Modeling glacial hydrology & implications for submarine meltwater discharge

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- Brief summary of (sub)glacial hydrology
- Basal lubrication
- Subglacial discharge to the ocean

Surface mass balance + surface water routing

- Temperature index models
- Energy balance models
- Refreezing / routing

Englacial drainage / storage

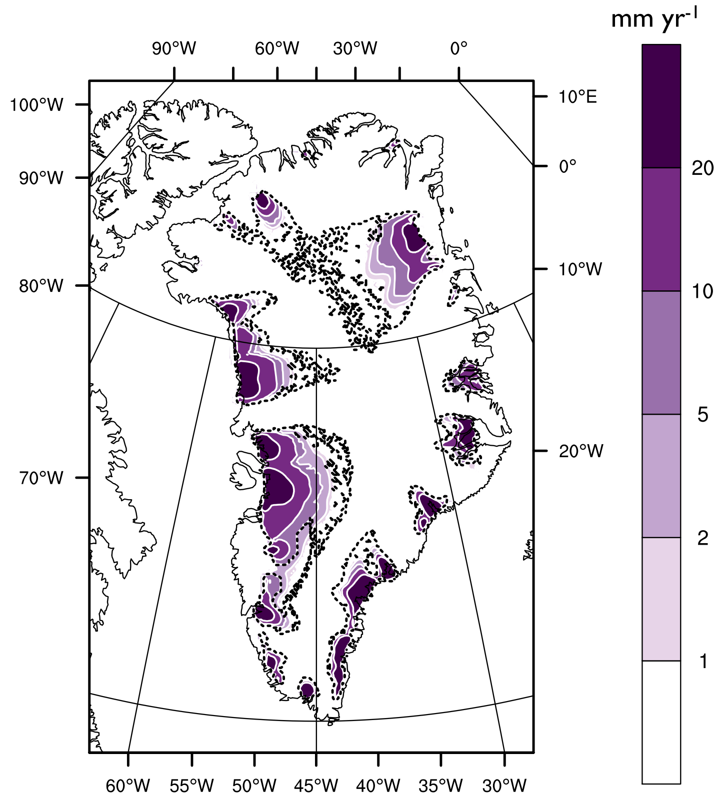
- Moulins
- Storage / refreezing

Subglacial drainage

- Basal melting / refreezing
- Storage
- Transport

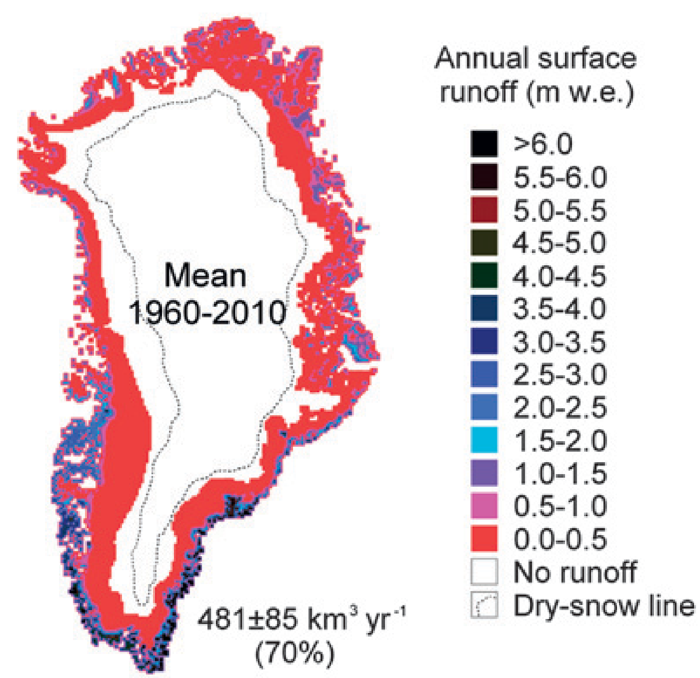
Water sources

- Basal melting ~5 mm yr^{-1}



Aschwanden et al 2012

- Surface runoff ~1000 mm yr^{-1}

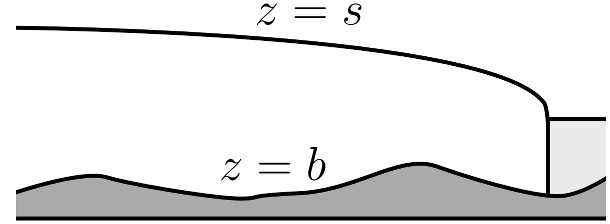


Mernild & Liston 2012

⇨ Discharge ~1 m^3s^{-1} per kilometre of margin

Individual drainage basins have summer meltwater discharge 1 m^3s^{-1} - 1000 m^3s^{-1}

Two key concepts



- Hydraulic potential $\mathbf{q} \propto -\nabla\phi$

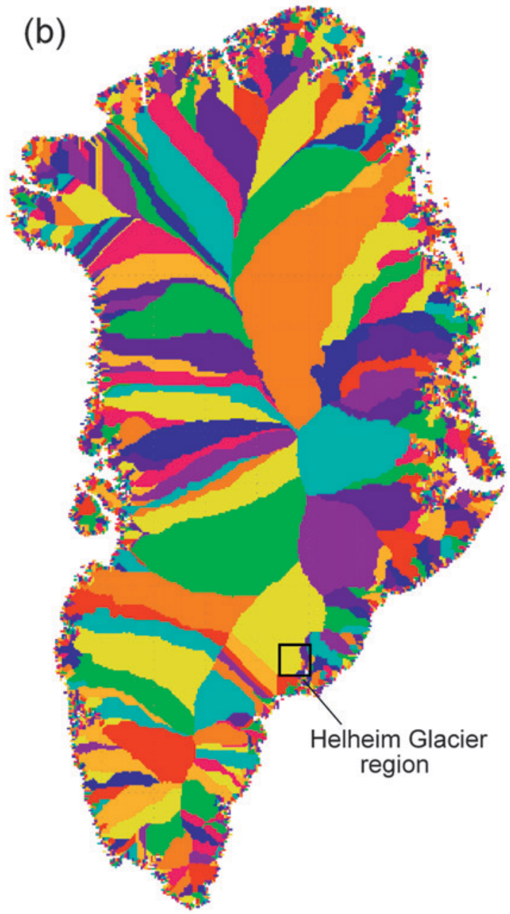
  - Surface water flows down surface gradient $\phi = \rho_w g s$

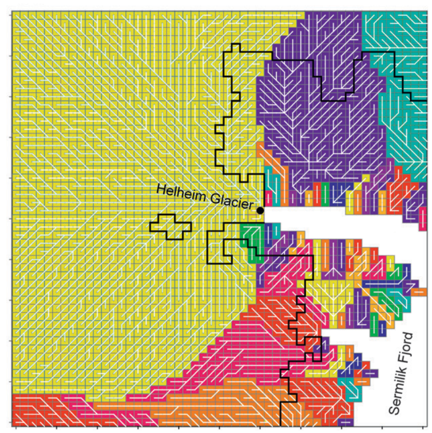
  - Basal water flows down surface gradient
    + small influence of bed gradient
    + small influence of effective pressure

  $$\phi = \rho_i g s + (\rho_w - \rho_i) g b + N$$

- Effective pressure $N = p_i - p_w$ (pressures taken to be suitable local averages)

