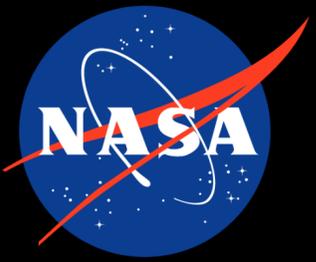
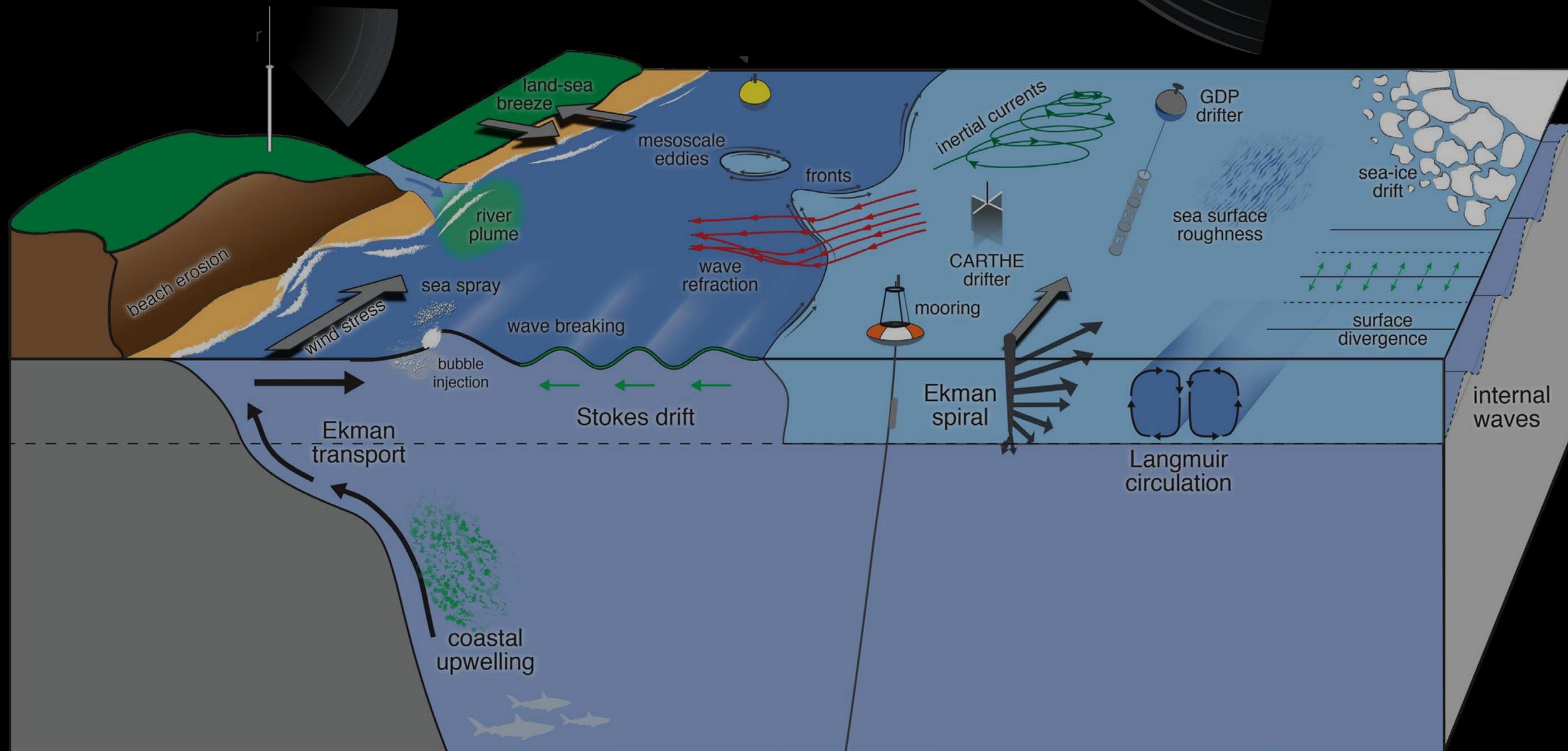
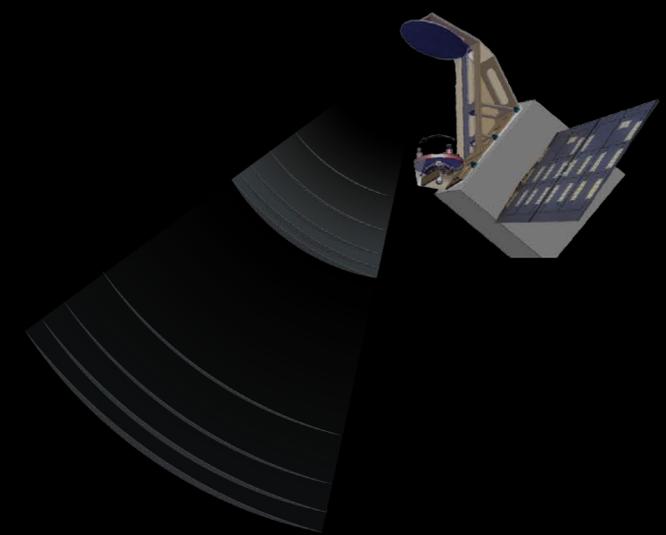


# Wind, wave, and current interactions

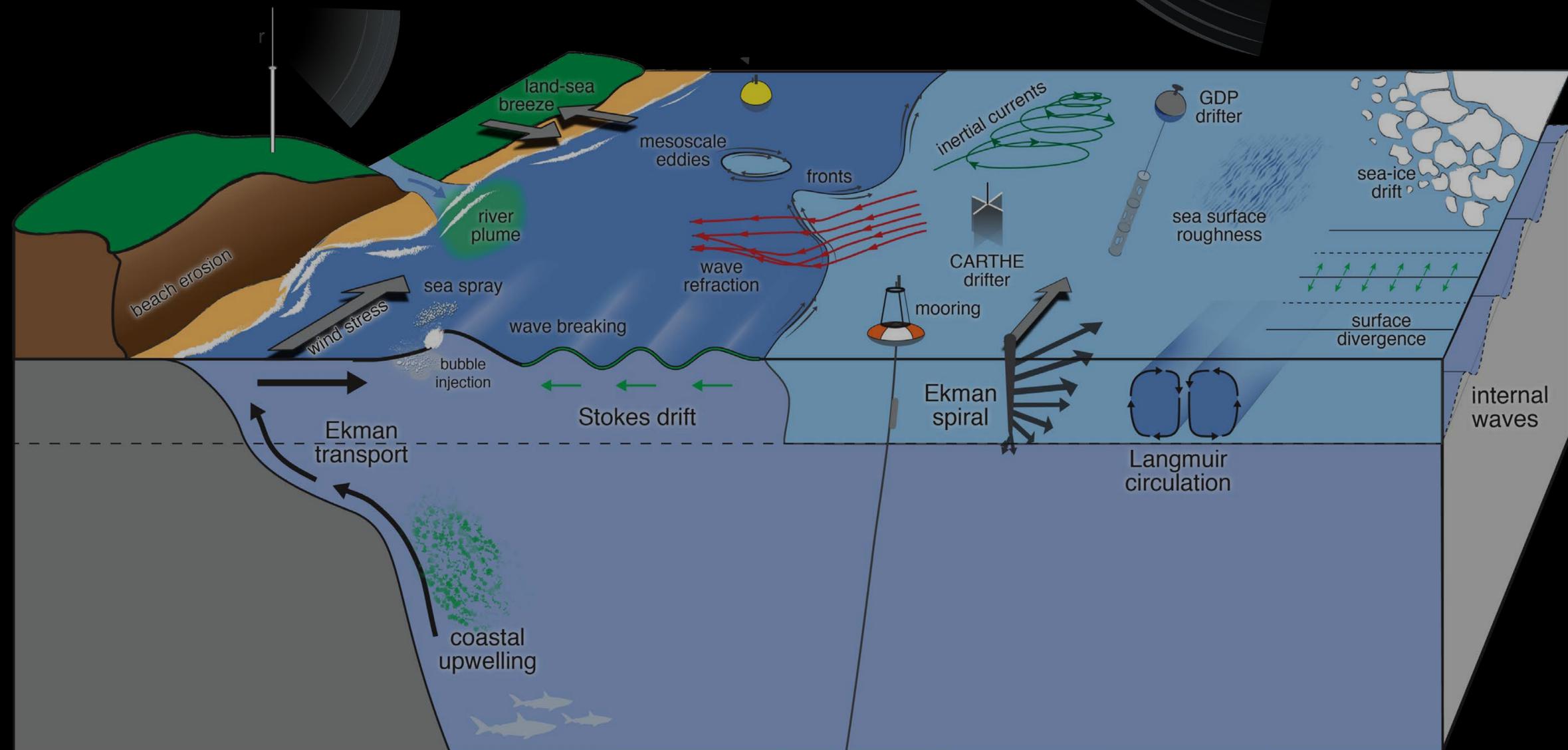
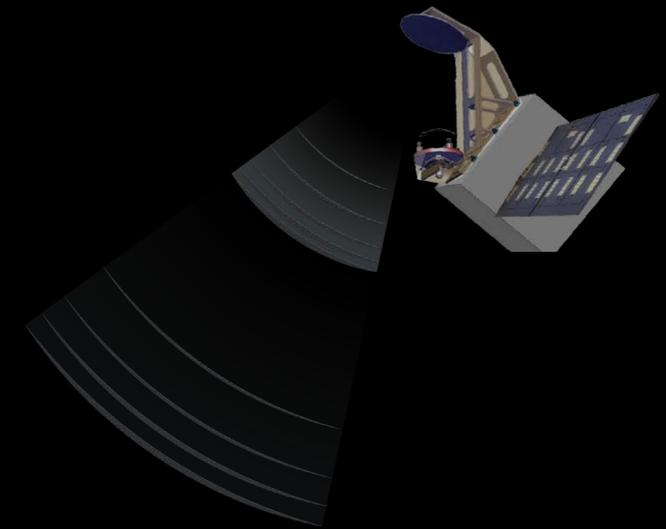
Bia Villas Bôas

Fabrice Ardhuin, Bruce Cornuelle, Sarah Gille, Matt Mazloff, Bill Young, Gwendal Maréchal, and many others.



From Villas Bôas et al. (2019) by Momme Hell.

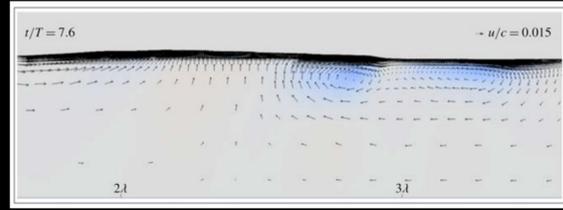
Improved understanding and representation of air-sea interactions **demand** a **combined** cross-boundary approach that can only be achieved through **integrated observations** and **modeling** of ocean **winds**, surface **currents**, and ocean surface **waves**.



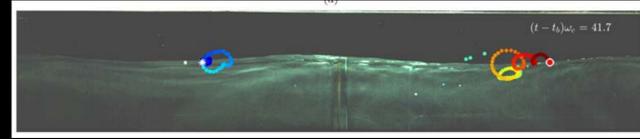
From Villas Bôas et al. (2019) by Momme Hell.

**wavy processes**

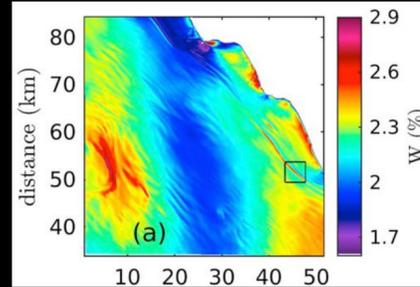
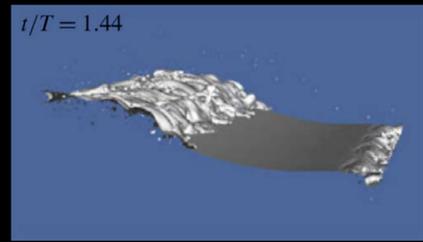
Pizzo et al., (2016)



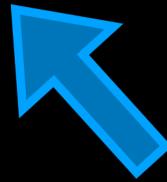
Lenain et al., (2019)



Deike et al., (2016)

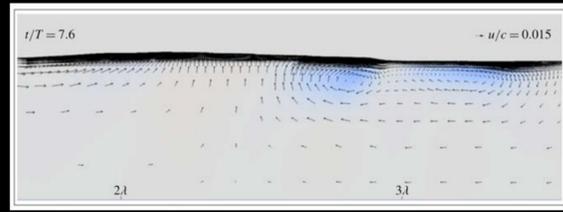


Romero (2019)

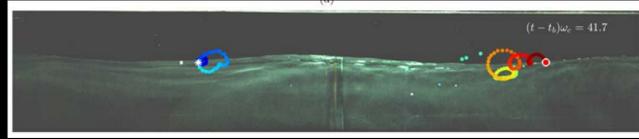


wavy processes

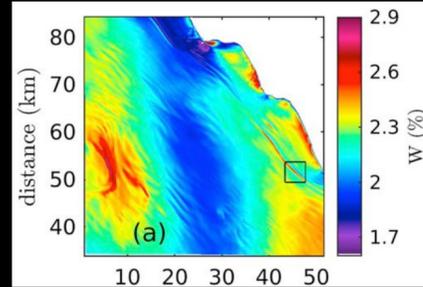
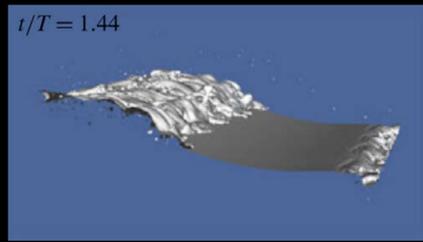
Pizzo et al., (2016)



Lenain et al., (2019)

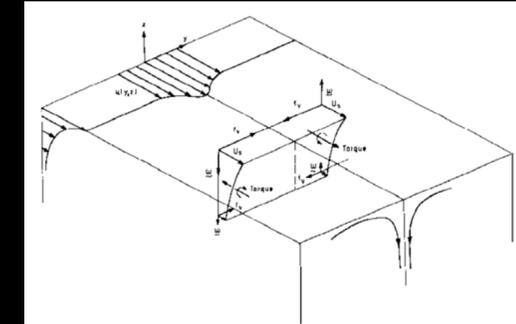
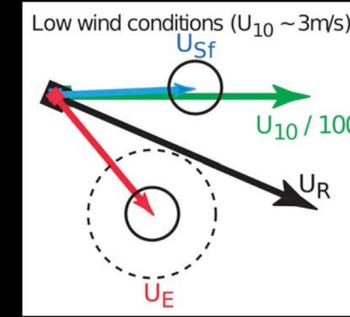


Deike et al., (2016)

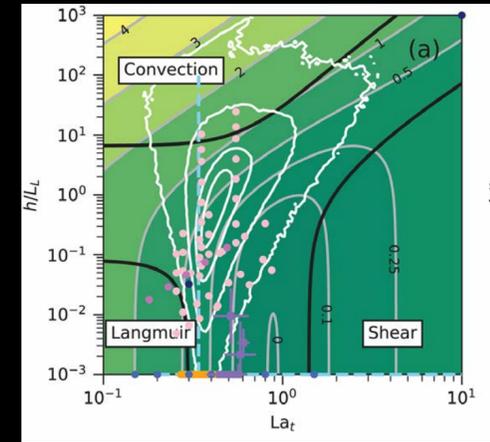


Romero (2019)

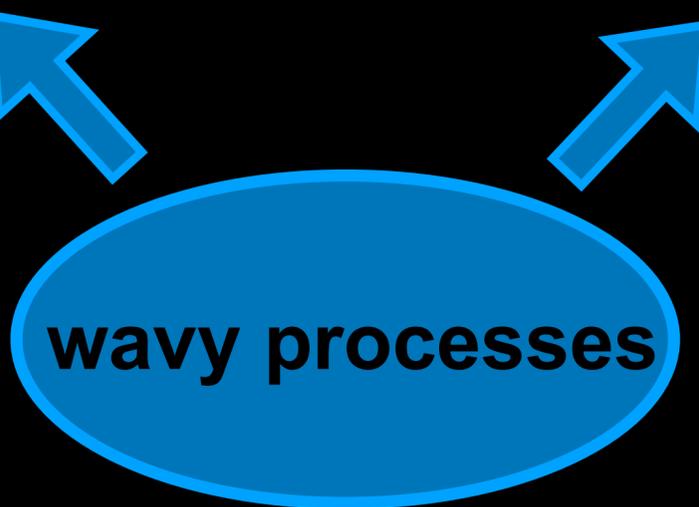
Ardhuin et al., (2009)



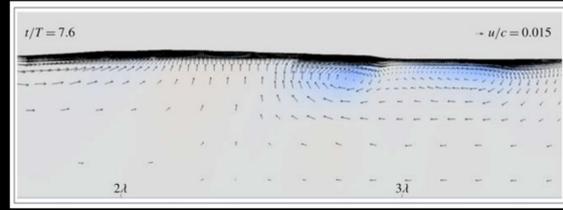
Leibovich (1983)



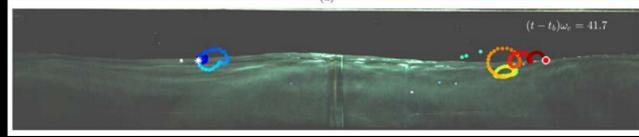
Belcher et al. (2012)  
Li et al., (2016)



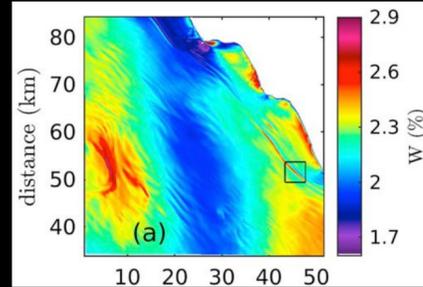
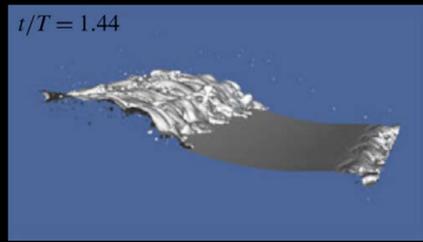
Pizzo et al., (2016)



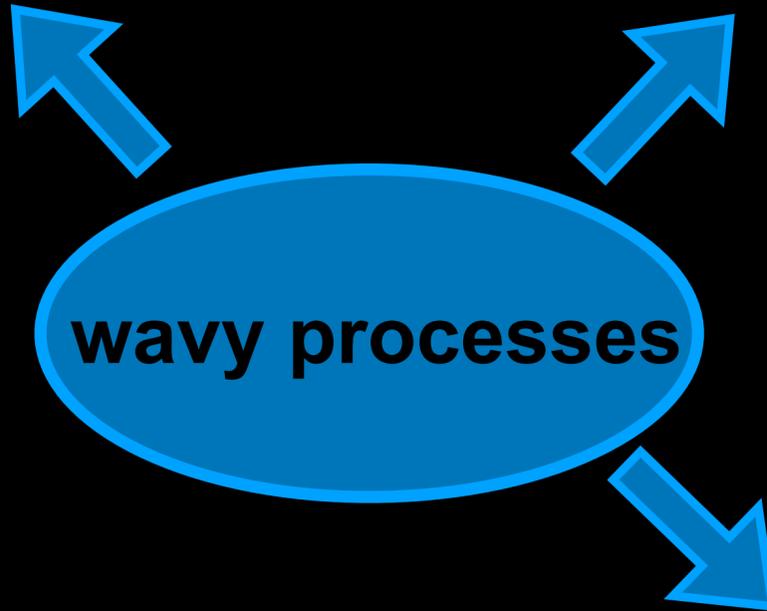
Lenain et al., (2019)



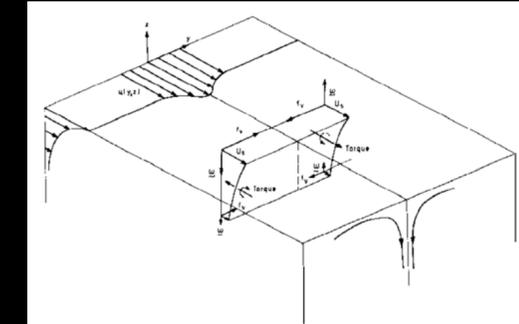
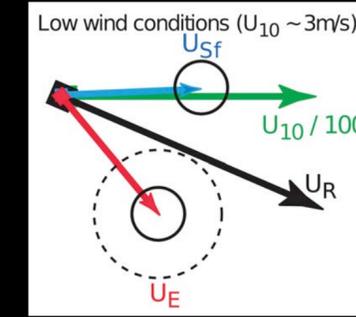
Deike et al., (2016)



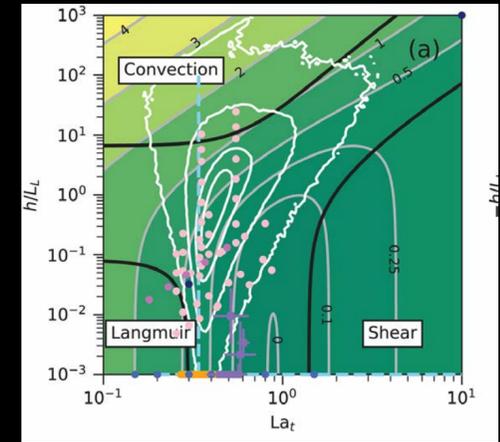
Romero (2019)



Ardhuin et al., (2009)

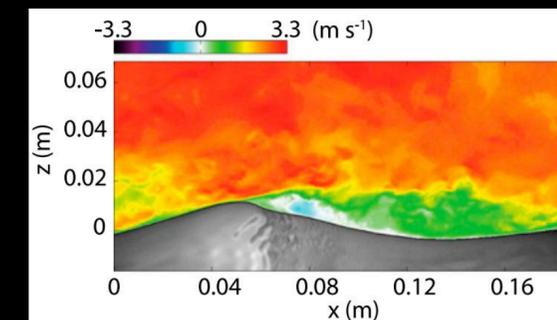
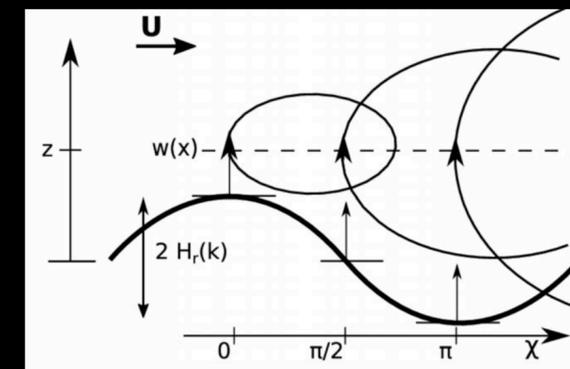


Leibovich (1983)

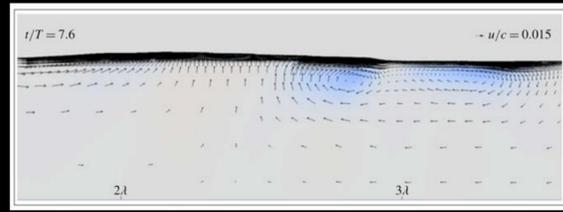


Belcher et al. (2012)  
Li et al., (2016)

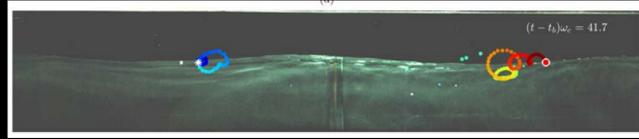
Gare et al. (2013), Buckley & Veron (2016),  
Ayet et al. (2019)



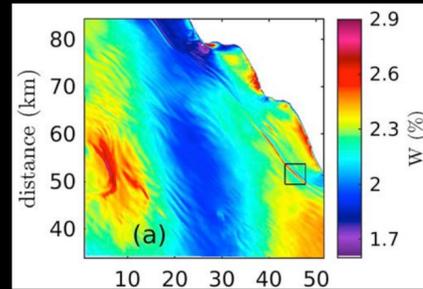
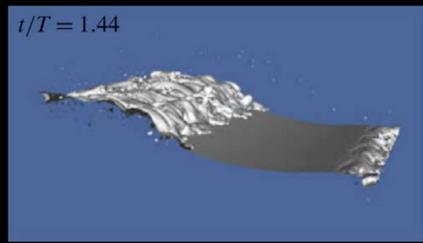
Pizzo et al., (2016)



Lenain et al., (2019)

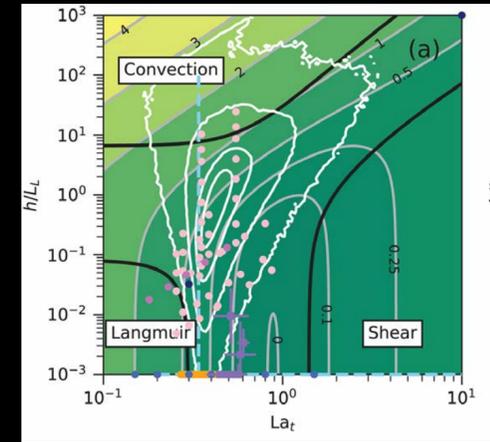
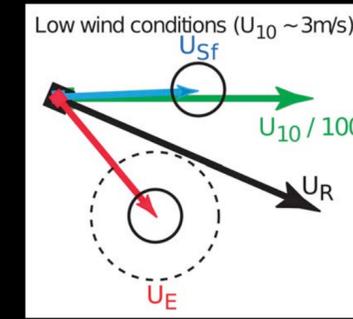


Deike et al., (2016)

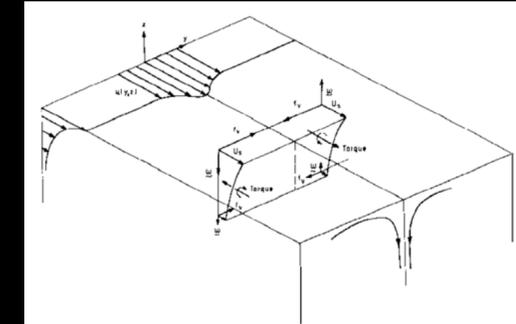


Romero (2019)

Ardhuin et al., (2009)



Belcher et al. (2012)  
Li et al., (2016)



