Using Power Spectra to Estimate Stochastic Model Parameters for ENSO and their connection to decadal variability

Maria Gehne, Richard Kleeman

Center for Atmosphere Ocean Science, Courant Institute, New York University

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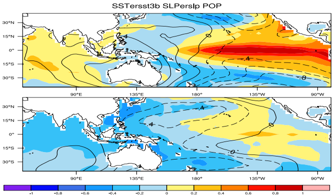
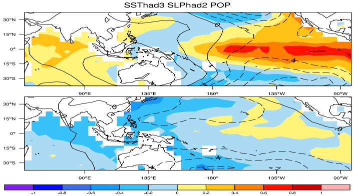
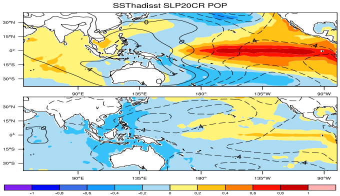
Principal Oscillation Patterns







NOAA SST and SLP POP

Leading SST POP for ERSST data. Contours are SLP POP for ERSLP.

Hadley SST and SLP POP

Leading SST POP for Hadley Centre SST data. Contours are SLP POP for Hadley Centre SLP.

HadISST and 20CR SLP POP

Leading SST POP for HadISST data. Contours are 20th C Reanalysis SLP POP.

Observed Power Spectra

Top: Nino3 and SO index power spectra.

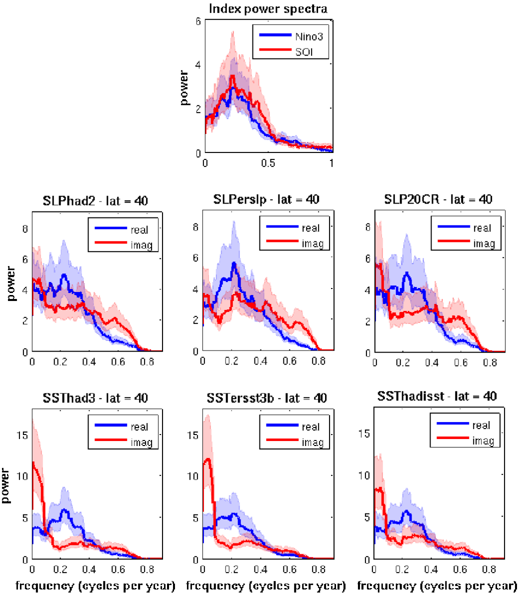
Peak at frequency 4-5 years, decay at low frequencies. Very similar for Nino3 and SOI.

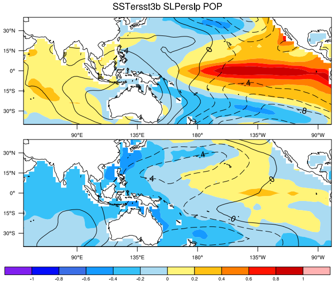
Center: SLP POP power spectra.

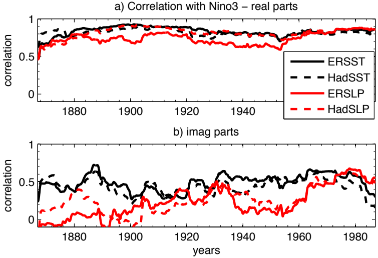
Peak spectrum (real part) is very similar to Nino3 or SOI spectrum. Precursor has less power at ENSO frequency but higher power at decadal frequencies.

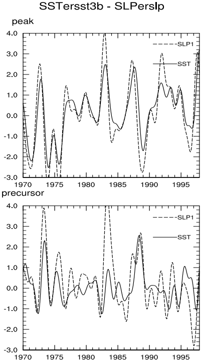
Bottom: SST POP power spectra.

Low frequency part of precursor spectrum has more power than the peak ENSO power.