Supraglacial drainage

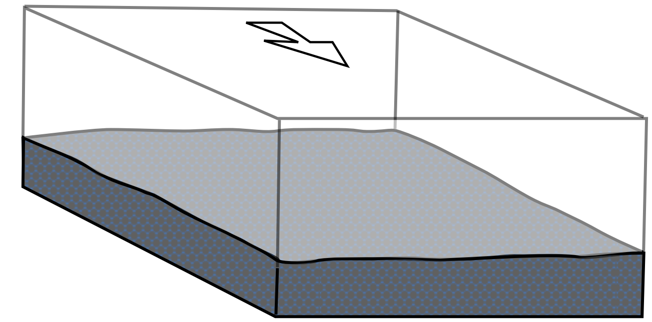




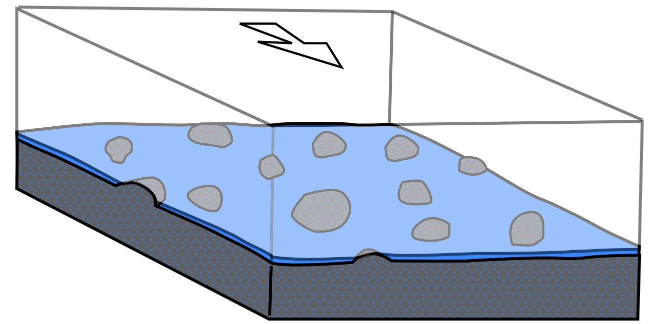
Mernild & Liston 2012

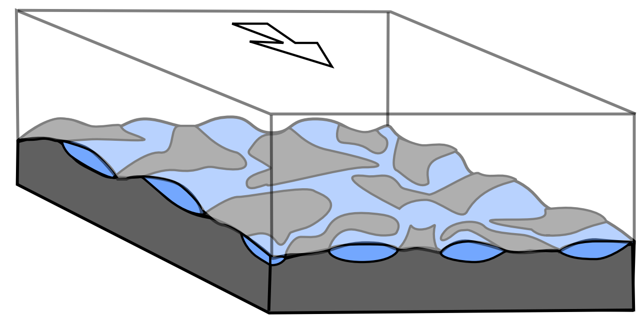
Subglacial drainage

- Saturated sediments
  - not capable of carrying required discharge

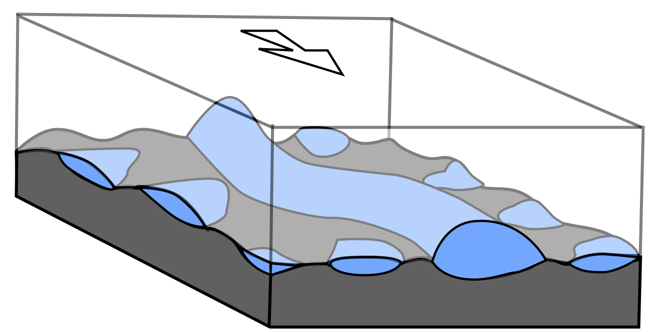


- ‘Distributed’ systems
  - uneven water films
    Weertman 1972, Walder 1982, Alley 1989, Creyts & Schoof 2009
  - micro-cavity networks
    Fountain & Walder 1998, Flowers & Clarke 2002
  - canals
    Walder & Fowler 1994
  - linked cavities
    Lliboutry 1976, Walder 1986, Fowler 1986, Kamb 1987
  - Nye channels
    Nye 1973

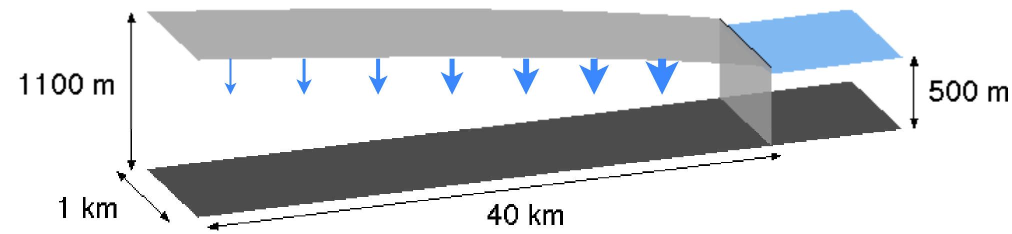




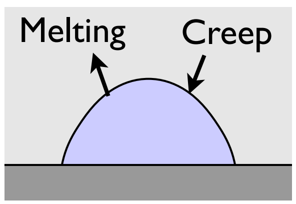
- ‘Channel’ systems
  Röthlisberger 1972, Nye 1976, Hooke et al 1990



Steady state theory

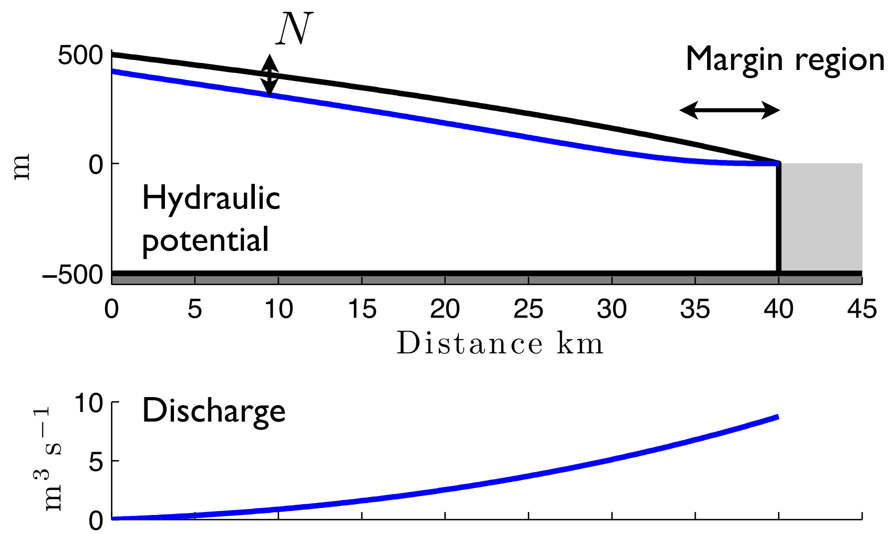


Channel theory

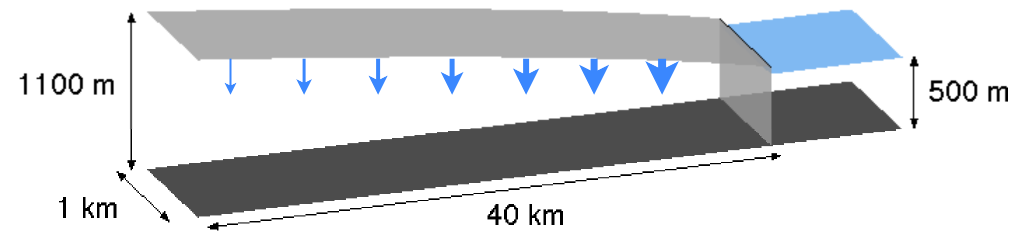


$$N \propto |\nabla \phi|^{11/24} Q^{1/12}$$

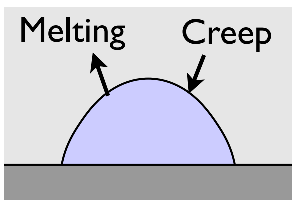
$$t \propto \frac{1}{|\nabla \phi|^{11/8} Q^{1/4}}$$

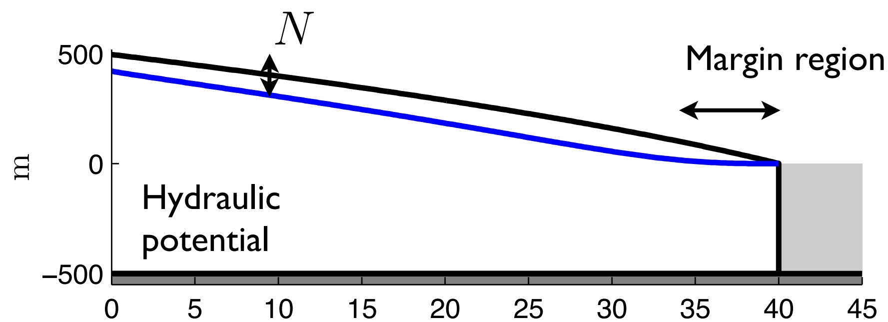


Steady state theory



Channel theory

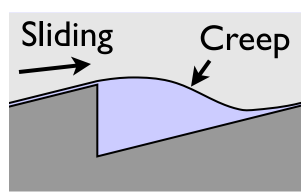




$$N \propto |\nabla\phi|^{11/24} Q^{1/12}$$

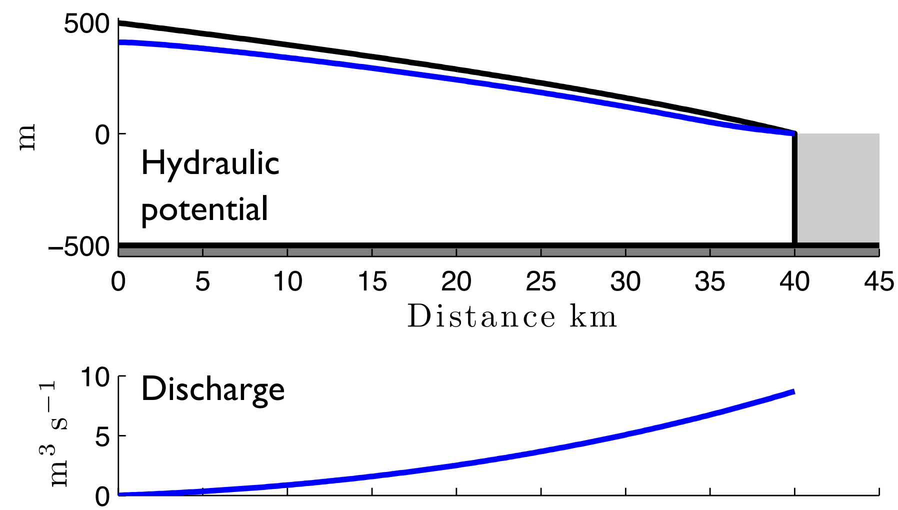
$$t \propto \frac{1}{|\nabla\phi|^{11/8} Q^{1/4}}$$