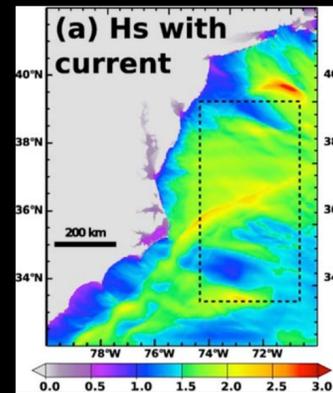
Leibovich (1983)

wavy processes

Romero et al (2017)

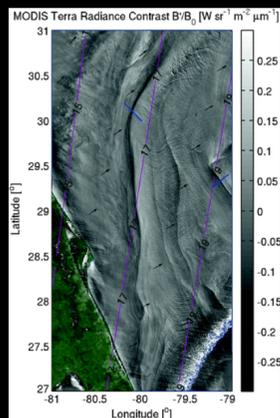


Ardhuin et al., (2017)

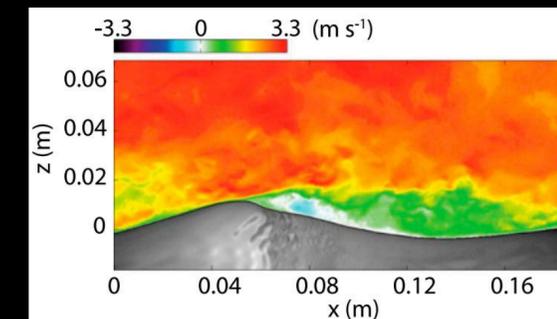
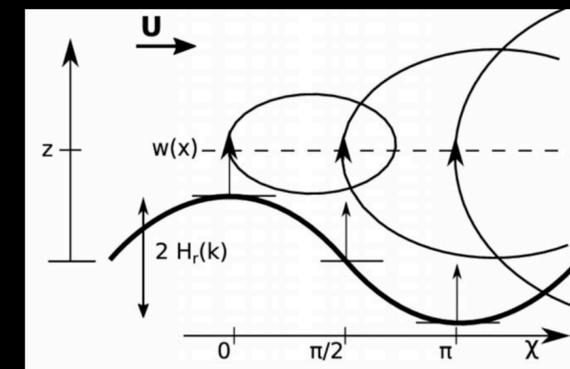


Gare et al. (2013), Buckley & Veron (2016),  
Ayet et al. (2019)

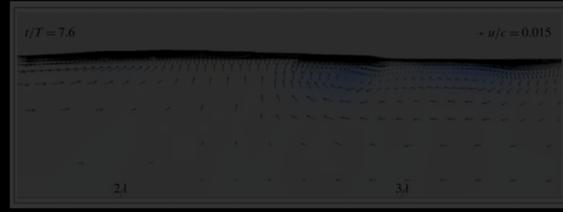
Zippel & Thomson (2016)



Rasclé et al. (2016)



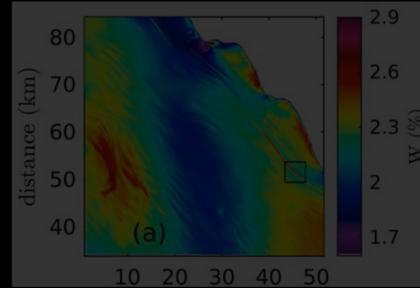
Pizzo et al., (2016)



Lenain et al., (2019)

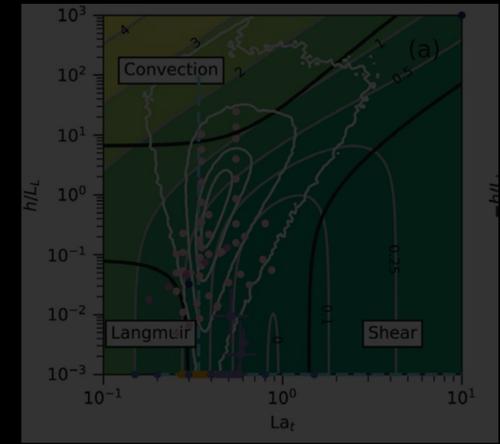
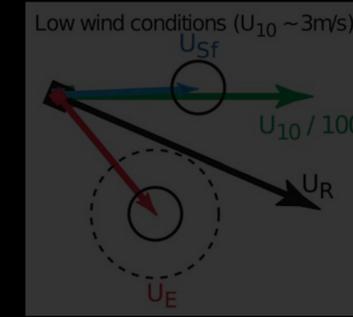


Deike et al., (2016)

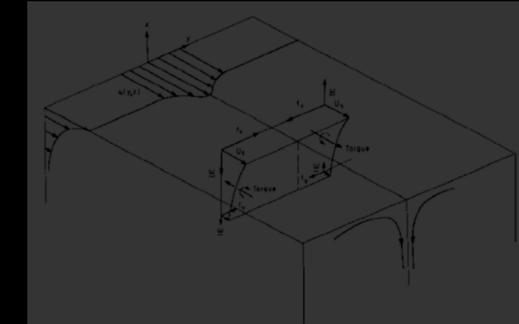


Romero (2019)

Ardhuin et al., (2009)



Belcher et al. (2012)  
Li et al., (2016)



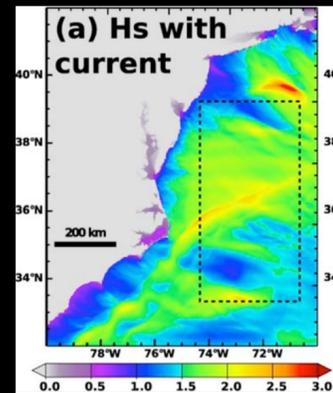
Leibovich (1983)

wavy processes

Romero et al (2017)

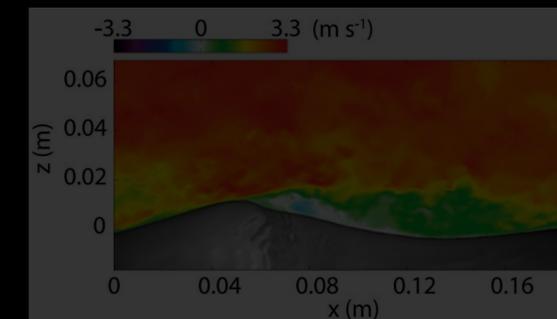
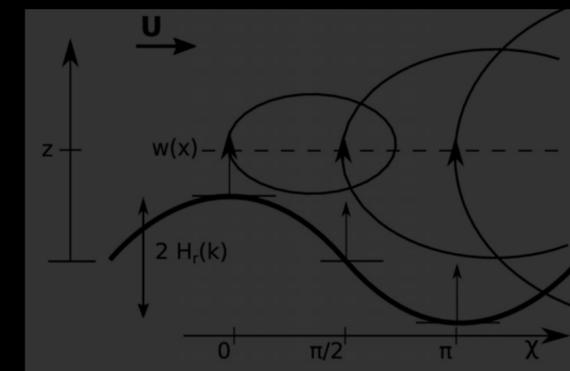
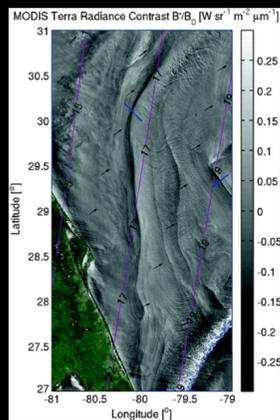


Ardhuin et al., (2017)



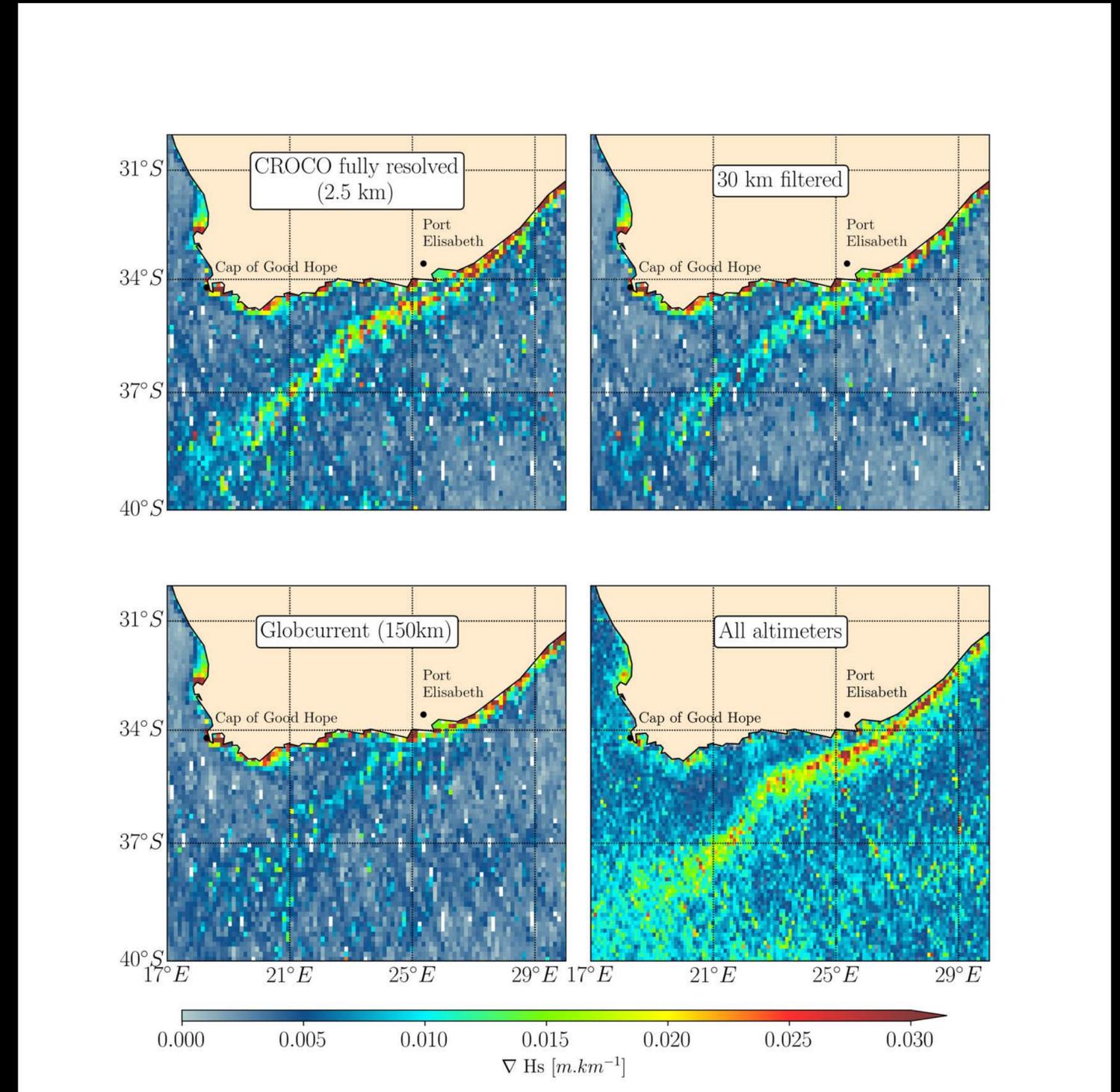
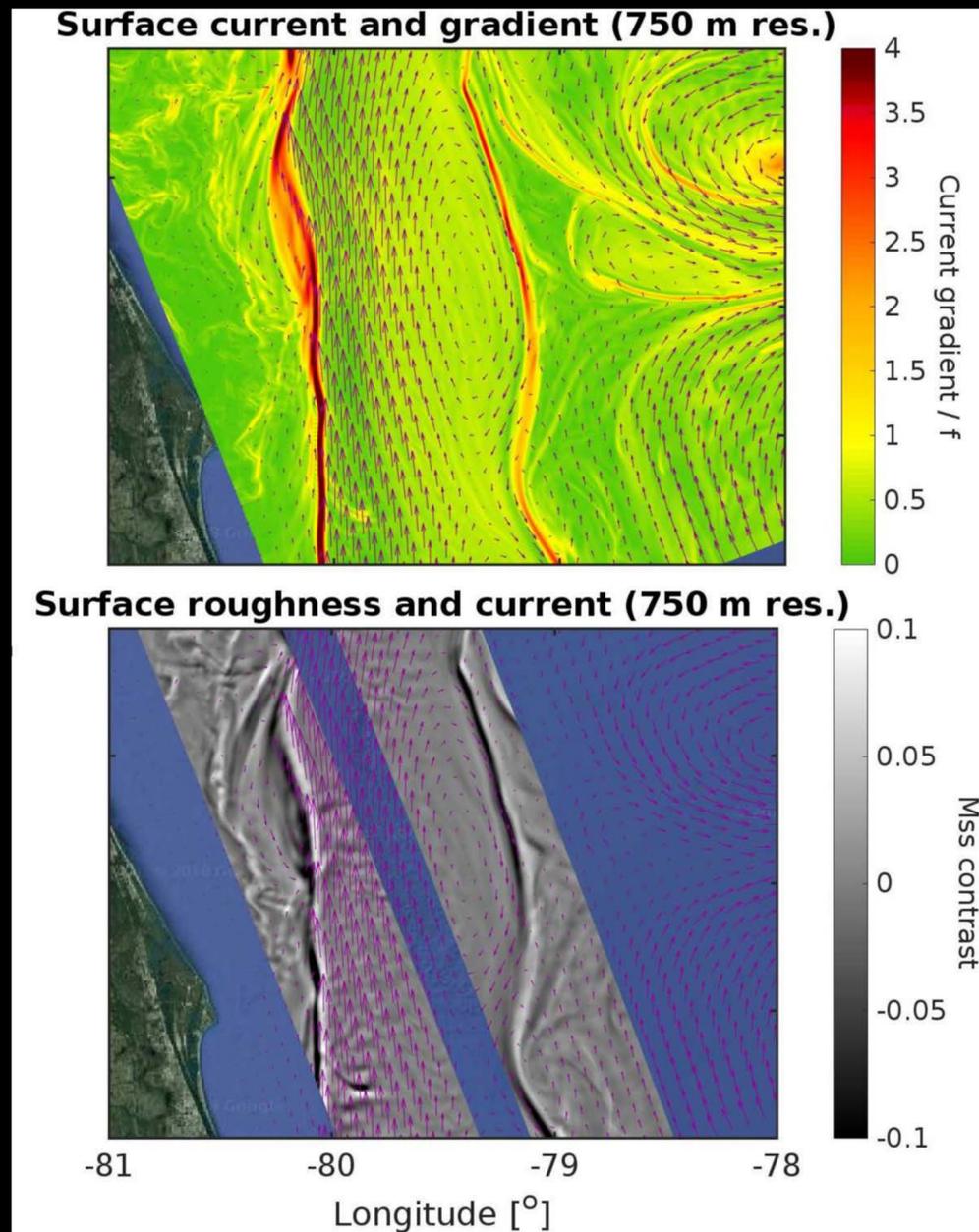
Gare et al. (2013), Buckley & Veron (2016),  
Ayet et al. (2019)

Zippel & Thomson (2016)



# What do we know? Wave properties vary on small scales!

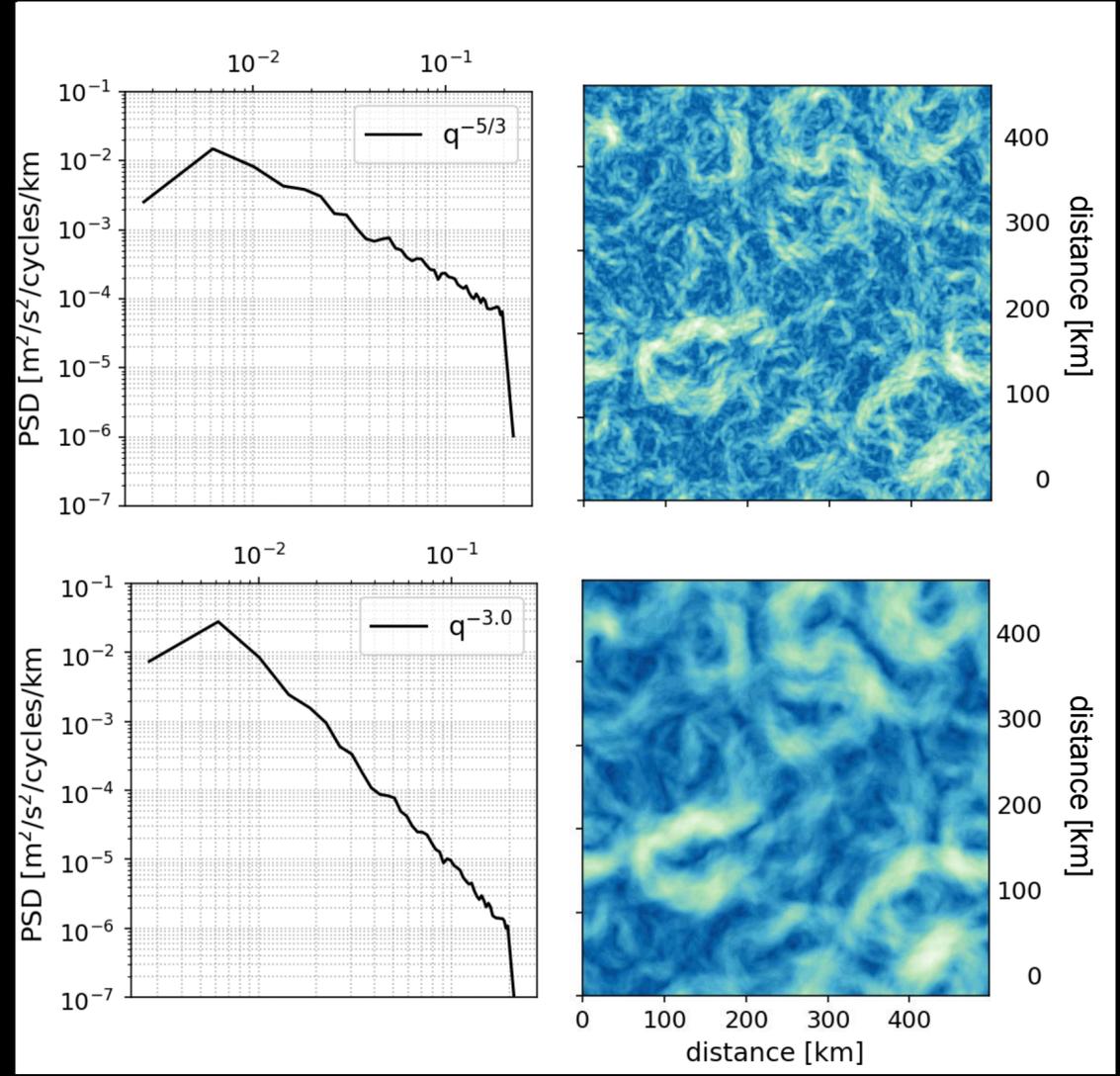
Morrow et al., 2019 and Raschle et al., 2016



Work from Gwendal Marechal and Fabrice Ardhuin

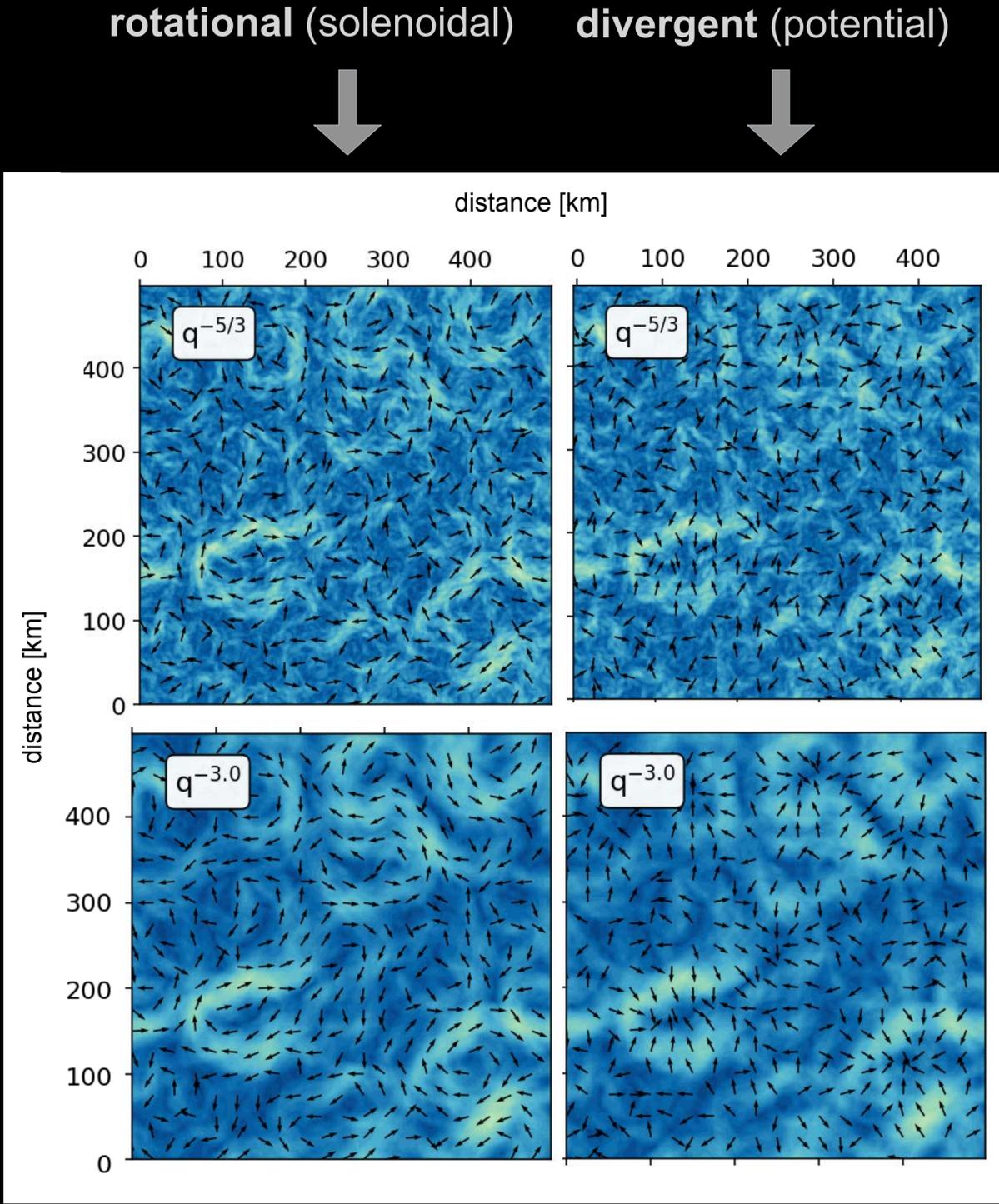
# How well do we understand these sea state gradients? surface wave response to vorticity and divergence.

## Synthetic surface currents



The variance of the flow is all contained in wavelengths between 5km and 300km.

Helmholtz  
→  
decomposition





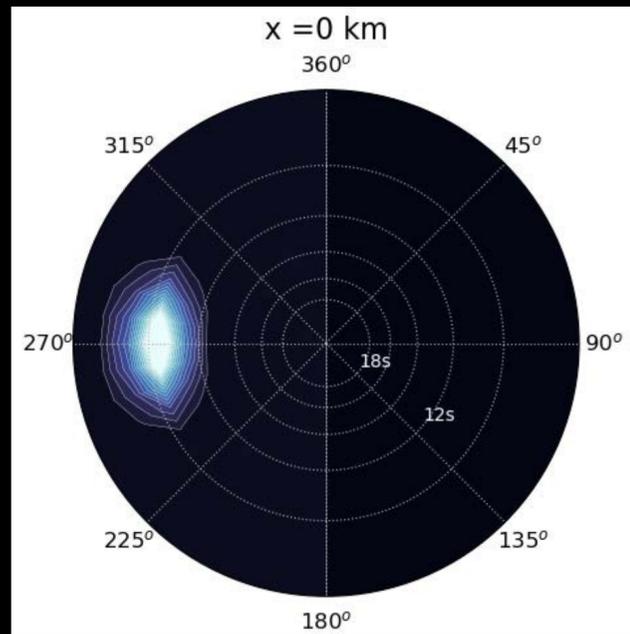
# Wave Model:

We use the wave model WaveWatch III (WW3) to integrate the action balance equation (with no source terms):

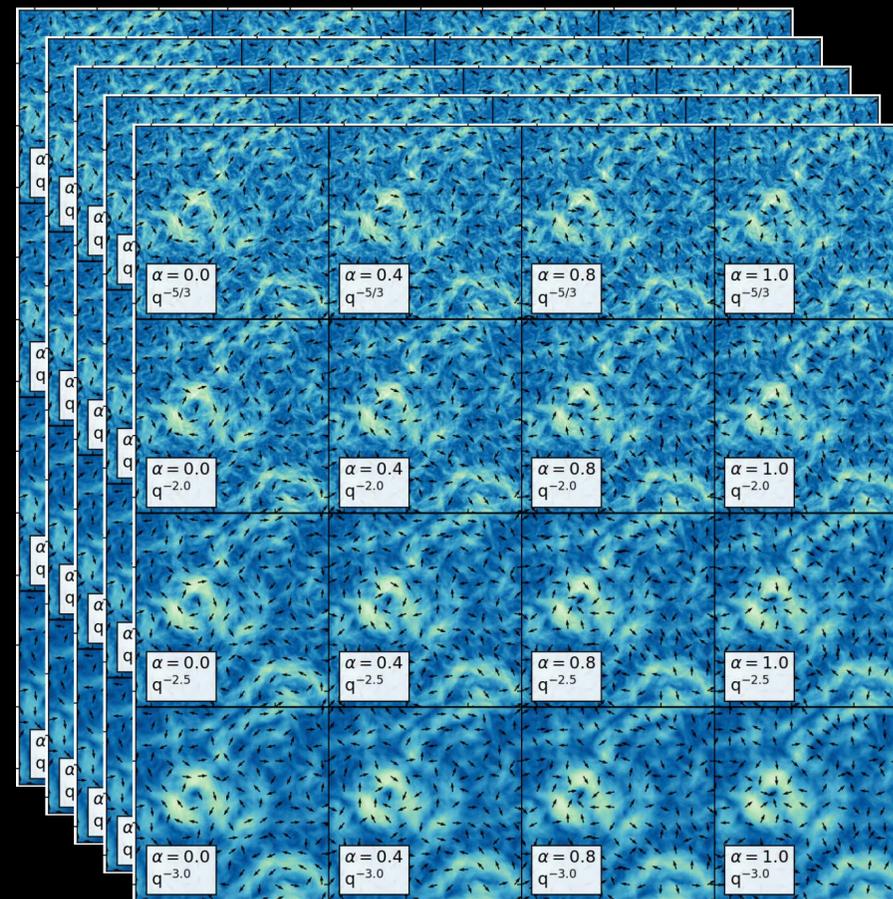
$$\frac{\partial N}{\partial t} + \nabla \cdot (\dot{\mathbf{x}}N) + \frac{\partial}{\partial k}(\dot{k}N) + \frac{\partial}{\partial \theta}(\dot{\theta}N) = 0$$

for an initially narrow-banded wave spectrum with waves propagating from the left side of the domain.

$$\sigma_{\theta} = 12^{\circ}; \sigma_f = 0.01\text{Hz}$$



$$\theta_0 = 270^{\circ}$$



Divergence Fraction	0.0	0.2	0.4	0.6	0.8	1.0
Spectral Slope	-5/3	-2.0	-2.5	-3.0		
Wave Period	7.0s	10.3s	16.6s			

For each member of the ensemble, there are 72 possible combinations of wave period, flow spectral slope and divergence fraction giving a total of 3600 simulations.

