Structure Functions as a Tool for Measuring the Mesoscale

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Baylor Fox-Kemper
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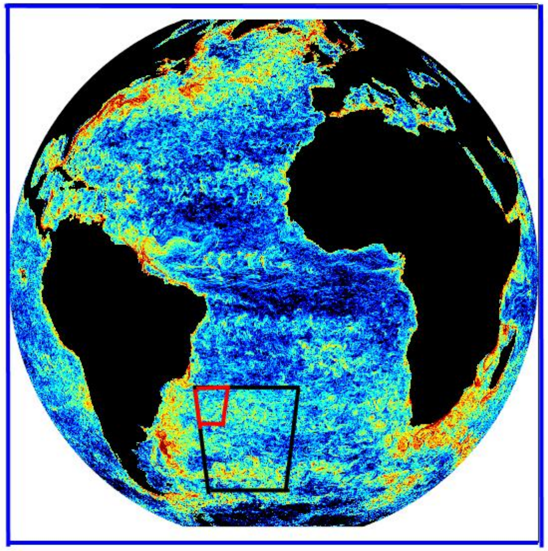
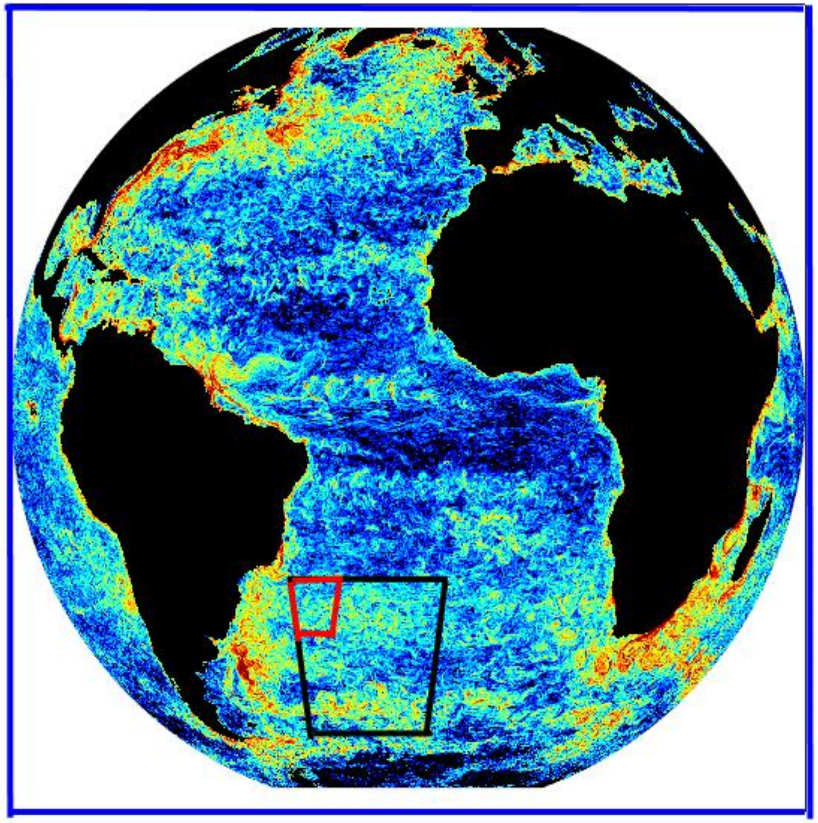