Cavity theory

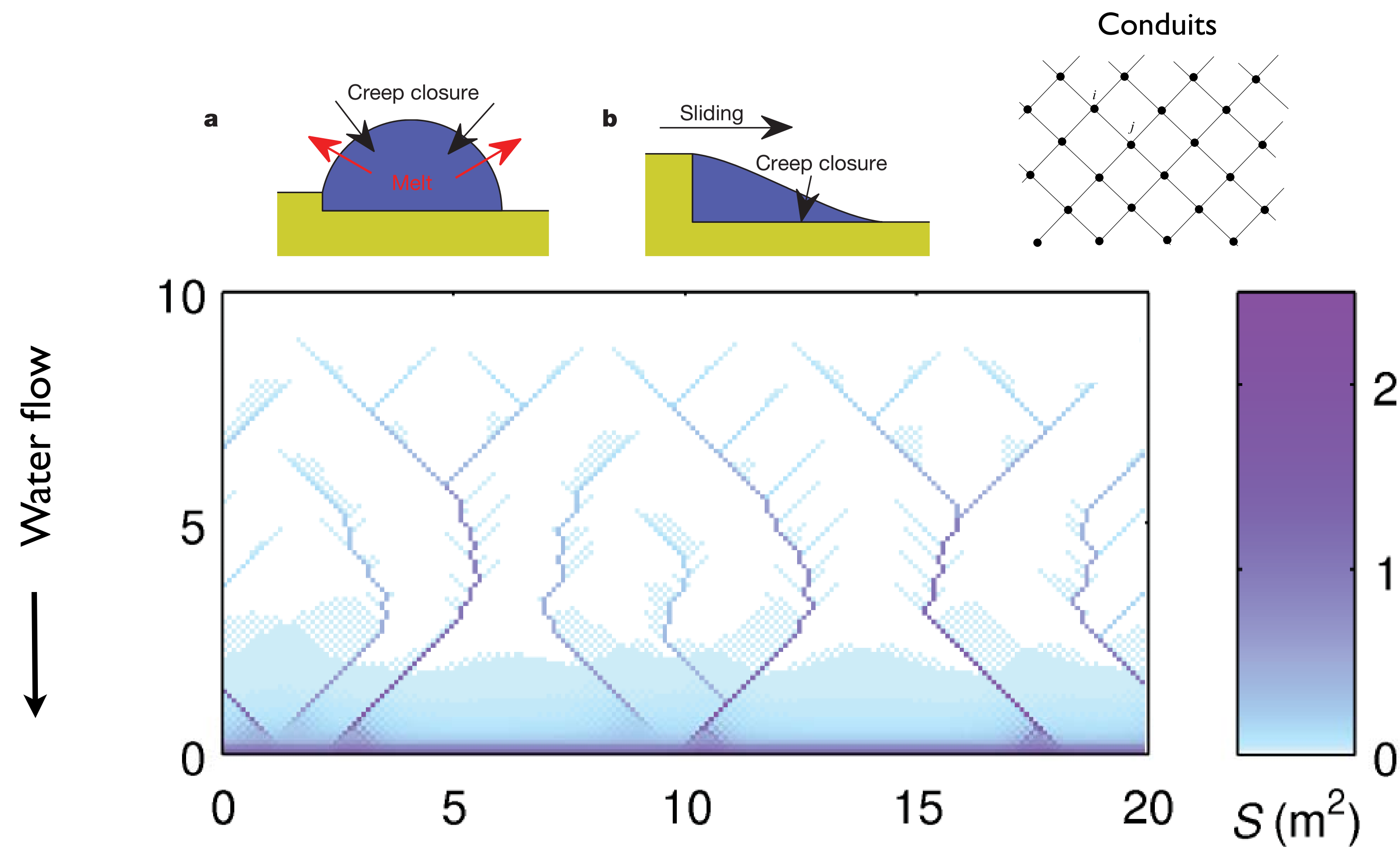


$$N \propto \frac{u_b^{1/3} |\nabla\phi|^{1/9}}{Q^{1/9}}$$

$$t \propto \frac{Q^{1/3}}{u_b |\nabla\phi|^{1/3}}$$

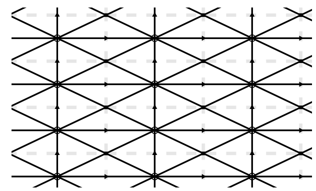


Conduit network

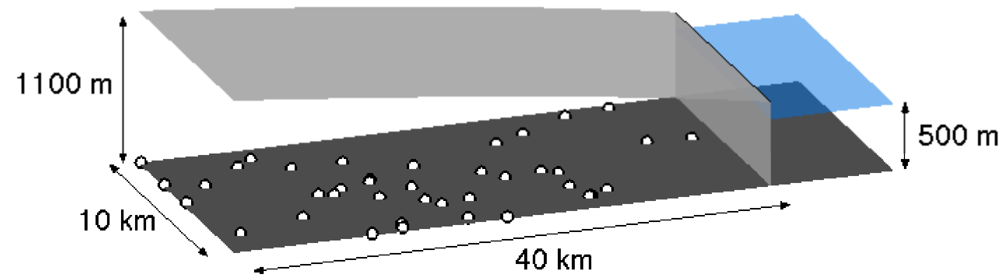


Schoof 2010

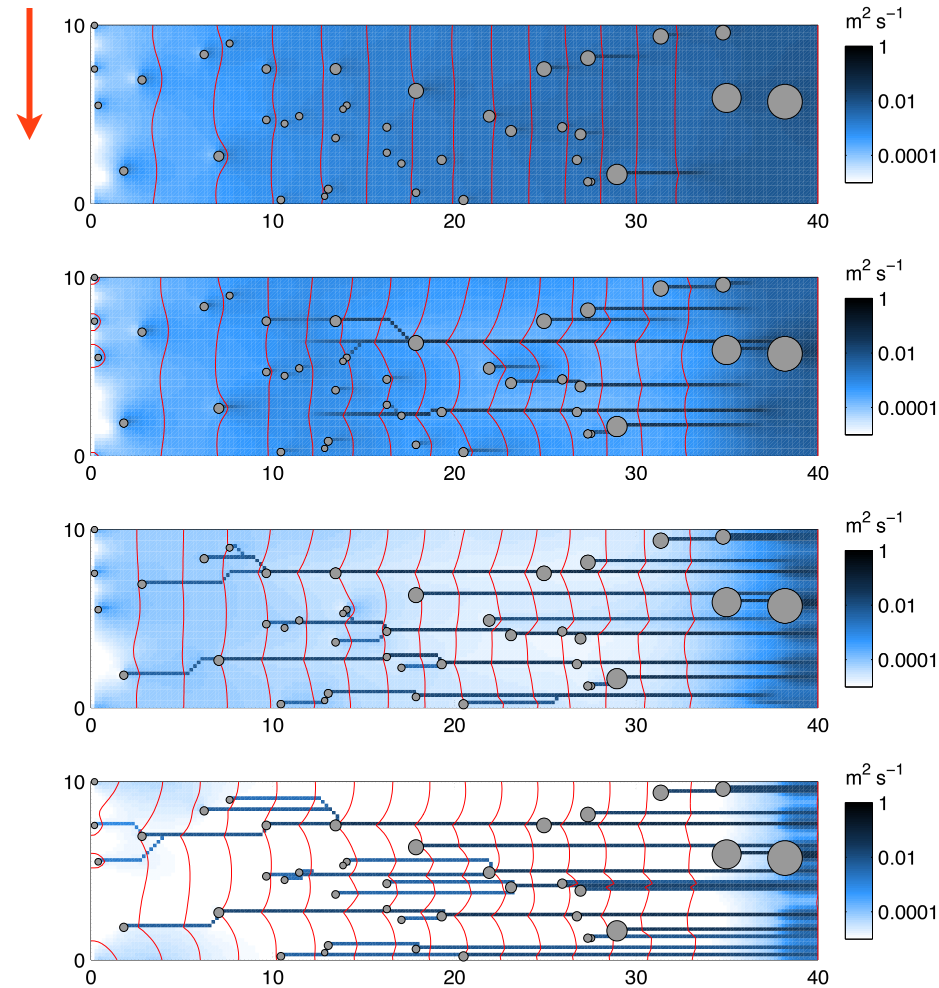
Channel spacing



Conduits + cavities

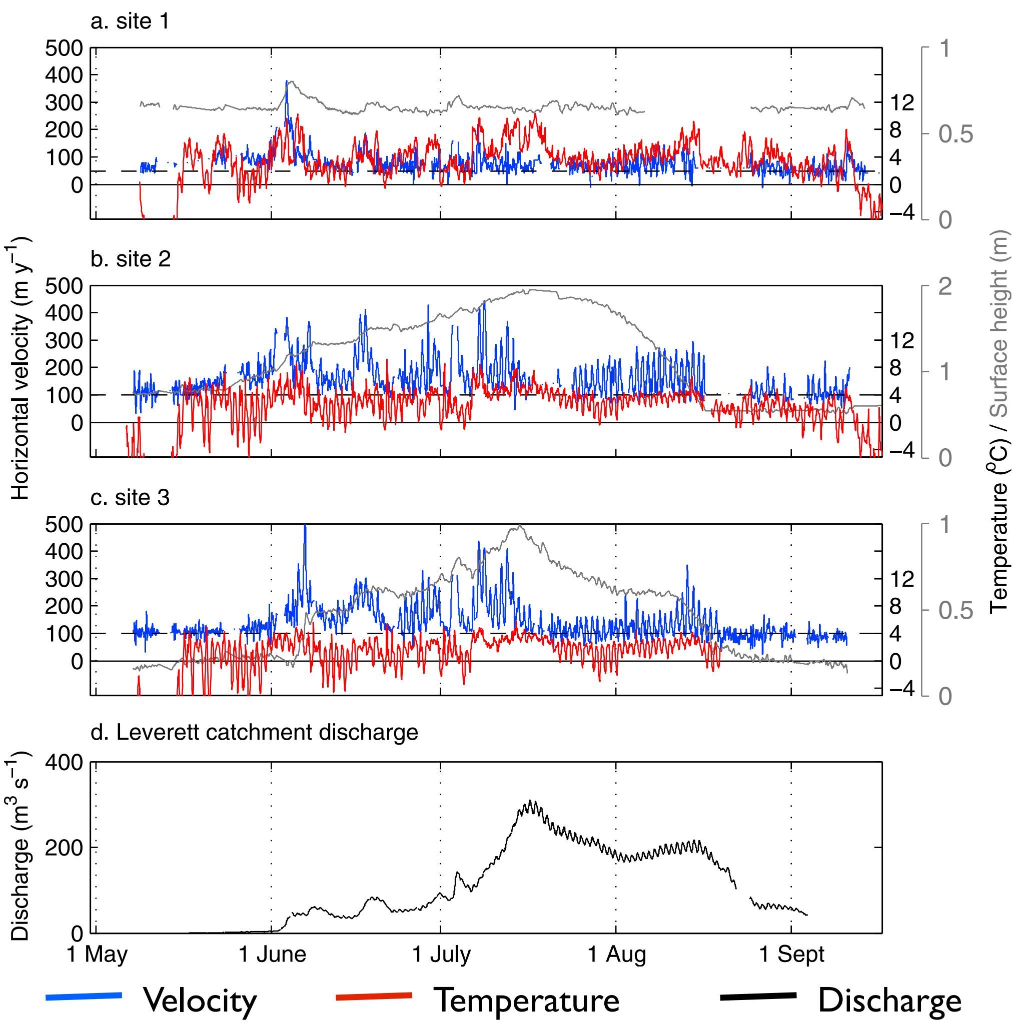


More poorly connected cavities



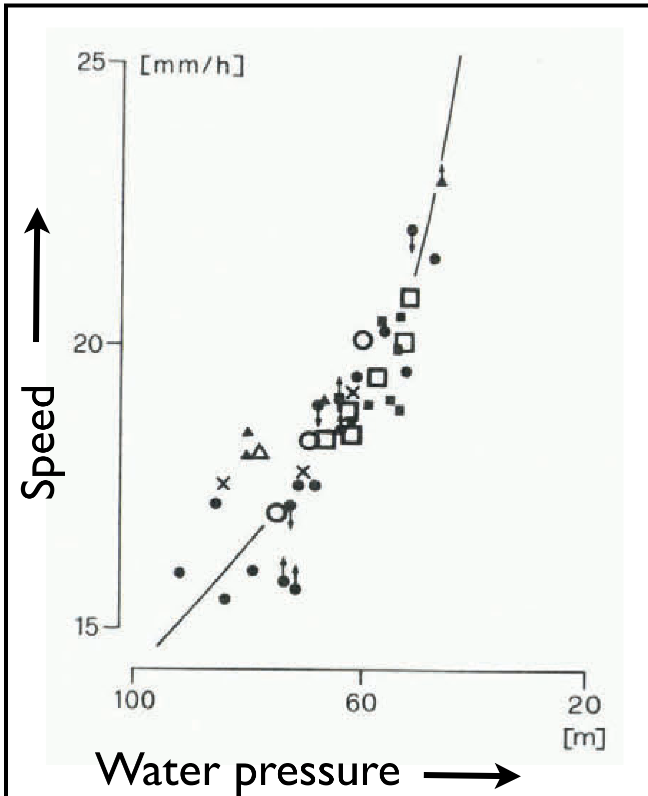
- More densely spaced smaller channels likely if
  - slope is large
  - distributed system is poorly connected
- Steady state may never occur in practice.
