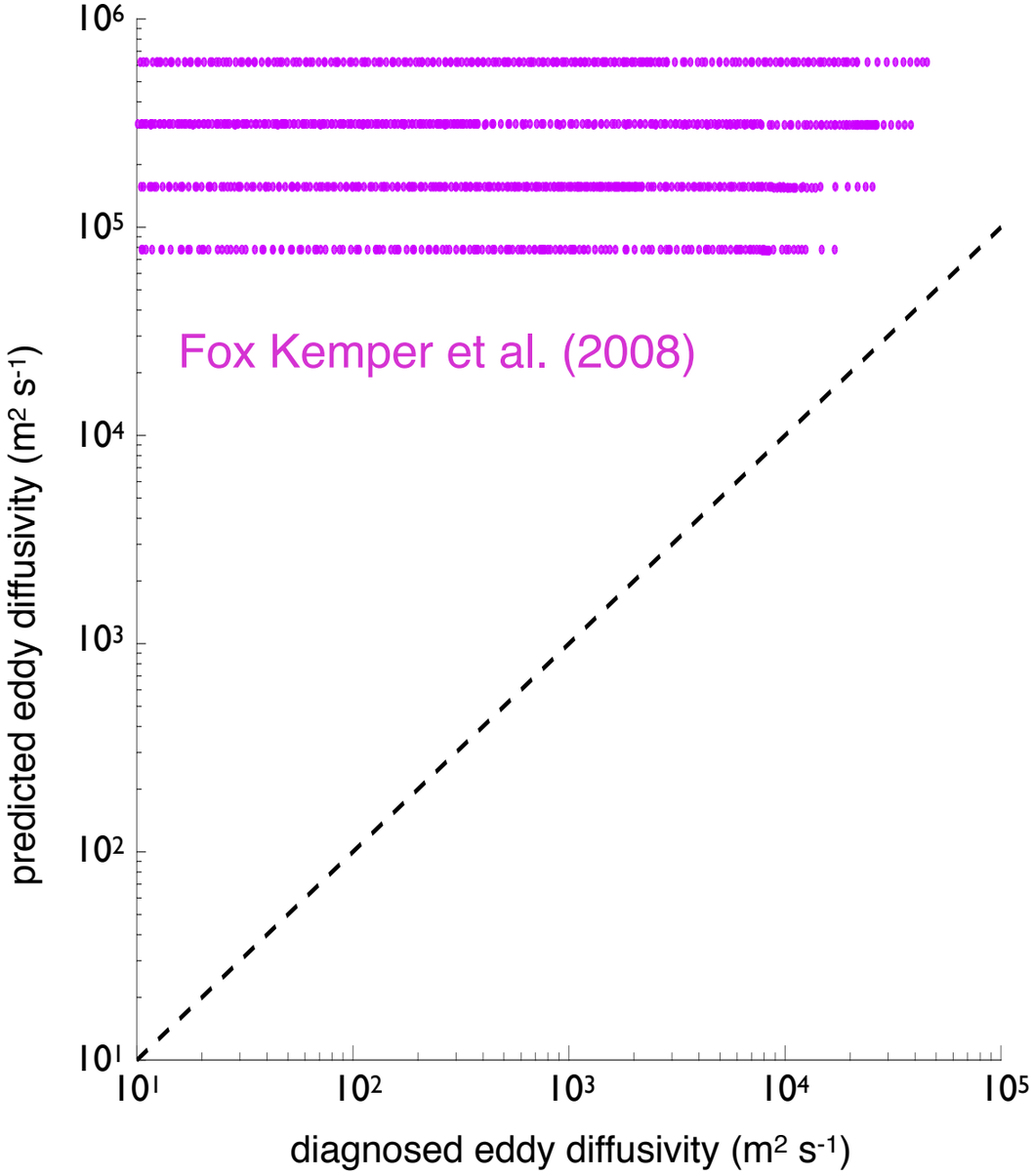