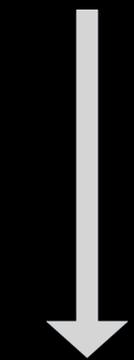
# Peak direction (refraction)

Changes in the peak wave **direction** are **larger** for **rotational** flows (left panels) than divergent (right panels).

This result is consistent with the predictions from **ray theory**: in the limit of weak current gradients one can approximate the curvature of individual rays by the ratio between the **vorticity** of the flow and the **group velocity** of the waves:

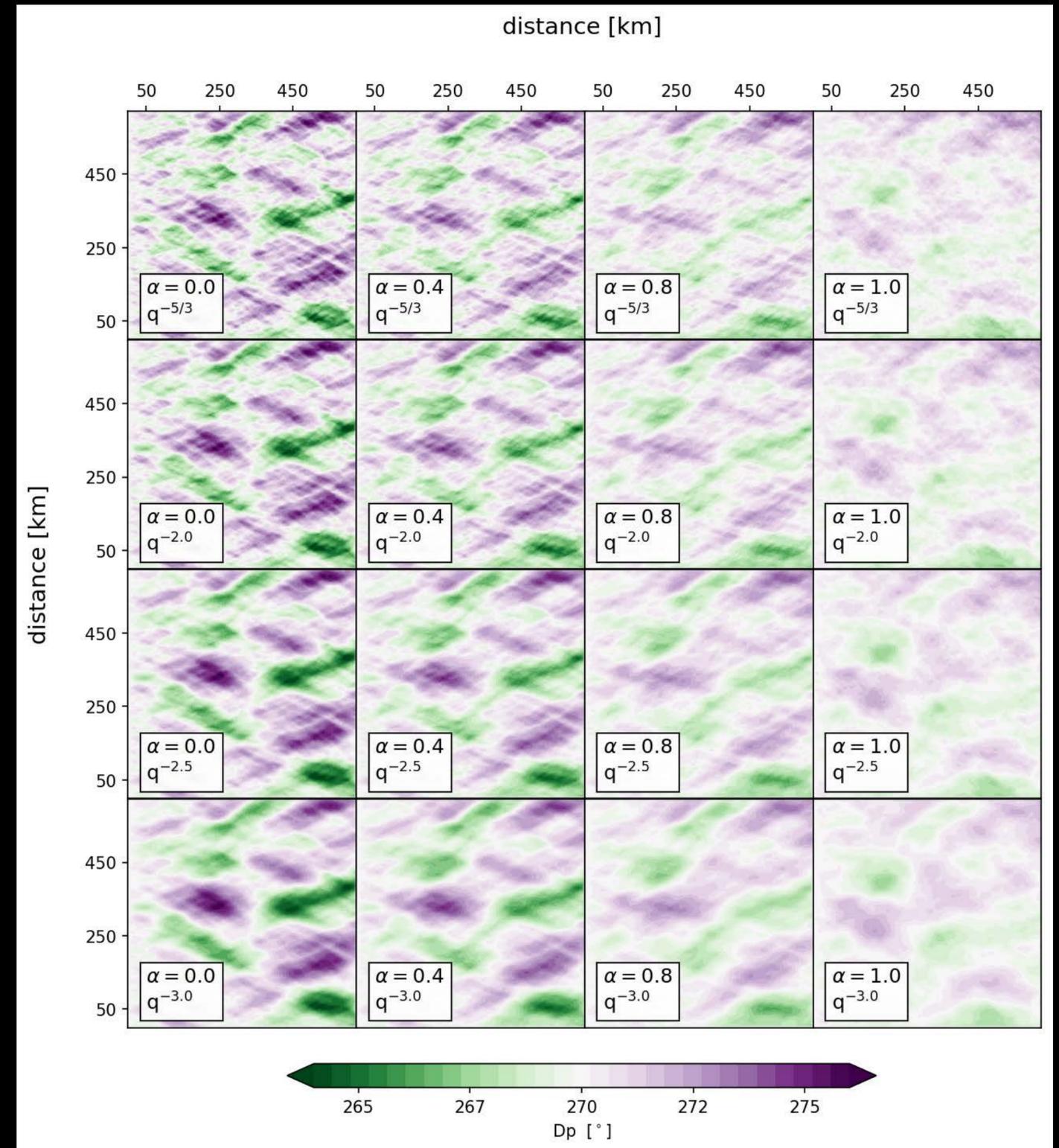
$$\chi = \frac{\zeta}{c_g}$$

Shallower  
spectral slope



Steeper  
spectral slope

More vorticity  $\longrightarrow$  More divergence



# Significant wave height ( $H_s$ )

The spatial variability of significant wave height is highly dependent on the nature of the flow

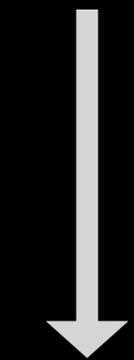
Strong **refraction** leads to strong convergence and divergence of **wave action**.

As a consequence, there is more **structure** in the significant wave height ( $H_s$ ) for the flow with more **vorticity**

**Changes** of up to 30% in  $H_s$  over scales of **tens of kilometers**.

Shallower KE spectral slope, are associated with finer structures in the  $H_s$  maps,

Shallower  
spectral slope

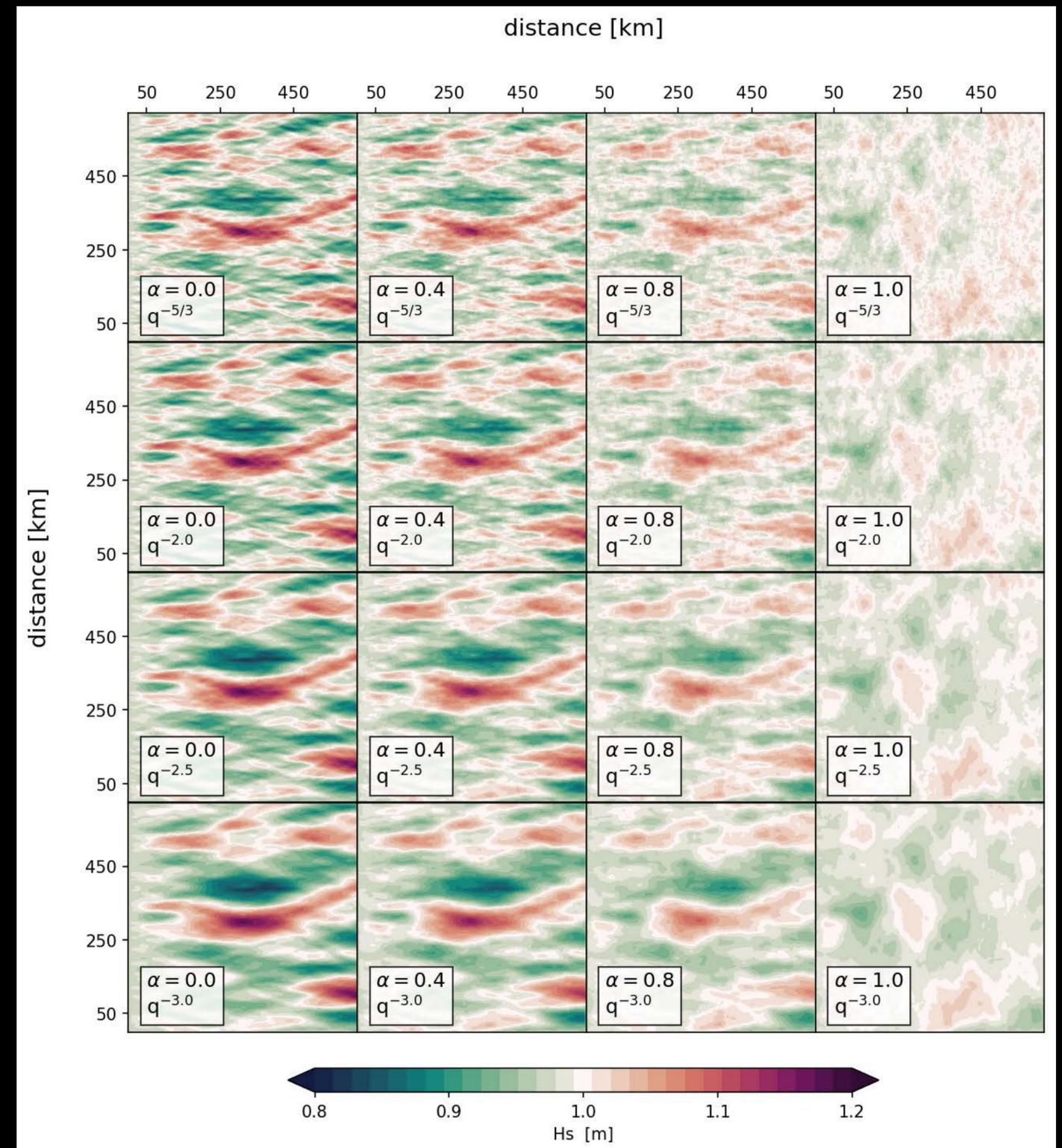


Steeper  
spectral slope

More vorticity



More divergence



More structure



Less structure

# Directional spreading

Directional spreading **increases** as the waves propagate through the domain.

More vorticity leads to more spreading

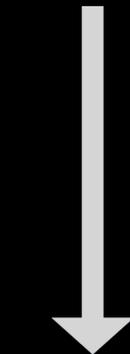
Shallower spectral slope leads to higher spreading

Virtually no spreading for purely divergent flows.

**The potential component of the flow has NO contribution to the directional diffusion of wave action.**

See the theoretical explanation in Villas Bôas and Young, *in press* in JFM (email me for a copy).

Shallower  
spectral slope

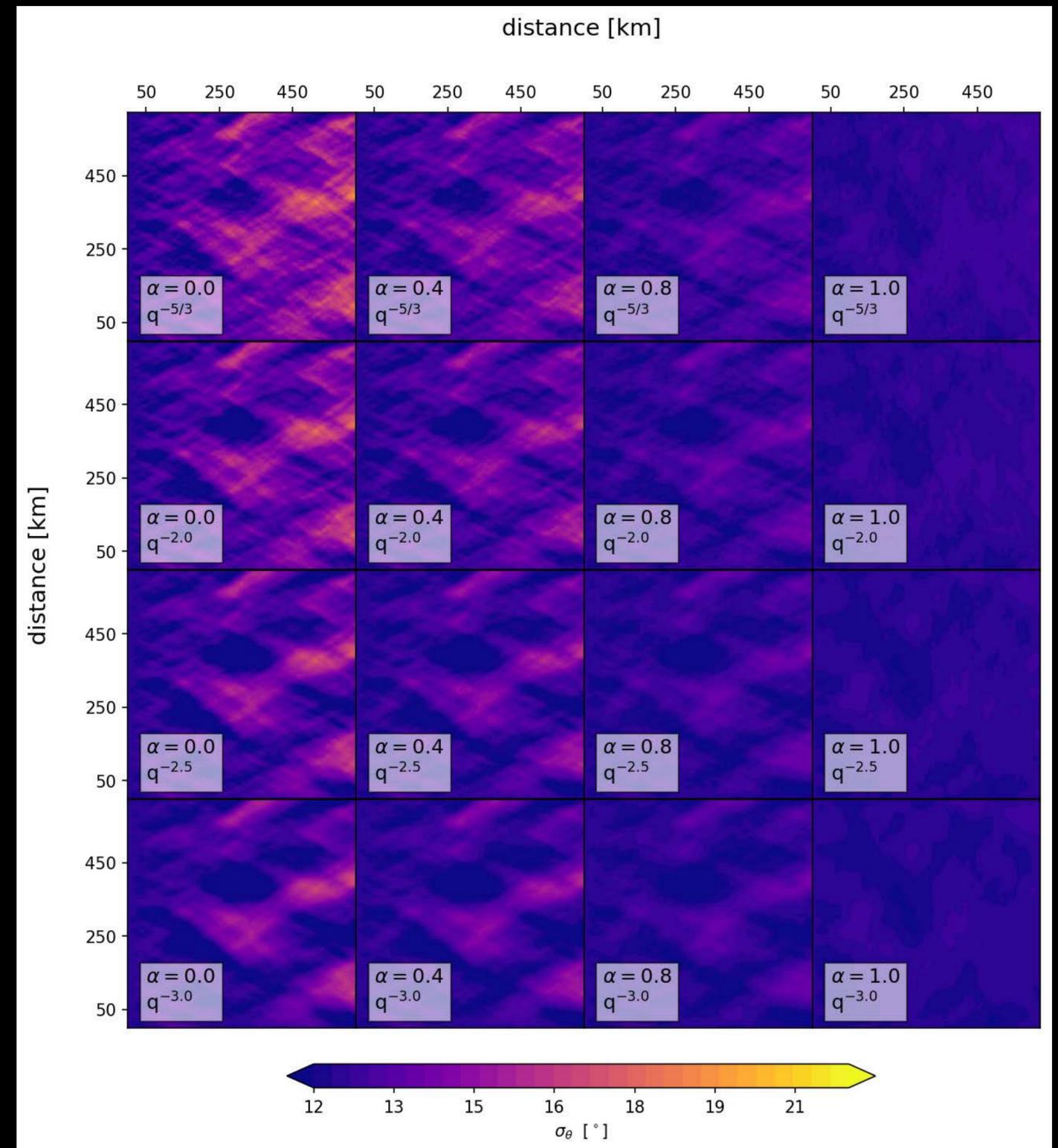


Steeper  
spectral slope

More vorticity



More divergence



# Directional spreading

Directional spreading **increases** as the waves propagate through the domain.

More vorticity leads to more spreading

Shallower spectral slope leads to higher spreading

Virtually no spreading for purely divergent flows.

The potential component of the flow has **NO** contribution to the directional diffusion of wave action.

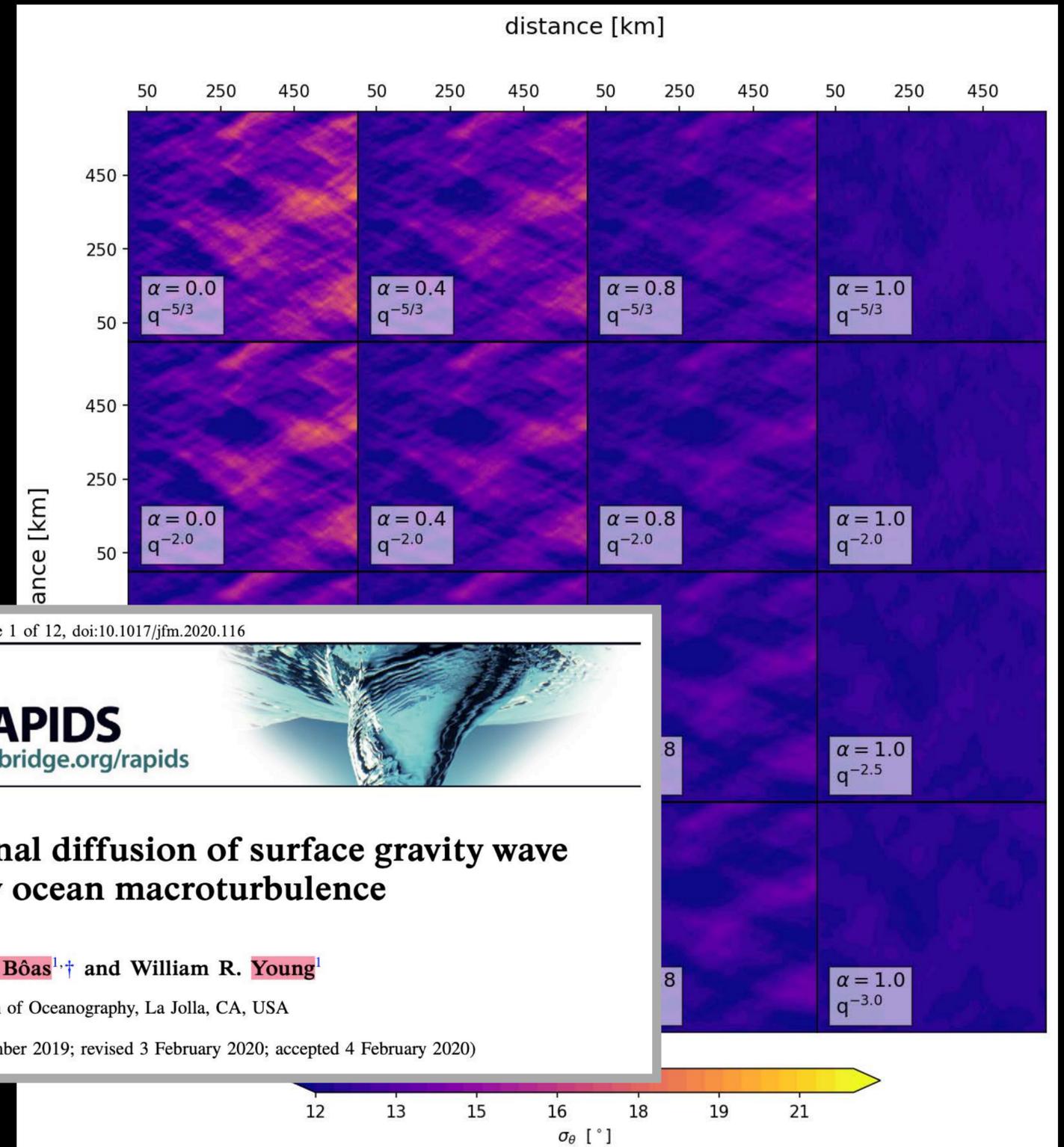
See the theoretical explanation in Villas Bôas and Young, *in press* in JFM (email me for a copy).

More vorticity



More divergence

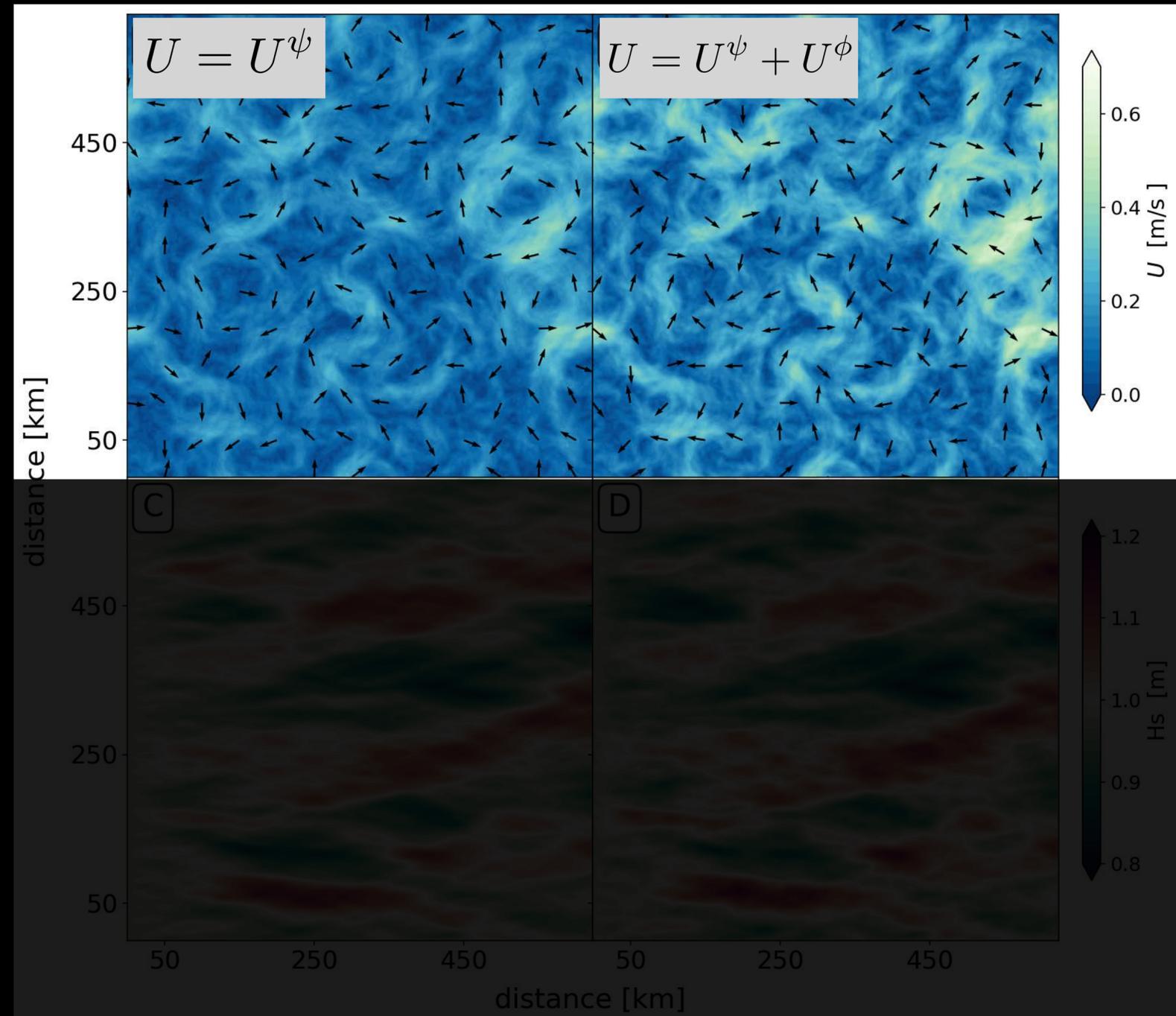
Shallower spectral slope



**Hypothesis:** The spatial variability of  $H_s$  is dominated by the spatial variability of the rotational component of the flow

**Hypothesis:** The spatial variability of  $H_s$  is dominated by the spatial variability of the rotational component of the flow

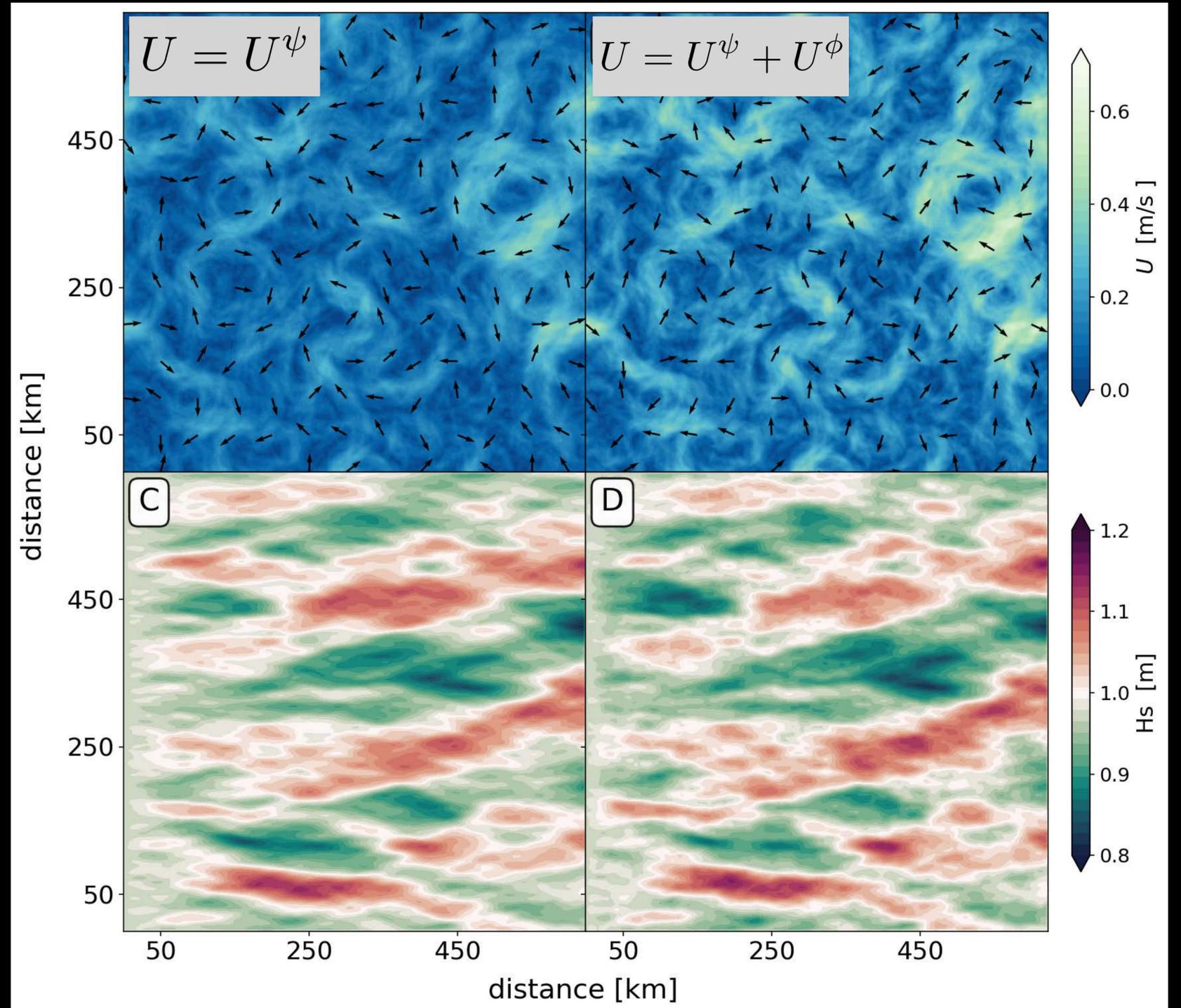
We **double** the kinetic energy of the purely rotational flow by adding a potential component.