- Continuum of scales, from orifices to channels.
- Modelled channels tend to coarsen over time.

Basal sliding



Bartholomew et al 2012

- Frequently observed that ice speed varies diurnally

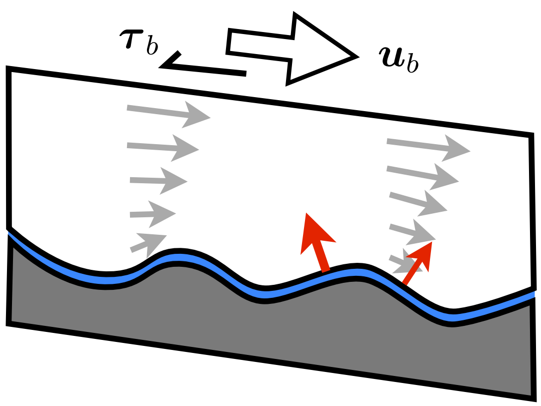


Iken & Bindschadler 1986

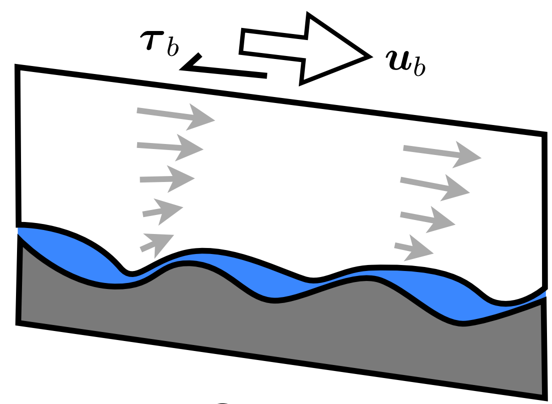
- Correlations between speed and borehole water pressure

Basal sliding

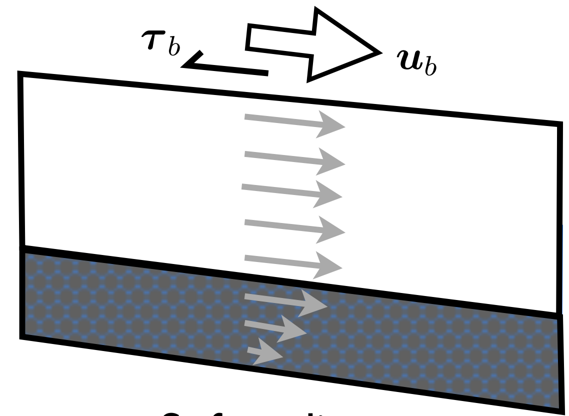
$$\boldsymbol{\tau}_b \approx -\rho_i g H \nabla s$$



Hard bedrock



Cavities



Soft sediments

$$\tau_b = RU_b^{1/m}$$

Water film facilitates sliding

$$\tau_b = CU_b^p N^q$$

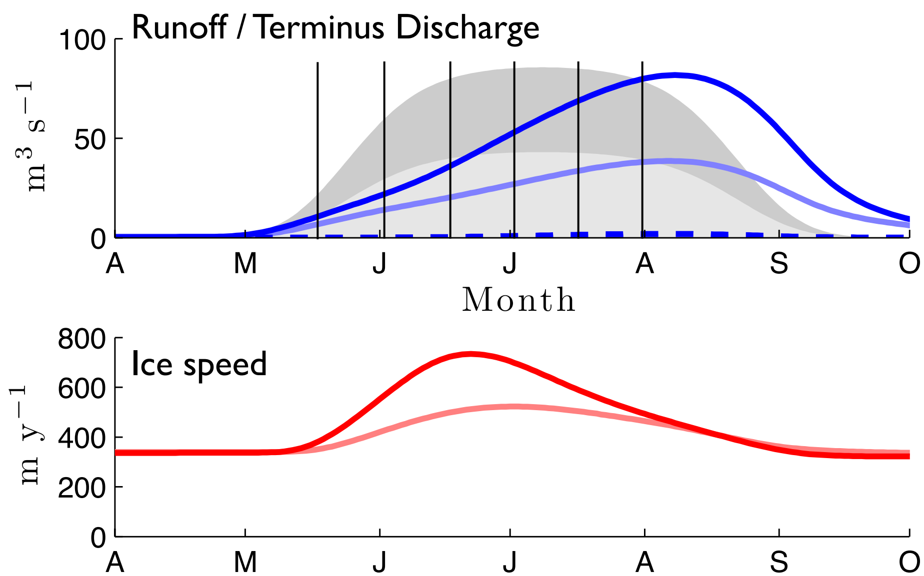
Lower effective pressure
Larger cavities