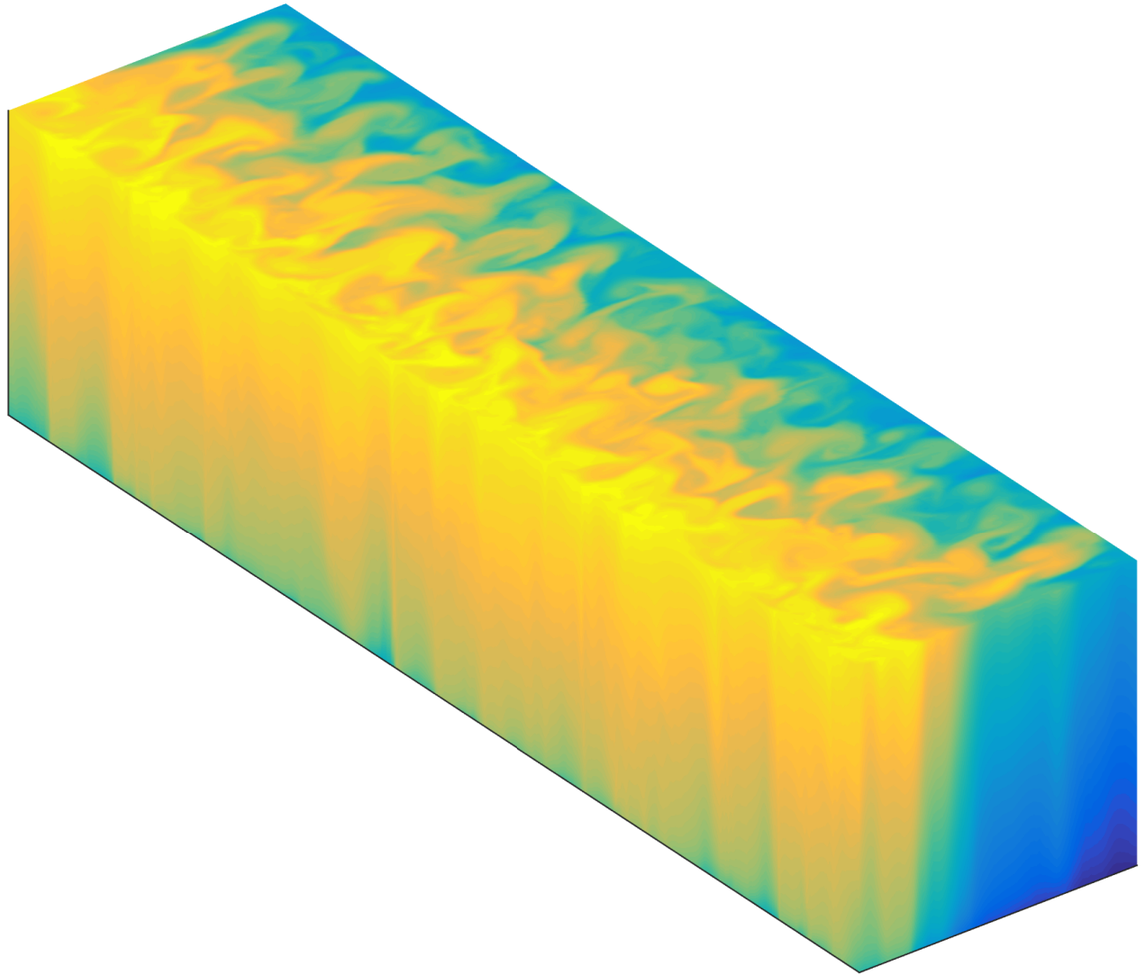


