Modeling Glacial Hydrology & Implications for Submarine Melting Ice Discharge

Ian Hewitt, Mathematical Institute, University of Oxford
Brief summary of (sub)glacial hydrology

- Subglacial discharge to the ocean
- Basal lubrication
Surface mass balance + surface water routing

- Energy balance models
- Englacial drainage
- Storage / refreezing
- Transport
- Storage / refreezing
- Moulins
- Refreezing / routing

Subglacial drainage

- Refreezing / routing
- Energy balance models
- Temperature index models

Englacial drainage / storage

- Storage
Individual drainage basins have summer meltwater discharge of 1 m$^3$ s$^{-1}$ per kilometre of margin. Discharge $\sim 1$ m$^3$ s$^{-1}$ to 572 m$^3$ s$^{-1}$

Mernild & Liston 2012

Figure 7. Thickness of the basal temperate ice layer for the control run (left) and the cold mode run (right). Values are in meters and the contour interval is 0.5 m. The dashed line indicates the cold–temperate transition surface. Dotted areas indicate where the bed is temperate but the ice immediately above is cold. The role of water content on the creep of polymers – a comparison of the Stefan problem with the enthalpy method. Aschwanden and others: An enthalpy formulation for glaciers and ice sheets.
(Pressures taken to be suitable local averages)

\[ N + q \delta^{(3d - m d)} + s b^m d = \phi \]

\[ s b^m d = \phi \]

\[ \phi \Delta - \infty \]

Two Key Concepts:

- Basal water flows down surface gradient
- Surface water flows down surface gradient
- Small influence of effective pressure
- Small influence of bed gradient
- Basal melting due to geothermal heat flux, frictional heating, turbulent dissipation. Typical rates 5 mm yr
- Surface melting during summer. Typical rates 1000 mm yr
- Surface melt water provides the dominant source of water ydissipation. Typical rates 5 mm yr
Figure 1. (a) Greenland simulation domain with topography (500-m contour interval), the location of the coastal and GrIS meteorological tower stations (red dots; station specifications are provided in Table 1), and names of the surrounding seas and oceans. Greenland has been divided into three regions (east, west, and north) based on what oceans and seas watershed runoff flow into (see dashed lines and circles). The GrIS is marked with a color scale from gray to white (related to elevation), and the area outside the GrIS with black. (b) Simulated individual Greenland drainage basins (represented by multiple colors). Also, a specific region is illustrated from where examples of catchment runoff and hydrographs are illustrated (see bold square). (c) A closeup example of the individual drainage basins and flow network for the Helheim Glacier region, at the innermost part of the Sermilik Fjord, southeast Greenland, including the location for the runoff values and hydrographs illustrated in Fig. 5.
Saturated sediments - not capable of carrying required discharge


‘Distributed’ systems
- linked cavities
- Nye channels
- micro-cavity networks
- canals

Walder & Fowler 1994


Nye 1973

Fountain & Walder 1998, Flowers & Clarke 2002


‘Channel’ systems

Subglacial drainage

not capable of carrying required discharge

Saturated sediments

Nye channels
- linked cavities
- Walder & Fowler 1994
- canals
- Fountain & Walder 1998, Flowers & Clarke 2002
- micro-cavity networks
- uneven water films


‘Distributed’ systems

...
Surface melt water provides the dominant source of water discharge. Typical rates are 1000 mm yr^{-1} from Antarctica — more like mountain glaciers). Basal melting due to geothermal heat flux, frictional heating, turbulent dissipation. Typical rates are 5 mm yr^{-1}.

What happens to water and how to model it.

Surface melting during summer. Typical rates are 1000 mm yr^{-1}.

Steady state theory

Channel theory
Discharge $Q$ averaged over some representative area.

$\phi$ is the volumetric water content, $\Delta$ is the potential difference between the bed and the surface, $\rho$ is the density of the water, $g$ is the acceleration due to gravity, and $N$ is a normalizing parameter.

At bed elevation $z_b$, the effective pressure is $p_{eff} = p - \rho g z_b$.

At surface elevation $z_s$, the effective pressure is $p_{eff} = p - \rho g z_s$.

Surface melting during summer. Typical rates 1000 mm yr$^{-1}$.

Basal melting due to geothermal heat flux, frictional heating, and turbulent dissipation. Typical rates 5 mm yr$^{-1}$.

Steady state theory
and less pronounced period of low effective pressure than the smaller jump to a critical lower discharge, a temporary jump in effective pressure that leads the system to shut down for $m$, that corresponds and a lower limit, causes (systems, and increases with water supply for channelized ones. For some decreases with water supply (and, hence, discharge) for stable unchannelized $c$.

The abrupt increase in, The seasonal evolution of the drainage system. a
Modelled channels tend to coarsen over time.