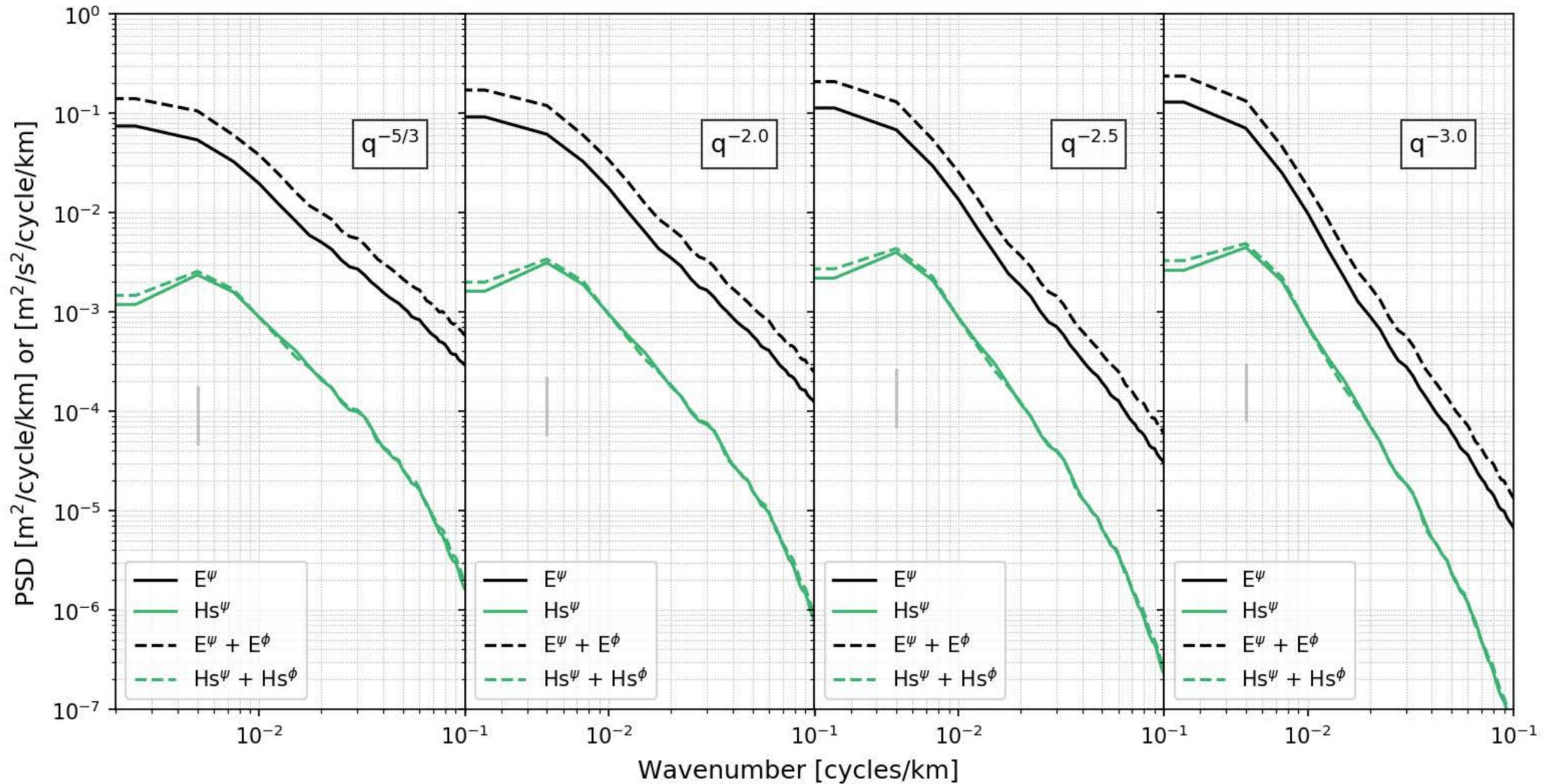
**Hypothesis:** The spatial variability of Hs is dominated by the spatial variability of the rotational component of the flow

We **double** the kinetic energy of the purely rotational flow by adding a potential component.

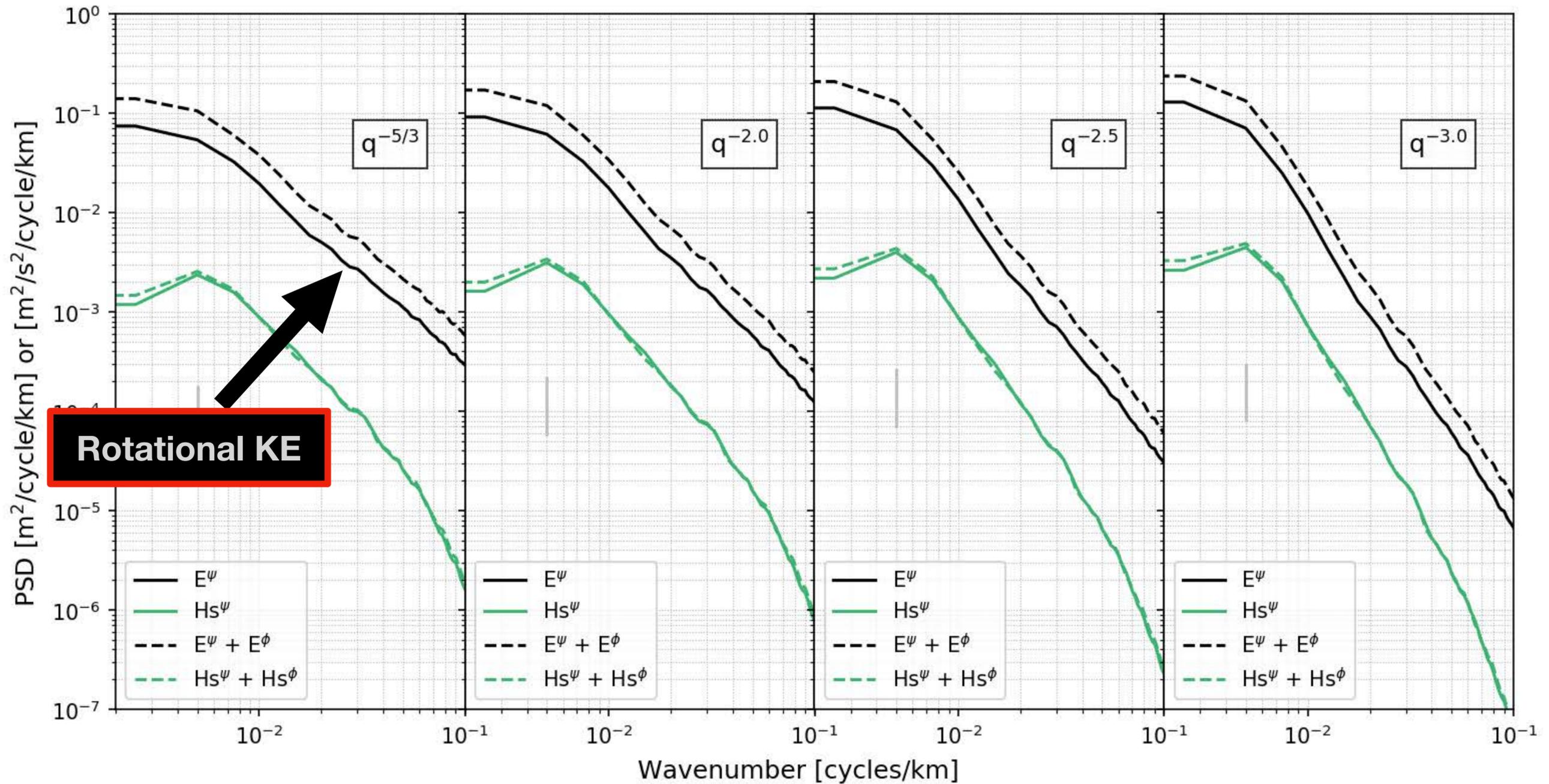
The Hs response is virtually the same:  
The spatial variability of Hs at these scales is not affected by the potential component of the flow.



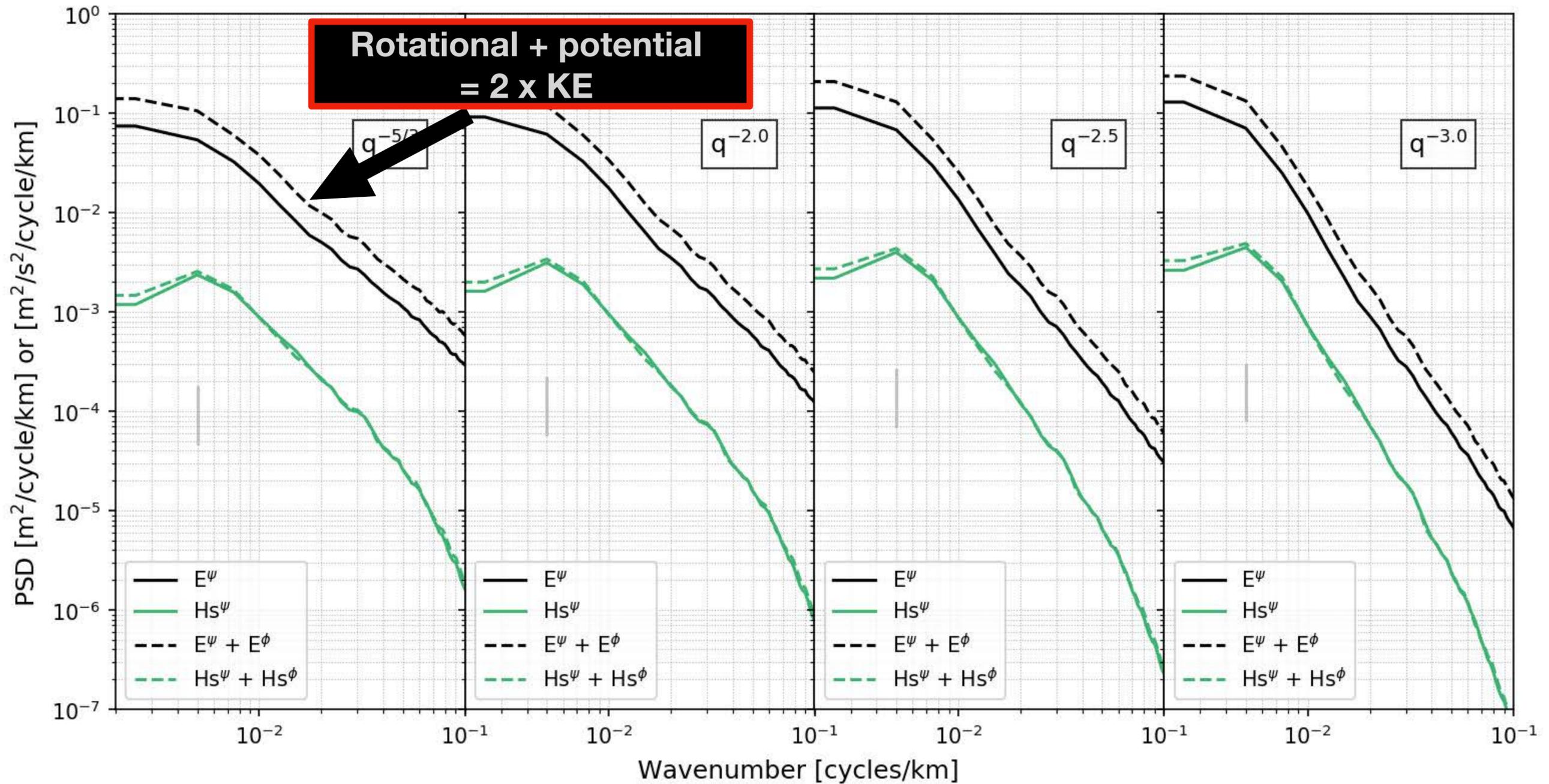
► The wavenumber spectra of Hs (averaged across the ensemble) is **not** affected by the **potential** flow



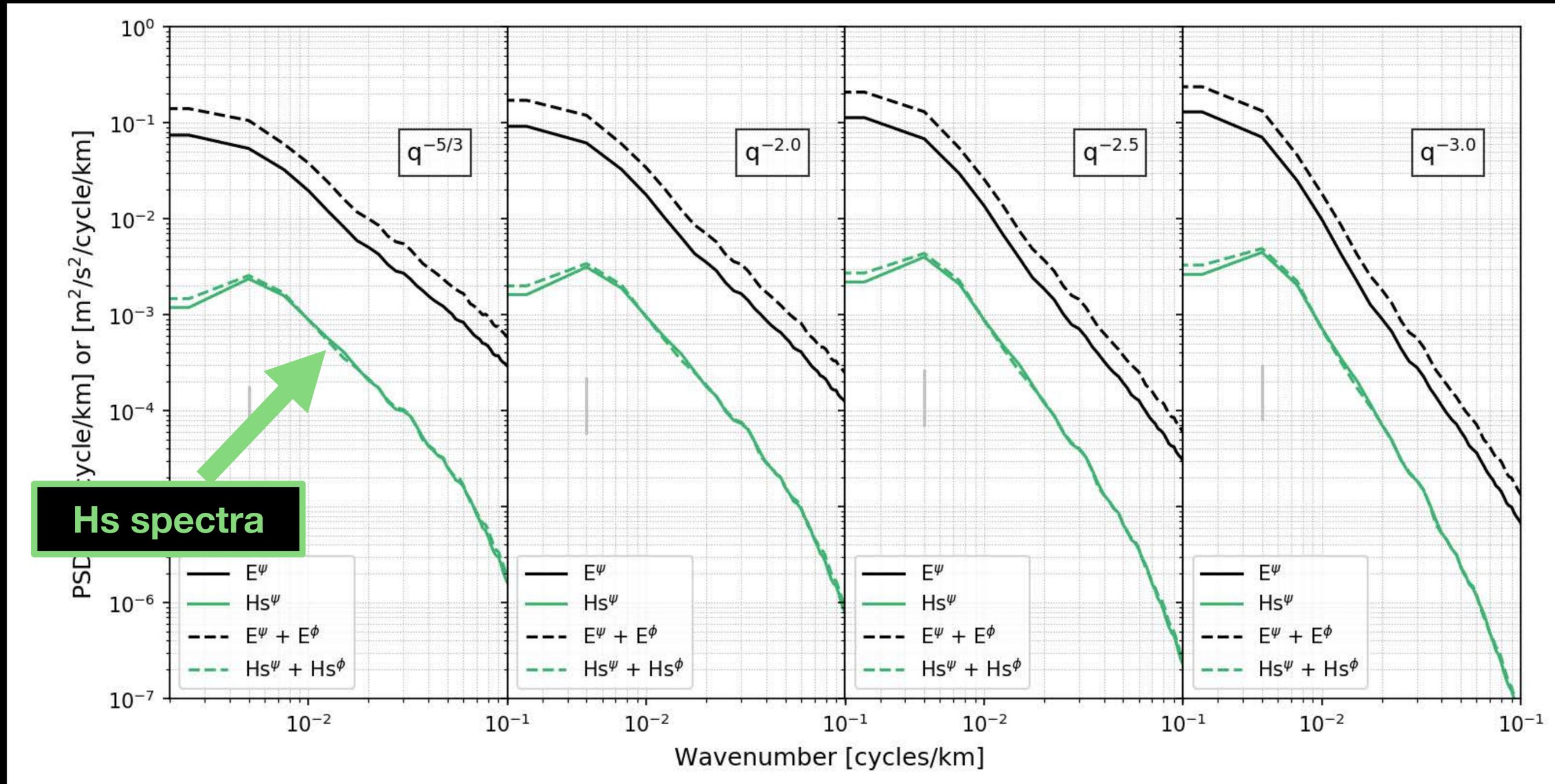
► The wavenumber spectra of Hs (averaged across the ensemble) is **not** affected by the **potential** flow



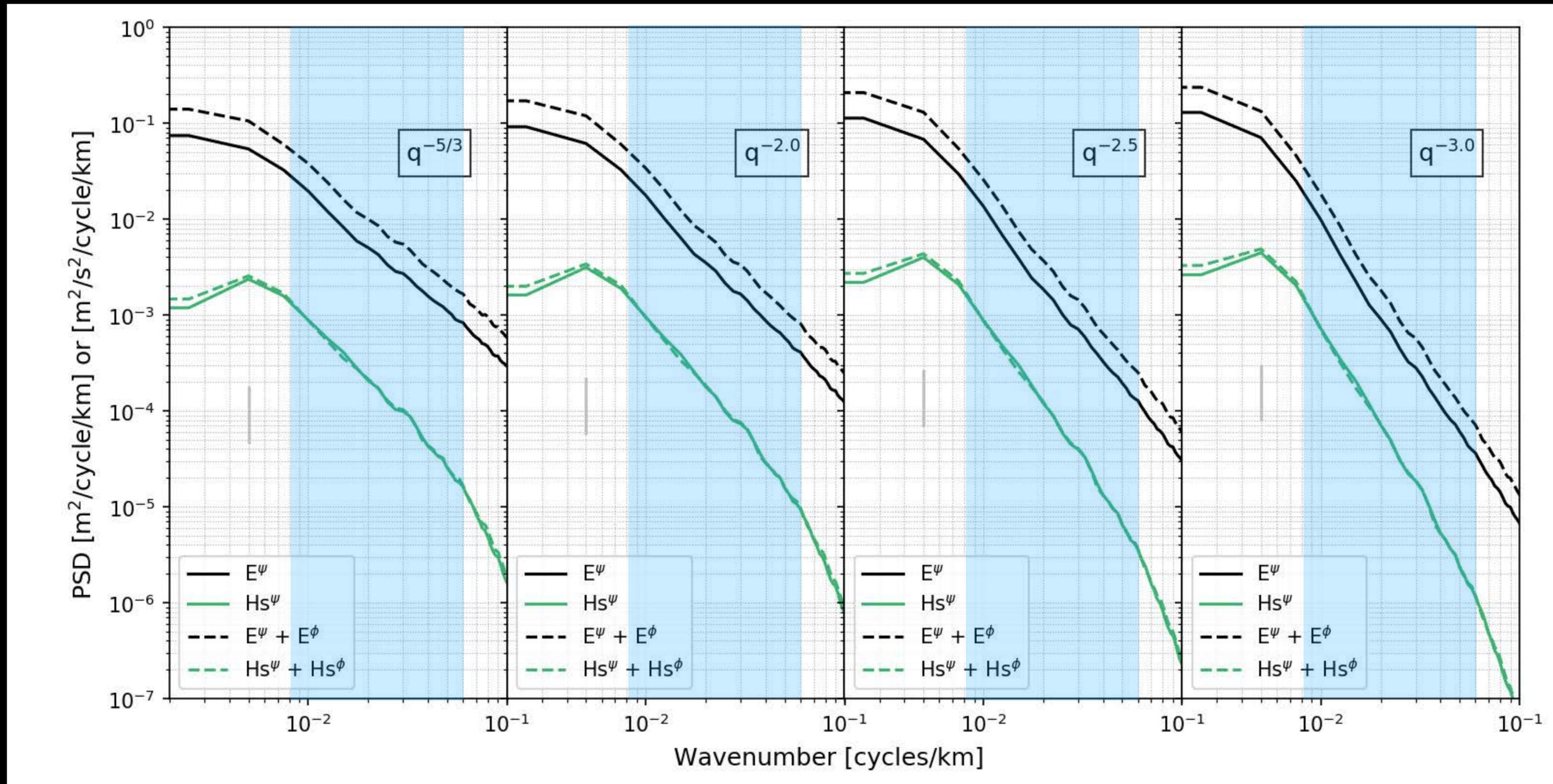
► The wavenumber spectra of Hs (averaged across the ensemble) is **not** affected by the **potential** flow



► The wavenumber spectra of Hs (averaged across the ensemble) is **not** affected by the **potential** flow



- The wavenumber spectra of Hs (averaged across the ensemble) is **not** affected by the **potential** flow

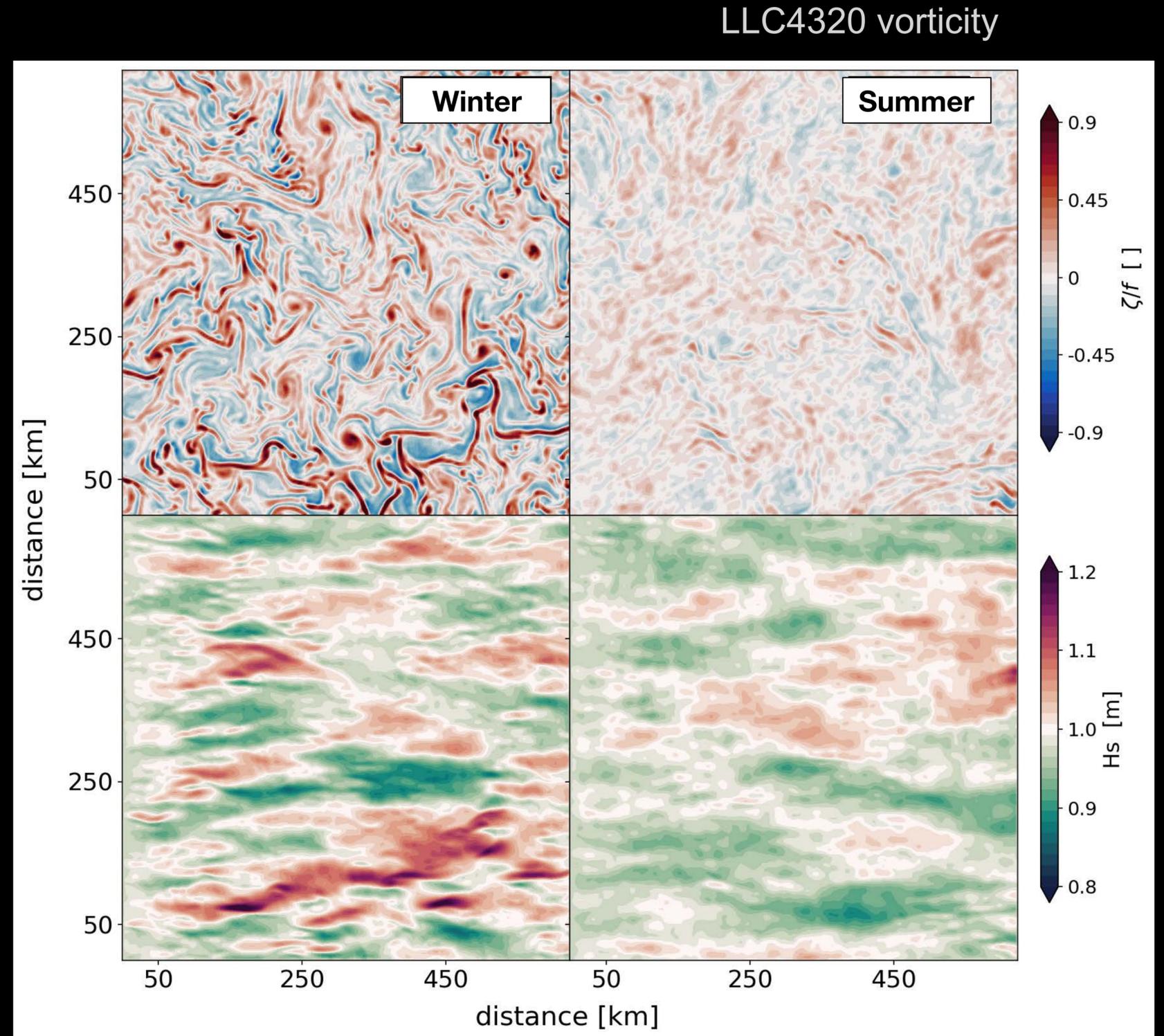


- Steeper KE spectral slopes lead to steeper Hs spectral slopes

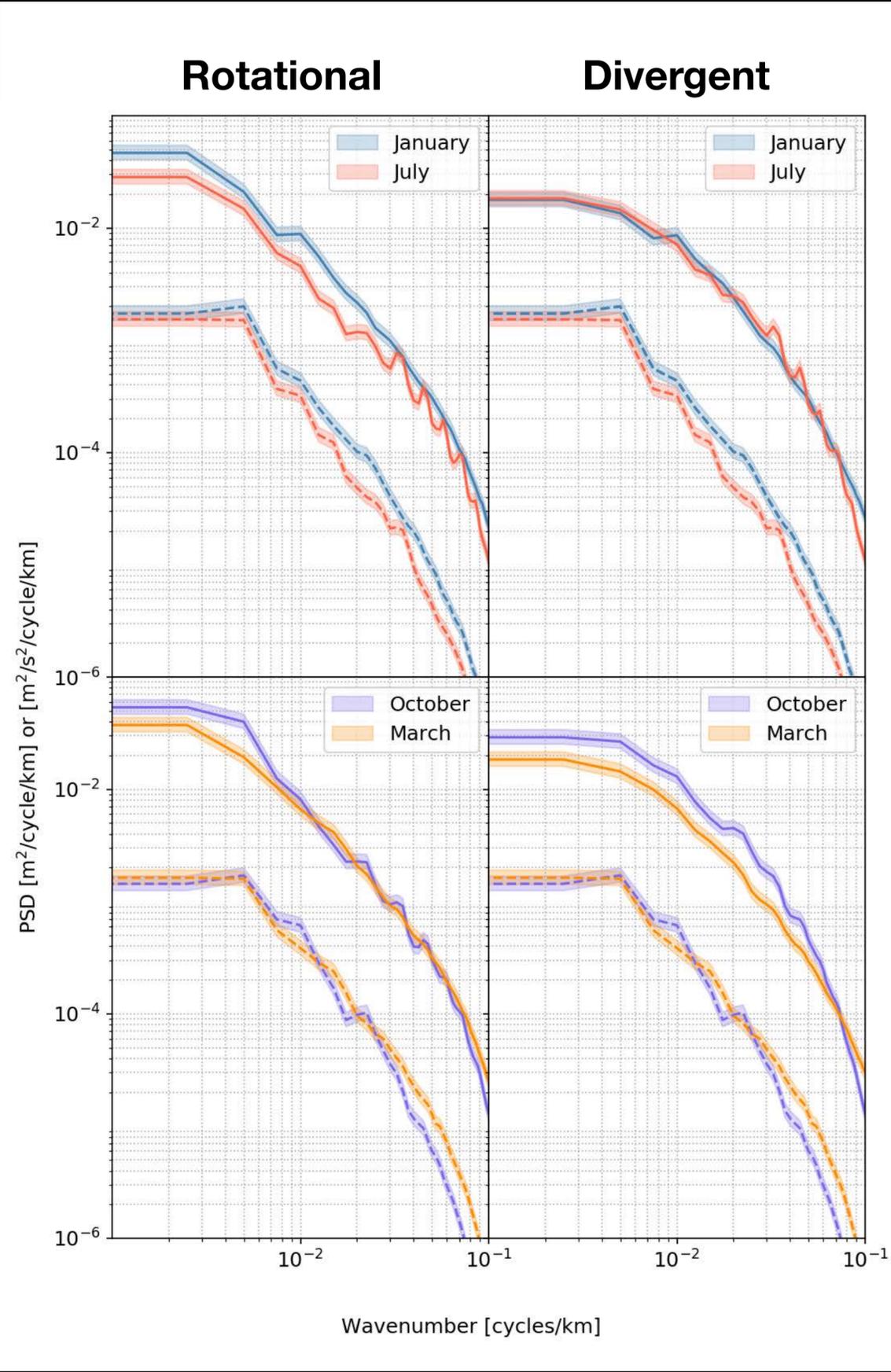
# Do these results hold for realistic currents?