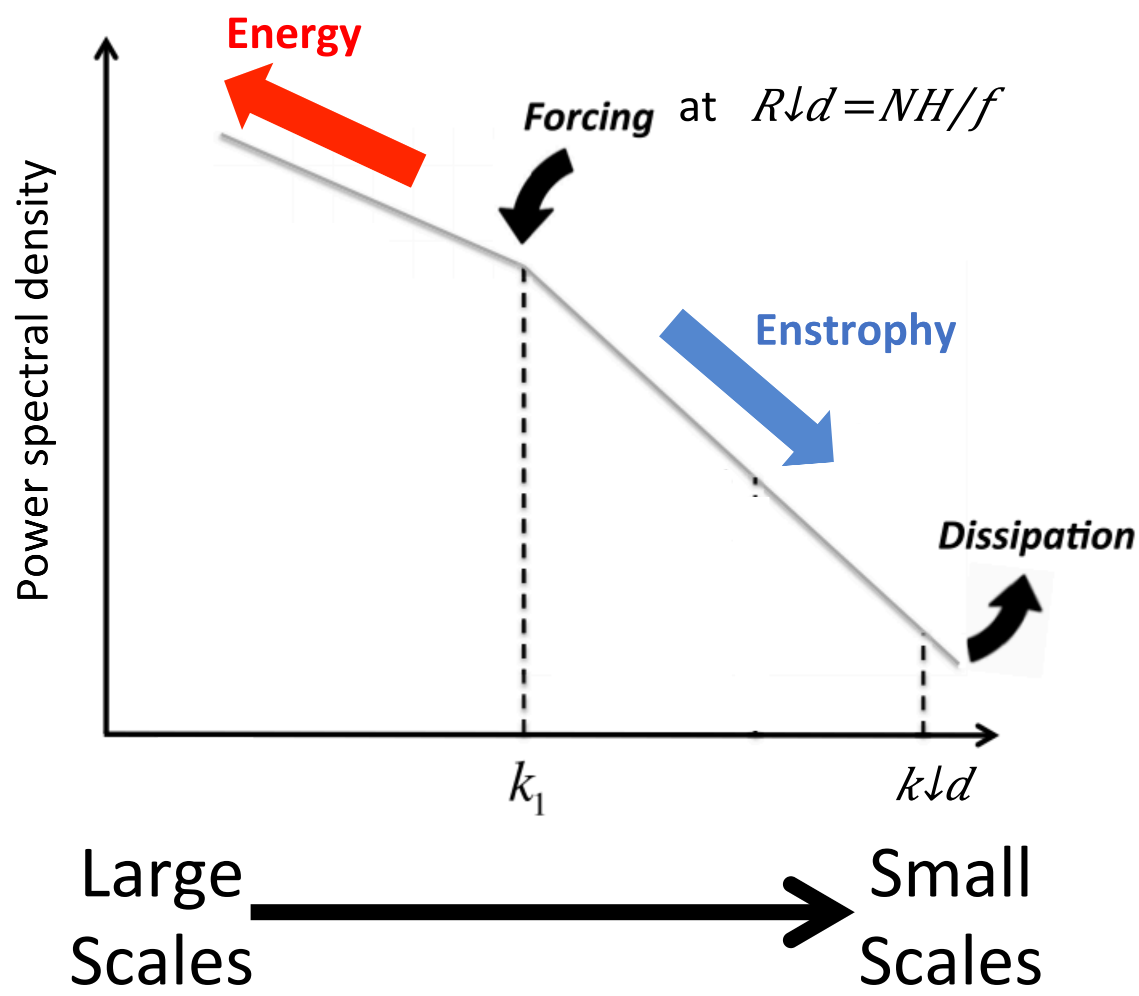
Mesoscale Turbulence

- The dynamics of the ocean are heterogeneous and spatially intermittent, even at small scales



Pearson & Fox-Kemper
Physical Review Letters (2018)

Inertial Cascades



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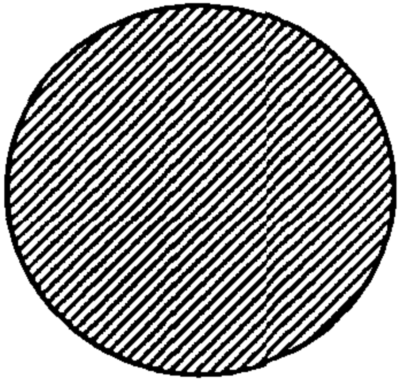

- Mesoscale eddy forced at deformation radius
- Eddy breaks into child eddies conserving properties

Adapted from Meneveau & Sreenivasan, *JFM* (1991)

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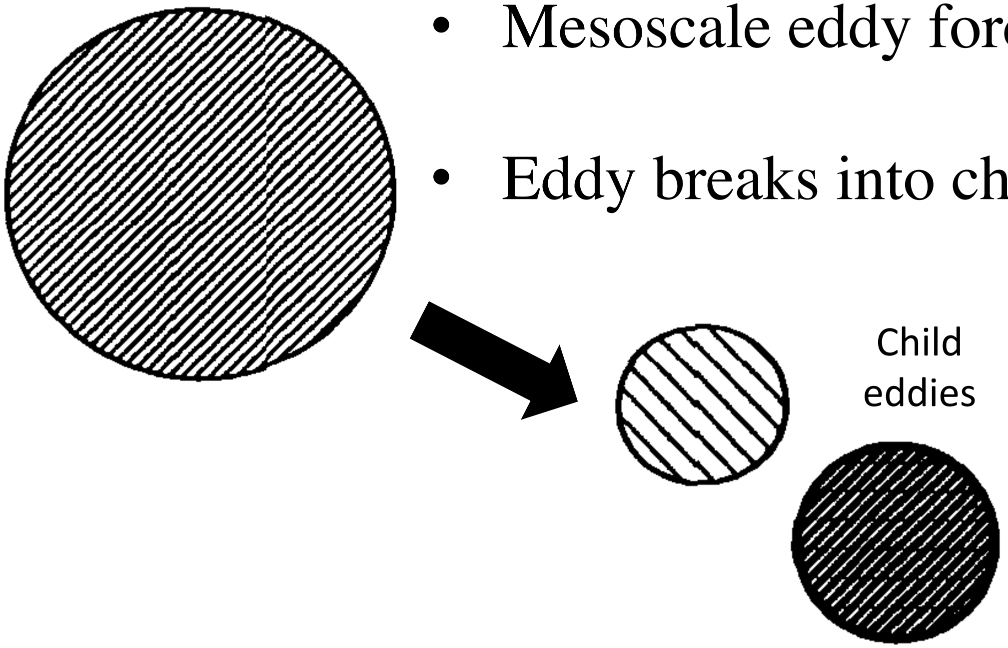

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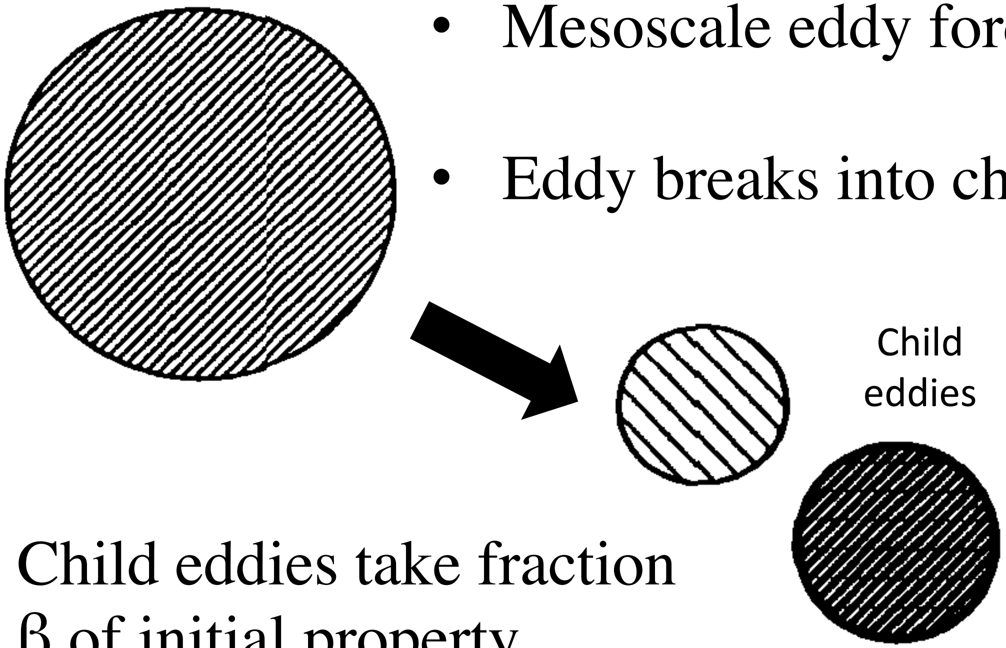