Proof of concept 2: implied eddy diffusivity

(Bachman et al., 2017)

$$\kappa = \alpha \frac{N}{\partial \bar{b}/\partial y} E \qquad (\alpha = 0.2)$$



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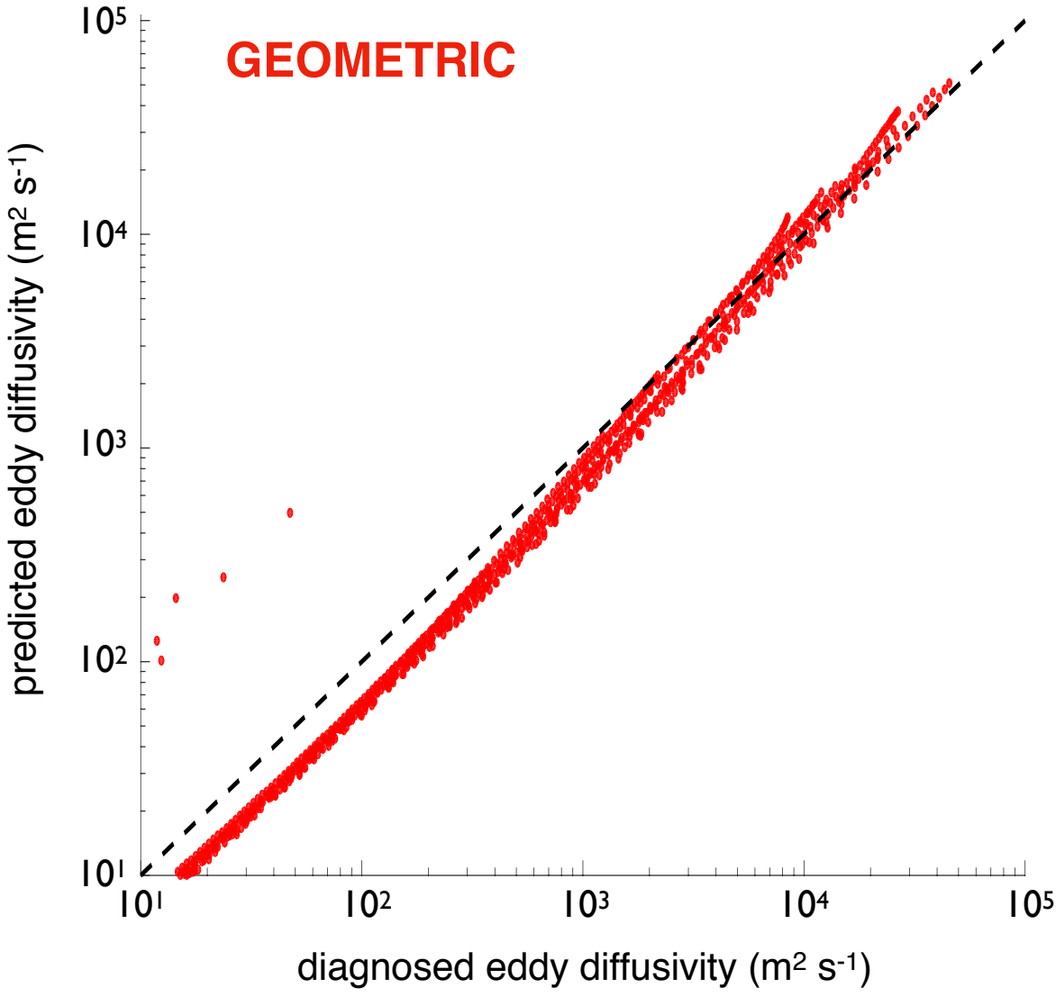
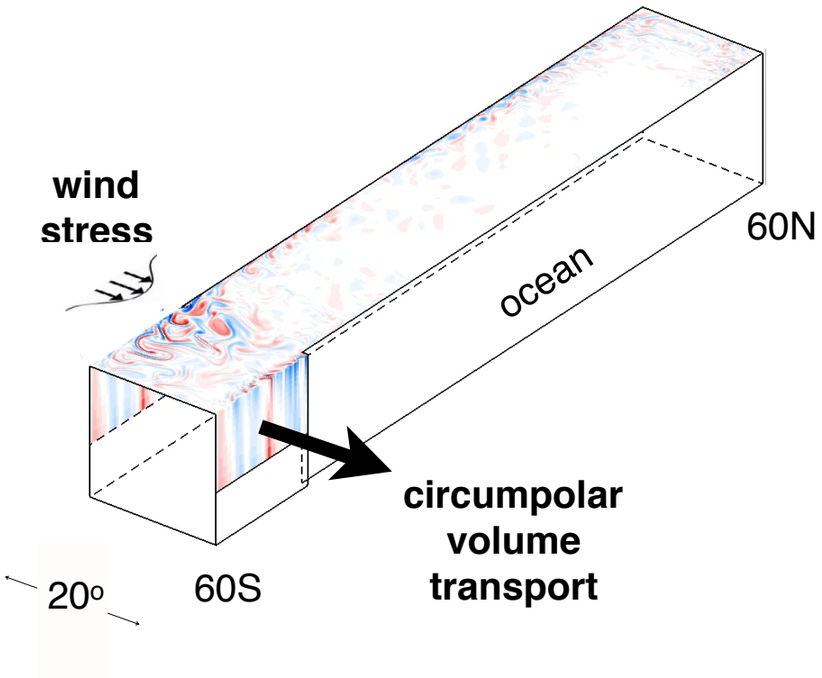
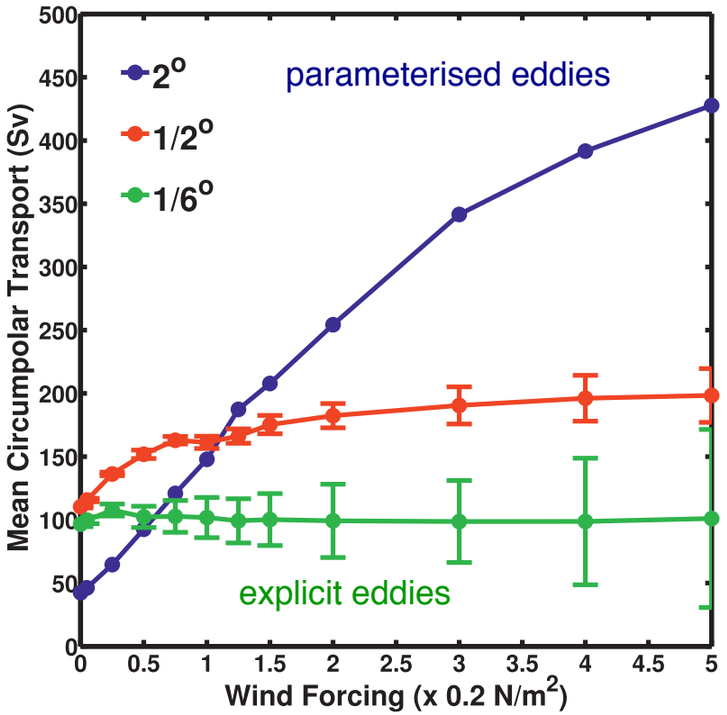