We used an equivalent setup to run WaveWatch III forced with realistic currents from the MITgcm LLC4320 in the CCS region.

This example illustrates how the **seasonality** of the **submesoscale** in the CCS affects the wave field leading to stronger/weaker gradients in  $H_s$ .

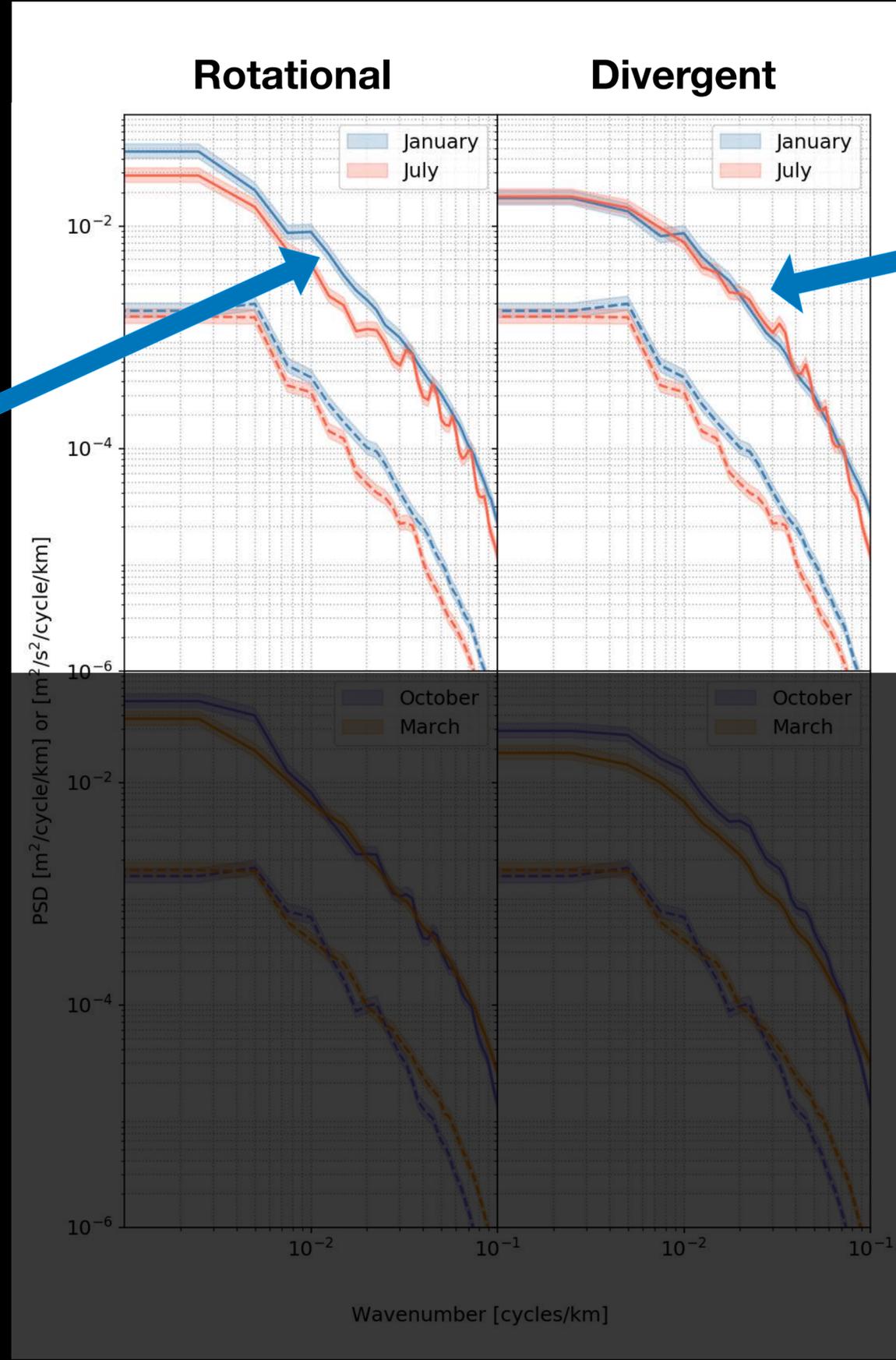


# Seasonality of the surface kinetic energy in the CCS



# Seasonality of the surface kinetic energy in the CCS

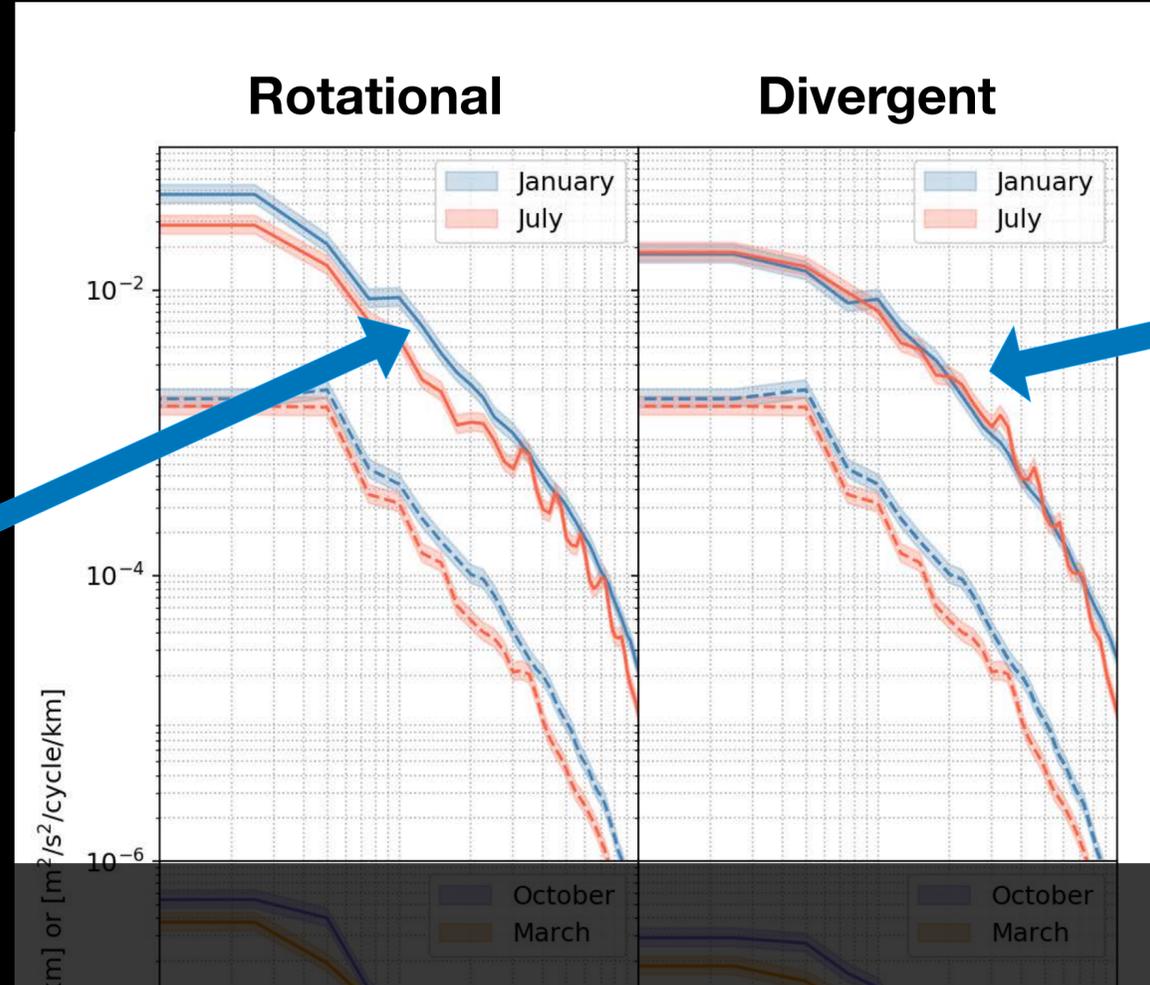
The surface kinetic energy at submesoscales in the CCS is dominated by balanced motions (rotational) in late winter/spring.



Between January and July the KE spectra of the **divergent** component do not change much.

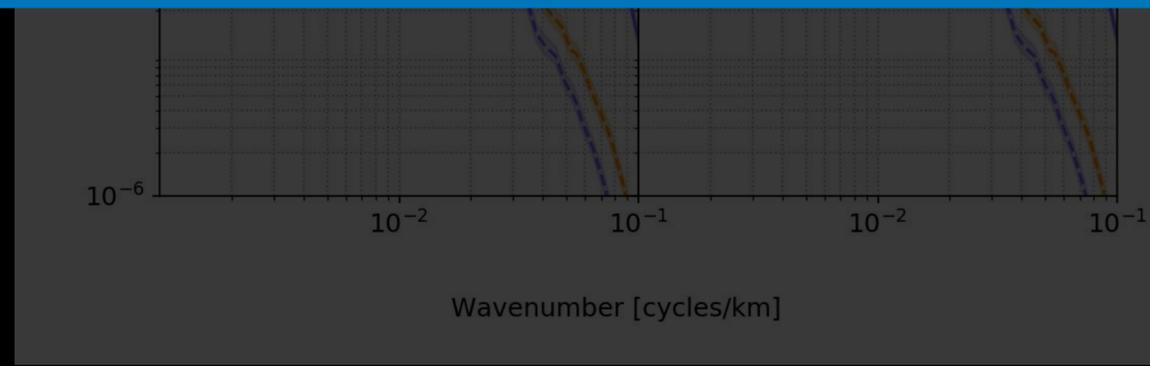
# Seasonality of the surface kinetic energy in the CCS

The surface kinetic energy at submesoscales in the CCS is dominated by balanced motions (rotational) in late winter/spring.

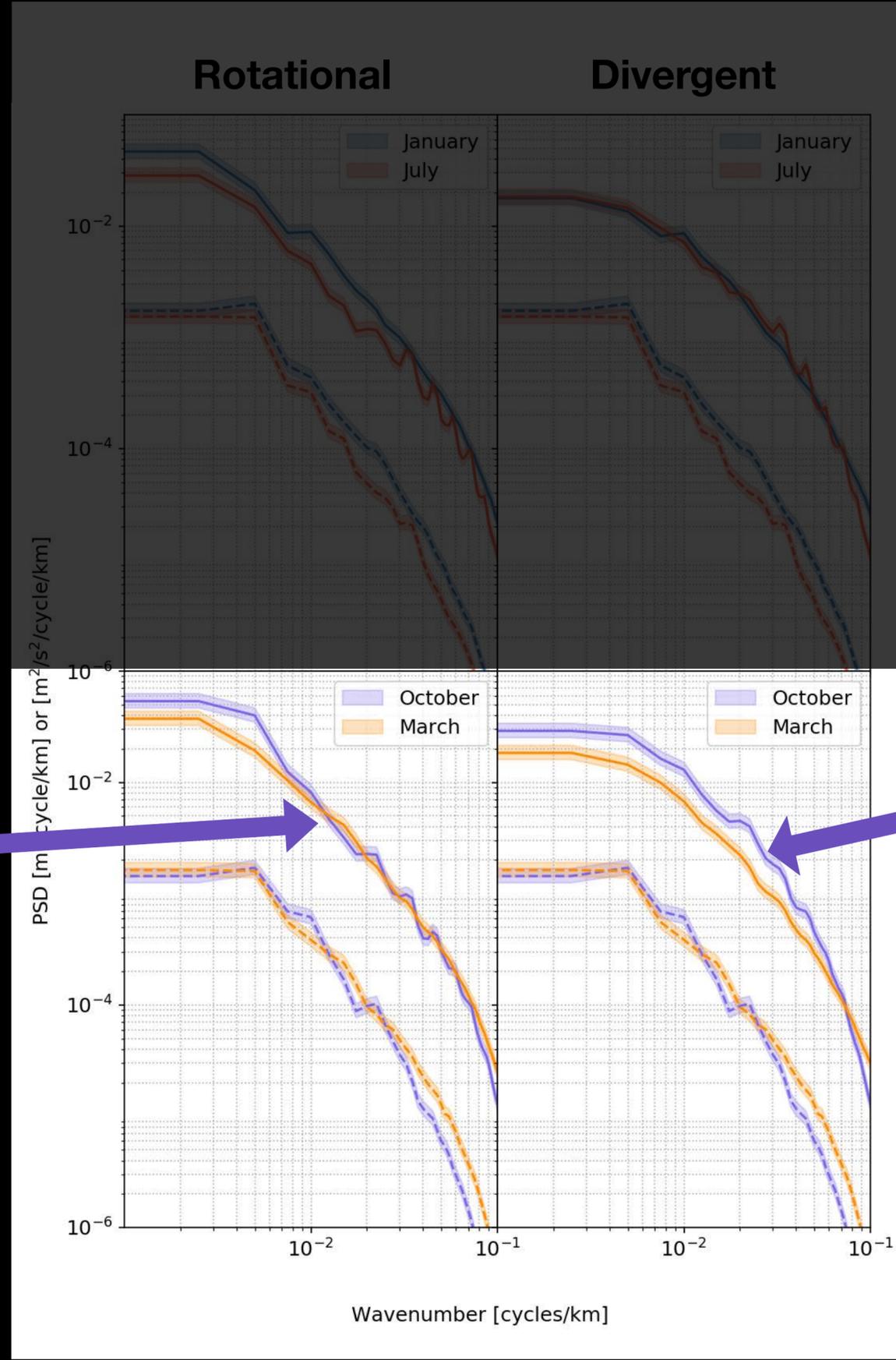


Between January and July the KE spectra of the **divergent** component **do not** change much.

The Hs spectrum is **more energetic** in the **winter** at scales between 200km and 50km in response to the seasonality of the **rotational** KE



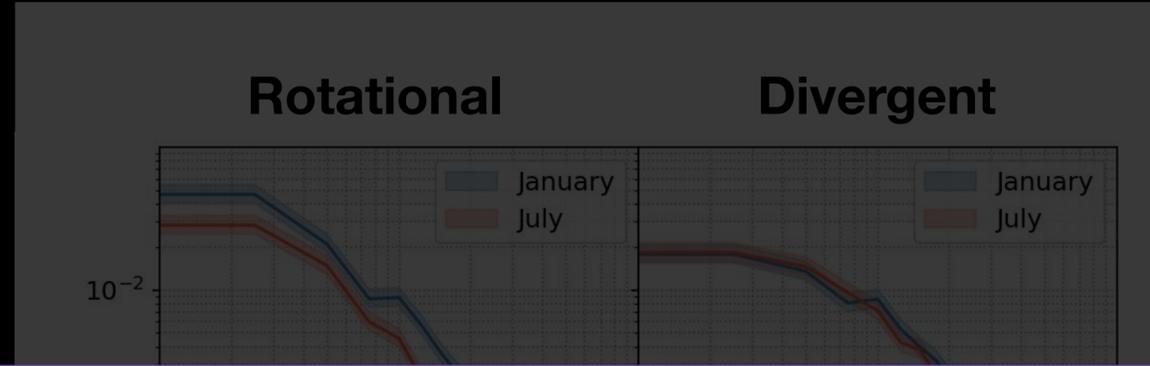
# Seasonality of the surface kinetic energy in the CCS



Between October and March the KE spectra of the **solenoidal** component **do not** change much at scales smaller than 200km.

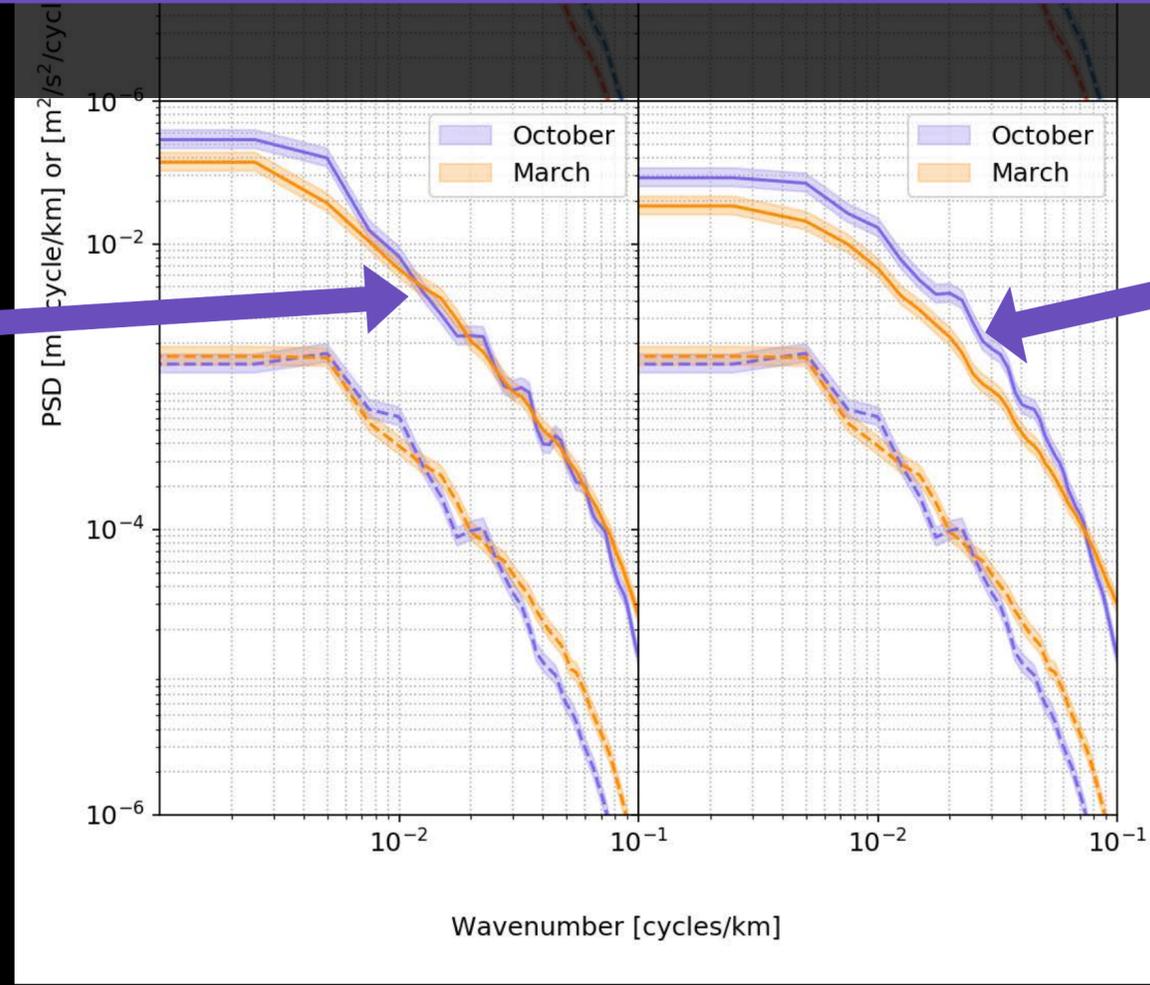
Between January and July the KE spectra of the **divergent** component **do not** change much.

# Seasonality of the surface kinetic energy in the CCS



The Hs spectra **do not change** between October and March at scales between 200km and 50km since the **divergent** component **does not** affect the spatial variability of Hs

Between October and March the KE spectra of the **solenoidal** component **do not** change much at scales smaller than 200km.



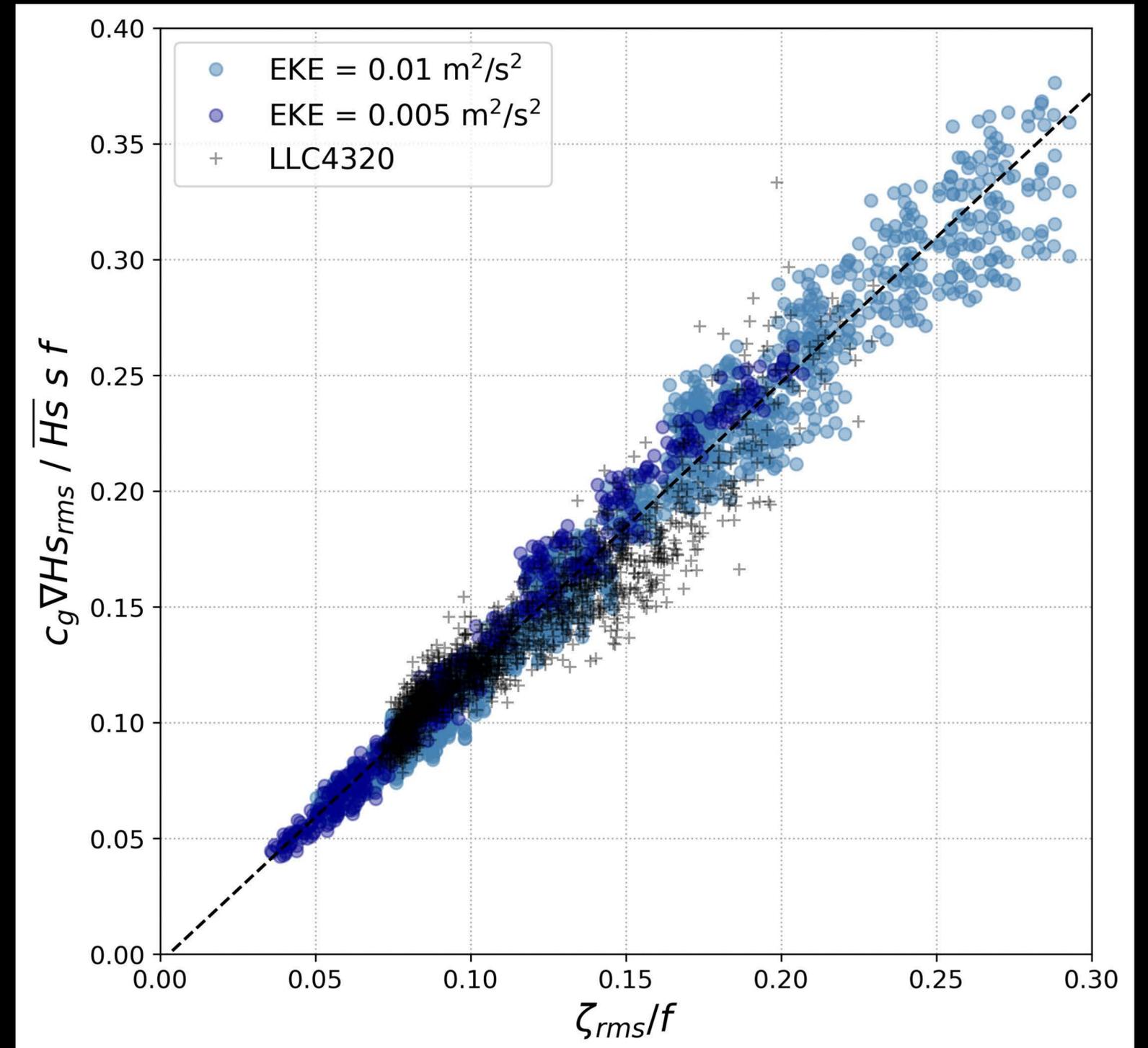
Between January and July the KE spectra of the **divergent** component **do not** change much.

# Spatial gradients of Hs are correlated with vorticity

- ▶ Direct relationship between spatial gradients of significant wave height and the vertical vorticity of the flow ( $r^2 > 0.9$ ):

$$c_g \frac{(\nabla H_s)_{rms}}{\overline{H_{ss}}} = \zeta_{rms}$$

- ▶ Good agreement between the idealized currents and the LLC4320 in the CCS region.

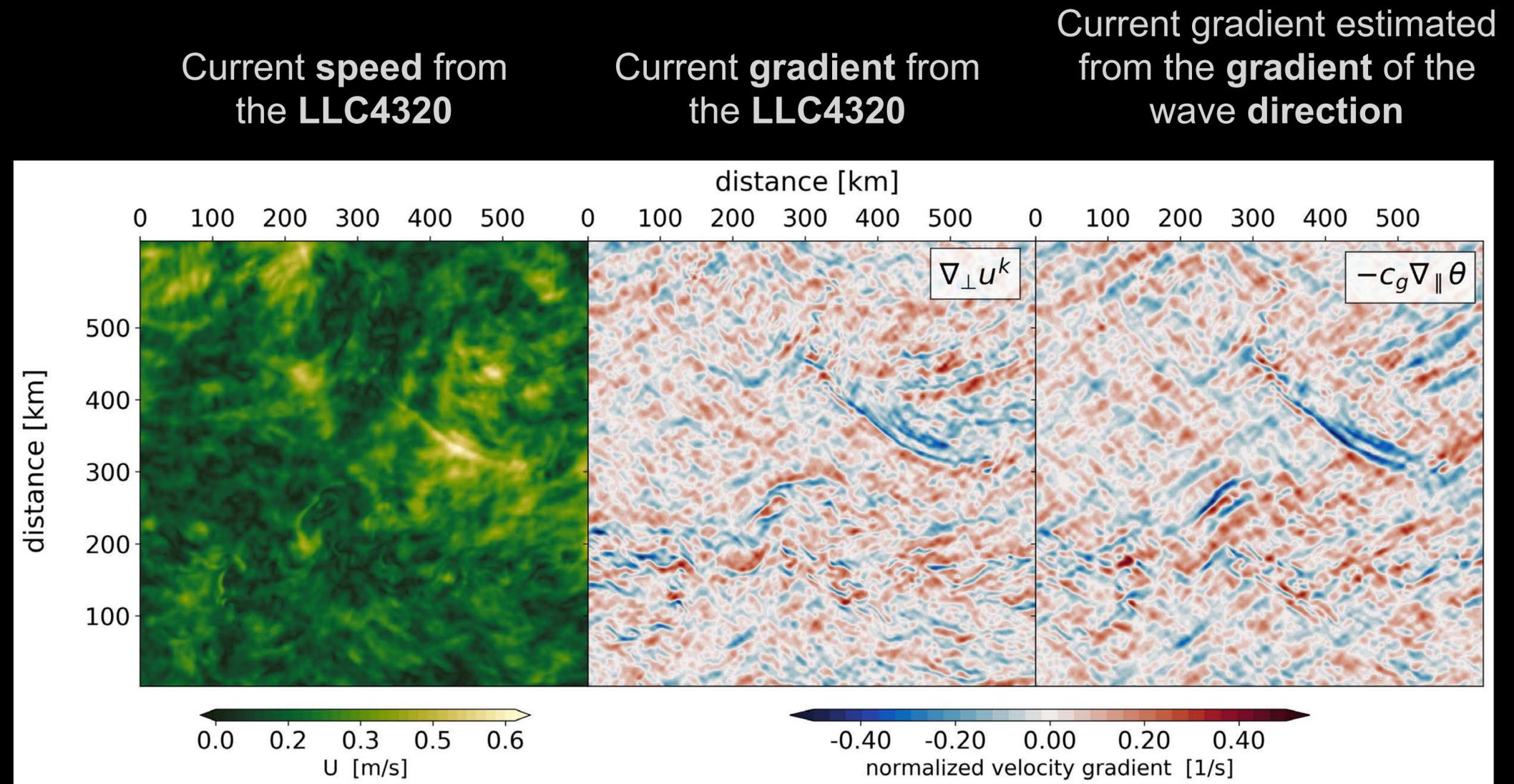


# Could the signature of currents on waves be used to infer properties of the flow?

Assuming that the current speed is small in comparison to the group velocity of the waves, the ray equation for changes in wave direction can be approximated by:

$$\hat{n} \cdot \nabla (\hat{k} \cdot \mathbf{U}) \approx -c_g (\hat{k} \cdot \nabla) \theta$$

such that the gradient of the current can be obtained from the gradient in the wave direction.