Evolution equation

forced, damped linear oscillator

$$d\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \epsilon & -\eta \\ \eta & \epsilon \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} dt + \mathbf{F} dB_t$$

$$\mathbf{R} = \mathbf{F}\mathbf{F}^{\mathbf{T}} = c \begin{bmatrix} 1 & r\sqrt{\alpha} \\ r\sqrt{\alpha} & \alpha \end{bmatrix}.$$

Analytical spectrum of linear system

For a linear system: $dx = \mathbf{A}xdt + \mathbf{F}dB_t$ with forcing covariance $\mathbf{R} = \mathbf{F}\mathbf{F}^\mathbf{T}$ the spectral matrix is

$$\mathbf{S}(\omega) = (\mathbf{A} + i\omega\mathbf{I})^{-1}\mathbf{R}(\mathbf{A}^* - i\omega\mathbf{I})^{-1}$$

and the spectrum of a certain linear combination $v = [a, b]$ of basis vectors is

$$\begin{aligned} S_v(\omega) &= v\mathbf{S}(\omega)v^T \\ &= \frac{l(1+\alpha)(a^2+b^2)}{8\pi}\left[\mathbf{S}_+(\omega) + \mathbf{S}_-(\omega) + 2f_{12}\cos\left(\theta\left(\omega\right)+\gamma\right)\sqrt{\mathbf{S}_+(\omega)\mathbf{S}_-(\omega)}\right] \end{aligned}$$

with $\mathbf{S}_\pm(\omega) = \frac{1}{\epsilon^2 + (\omega \pm \eta)^2}$. [1]

[1] Kleeman, Richard, 2011: Spectral Analysis of Multidimensional Stochastic Geophysical Models with an Application to Decadal ENSO Variability. J. Atmos. Sci., 68, 1325.

Power spectra as constraint for model parameters

- Idealized linear stochastic models to capture essential dynamical properties.
- Main technique: estimate model parameter values by fitting theoretical to observed power spectra.
- Model: observed statistical properties, reproduces several key dynamical features.

Why power spectra?

- robust statistical quantities (given enough data)
- relate directly to (some) model parameters

Goals:

- Test the feasibility of estimating model parameters from power spectra.
- Investigate the physical relevance of the fitted parameter values.

Analytical spectrum of linear system

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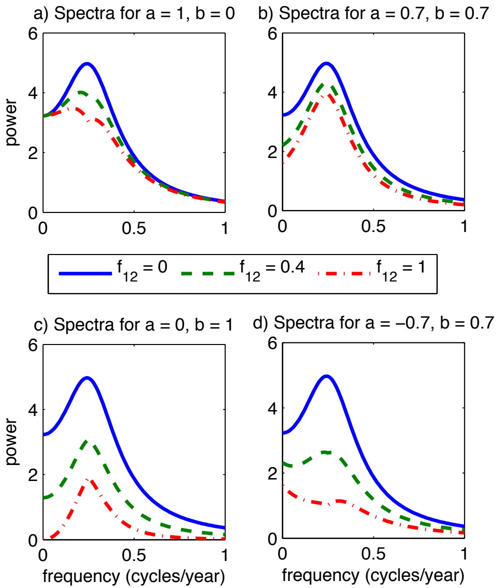
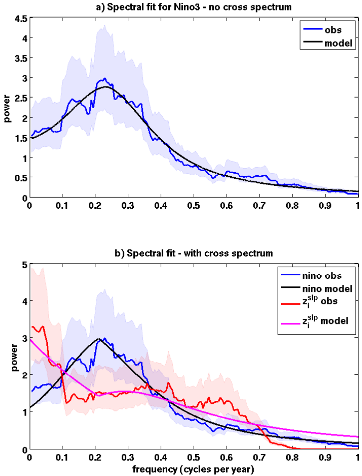
So what does this actually look like?