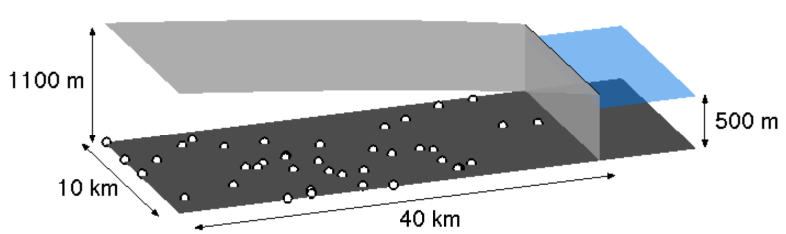
$$\tau_b = \mu N$$

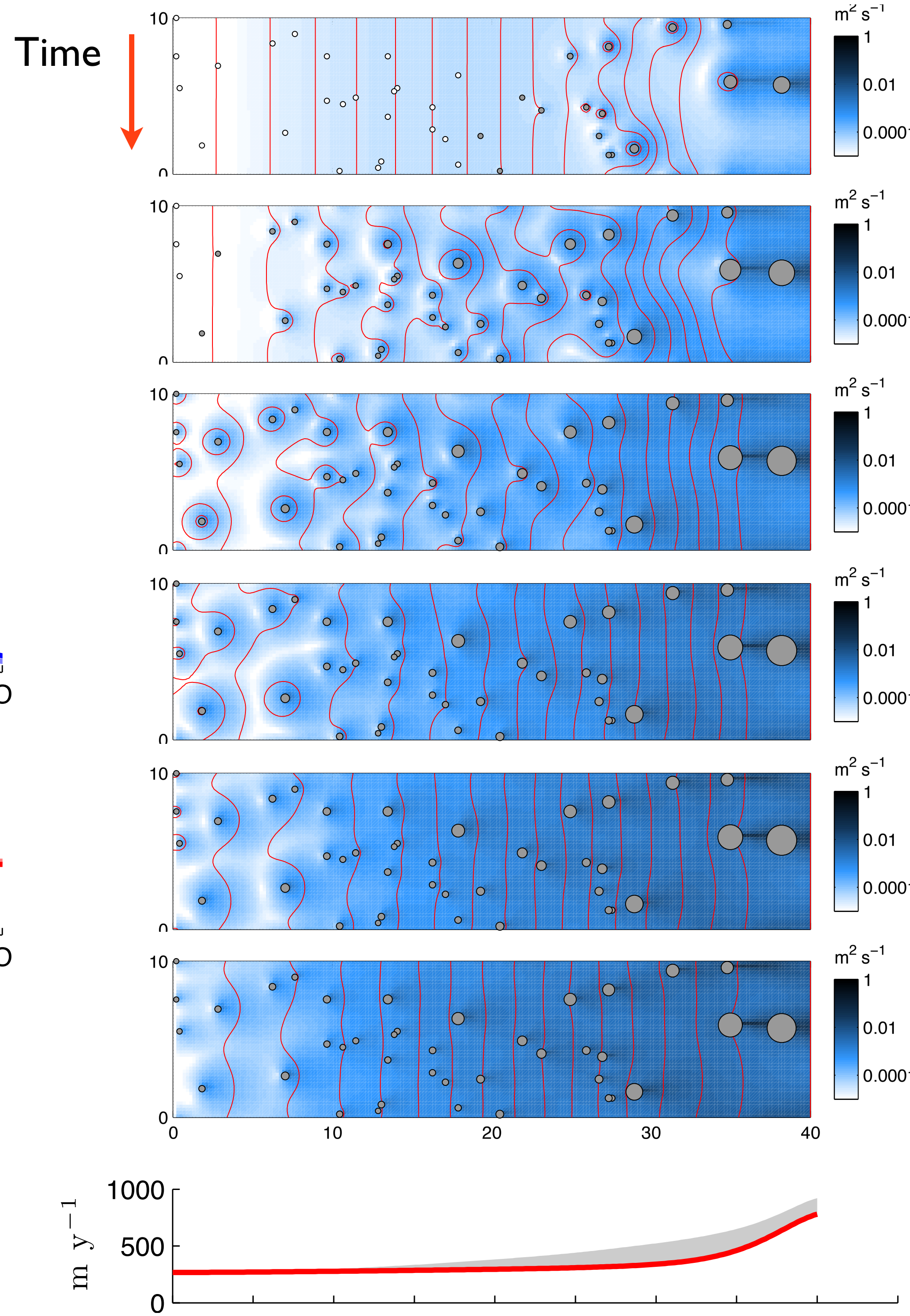
Lower effective pressure
Lower yield stress

Modelled sliding variations

- Conduit + cavity drainage
- Sliding law $\tau_b = C U_b^{1/3} N^{1/3}$
- Ice flow due to sliding only (SSA)



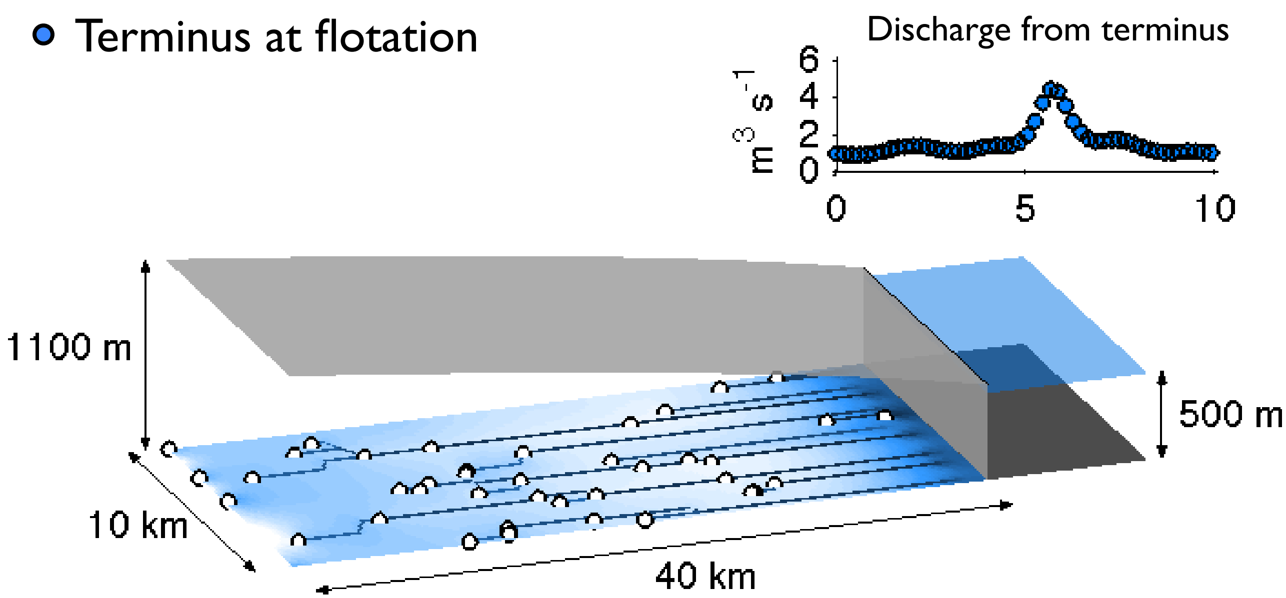




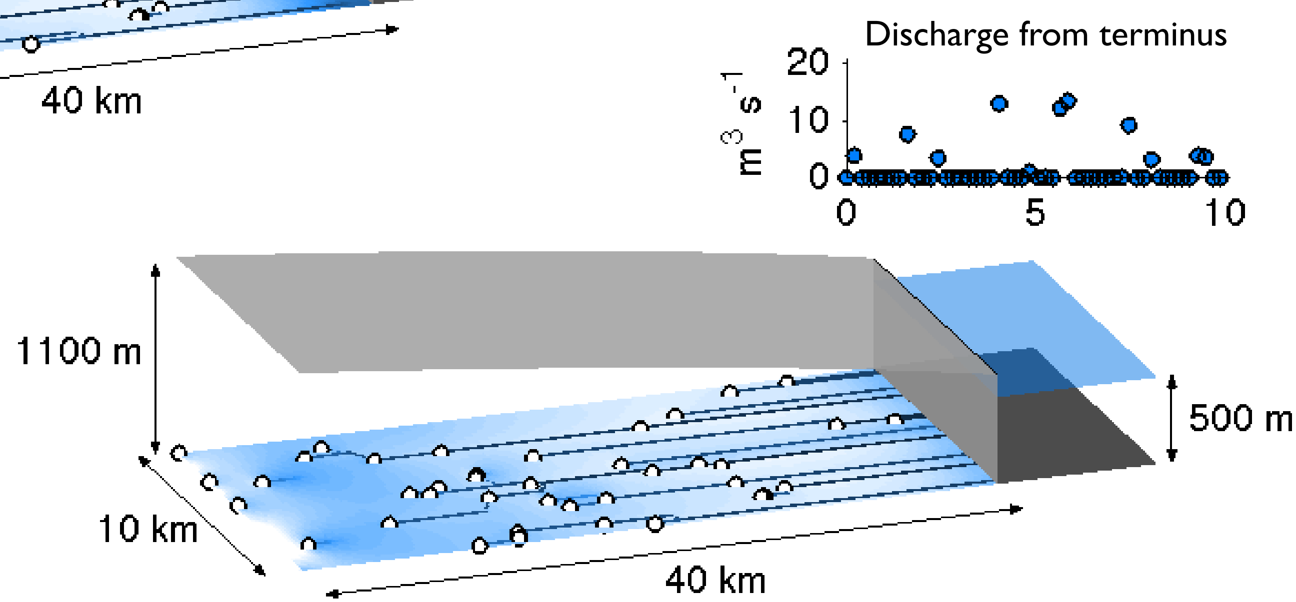
Discharge to the ocean

- How is discharge to the ocean distributed?