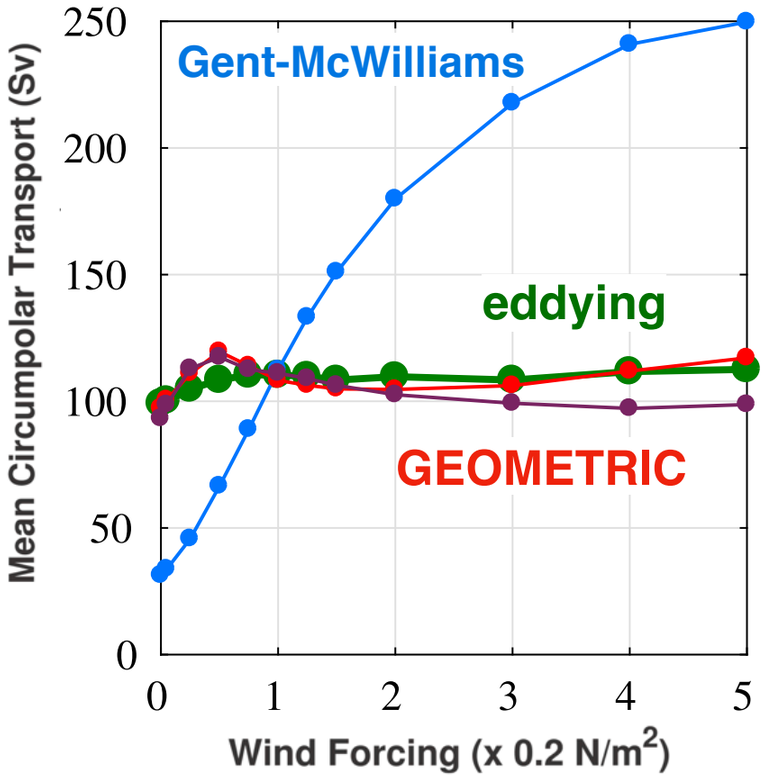
Proof of concept 3: eddy saturation

(Mak et al., 2017; Mak et al, 2018)





(Munday et al., 2013)



(Mak et al., 2018)