Analytical spectrum of linear system



Spectral fit to observations



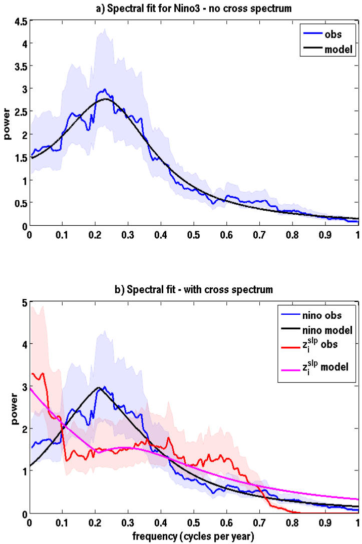
Spectral fit to observations

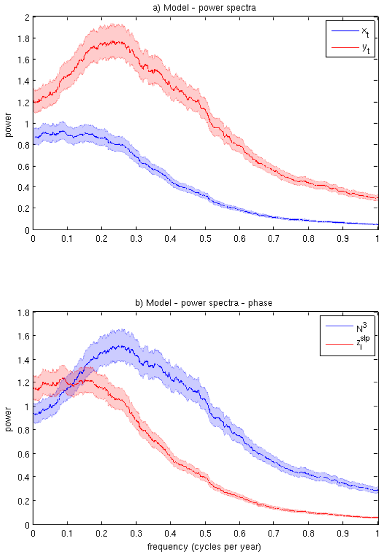
Fitted parameter values

Table: List of the parameter values and confidence intervals obtained from spectral fit for Nino3 and SOI mature phase data and z_i^{slp} and z_i^{sst} data for the precursor. Note that values for ϵ and η are given in months and years respectively. Confidence intervals are at the 95% level.

Par	Nino3 - SLPhad2	confidence interval	Nino3 - SLPerslp	confidence interval	Nino3 - SLP20CR	confidence interval
α	2.09	(1.88, 2.29)	1.5	(1.35, 1.65)	2.17	(1.87, 2.48)
c	0.45	(0.41, 0.49)	1.07	(0.76, 1.38)	0.60	(0.35, 0.86)
ϕ	3.14	(3.14, 3.14)	−0.55	(−0.63, −0.47)	−0.18	(−0.38, 0.02)
r	−0.63	(−0.7, −0.55)	−0.62	(−0.71, −0.53)	−0.72	(−0.84, −0.61)
ϵ	−7.99	(−8.24, −7.47)	−6.97	(−7.66, −6.40)	−7.00	(−7.55, −6.53)
η	4.41	(4.40, 4.63)	4.50	(4.28, 4.74)	4.77	(4.55, 5.02)
Par	SOI SLPhad2	confidence interval	SOI - SLPerslp	confidence interval	SOI - SLP20CR	confidence interval
α	2.05	(1.91, 2.19)	1.73	(1.59, 1.88)	2.24	(2.09, 2.40)
c	0.81	(0.58, 1.05)	1.65	(1.25, 2.06)	0.96	(0.65, 1.27)
ϕ	−0.29	(−0.47, −0.12)	−0.61	(−0.66, −0.57)	−0.34	(−0.49, −0.20)
r	0.57	(0.53, 0.62)	−0.72	(−0.78, −0.67)	0.71	(0.67, 0.76)
ϵ	−7.47	(−7.96, −7.03)	−6.47	(−6.98, −6.04)	−6.69	(−7.17, −6.27)
η	4.34	(4.17, 4.52)	4.48	(4.30, 4.67)	4.63	(4.43, 4.85)
Par	Nino3 SSThad3	confidence interval	Nino3 - SSTersst3b	confidence interval	Nino3 - SSThadisst	confidence interval
α	2.76	(2.46, 3.06)	3.03	(2.61, 3.45)	2.72	(2.49, 2.94)
c	0.71	(0.43, 0.99)	0.63	(0.34, 0.93)	0.47	(0.34, 0.59)
ϕ	−0.20	(−0.30, −0.11)	−0.23	(−0.33, −0.13)	−0.08	(−0.16, −0.01)
r	−1.0	(−1.0, −1.0)	−1.0	(−1.0, −1.0)	−1.0	(−1.0, −1.0)
ϵ	−6.61	(−7.60, −6.01)	−6.92	(−7.97, −6.04)	−7.07	(−7.83, −6.44)
η	5.59	(5.07, 5.83)	6.03	(5.55, 6.61)	5.37	(5.07, 5.70)