- Terminus at flotation



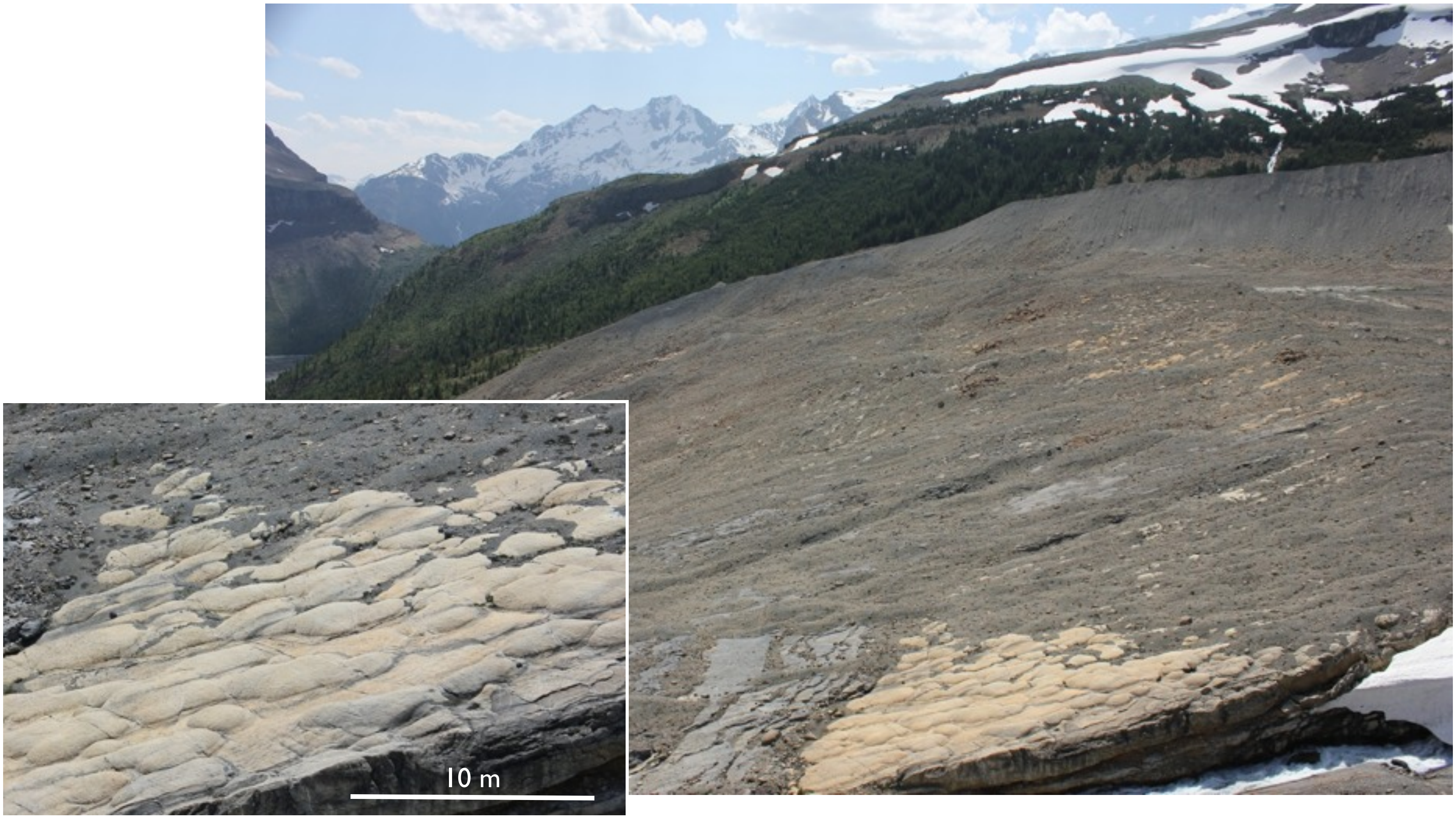
- Terminus above flotation

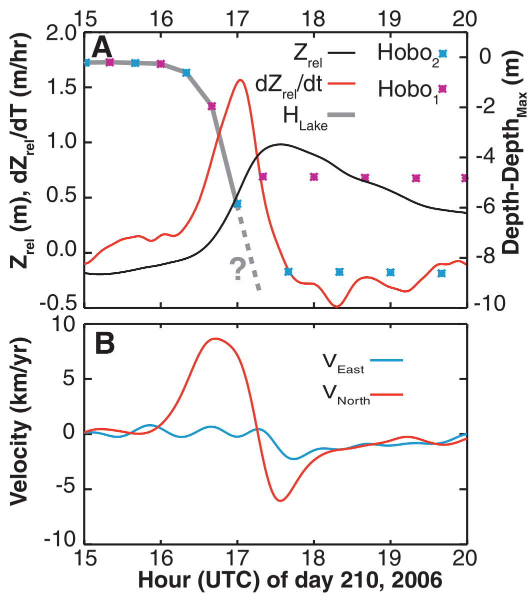


Summary

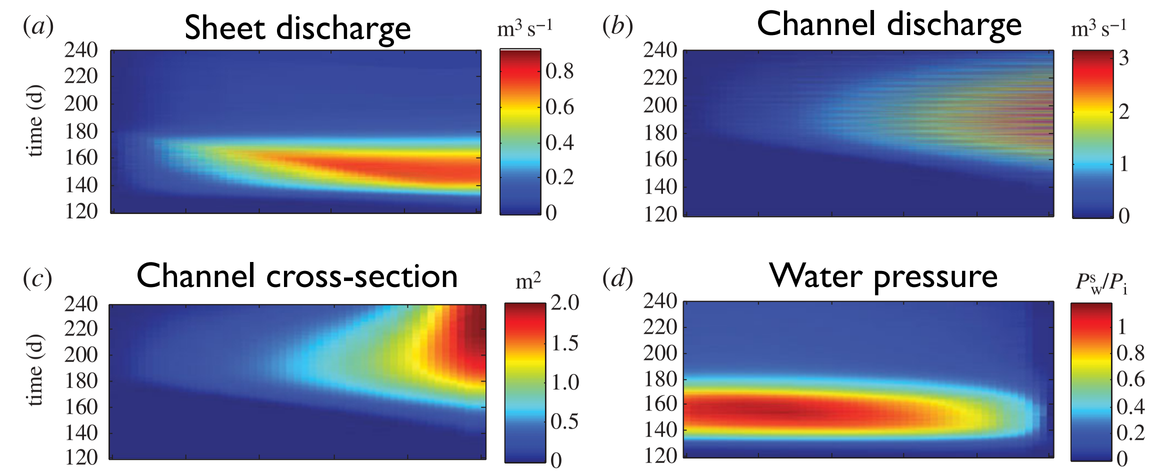
- Greatly Increased quantity of surface runoff in recent years.
- Little data on subglacial hydrology. Simple first order approaches required for modelling.
- Evolution of the drainage system is important.
- Any channelized flow may fan out near a terminus.

- How appropriate are effective pressure-dependent sliding laws?
- What *is* basal water pressure?
- Ice dynamics; may be better to use statistical links between surface melting and sliding speed.