Physics of eddy saturation (*Marshall et al., 2017*)

momentum budget *(Johnson and Bryden, 1989)*

wind stress ≈ eddy form stress

eddy energy (Marshall et al., 2012)

eddy energy budget

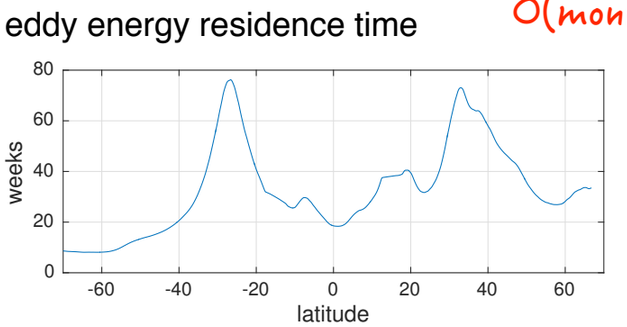
eddy energy source ≈ eddy energy sink

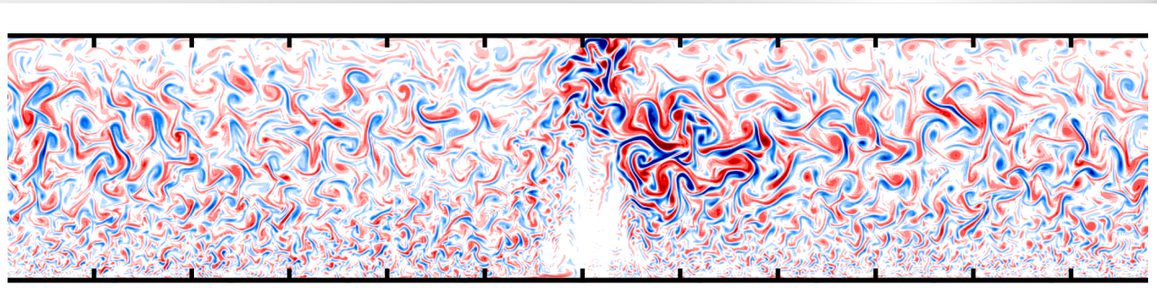
transport × ~~eddy energy~~ ~~eddy energy~~ × damping rate