- Child eddies take fraction β of initial property

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- This cascade to smaller scales repeats
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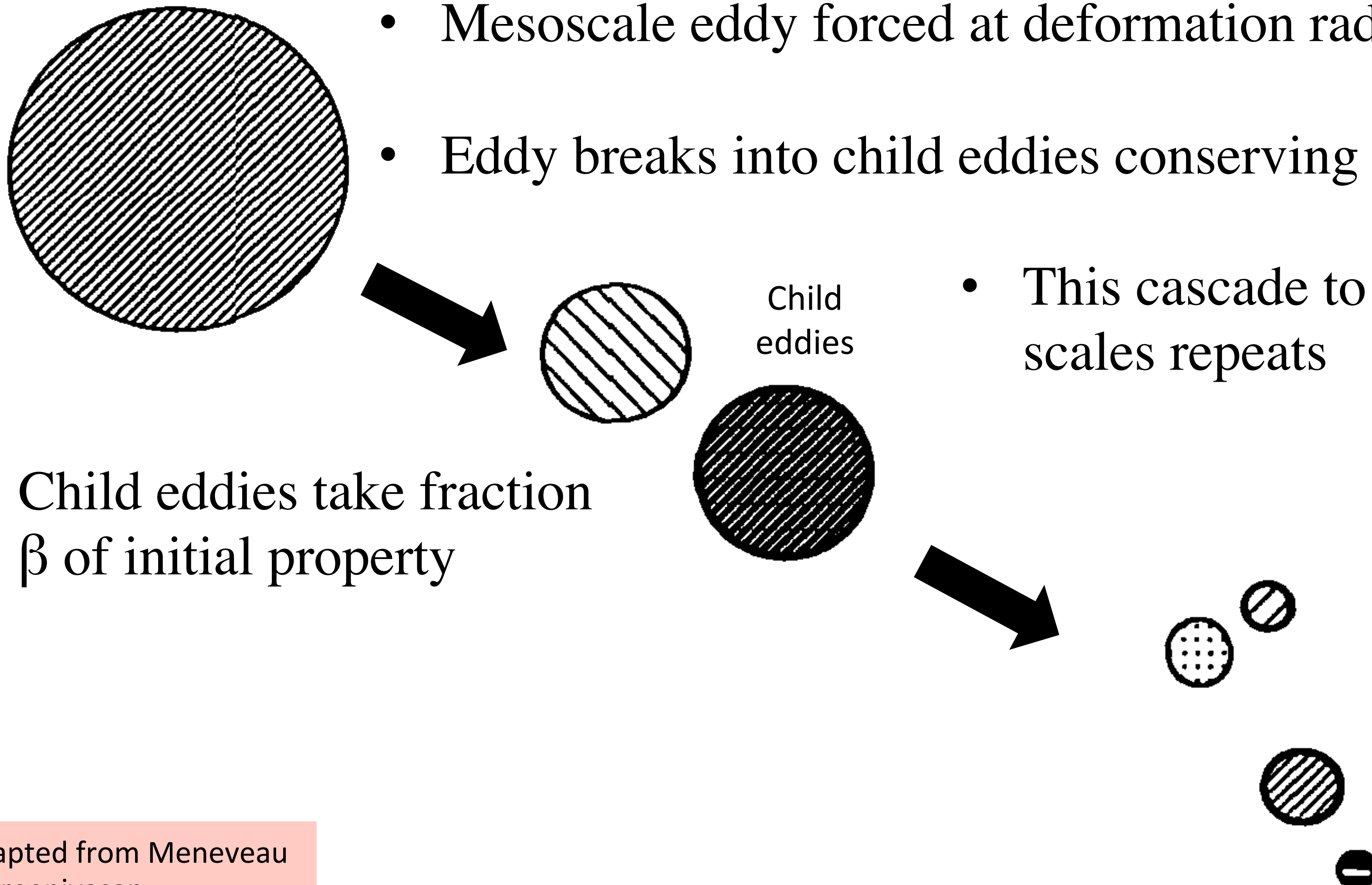

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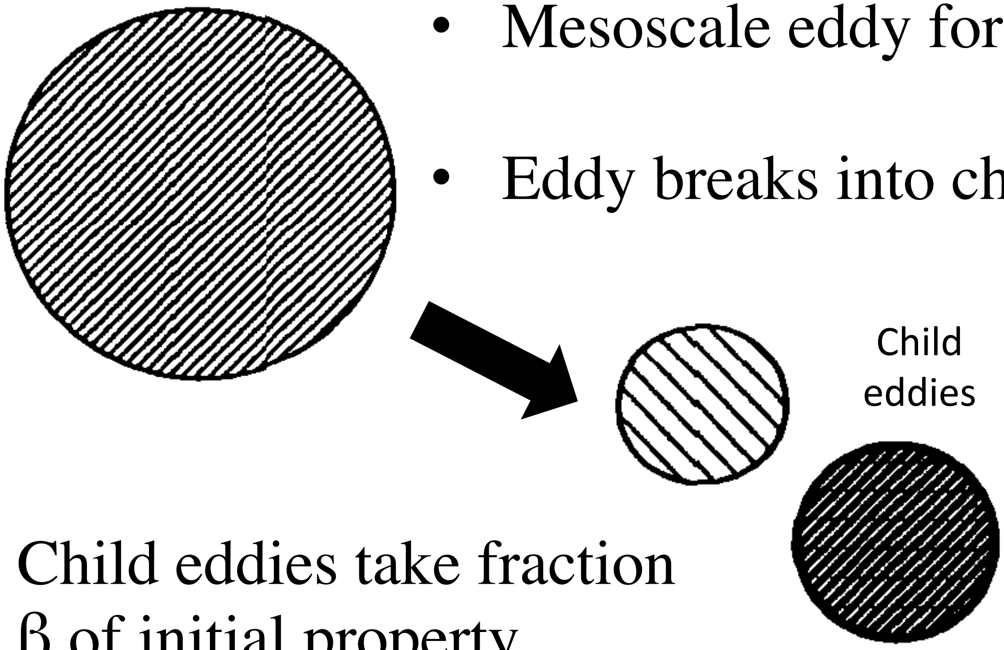