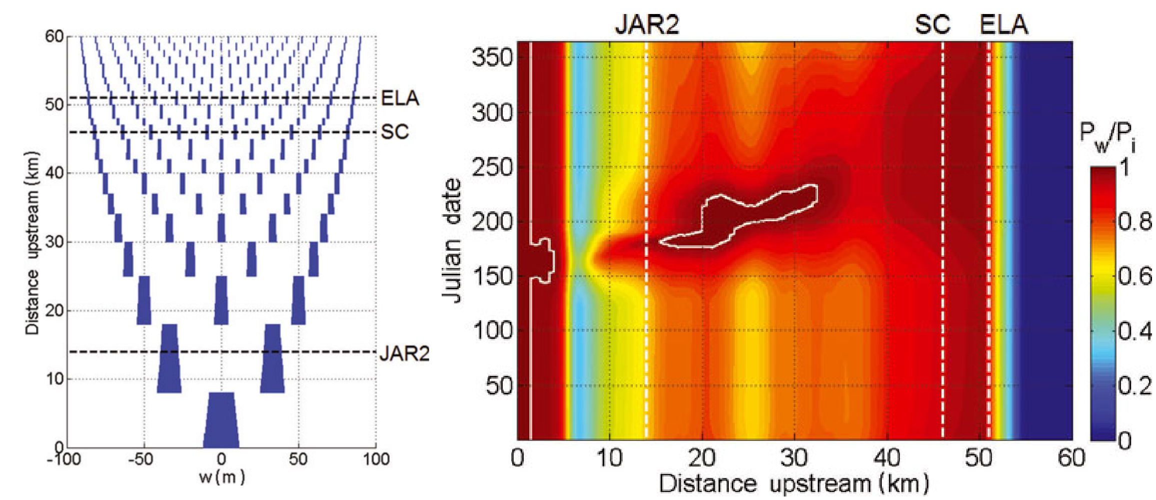

10 m



Das et al 2008



Pimentel & Flowers 2010



Colgan et al 2012

Models

- Mass conservation fundamental.
- Temporal evolution of the drainage system is important.
  - Effective pressure / ice dynamics
  - Delivery to margin
- Channelized flow expected if discharge sufficiently large.

- Simplest models are film / diffusion models.

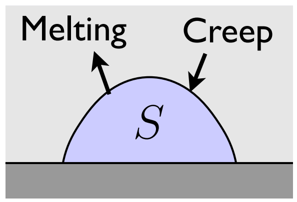
  Arnold & Clarke 2002, Johnson & Fastook 2002, Le Brocq et al 2009, van Pelt & Oerlemans 2012
- A number of models impose a distribution of channels.

  Flowers & Clarke 2004, Kessler & Anderson 2004, Pimentel & Flowers 2011, Colgan et al 2012
- Some recent models allow a dynamically evolving channel network. They impose a seedling network of 'conduits' which compete with each other to grow into channels.

  Schoof 2010, Werder et al 2012, Hewitt 2013

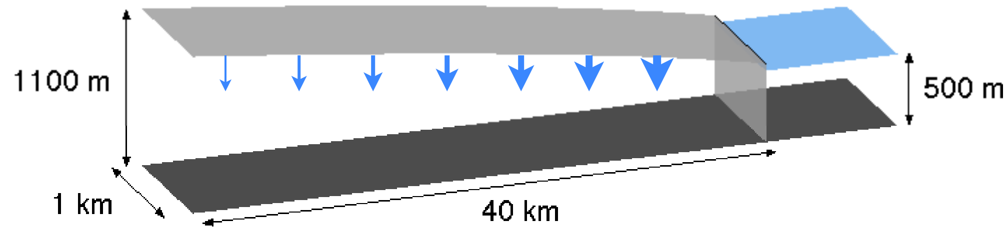
Channel theory

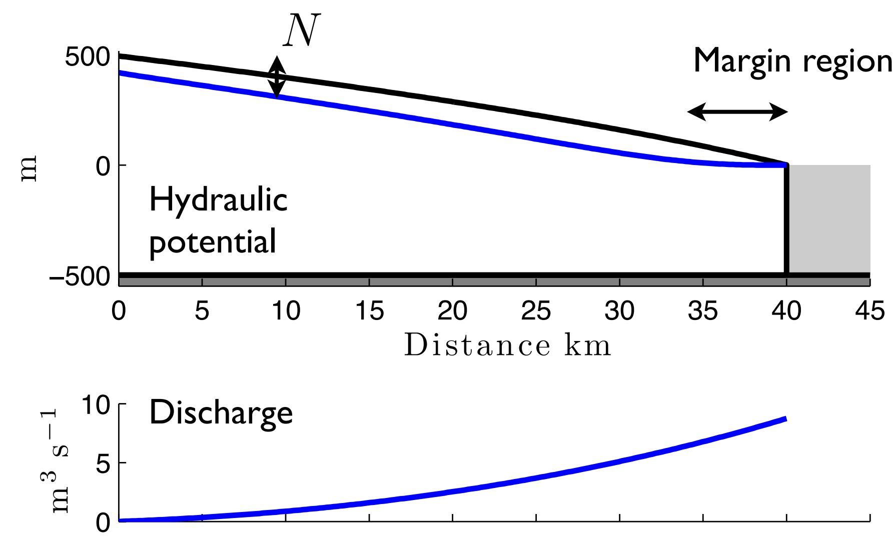
Röthlisberger 1972, Nye 1976, Hooke et al 1990



Steady state $N \propto |\nabla\phi|^{11/24} Q^{1/12}$

Time scale $t \propto \dfrac{1}{|\nabla\phi|^{11/8} Q^{1/4}}$





$$Q = K_c S^{4/3} |\mathbf{s} \cdot \nabla\phi|^{1/2}$$

Water flow

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s} = \frac{Q}{\rho_w L} \left[(1-\beta)|\mathbf{s} \cdot \nabla\phi| + \beta\rho_w g|\mathbf{s} \cdot \nabla b|\right] + R$$

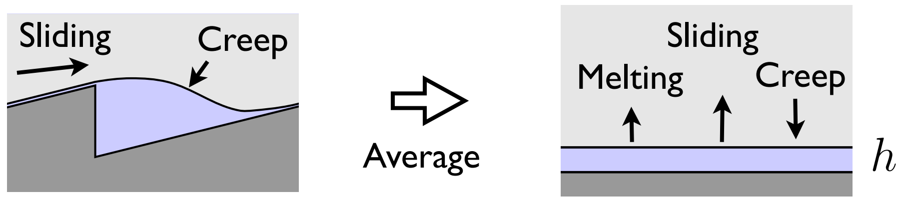
Conservation of water

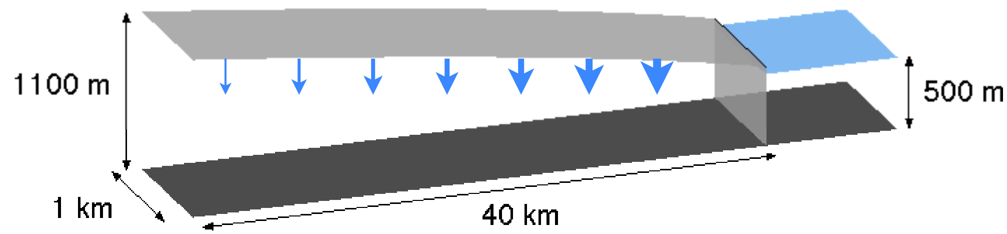
$$\frac{\partial S}{\partial t} = \frac{Q}{\rho_i L} \left[(1-\beta)|\mathbf{s} \cdot \nabla\phi| + \beta\rho_w g|\mathbf{s} \cdot \nabla b|\right] - \gamma_c A S N^n$$

Channel evolution

Cavity theory

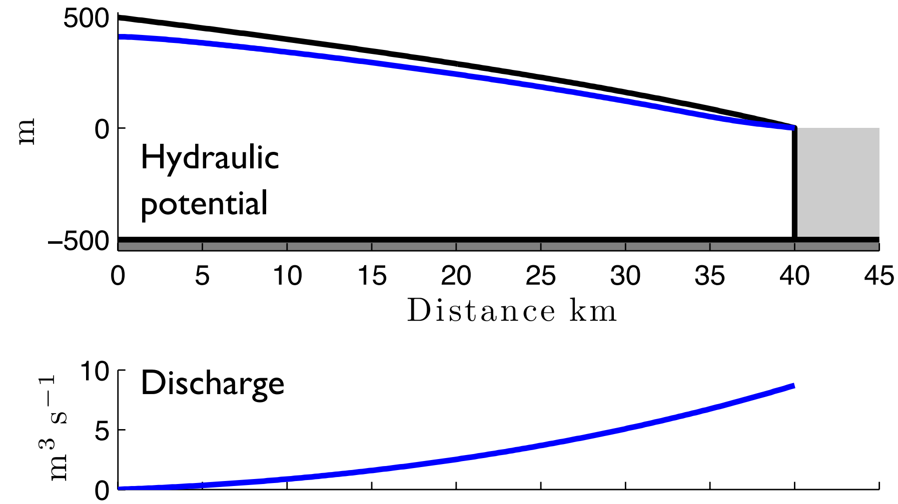
Walder 1986, Folwer 1986, Kamb 1987, Hewitt 2011