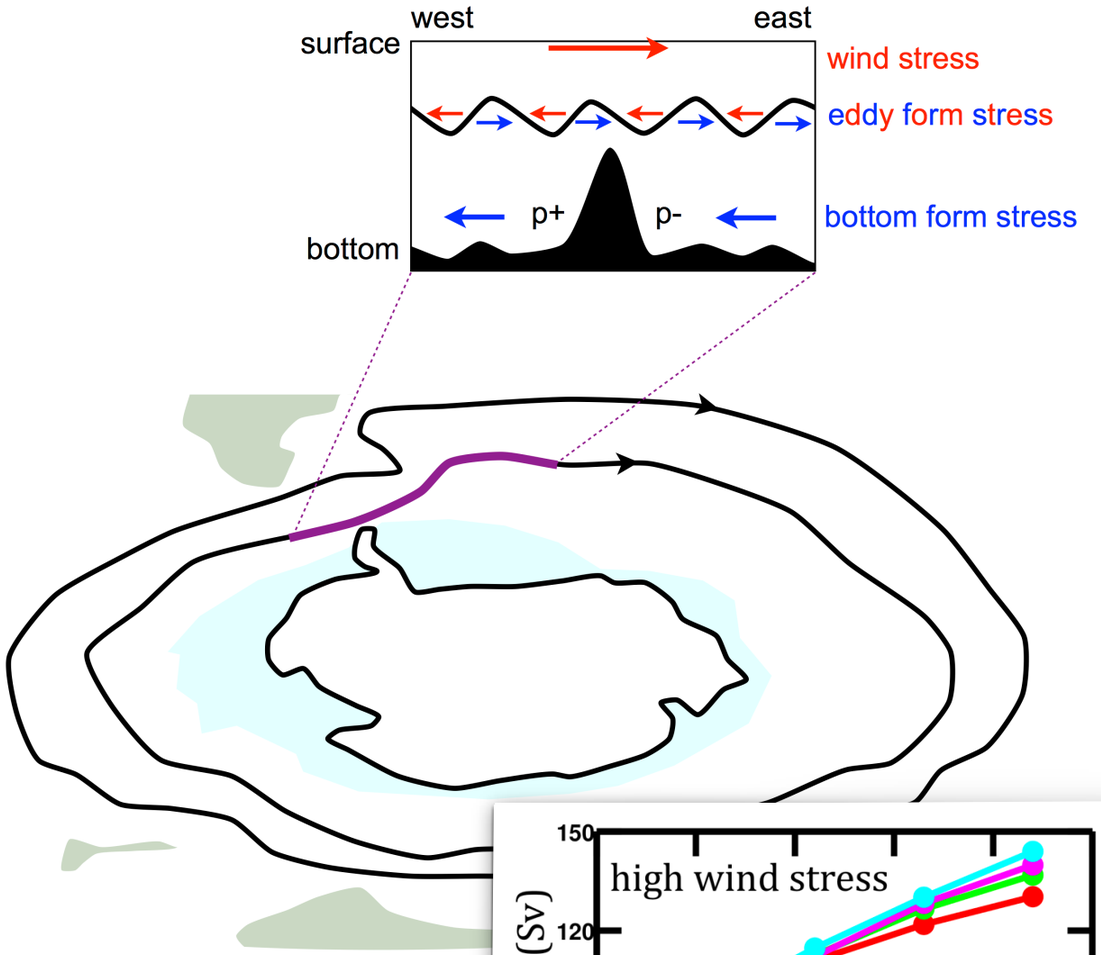
eddy energy ~ wind stress

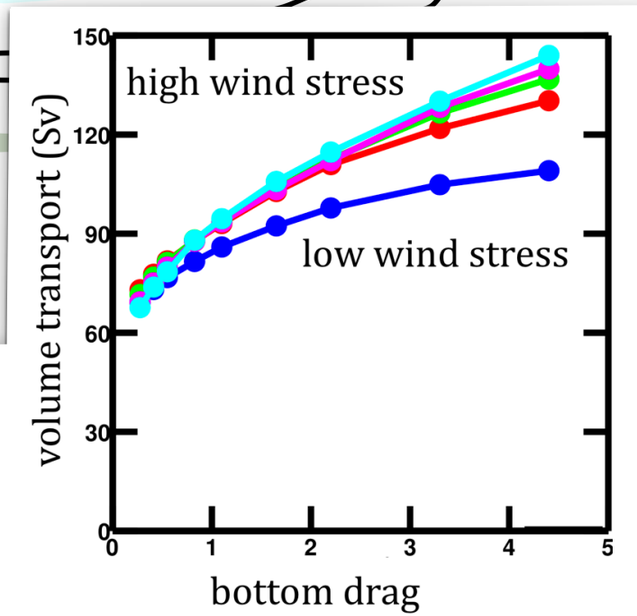
volume transport ~ damping rate

$O(\text{months}^{-1})$ (Zhai and Marshall, in prep.)