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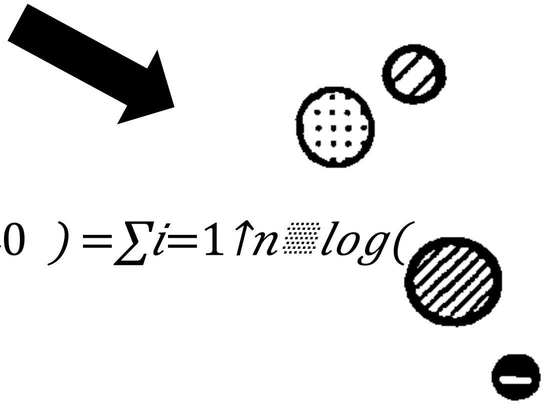

- After n cascade steps then

$$\varepsilon_n = \varepsilon_0 \prod_{i=1}^{n} \beta_i \quad \Rightarrow \quad \log(\varepsilon_n / \varepsilon_0) = \sum_{i=1}^{n} \log(\beta_i)$$

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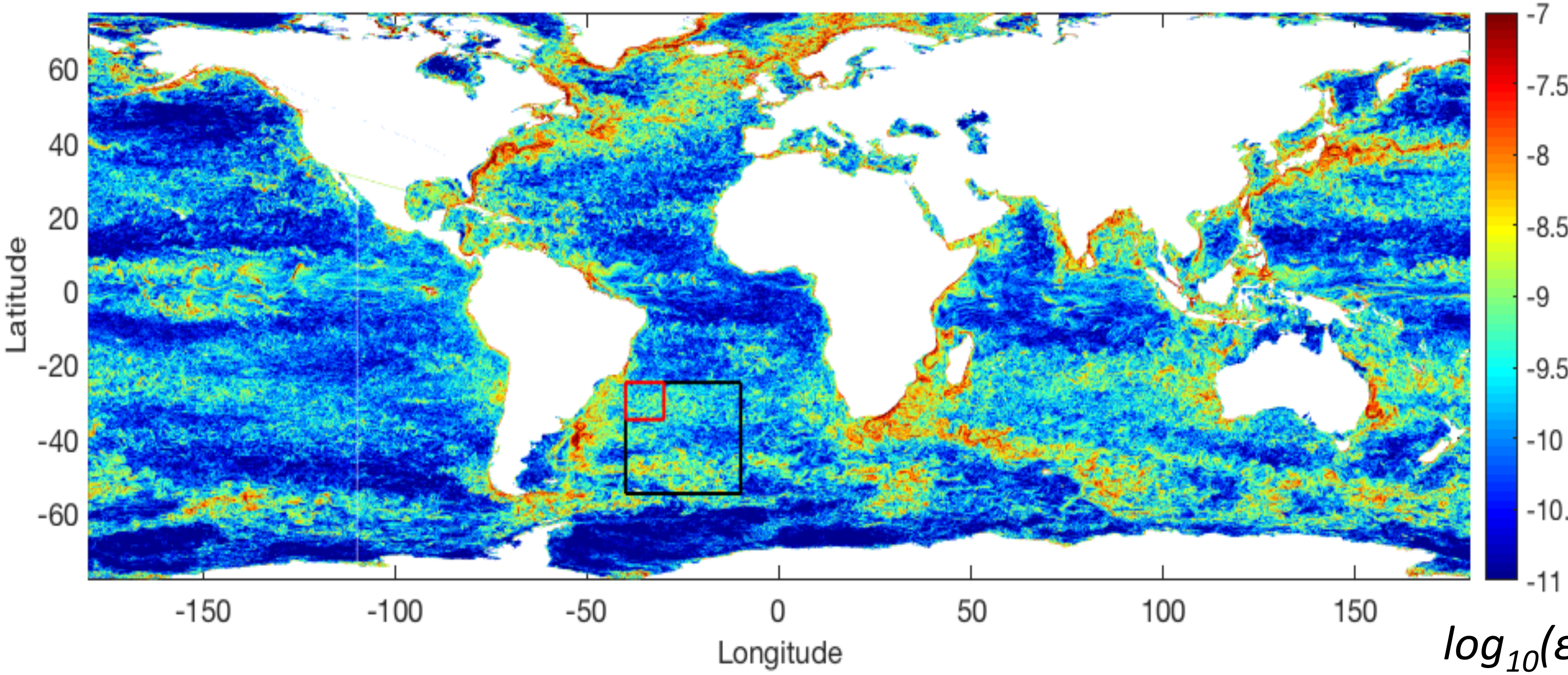
Spatial Distribution of Turbulence Statistics

