Mesoscale Eddy Energy Transport
Theory and Idealized Experiments

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Sources and Sinks (*and Transport*) of Ocean Mesoscale Eddy Energy

In this talk **ocean mesoscale eddies** are geostrophic, hydrostatic, and completely unresolved.

The ultimate goal is to parameterize energy transport in non-eddying ocean models.

Including a non-local energy budget as part of a subgrid-scale model is an old and well-developed idea in turbulence modeling.

- Non-eddying OGCM: Eden & Greatbatch (2008)
- Eddy-permitting OGCM: Jansen et al. (2015), and Juricke et al. (2019).

Most models (incl. non-ocean) parameterize subgrid-scale energy transport as a mix of harmonic diffusion and advection by the resolved flow.

There is some theoretical and experimental justification for this model in engineering-scale applications; essentially none for ocean mesoscale eddy energy transport.

IMPORTANCE

Brief and probably incomplete review of papers showing non-negligible mesoscale eddy energy transport:

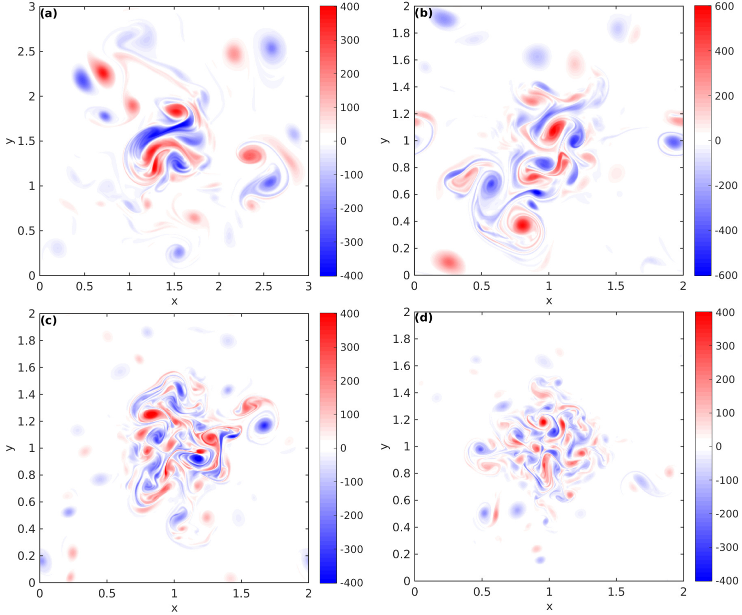
- Lots of regional Lorenz energy cycle papers.
- SSH Altimetry (e.g. Chelton et al. 2011) clearly shows that coherent eddies move long distances. The connection to mesoscale energy transport is indirect, but suggestive.
- Grooms et al. (DAO 2012) multiple-scales asymptotics.
- Grooms et al. (JPO 2013) ran a QG gyre model and diagnosed an eddy energy budget.
- Chen et al. (JPO 2014) nonlocality in an eddy-permitting state estimate.
- Yang et al. (JPO 2017) in the Kuroshio in ECCO II.

EXPERIMENTS

Lacking theory we can at least ask whether mesoscale eddy energy diffuses in simulations, and at what rate.

I start with the simplest GFD model: barotropic dyamics on an f-plane.

I use stochastic white noise forcing with a characteristic length scale L, localized in the circle of a square periodic domain.



Vorticity with different forcing amplitudes and scales.

The actual time-mean energy budget has the form

$$\nabla \cdot (\boldsymbol{F}_E) = F - 2E - 2\nu Z$$

I track all terms from the simulation.

I will fit a flux-divergence model of the form

$$\nabla \cdot (\boldsymbol{F}_E^M) = (-\kappa\Delta)^\alpha E^M$$

which includes harmonic diffusion $\alpha = 1$ and biharmonic $\alpha = 2$.

Time-mean parameterized energy solves

$$\nabla \cdot (\boldsymbol{F}_E^M) = F - 2E^M - 2\nu Z$$

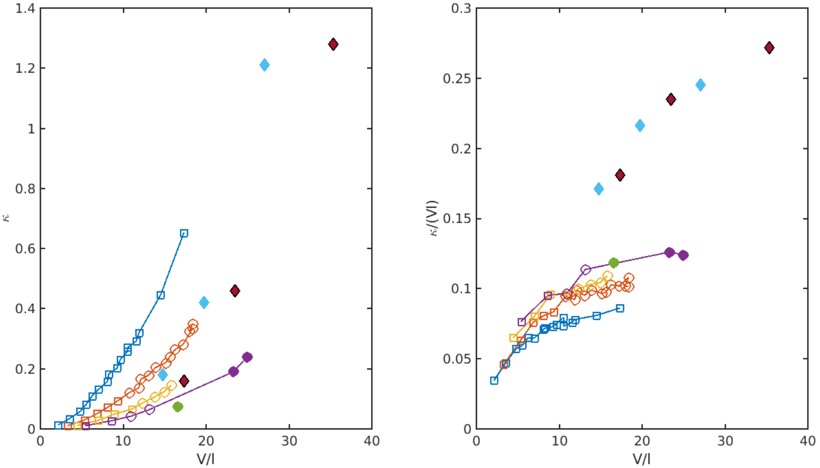
where ^{M} denotes 'modeled' (vs diagnosed/true).