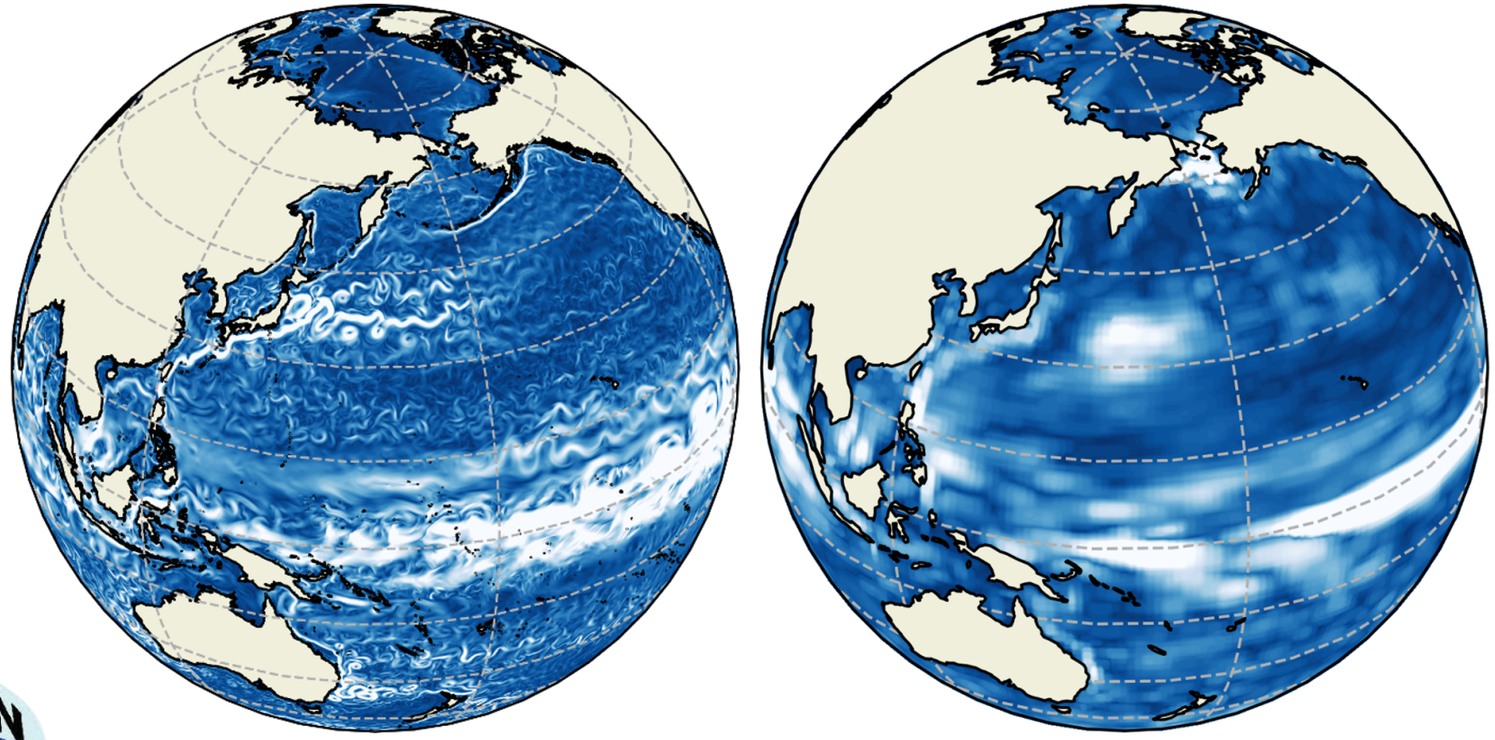








NERC-funded GEOMETRIC project: implementation in NEMO

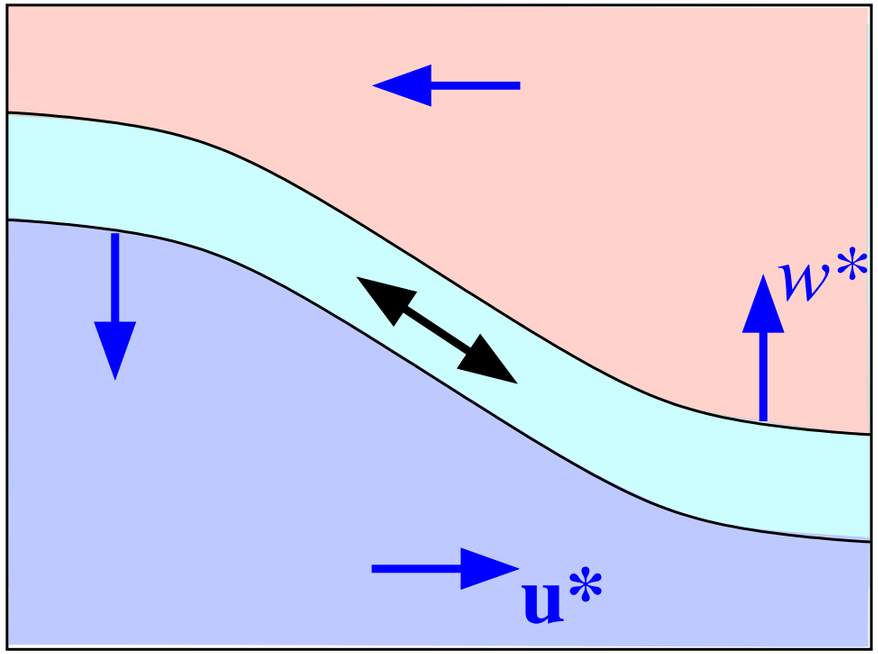


Recipe for implementation of GEOMETRIC in an ocean model

1. Employ existing Gent and McWilliams code with prescribed eddy diffusivity profile

2. Solve a prognostic equation for the depth-integrated eddy energy

3. Rescale eddy diffusivity profile at each latitude/longitude to match energetic constraint



$$\boldsymbol{\psi}_0 = \kappa_0(z)\frac{\nabla b}{N^2} \qquad \lambda = \frac{\alpha \int E\, dz}{\int N|\boldsymbol{\psi}_0|\, dz}$$