- Steady state may never occur in practice.
- More densely spaced smaller channels likely if:
  - slope is large
  - distributed system is poorly connected

Continuum of scales, from orifices to channels.
Correlations between speed and porehole water pressure

Basal sliding

Variance diurnally

Frequently observed that ice speed

Water pressure

Speed

Water pressure [m]

20

60

100

25 [mm/h]

Lichen Bindschadler 1986

Bartholomew et al. 2012

Discharge

Temperature

Velocity

1 May

1 June

1 July

1 Aug

1 Sept

Discharge (m³ s⁻¹)

Temperature (°C)

Horizontal velocity (m s⁻¹)

Temperature (°C) / Surface height (m)

Temperature (°C) / Horizontal velocity (m s⁻¹)

Temperature (°C) / Surface height (m)

Temperature (°C) / Horizontal velocity (m s⁻¹)

Temperature (°C) / Surface height (m)

Temperature (°C) / Horizontal velocity (m s⁻¹)
Basal sliding

\[ \tau_b \approx -\rho_i g H \nabla s \]

- **Hard bedrock**
  - \( \tau_b = RU_b^{1/m} \)
  - Water film facilitates sliding

- **Cavities**
  - \( \tau_b = CU_b^p N^q \)
  - Lower effective pressure
  - Larger cavities

- **Soft sediments**
  - \( \tau_b = \mu N \)
  - Lower effective pressure
  - Lower yield stress
Ice flow due to sliding only (SSA)

Sliding law

Conduit + cavity drainage

Modelled sliding variations
How is discharge to the ocean distributed?
How appropriate are effective pressure-dependent sliding laws?

What is basal water pressure?

How appropriate are effective pressure-dependent sliding laws?

Any channelized flow may fan out near a terminus.

Evolution of the drainage system is important.

Little data on subglacial hydrology. Simple first order approaches required for modelling.

Greatly increased quantity of surface runoff in recent years.

Summary
Channelized flow expected if discharge sufficiently large.

Temporal evolution of the drainage system is important.

Mass conservation fundamental.

Effective pressure / ice dynamics

Delivery to margin

A number of models impose a distribution of channels.

Some recent models allow a dynamically evolving channel network. They impose a seedling network of conduits which compete with each other to grow into channels.

A number of models impose a distribution of channels.


Simplest models are film / diffusion models.

Channelized flow expected if discharge sufficiently large.

Models
Channel evolution

Water flow

Conservation of water

\[ uN SV^\gamma - [q \Delta \cdot s|b^m d g| + \phi \Delta \cdot s|(g - 1)|] \frac{T^d}{\partial} = \frac{4\varepsilon}{SE} \]

\[ H + [q \Delta \cdot s|b^m d g| + \phi \Delta \cdot s|(g - 1)|] \frac{T^m}{\partial} = \frac{sE}{\partial E} + \frac{4\varepsilon}{SE} \]

\[ \frac{1}{1} \phi \Delta \cdot s|_{S}^{3/4} S^2 \eta = \partial \]

Time scale

Steady state

\[ \frac{1}{1} \partial_{y/11} \phi \Delta \]

\[ \frac{1}{1} \partial_{y/12} \phi \Delta \]


Channel theory
Alternative

Conservation of water

Water flow

\[ u_N \frac{\partial}{\partial y} - q \frac{\partial}{\partial y} + w \frac{\partial}{\partial d} = \frac{\partial \rho}{\partial y} \]

\[ \mu + \omega = b \cdot \Delta + \frac{\partial \rho}{\partial y} \]

\[ \phi \Delta \xi h \phi K - = b \]

\[ \frac{6}{1} \phi \Delta \xi h \rho_n \propto t \]

\[ \frac{6}{1} \phi \Delta \xi h \rho_n \propto N \]

Ways in which water a

Basal melting due to geothermal heat flux, frictional heating, turbulent
Six lakes were identifiable before snow cover became patchy, dilation is negligible (see Appendix A). For these reasons, regions began shortly after the onset of melt and nearly all for calculating strain rates is appropriate, and uplift due to till had a diameter greater than 0.50 km. Lake filling in all regions, and 19 in the upglacier region. Of these 62 lakes, 45 (73%) had a diameter greater than 0.25 km, and 25 (40%) uncertainty is introduced by assumptions that vertical strain bed separation (see Appendix A). Additional unquantifiable this results in 10 cm and, by day 223, 20 cm of uncertainty in ...
Lower yield stress
Lower effective pressure

Kamb 1991, Tulaczyk 2000

$N^r T = q$

Larger cavities
Lower effective pressure

Liboutry 1979, Budd et al. 1979, Fowler 1986

$\frac{v N X + q n}{q n} N^r T = q$

Water film facilitates sliding

Weereman 1957, Nye 1969, Kamb 1970

$\frac{v R H}{w / 1} N^r T = q$

Soft sediments

Cavities

Hard bedrock

$s \Delta H^i d - q$

Basal sliding
\[ a \frac{db^a}{da} = \bar{\zeta} \]

\[
T^{ad} \frac{\phi \Delta \cdot b^\gamma}{\phi \delta / \phi \delta} + |s \phi / \phi \phi | = \mathcal{W}
\]

\[
\frac{T^{ad}}{\mu \cdot \eta} + \mathcal{G} = u
\]

\[
N_{I-u} N |R^u \frac{u^k}{V^2} - W^r \frac{V}{a^d} = \frac{\mathcal{E}}{S^2}
\]

\[
N_{I-u} N |J^u \frac{u^k}{V^2} - \frac{\mathcal{E}}{S^2} (\eta - \eta)^{v} + u \frac{v}{a^d} = \frac{\mathcal{E}}{\eta^2}
\]

\[
\frac{s \phi}{\phi \phi} \frac{v}{v} \frac{1}{1} \left| \frac{s \phi}{\phi \phi} \right|_{v/v} S^2 \delta Y = \delta
\]

\[
\phi \Delta \frac{b^a d}{\varepsilon} - = b
\]

\[
(u x)^g \mathcal{W} + (v x)^g \mathcal{W} + u = \frac{\mathcal{E}}{S^2} + (v x) \left[ \frac{s \phi}{\phi \phi} + \frac{\mathcal{E}}{S^2} \right] + b \cdot \Delta + \frac{\mathcal{E}}{\eta^2}
\]