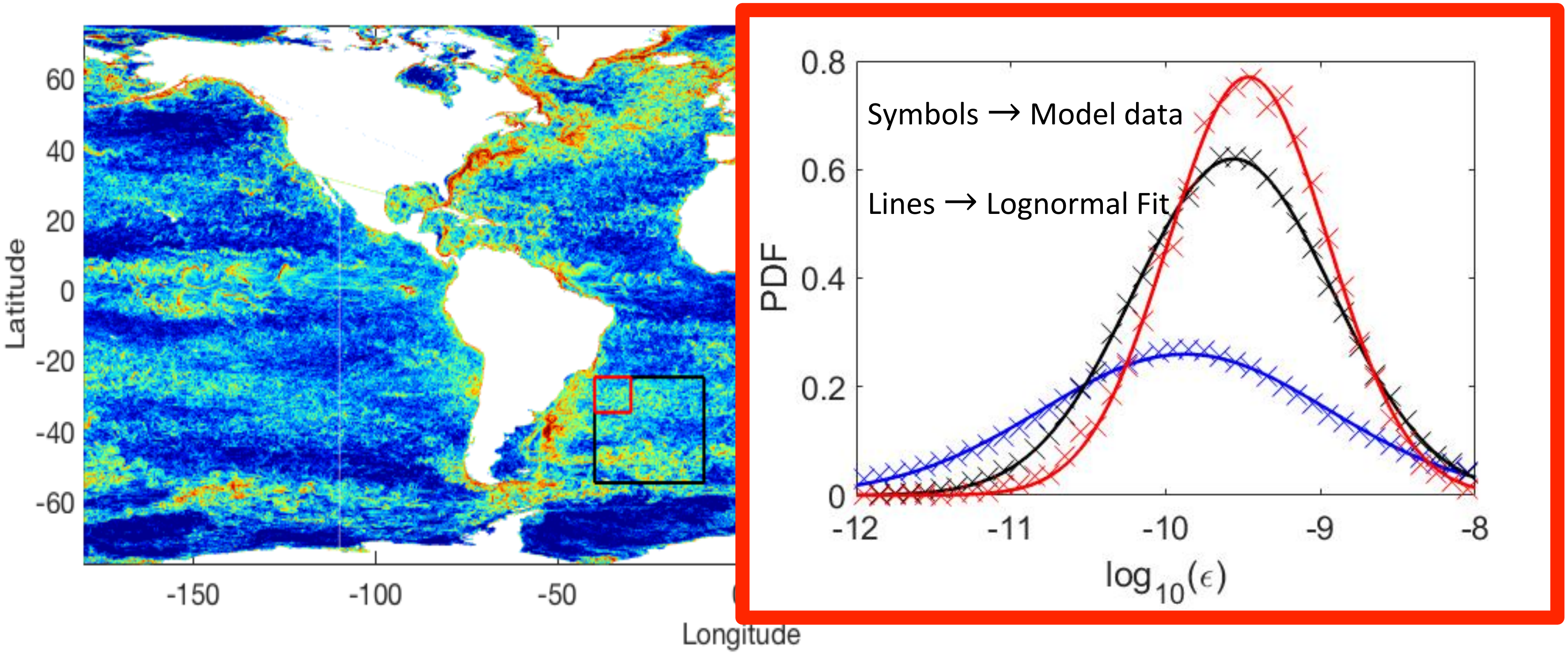
- KE dissipation in Parallel Ocean Program (POP) $1/10^{o}$ model



$log_{10}(\varepsilon)$
[$m^2\ s^{-3}$]

Pearson & Fox-Kemper,
Physical Review Letters (2018)

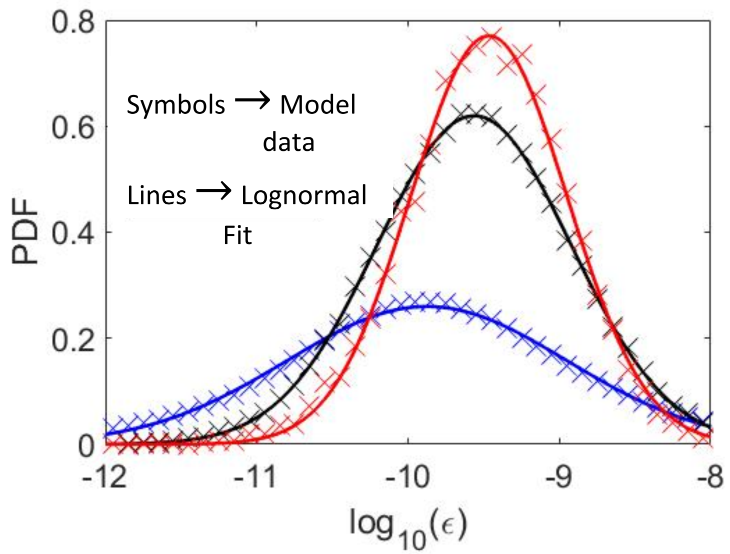
Spatial Distribution of Turbulence Statistics



Pearson & Fox-Kemper,
Physical Review Letters (2018)

Spatial Distribution of Turbulence Statistics

- Modelled mesoscale turbulence has log-normal statistics
- How can we observe and diagnose the spatial variations in mesoscale spectral fluxes?



Pearson & Fox-Kemper,
Physical Review Letters (2018)

Structure Functions

- Structure functions depend on spatial differences in variables

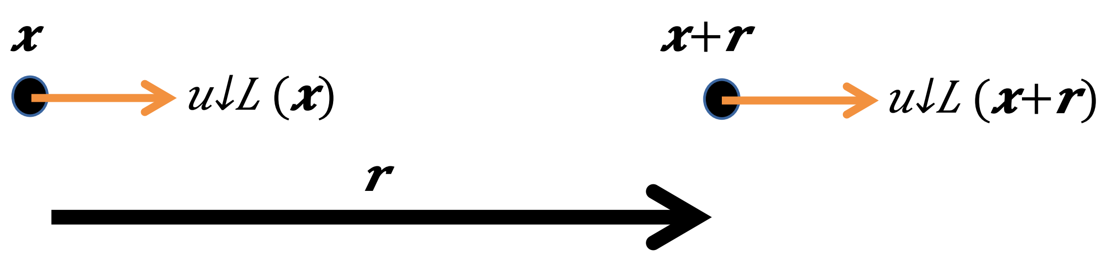
$$\delta\phi = \phi(\boldsymbol{x}+\boldsymbol{r}) - \phi(\boldsymbol{x})$$

- Exact laws relate structure functions to spectral fluxes in inertial cascades of isotropic, homogeneous turbulence