Steady state $N \propto \frac{u_b^{1/3} |\nabla \phi|^{1/9}}{Q^{1/9}}$

Time scale $t \propto \frac{Q^{1/3}}{u_b |\nabla \phi|^{1/3}}$



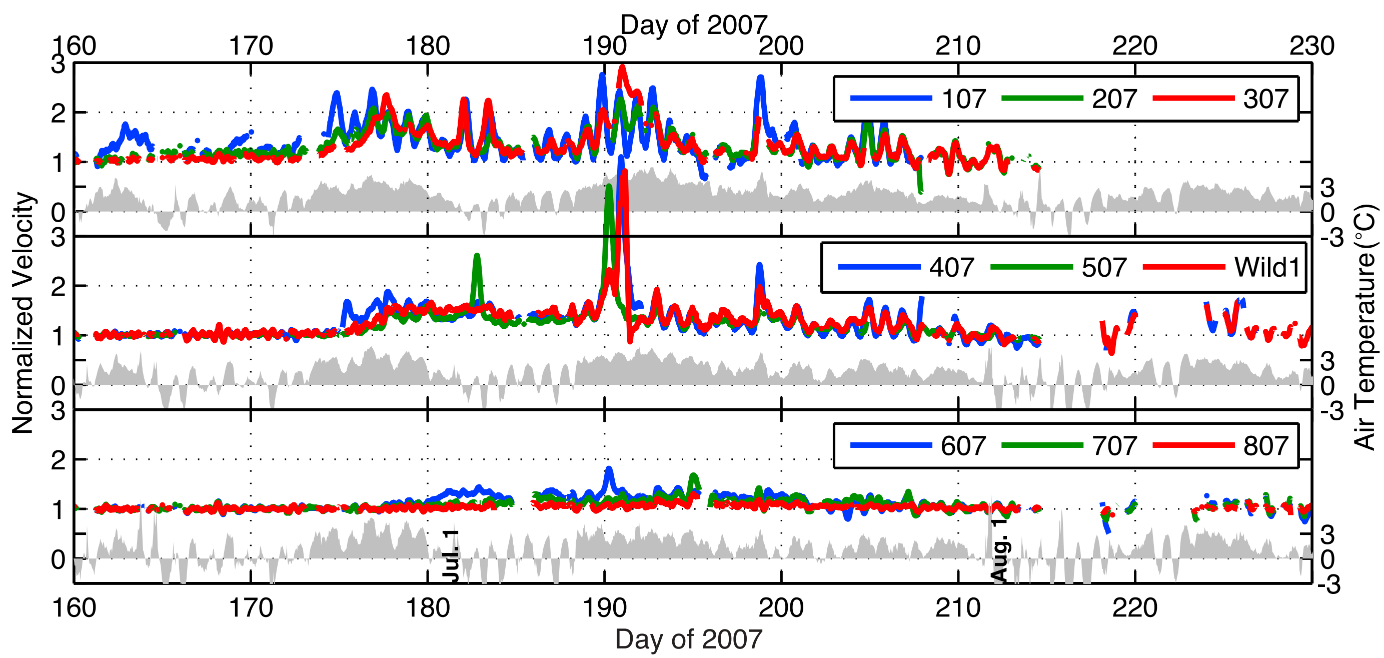
$$\mathbf{q} = -K h^3 \nabla \phi$$ Water flow

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = m + r$$ Conservation of water

$$\frac{\partial h}{\partial t} = \frac{\rho_w}{\rho_i} m + \frac{h_r}{l_r} U_b - \gamma A h N^n$$ Cavity evolution

Alternative $h = h_e(N)$

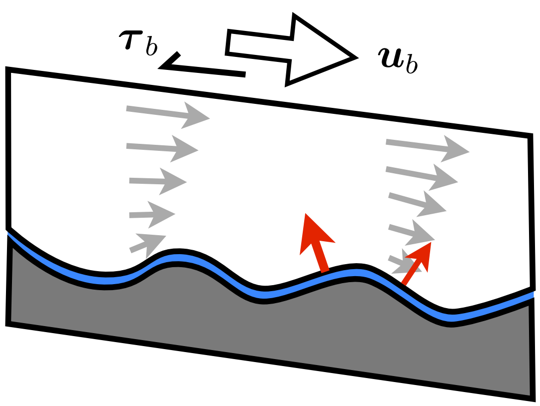
Flowers & Clarke 2002



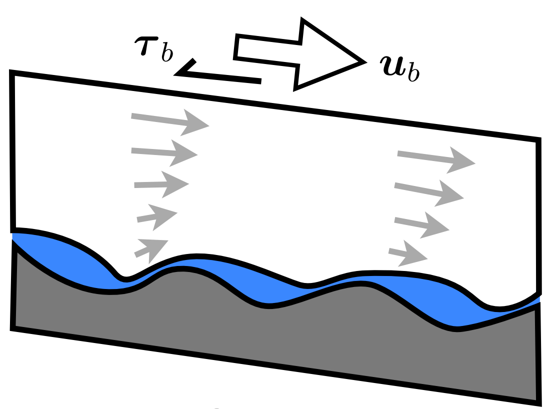
Hoffman et al 2011

Basal sliding

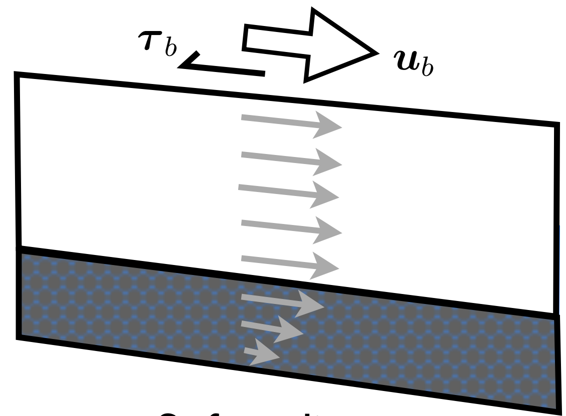
$$\boldsymbol{\tau}_b \approx -\rho_i g H \nabla s$$



Hard bedrock



Cavities



Soft sediments

Hard bedrock

$$\tau_b = R U_b^{1/m}$$

Weertman 1957, Nye 1969, Kamb 1970

Water film facilitates sliding

Cavities

$$\tau_b = C U_b^p N^q$$

Lliboutry 1979, Budd et al 1979, Fowler 1986

Lower effective pressure

Larger cavities

$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n}$$

Schoof 2005, Gagliardini et al 2007

Soft sediments

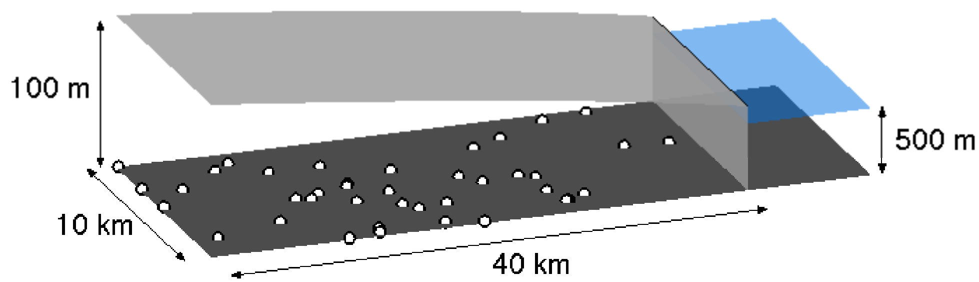
$$\tau_b = \mu N$$

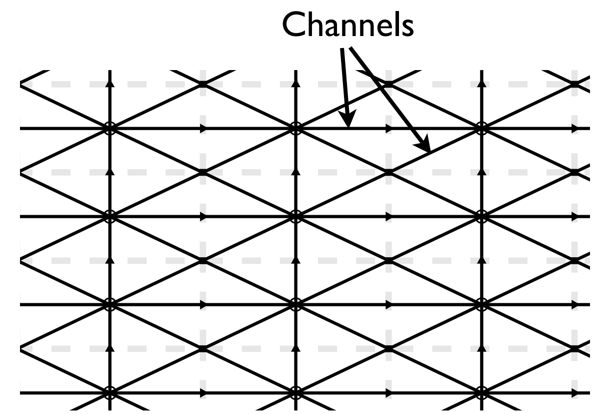
Kamb 1991, Tulaczyk 2000

Lower effective pressure

Lower yield stress

Subglacial drainage model





Mass conservation

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} + \left[\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s}\right]\delta(\mathbf{x}_c) + \frac{\partial \Sigma}{\partial t} = m + M\delta(\mathbf{x}_c) + R\delta(\mathbf{x}_m)$$

Water flow parameterizations

$$\mathbf{q} = -\frac{Kh^3}{\rho_w g}\nabla\phi$$

$$Q = -K_c S^{5/4}\left|\frac{\partial\phi}{\partial s}\right|^{-1/2}\frac{\partial\phi}{\partial s}$$

Evolution of drainage space

$$\frac{\partial h}{\partial t} = \frac{\rho_w}{\rho_i}m + U_b(h_r - h)/l_r - \frac{2A}{n^n}h|N|^{n-1}N$$

$$\frac{\partial S}{\partial t} = \frac{\rho_w}{\rho_i}M - \frac{2A}{n^n}S|N|^{n-1}N$$

Energy conservation

$$m = \frac{G + \boldsymbol{\tau}_b \cdot \mathbf{u}_b}{\rho_w L} \qquad M = \frac{|Q\partial\phi/\partial s| + \lambda_c|\mathbf{q}\cdot\nabla\phi|}{\rho_w L}$$

Englacial storage

$$\Sigma = \sigma\rho_w g p_w$$