Spectral fit to observations





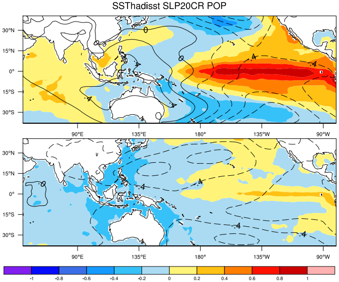
Relation to ENSO phases

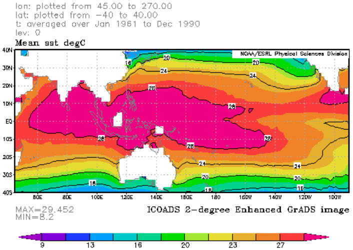
Stronger forcing onto mature phase of ENSO than precursor phase:

Covariance matrix of modeled mature and precursor phase (Nino3 - SLP20CR):

$$\mathbf{R} = 1.51 \begin{bmatrix} 1 & -0.31 \\ -0.31 & 0.26 \end{bmatrix}$$

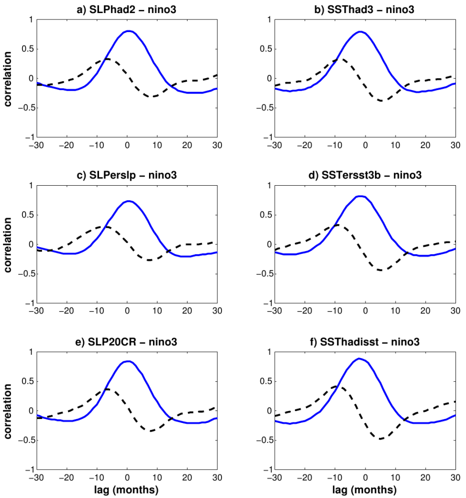
Weaker forcing of the precursor phase.



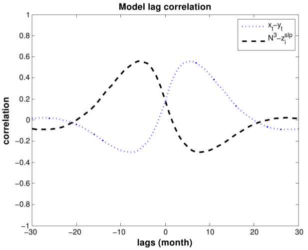


Relation to ENSO phases

Lag correlation: data



Lag correlation: model



- Model phases show similar lag behavior and correlation strength as observed ENSO phases.

Summary

- Simple model is able to reproduce several characteristics of ENSO: irregularity, oscillation, low-frequency component.
- Spectra give robust estimate of decay time: $\approx$ 8 months.
- Connection between ENSO precursor and Pacific decadal variability pattern - spectral weight of precursor and peak phase changes for low frequencies.
- Quantify difference in forcing and correlation between the precursor and peak ENSO phase.
- To reproduce the decadal spectral peak in the theoretical/fitted spectra we need: 1) unequal stochastic forcing, 2) non-zero correlation between the phase forcings.

- Other important features of ENSO are missing in this simple model setup: effects of non-linearity, non-normality, phase locking to the annual cycle, and skewness.
- Assume a stationary spectrum exists and therefore stationary statistics of ENSO.

Future/Current work

- Identify possible physical mechanisms behind the enhancement of the decadal peak of the precursor.
- Which kind of stochastic forcing is most likely to project onto this mode?
- The forcing statistics change with latitudinal extent; what does that mean for possible candidates for the stochastic forcing (e.g. MJO, higher latidude noise, wind stress)?
- Can we identify similar POP modes in GCMs, and do the forcing statistics derived from those differ from observations? What happens to these modes and their forcing statistics in climate change scenarios?

Thank you

Questions?