3D turbulence:

$$Spectral\ KE\ Flux = \varepsilon = -5/4 \cdot \delta u_L \, \delta u_L \, \delta u_L \ / r$$

Kolmogorov (1941)



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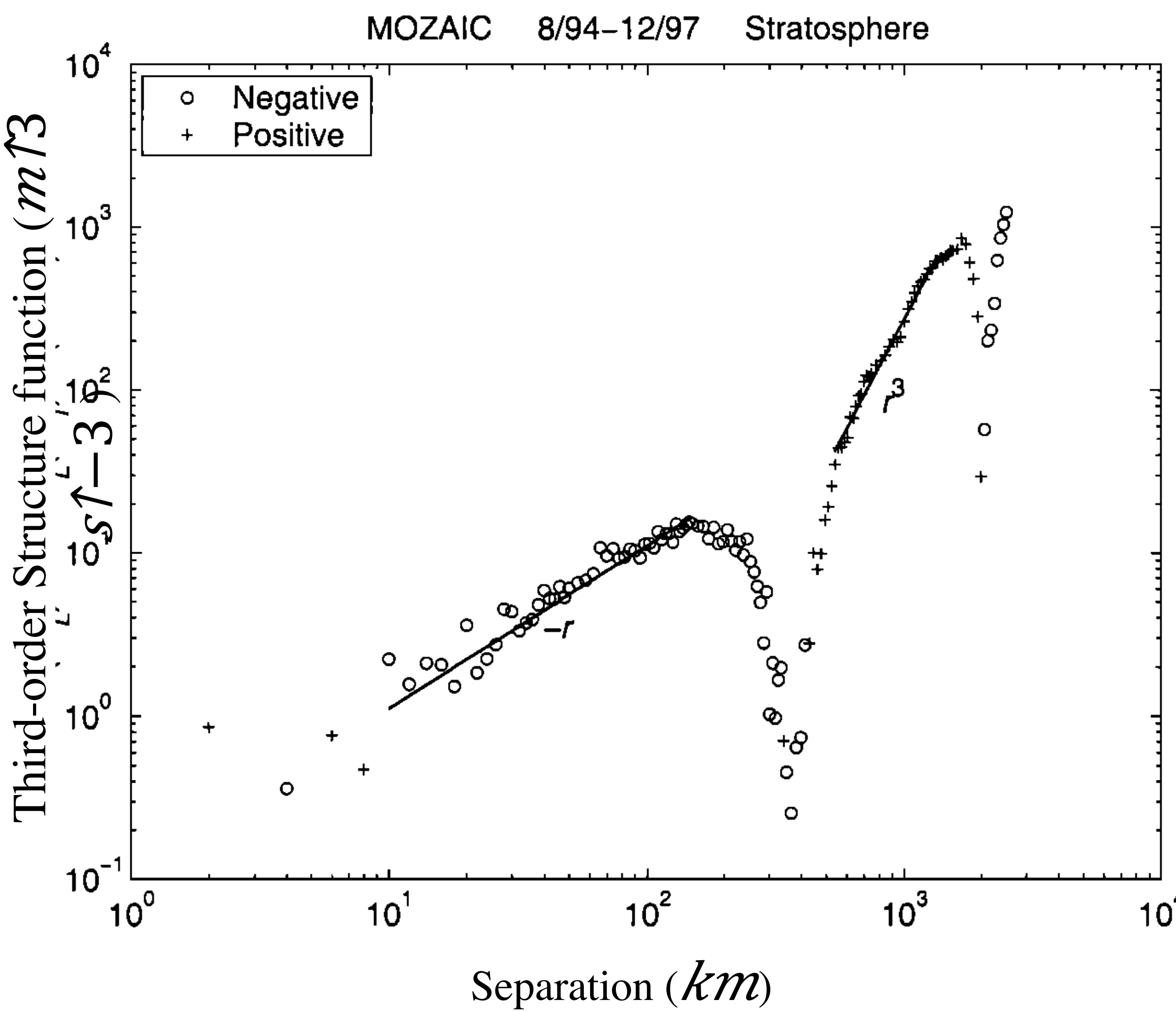
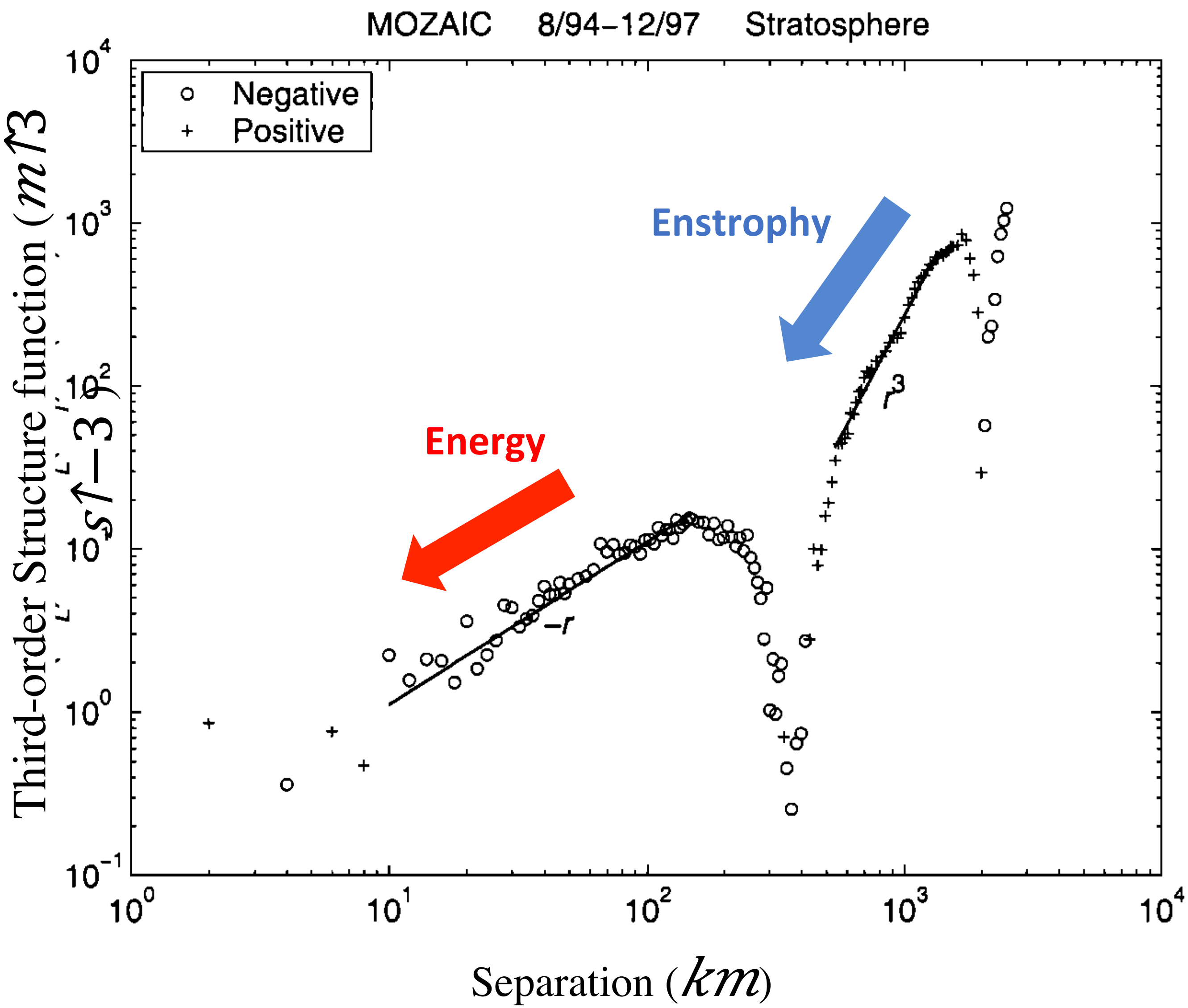
Lindborg (2007)