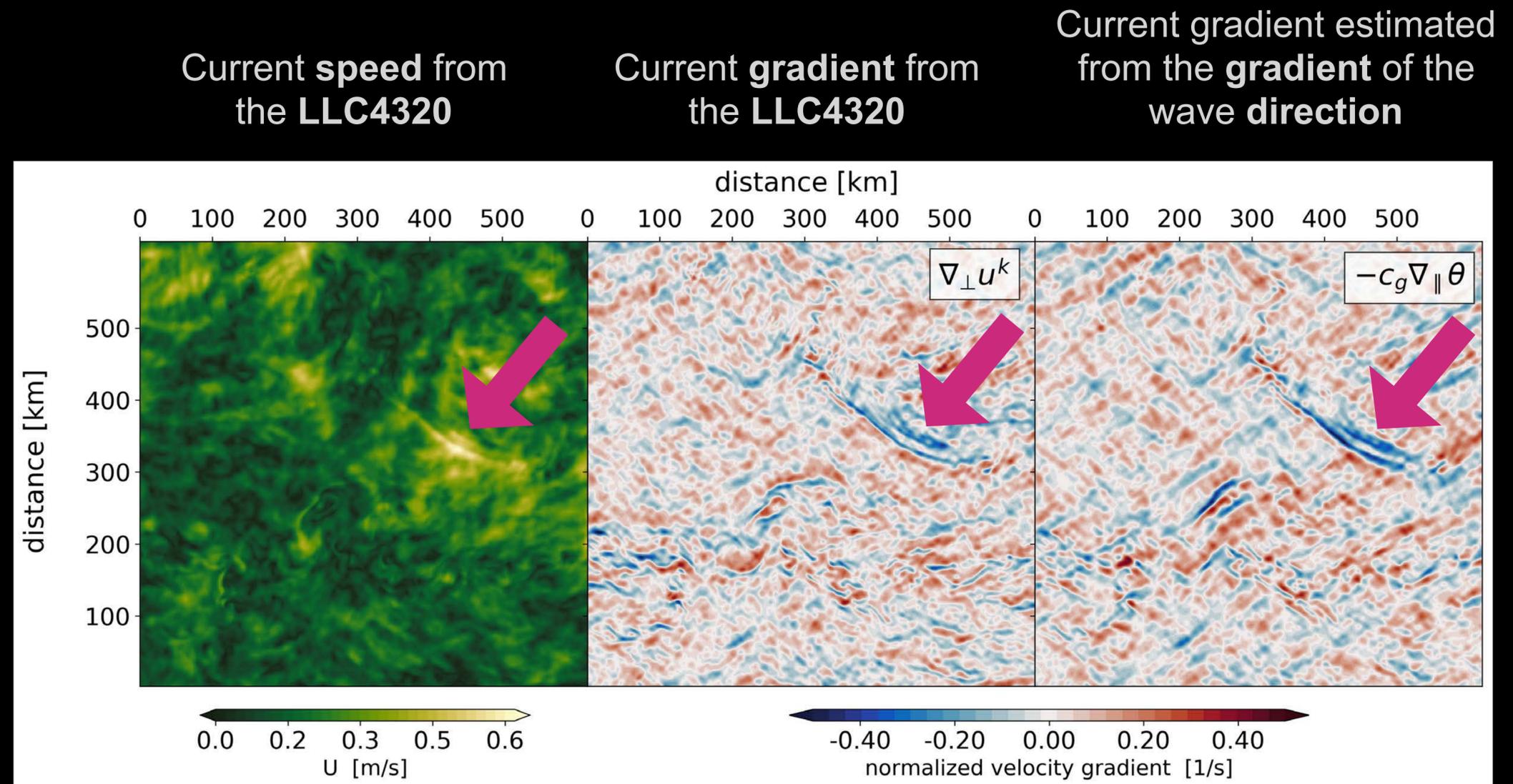


# Could the signature of currents on waves be used to infer properties of the flow?

Assuming that the current speed is small in comparison to the group velocity of the waves, the ray equation for changes in wave direction can be approximated by:

$$\hat{n} \cdot \nabla (\hat{k} \cdot \mathbf{U}) \approx -c_g (\hat{k} \cdot \nabla) \theta$$

such that the gradient of the current can be obtained from the gradient in the wave direction.



## Climate modeling

- ▶ Surface wave physics is key to improving climate models and better representing the coupling between the ocean and the atmosphere

## Climate modeling

- ▶ Surface wave physics is key to improving climate models and better representing the coupling between the ocean and the atmosphere

## Wave modeling

- ▶ Wave models without currents do not capture the small-scale and high-frequency variability of wave heights (and other wave properties).
- ▶ Having current forcing in numerical wave models could help reduce directional and arrival time biases, but doing that globally is somewhat impractical: it is computationally costly and surface current observations at scales shorter than 100 km are rare.

## Climate modeling

- ▶ Surface wave physics is key to improving climate models and better representing the coupling between the ocean and the atmosphere

## Wave modeling

- ▶ Wave models without currents do not capture the small-scale and high-frequency variability of wave heights (and other wave properties).
- ▶ Having current forcing in numerical wave models could help reduce directional and arrival time biases, but doing that globally is somewhat impractical: it is computationally costly and surface current observations at scales shorter than 100 km are rare.

## Remote sensing

- ▶ Surface waves and their spatial gradients are often a source of error for remote sensing measurements (e.g., sea state bias, layover, wave-induced Doppler...).
- ▶ How well do we understand sea state gradients? Waves respond very differently to vorticity and divergence.
- ▶ With **present altimetry** it's straight forward to get **geostrophic** currents from **SSH** measurements. **SWOT** we will be measuring at **scales** where the **SSH** signal might **not** be associated motions that are in **geostrophic** balance
- ▶ The signature of currents on waves could potentially be used to infer properties of the flow (e.g. transition from balanced unbalanced).

Extra

# Peak period (Doppler)

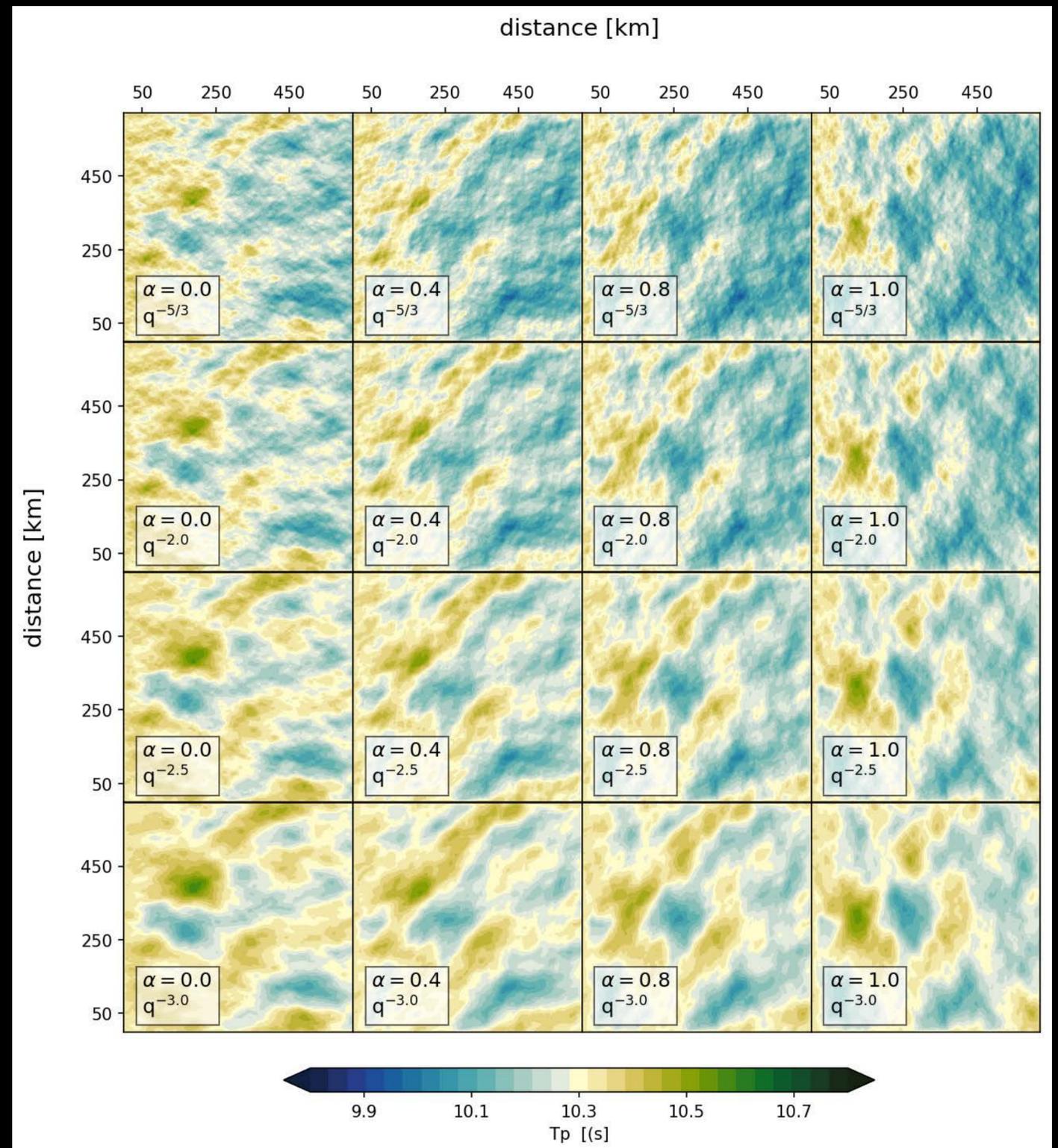
The effect of random surface currents on the peak period is relatively small ( $< 3\%$ ).

The spatial pattern of  $H_s$  in the purely divergent case (last column) nearly matches the spatial pattern of the peak period  $\rightarrow$

Changes in  $H_s$  for the purely divergent cases are direct response to changes in period (frequency)

$$N = \frac{E}{\sigma} = \text{const}$$

More vorticity  $\rightarrow$  More divergence



# Peak period (Doppler)

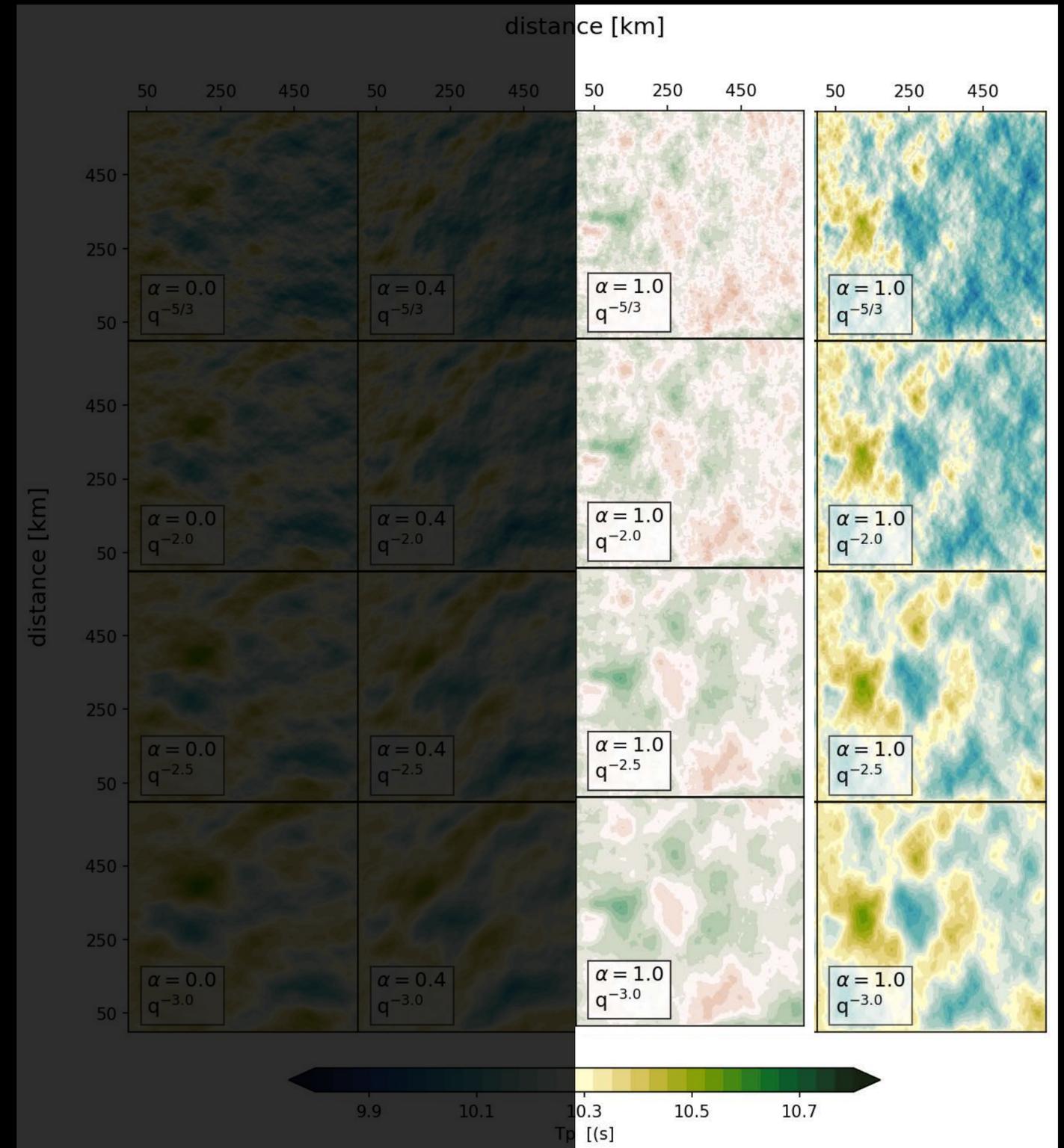
The effect of random surface currents on the peak period is relatively small (< 3%).

The spatial pattern of Hs in the purely divergent case (last column) nearly matches the spatial pattern of the peak period ->

Changes in Hs for the purely divergent cases are direct response to changes in period (frequency)

$$N = \frac{E}{\sigma} = \text{const}$$

More vorticity  $\rightarrow$  More divergence



# Peak period (Doppler)

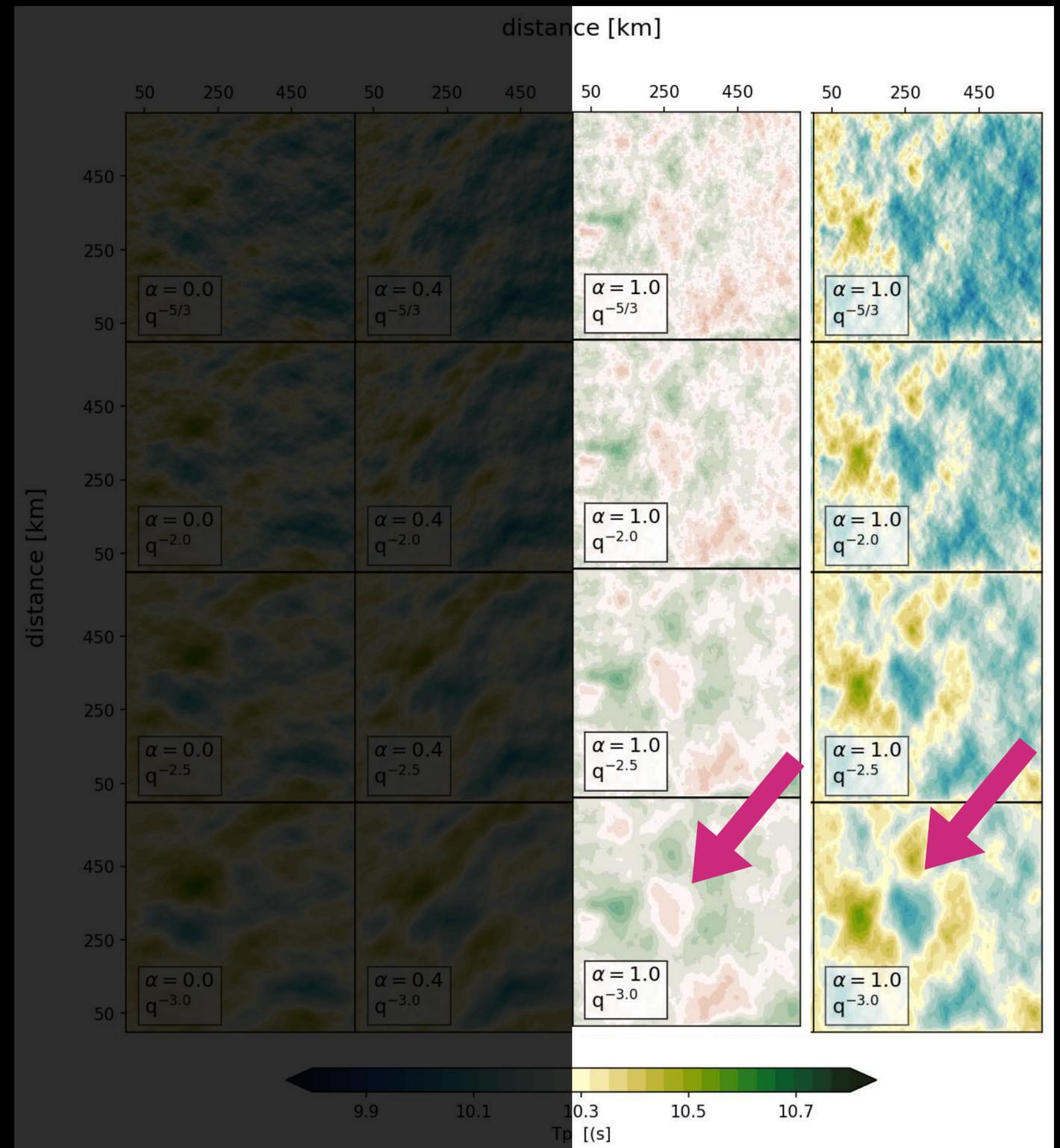
The effect of random surface currents on the peak period is relatively small (< 3%).

The spatial pattern of Hs in the purely divergent case (last column) nearly matches the spatial pattern of the peak period ->

Changes in Hs for the purely divergent cases are direct response to changes in period (frequency)

$$N = \frac{E}{\sigma} = \text{const}$$

More vorticity  $\rightarrow$  More divergence

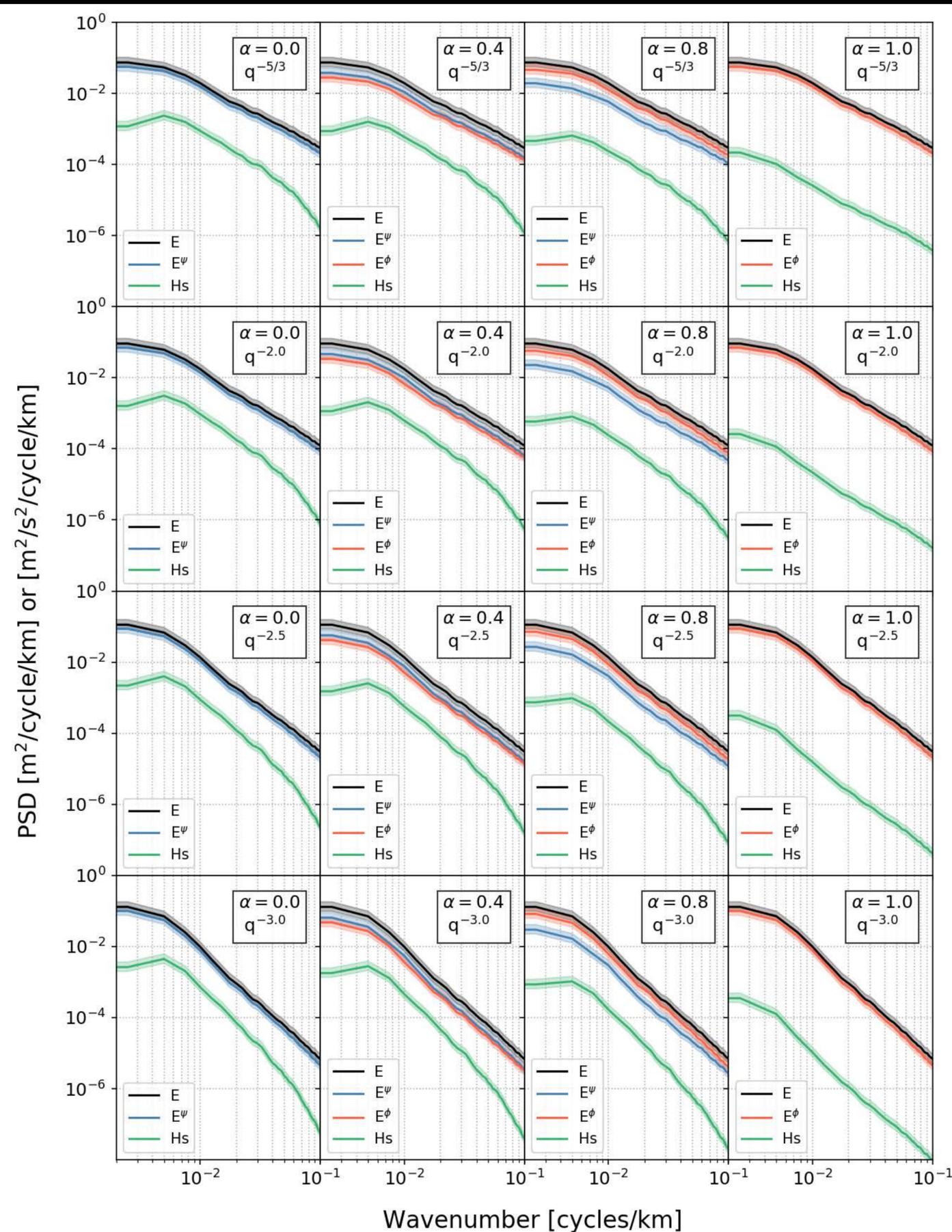


# Wavenumber spectra of Hs

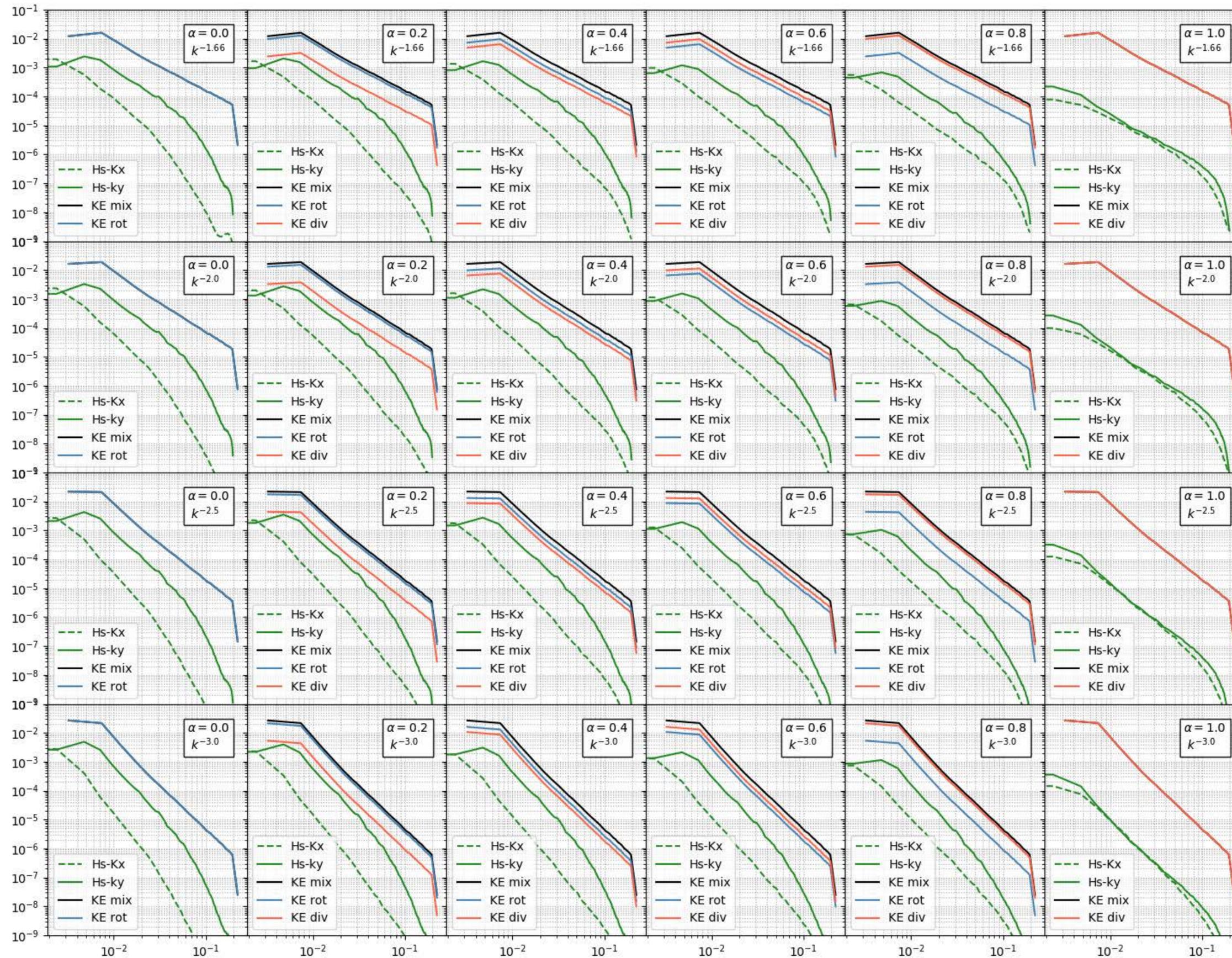
Varying the ratio of rotational to divergent flow while keeping the same EKE wavenumber spectrum (fixed spectral slope and variance) leads to strikingly different responses in the **Hs wavenumber spectra**.