The goal is for the energy produced by the parameterized model to match the true energy; matching transport term is subsidiary.

I choose the parameters α and κ so that the models energy E^M matches the true energy E as closely as possible.

The optimal α is typically very close to 1 (harmonic diffusion), so I set it to 1 and then just optimize κ.

The foregoing describes my setup from Grooms *Phys. Fluids* 2015. I later added a configuration where the forcing is homogeneous in x and localized in y (Grooms, *Phys. Rev. Fluids* 2017).



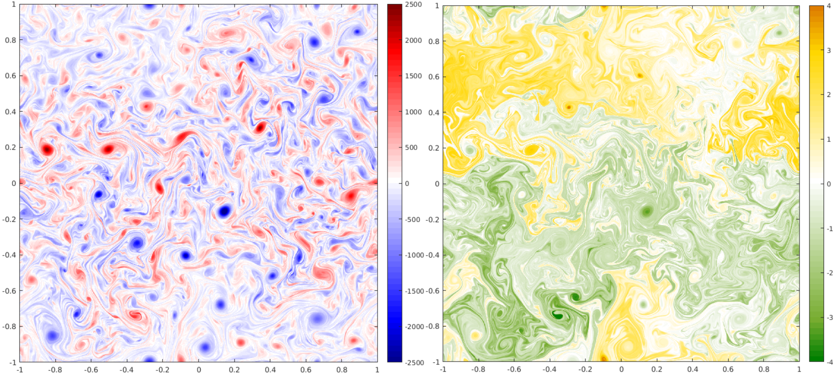
Left: Raw diffusivities (nondimensional); Right: Diffusivities scaled by a mixing length VL. Diamonds have x-homogeneous forcing, others have circular forcing; colors denote forcing length scales. Horizontal axis is a measure of nonlinearity: The turnover time divided by the Ekman damping time.

Initial conclusion: Eddy energy does seem to spread on average like harmonic diffusion with a mixing-length diffusivity.

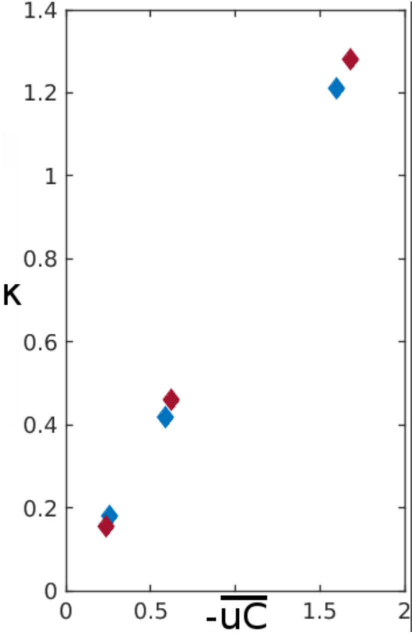
How does energy diffusivity relate to tracer diffusivity?

To study this I used the x-homogeneous forcing setup, and added a passive tracer with a mean gradient in the x direction.

I measured tracer diffusivity and compared it to energy diffusivity.



Left: Vorticity in simulation with x-homogeneous forcing.
Right: Passive tracer from same simulation.



Out of 6 experiments with different forcing length scales and amplitudes, the energy diffusivity is 75% of the tracer diffusivity.

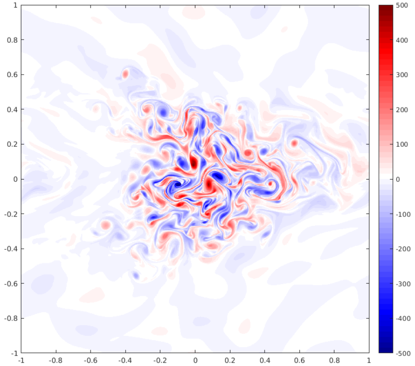
This is more robust than fitting the energy diffusivity to a mixing-length.

How does β effect the results?

β introduces anisotropy, so I go back to the circular forcing.

β also causes the eddies to rectify a time-mean flow that has to be included in the analysis.

I update my diffusion model to allow anisotropic diffusion.



Constant-coefficient harmonic energy diffusion is no longer a great fit, though still reasonable.

Diffusion becomes anisotropic, reduced in y direction.

No scaling laws available at this time.

Outlook

- There is still no theoretical basis for modeling mesoscale eddy energy transport as diffusion, even in 2D…
- On an f-plane, energy transports diffusively *on average*, but the average is not very representative – it takes up to 13,000 eddy turnover times for the energy average to equilibrate. Maybe a stochastic model?
- Further work needed to understand and model the impact of β (and topography or large-scale PV gradients)
- Further work needed to understand and model the impact of stratification
- What happens to the eddy energy at lateral boundaries?