Structure Functions



Cho & Lindborg (2001)

Structure Functions



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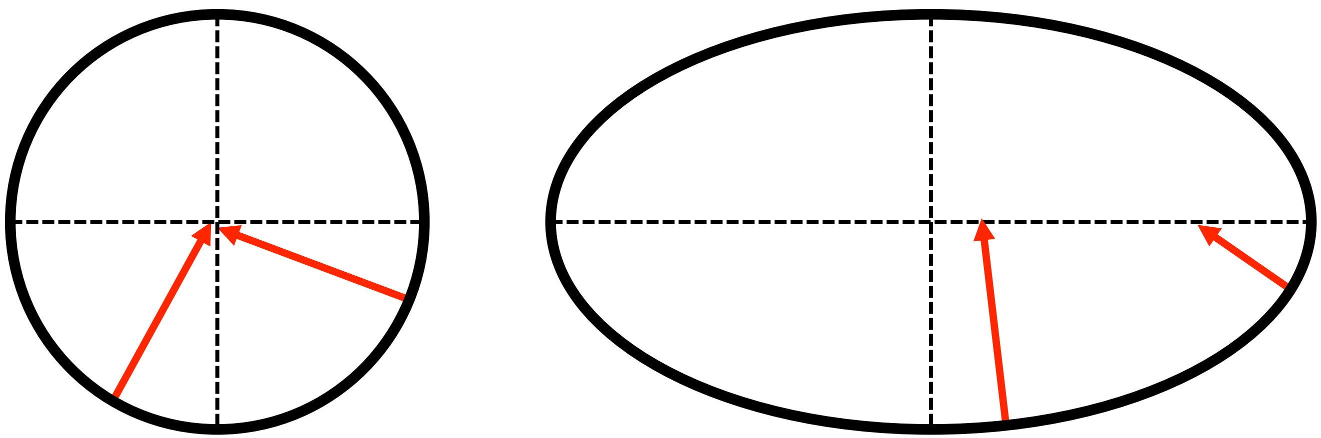
Anisotropic Structure Functions

Isotropic QG: $\varepsilon_q = -1/2 \cdot \delta u_L \, \delta q \delta q / r$

Anisotropic QG: $\varepsilon_q = -1/4 \; \nabla \cdot (\delta \boldsymbol{u} \delta q \delta q)$



Augier et al (2012)

- Requires complete prior knowledge of anisotropy to quantify ε_q

New Anisotropic Structure Functions

$$\varepsilon_q = -\frac{1}{2}\,\delta q \delta A_q \qquad \textit{where} \qquad A_q = -\boldsymbol{u}_g \cdot \nabla q$$

New statistic has the following benefits over $\varepsilon_q = -\frac{1}{4}\,\nabla\cdot(\delta\boldsymbol{u}\delta q\delta q)$**:**

1. Does not require integration

2. Can be evaluated without prior quantification of anisotropy

3. Has two, rather than three, differences (potential convergence benefits)

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However, it does require knowledge of local flow gradients (amenable to numerical models, and future data [i.e. SWOT])

Extension to other dynamical regimes

$\varepsilon_q = -1/2 \; \delta q \delta A_q$ *in the QG enstrophy cascade*

Analogous relations for diagnosing spectral fluxes in the;