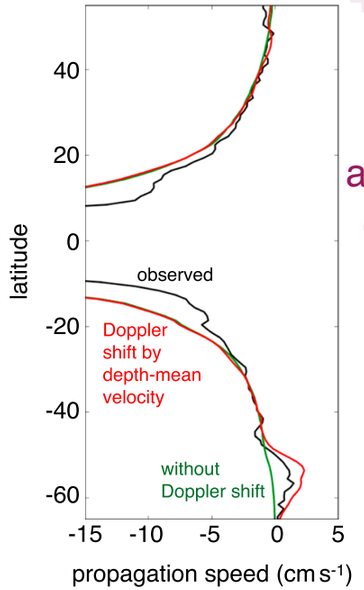
$$\boldsymbol{\psi} = \lambda\, \kappa_0(z)\frac{\nabla_h b}{N^2}$$

$$\mathbf{u}* = \frac{\partial \boldsymbol{\psi}}{\partial z}, \qquad w* = -\nabla_h \cdot \boldsymbol{\psi}$$

Prognostic equation for the depth-integrated eddy energy

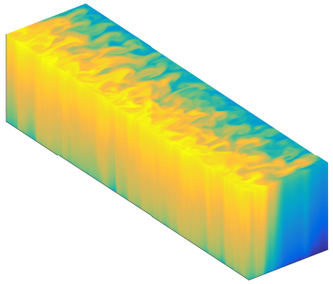
(Mak et al., 2019, in prep.)

$$\frac{\partial}{\partial t}\int E\,dz + \nabla\cdot\left((\overline{\mathbf{u}}^{z} - c_r\mathbf{i})\int E\,dz\right) = \int \kappa_{gm}\frac{|\nabla b|^2}{N^2}\,dz - \lambda\int E\,dz + \kappa_E\nabla^2\int E\,dz$$



westward propagation
(Chelton et al., 2007)
+
advection by depth-mean flow
(Klocker and Marshall, 2014)

eddy energy source:
baroclinic instability



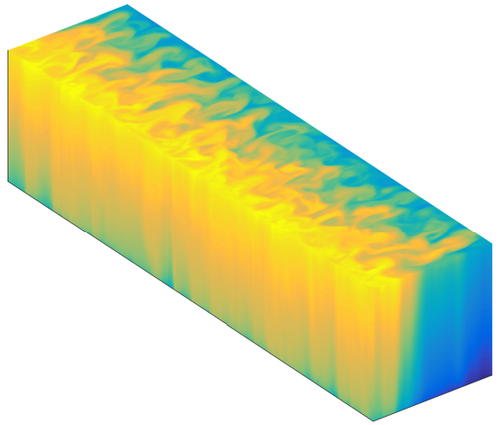
diffusion of eddy energy
(Eden and Greatbatch, 2008)

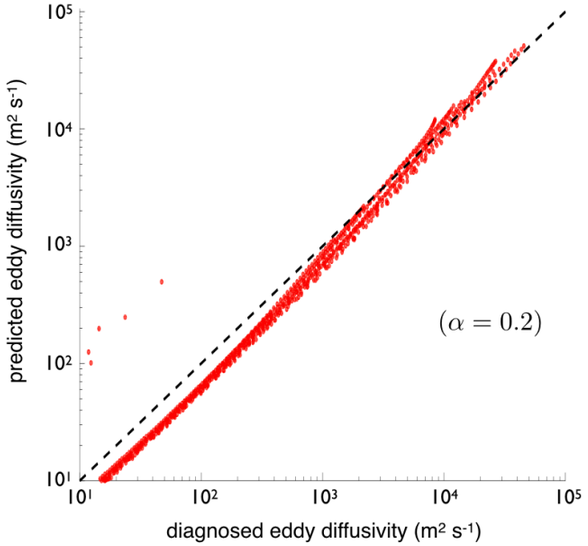
eddy energy sink: bottom drag (Sen et al., 2008);
lee wave radiation (Naveira Garabato et al., 2004);
….

(NEMO results removed from public copy)

Summary of key points

- GEOMETRIC eddy parameterisation framework:
  - based on conservation of energy and momentum
  - preserves dimensional Eady growth rate
  - reproduces realistic eddy diffusivities
  - reproduces and explains “eddy saturation”





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marshallocean.net/projects/geometric

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