In agreement with the cases illustrated in the snapshots, the **variance of Hs is larger** for purely **rotational** flows, in particular at lower wavenumbers.

For cases where the flow is predominantly **divergent**, the Hs wavenumber spectra have a more uniform slope that nearly **follows** the **spectral slope of the current spectrum**.

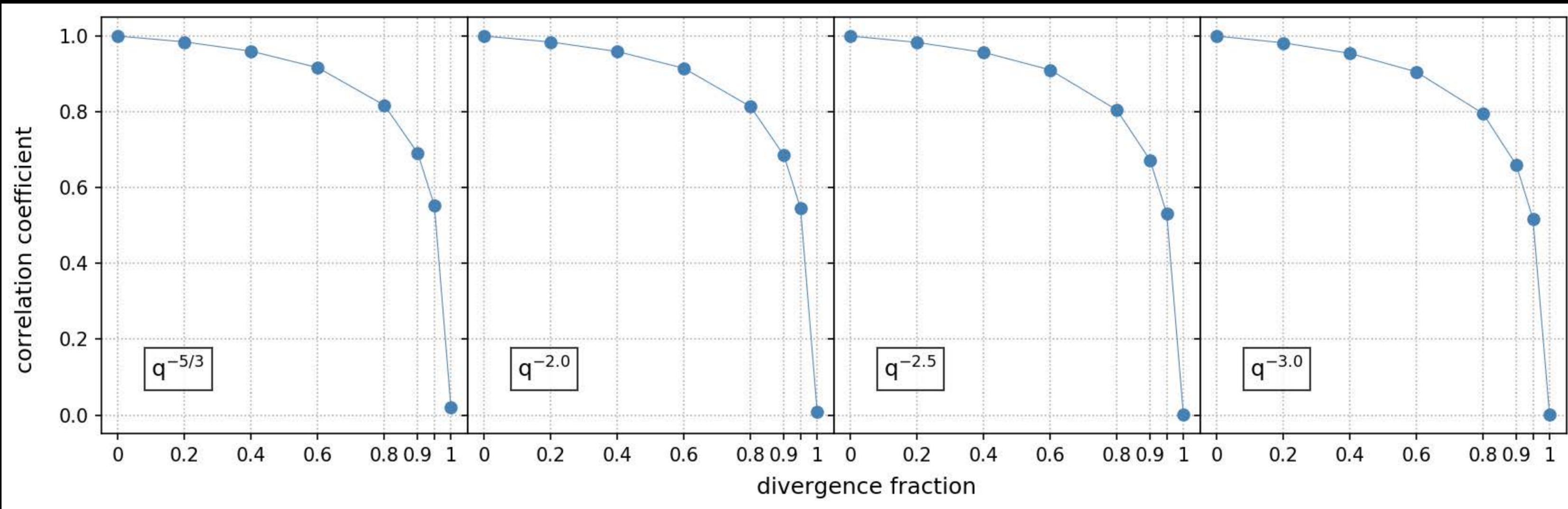


PSD [ $\text{m}^2/\text{cycle}/\text{km}$ ] or [ $\text{m}^2/\text{s}^2/\text{cycle}/\text{km}$ ]



Wavenumber [cycles/km]

# As long as there is some vorticity...



- ▶ The spatial correlation drops from 0.6 for 95% of divergence to nearly 0 for 100% of divergence.
- ▶ This holds for all spectral slopes.

# Current effects on deep-water linear waves

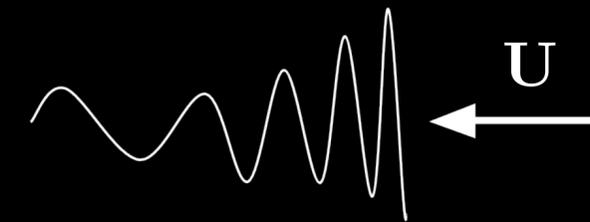
From a geometrical optics approximation framework, the effects of currents on the kinematics of the waves can be described by the ray equations:

$$\dot{\omega} = \frac{d}{dt}(\mathbf{U} \cdot \mathbf{k}) \quad (\text{conservation of abs. freq.})$$

$$\dot{\theta} = -\frac{1}{k} \hat{\mathbf{n}} \cdot \nabla (\mathbf{k} \cdot \mathbf{U}) \quad (\text{Refraction})$$

$$\dot{k} = -\hat{\mathbf{k}} \cdot \nabla (\mathbf{k} \cdot \mathbf{U}) \quad (\text{Change in wavenumber})$$

$$\dot{\mathbf{x}} = -\mathbf{c}'_g + \mathbf{U} \quad (\text{Advection})$$



While, wave dynamics is governed by the conservation of wave action density:

$$\frac{\partial N}{\partial t} + \nabla \cdot (\dot{\mathbf{x}}N) + \frac{\partial}{\partial k}(\dot{k}N) + \frac{\partial}{\partial \theta}(\dot{\theta}N) = S_{in} + S_{ds} + S_{nl}$$

# Current effects on deep-water linear waves

$$\frac{\partial N}{\partial t} + \nabla \cdot (\dot{\mathbf{x}}N) + \frac{\partial}{\partial k}(\dot{k}N) + \frac{\partial}{\partial \theta}(\dot{\theta}N) = S_{in} + S_{ds} + S_{nl}$$

change in wavenumber (magnitude)  $\downarrow$

change in the wind forcing  $\swarrow$

speed at which action is advected  $\uparrow$

change in wave direction  $\uparrow$

change in dissipation through breaking  $\swarrow$

# Current effects on deep-water linear waves

The right-hand side relies on parametrizations:

lack of observations  $\longrightarrow$  not so good parametrizations  $\longrightarrow$  not so good modeling

Left-hand side:

- ▶ Ok without currents for bulk quantities, BUT adding currents could improve
  - ▶ Delayed arrival times
  - ▶ Directional biases
  - ▶ Spatial gradients of significant wave height

$$\frac{\partial N}{\partial t} + \nabla \cdot (\dot{\mathbf{x}}N) + \frac{\partial}{\partial k}(\dot{k}N) + \frac{\partial}{\partial \theta}(\dot{\theta}N) = S_{in} + S_{ds} + S_{nl}$$

Annotations for the equation:

- change in wavenumber  $\downarrow$  (points to  $\frac{\partial}{\partial k}$ )
- change in the wind forcing  $\swarrow$  (points to  $S_{in}$ )
- change in wave direction  $\uparrow$  (points to  $\frac{\partial}{\partial \theta}$ )
- change in dissipation through breaking  $\swarrow$  (points to  $S_{ds}$ )
- speed at which action is advected  $\uparrow$  (points to  $\nabla \cdot (\dot{\mathbf{x}}N)$ )

# Diffusion of surface gravity wave action by mesoscale turbulence at the sea surface

Villas Bôas and Young

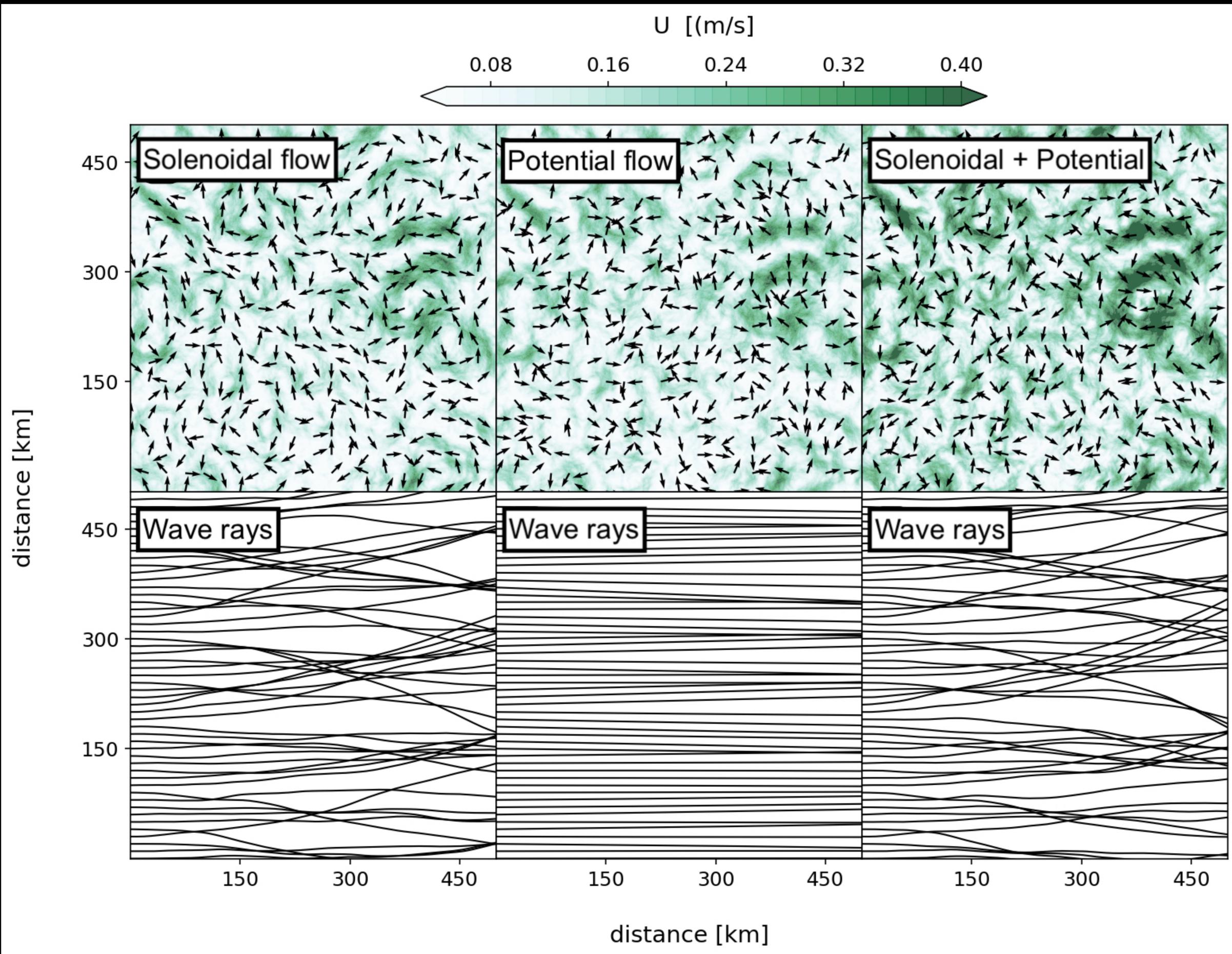
$$\partial_t A + \dot{x}_n \partial_{x_n} A + \dot{k}_n \partial_{k_n} A = 0$$

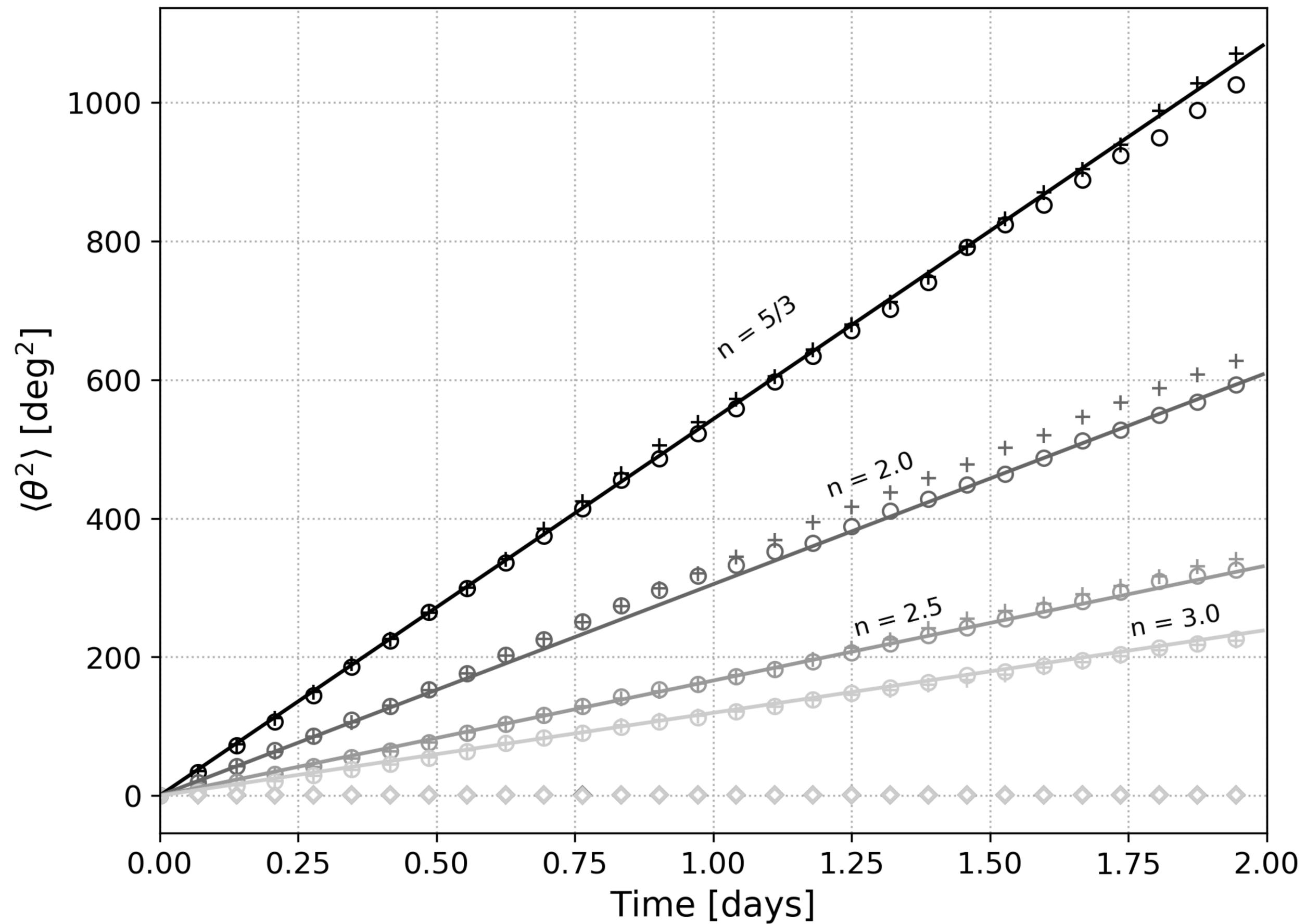
We apply a multiple-scale expansion approach to average the wave action balance equation over an ensemble of sea-surface velocity fields.

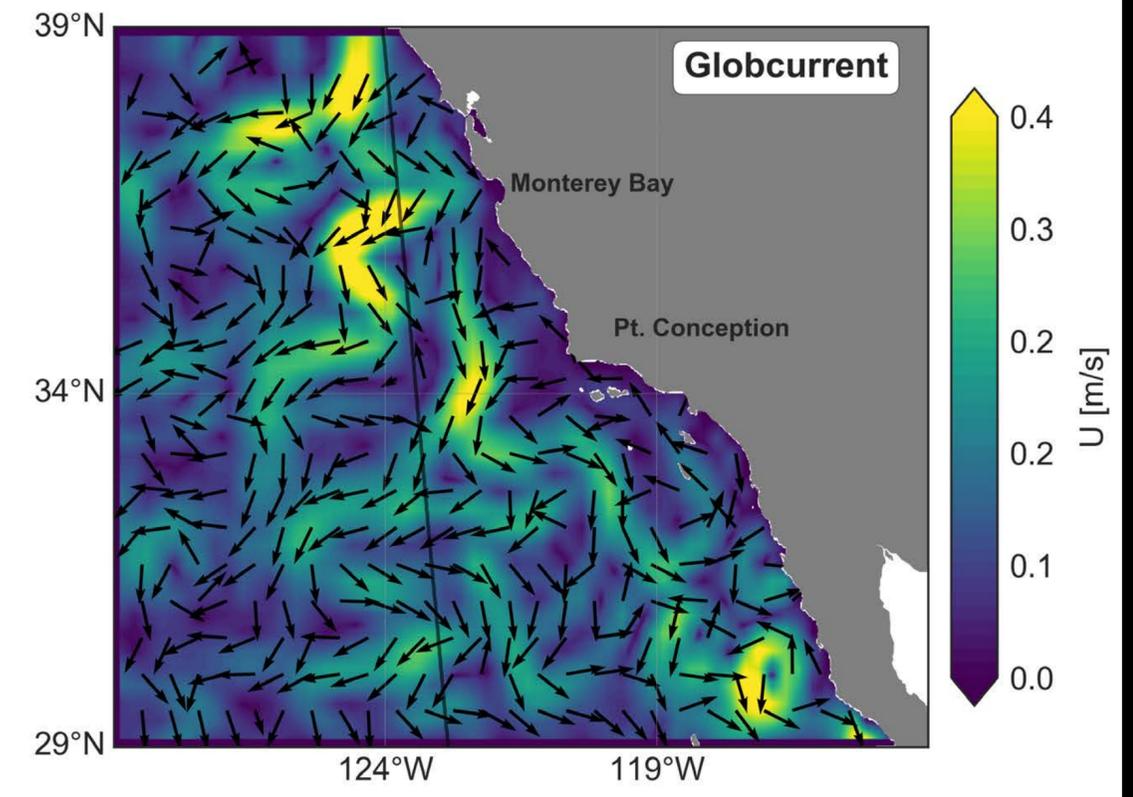
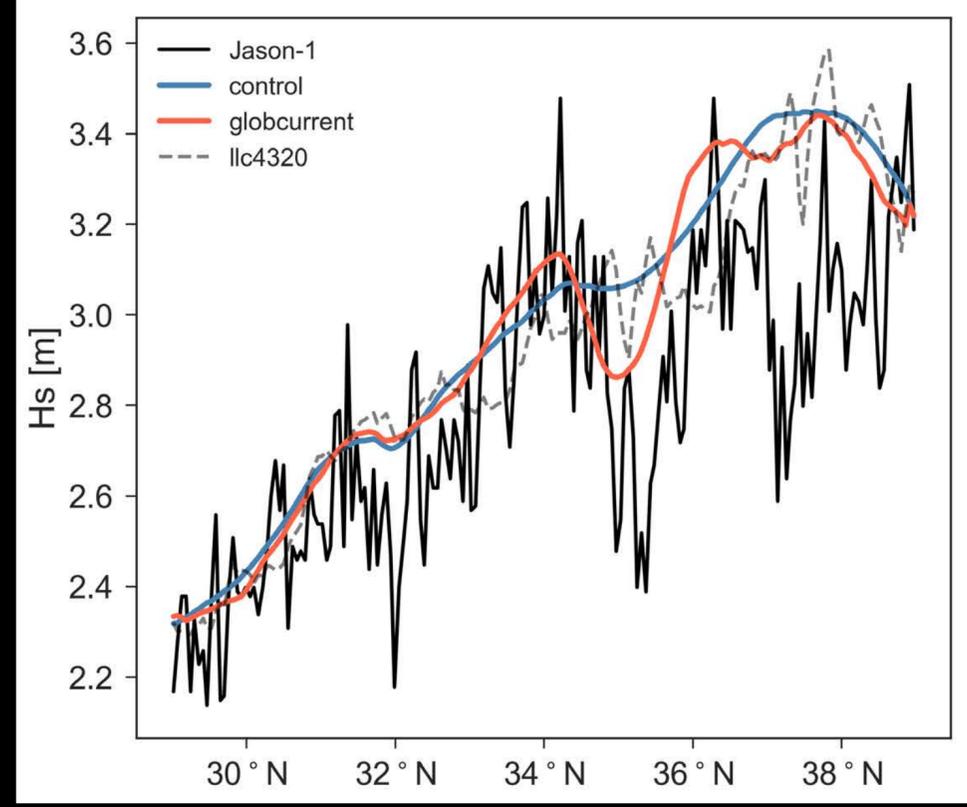
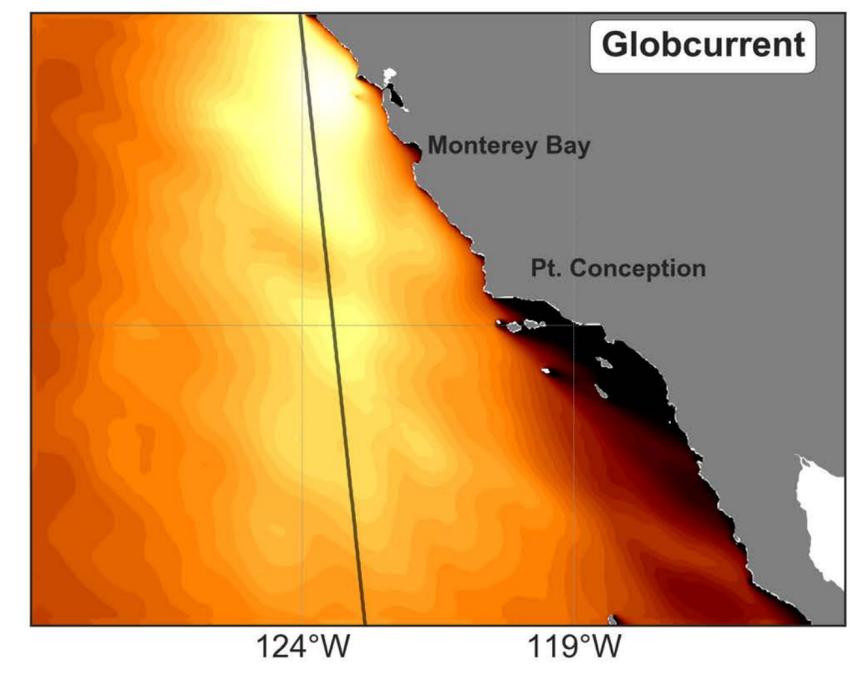
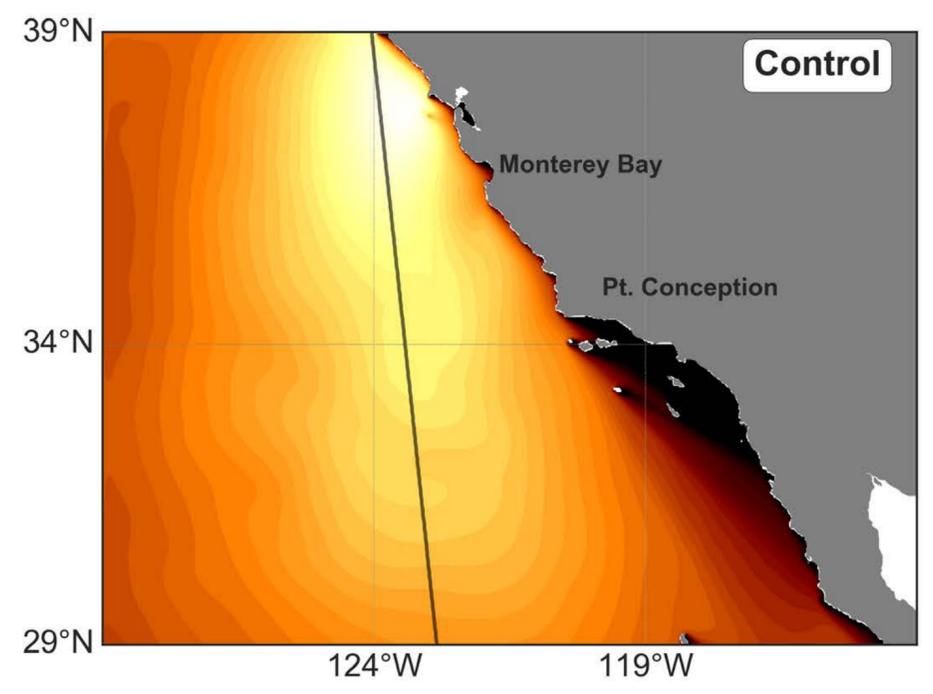
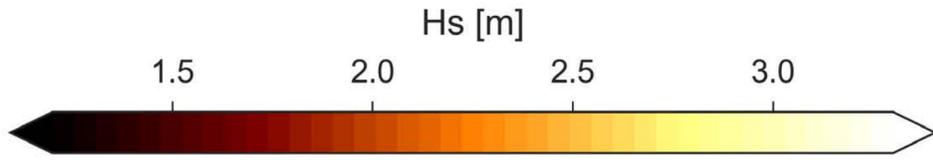
$$\bar{A}_t + c \cos \theta \bar{A}_x + c \sin \theta \bar{A}_y = \alpha \bar{A}_{\theta\theta}$$

For isotropic velocity fields, the diffusion of wave action can be written in terms of the energy spectrum of the rotational component of the flow:

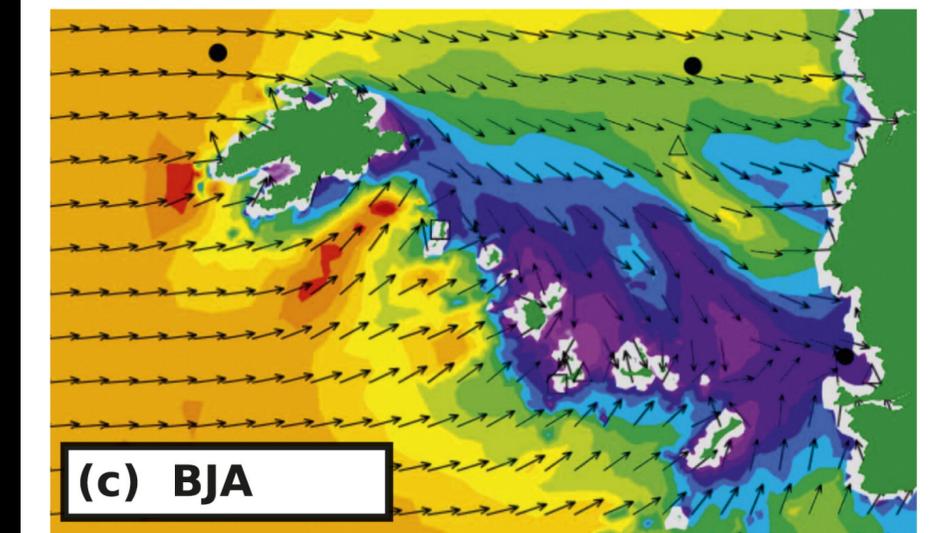
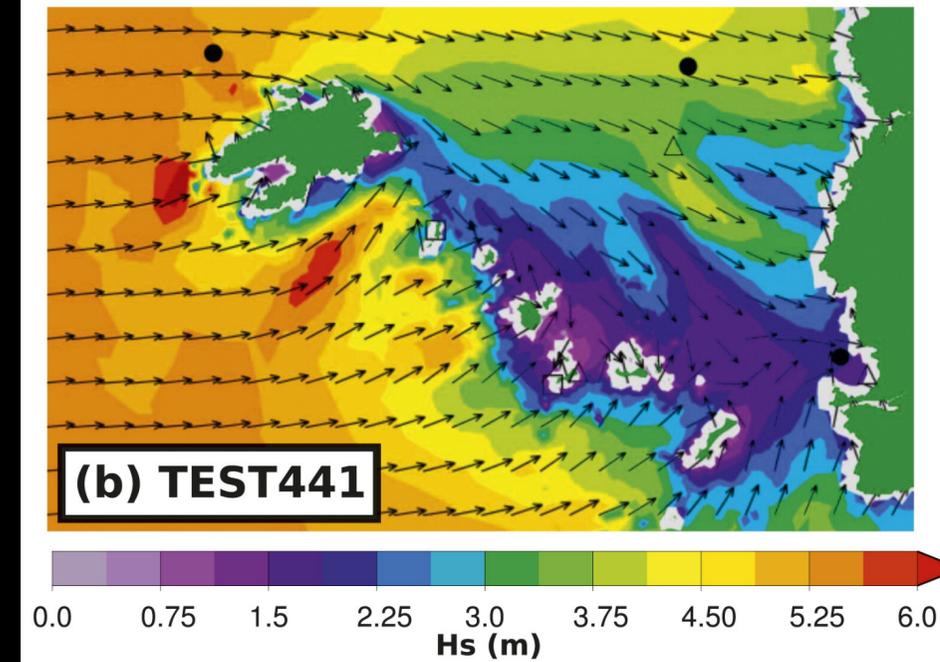
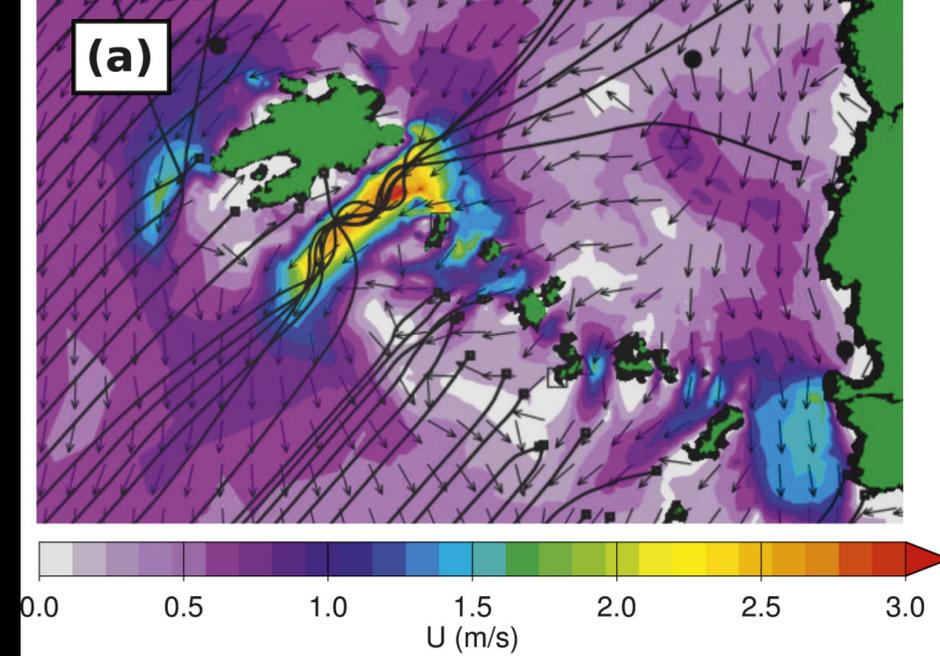
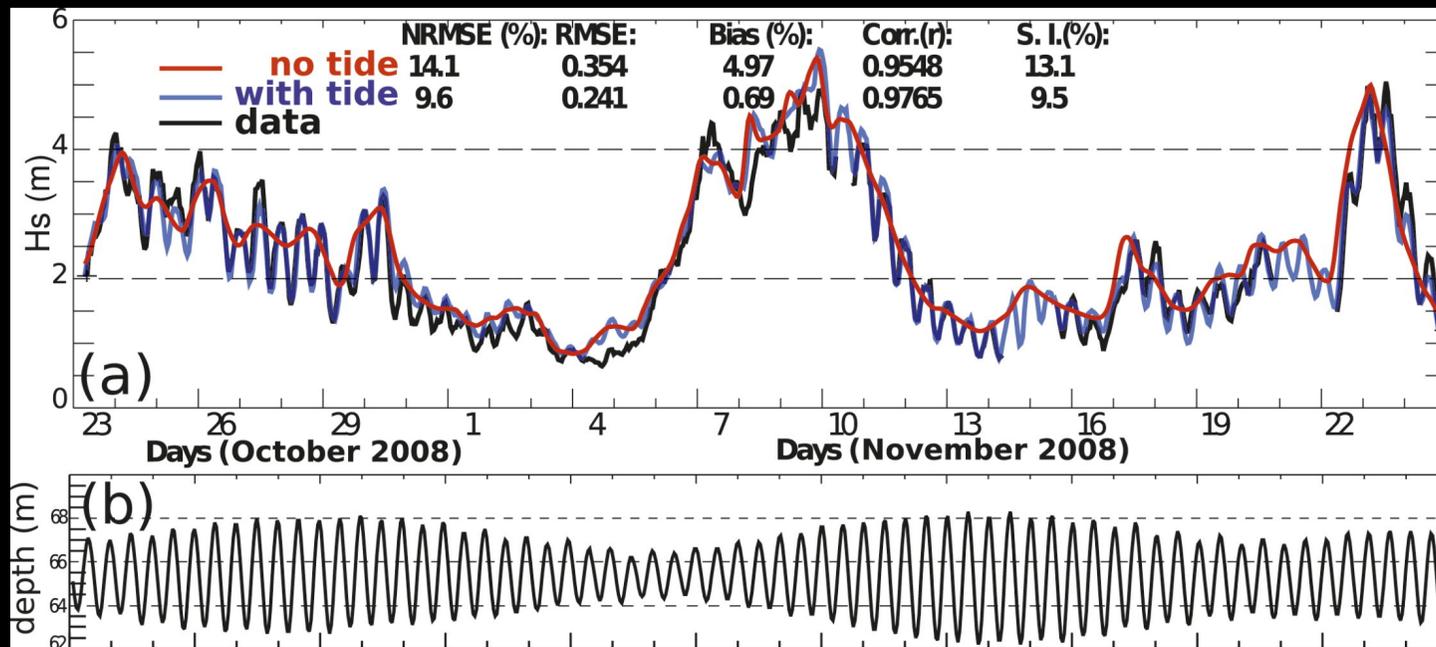
$$\alpha(k) = \frac{2}{c} \int_0^\infty q \tilde{E}^\psi(q) q$$

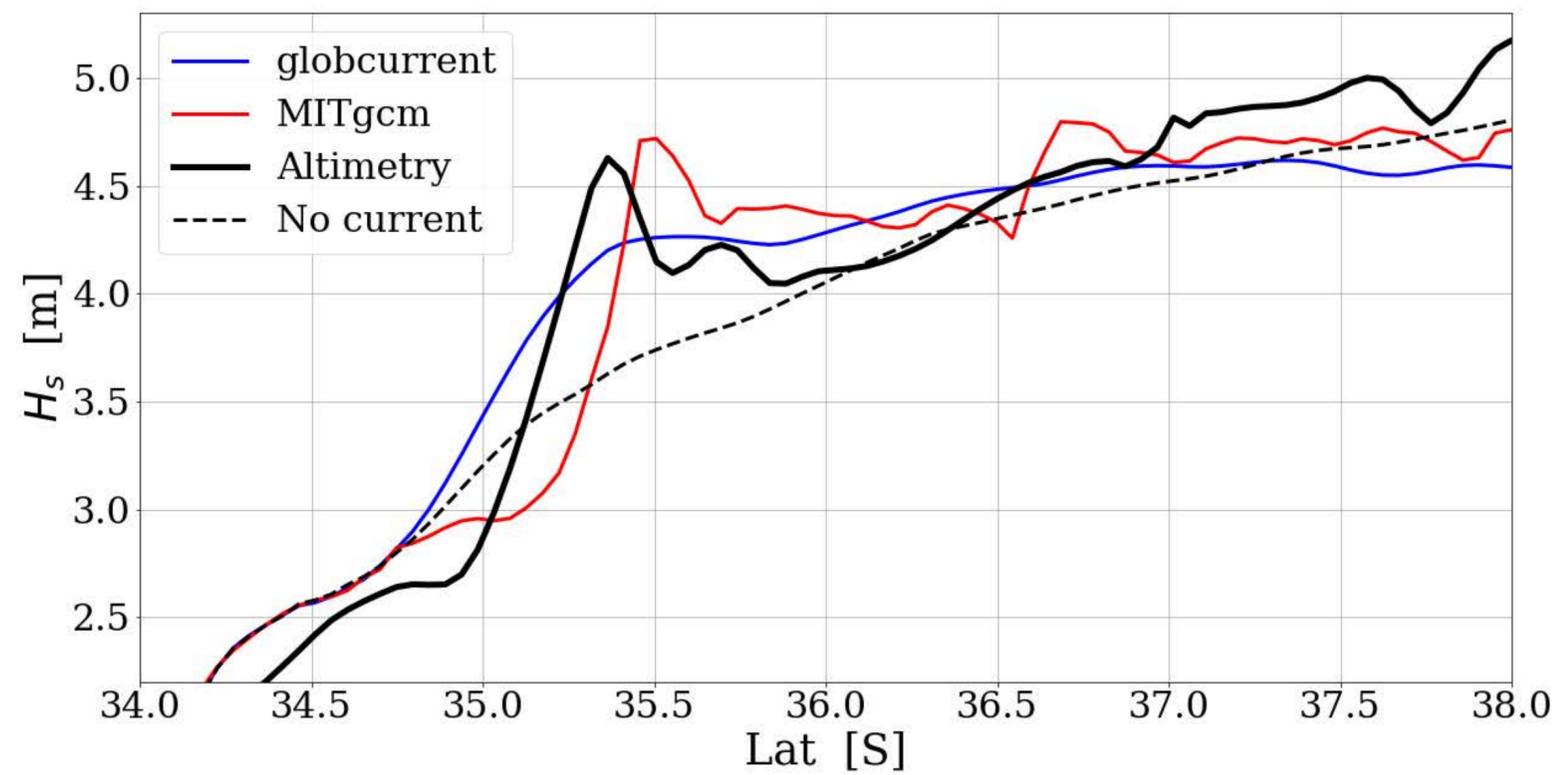
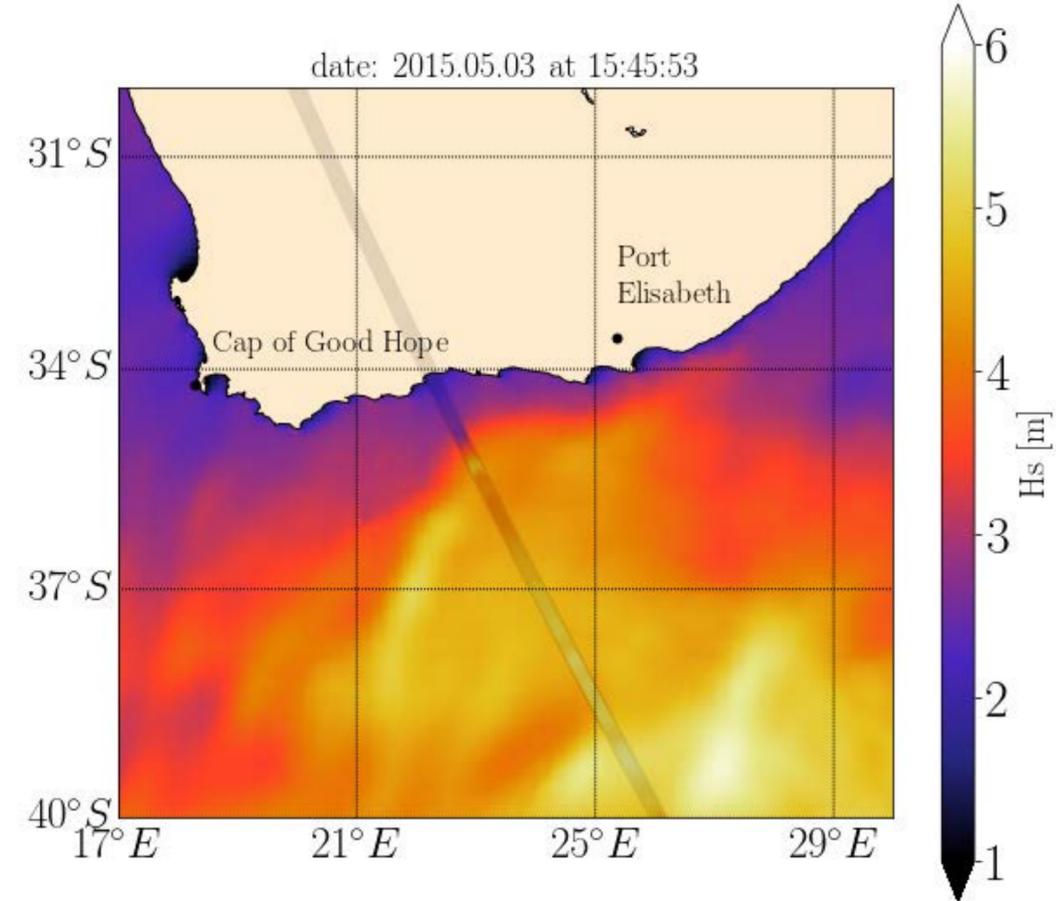
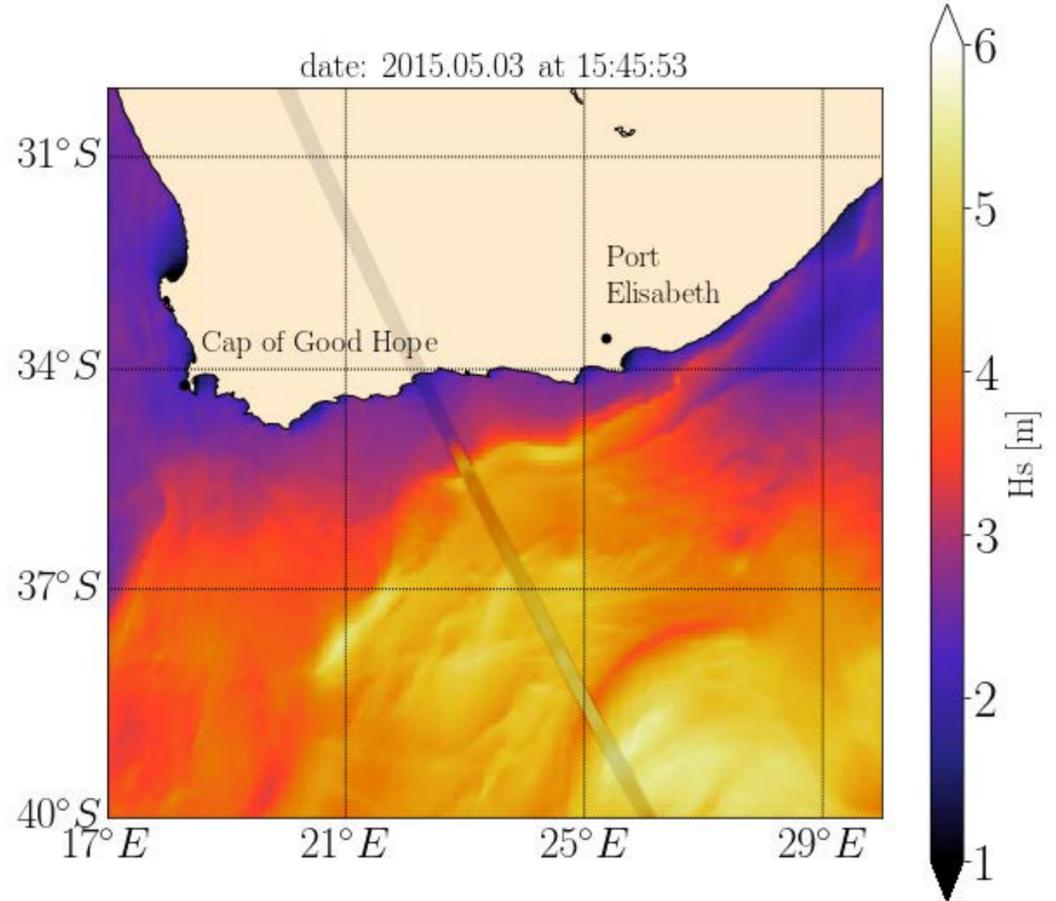






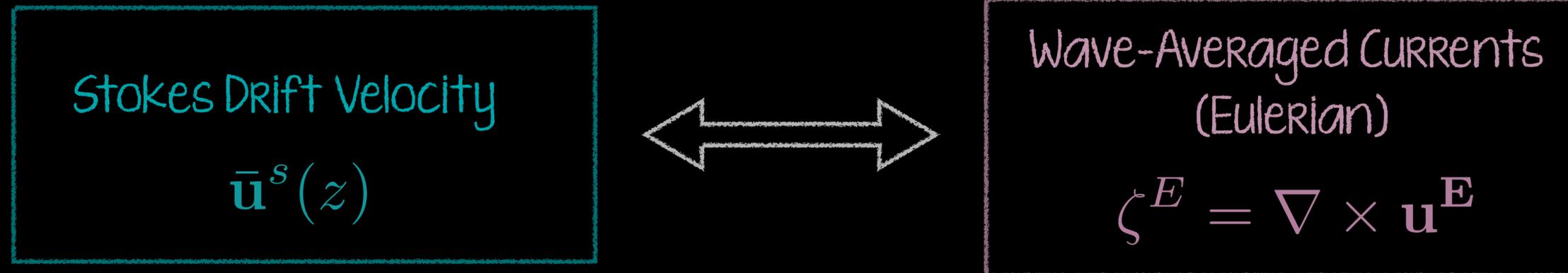
► The high-frequency variability of Hs is completely missed without currents





## 2. Langmuir turbulence

Craik and Leibovich (1976):



The Stokes drift velocity interacts with the mean Eulerian flow.  
This interaction shows up in the momentum equation as a “vortex force”

Vortex Force

$$\mathbf{F}^S = \bar{\mathbf{u}}^S \times \zeta^E$$

- ▶ The relative importance between the shear instability of the wind-driven currents and the vortex force is given by the **turbulent Langmuir number**:

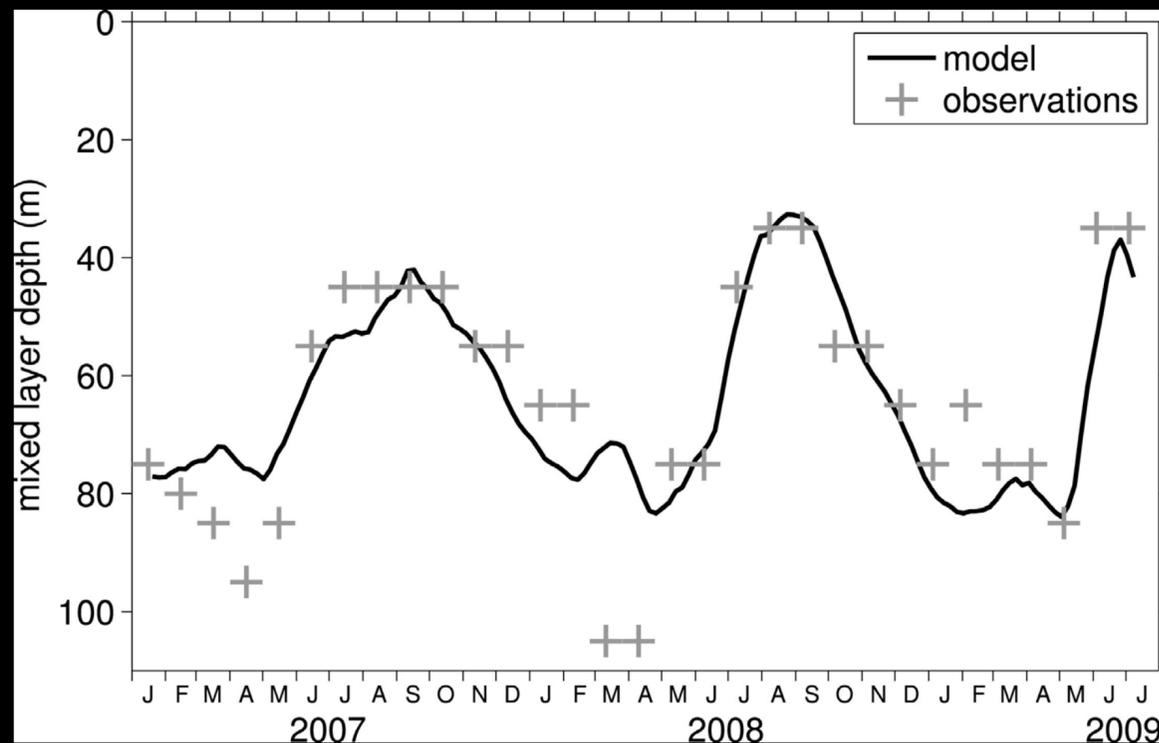
$$La_t = \left( \frac{u^*}{u_s} \right)^{1/2}$$

$$La_t = \mathcal{O}(1) \quad \Rightarrow \quad \text{Langmuir Turbulence}$$

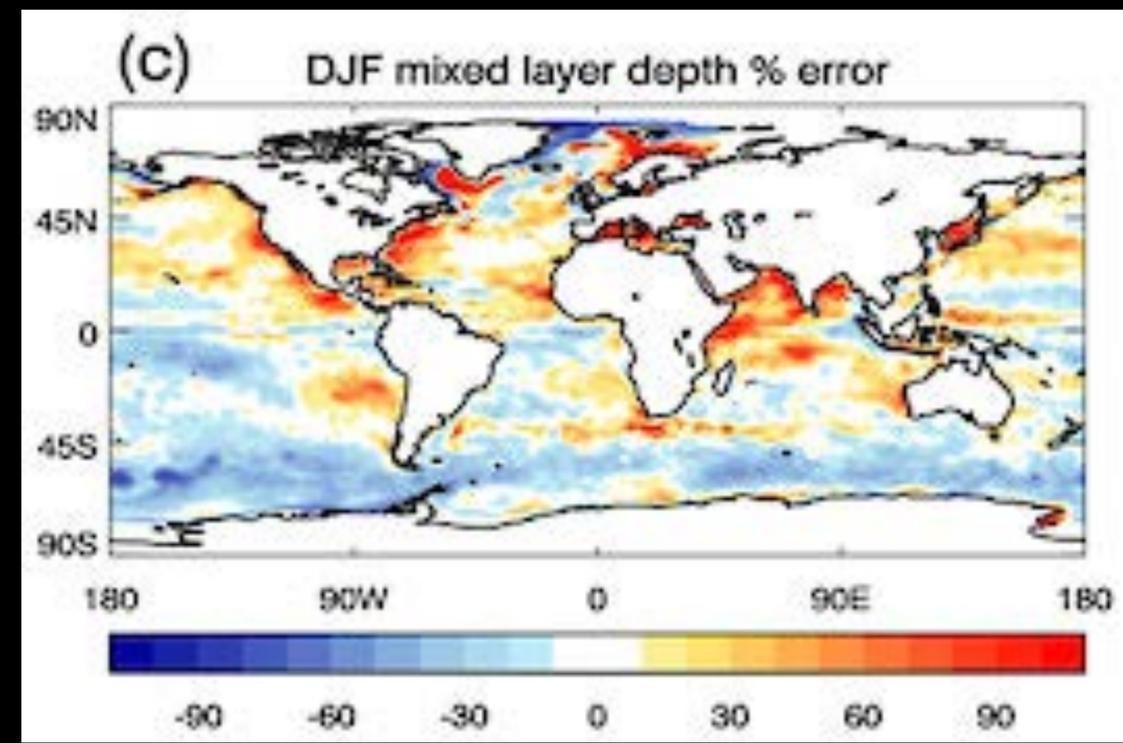
(After McWilliams et al. 1997)

- ▶ Langmuir turbulence penetrates deeper than the layer directly affected by Stokes drift
- ▶ Large Eddy Simulations (LES) of the wave-averaged momentum equations (Moeng, 1984)
  - Have inspired multiple scalings for the vertical turbulent kinetic energy [e.g., McWilliams and Sullivan (2000), Harcourt and D'Asaro (2008), Van Roekel et al. (2012) ]
- ▶ Only a few studies have tested these scalings in climate models

- ▶ Persistent **biases** in the modeled **mixed layer** suggests there could be processes relevant for **turbulent mixing** that have been **ignored** in most parameterizations of the mixed layer in climate models



Verdy et al. (2013)



Belcher et al. (2012)

1. Do we have good observations for wind-wave-current interactions? If not, what more/else do we need?

2. How well do models represent wind-wave-current interactions?

3. What additional observations/information do we need to improve the parameterization and data assimilation of wind-wave-current coupled interactions?