1. QG inverse energy cascade
2. Surface QG cascades
3. Two-dimensional cascades

Pearson, Pearson & Fox-Kemper *In prep. (GRL)*

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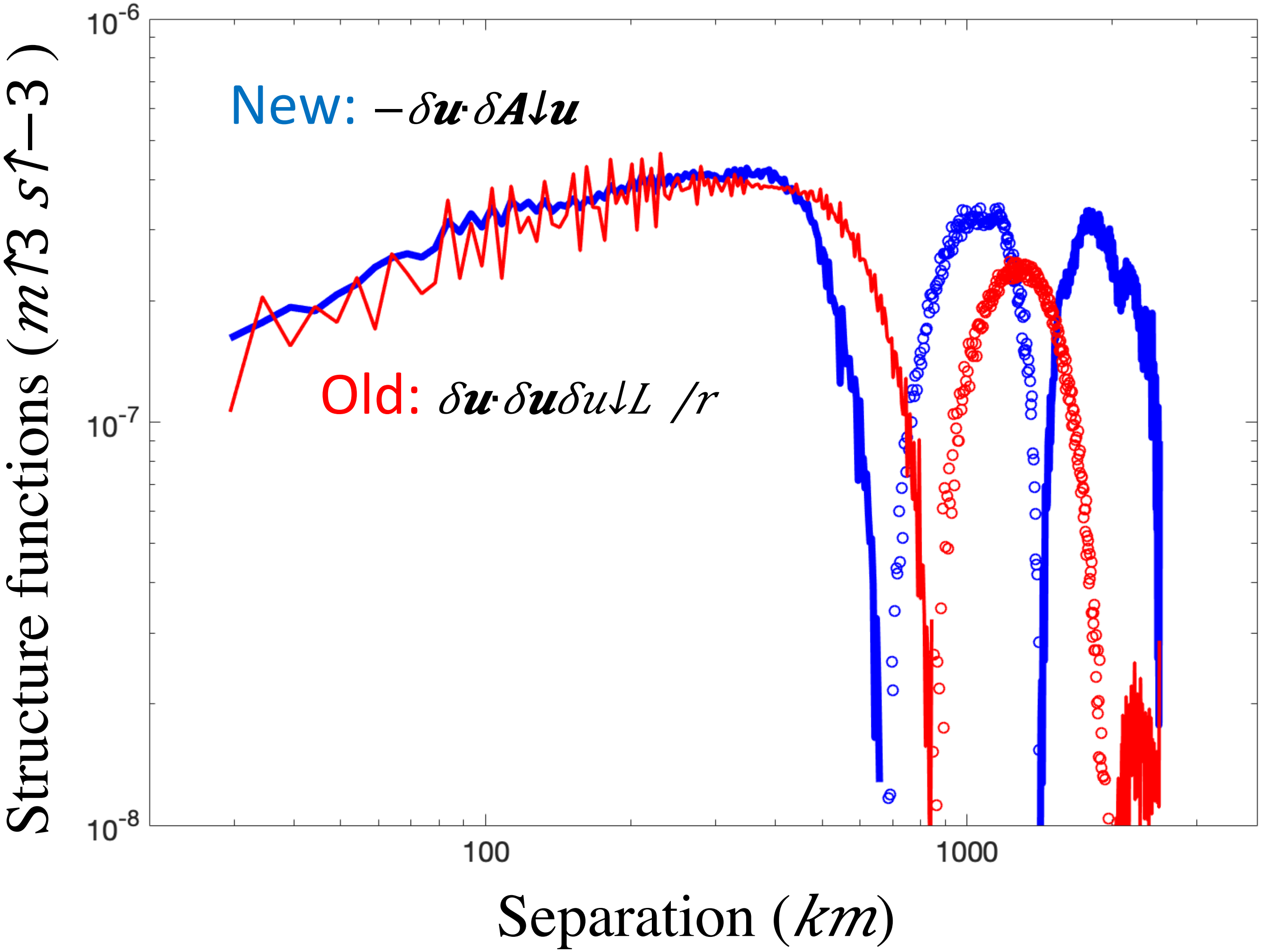
In isotropic limit, new and old structure functions converge

New comparable to old under anisotropic conditions



Surface QG dynamics
Simulated by **pyQG**
Abernathey et al (2015)

Summary & Conclusions

- **Mesoscale turbulence has log-normal statistics** in numerical models

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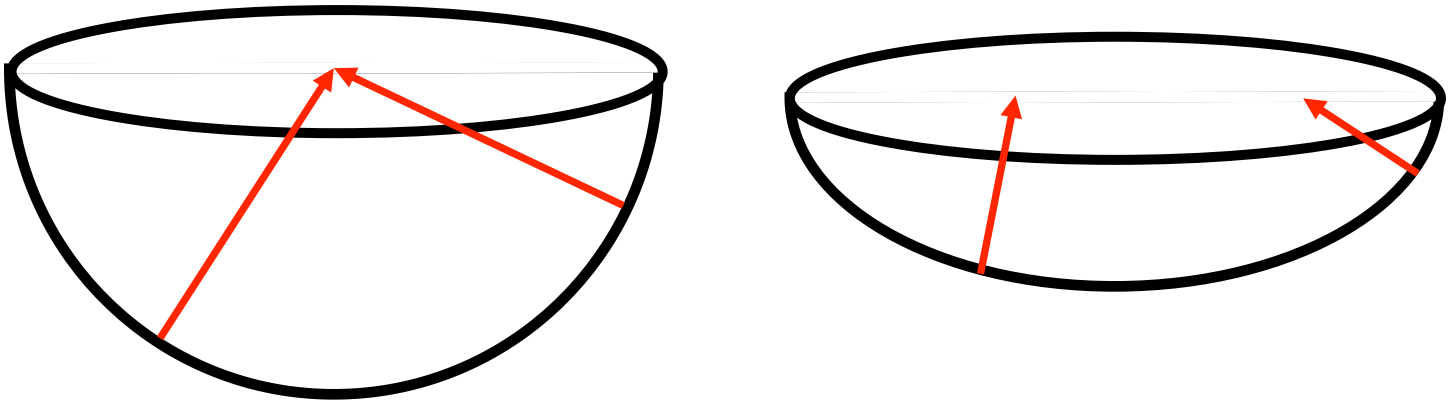
- Log-normality allows us to accurately quantify observational data and develop more precise turbulence parameterizations
- New structure functions could diagnose spectral fluxes from patchy or irregular data
- New structure functions for **anisotropic QG, SQG,** and **2D** inertial cascades

Anisotropic Structure Functions

$$\varepsilon_q = -1/4\ \nabla\cdot(\delta \boldsymbol{u} \delta q \delta q)$$

Anisotropic QG:



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