On the relationship between PI and CAPE

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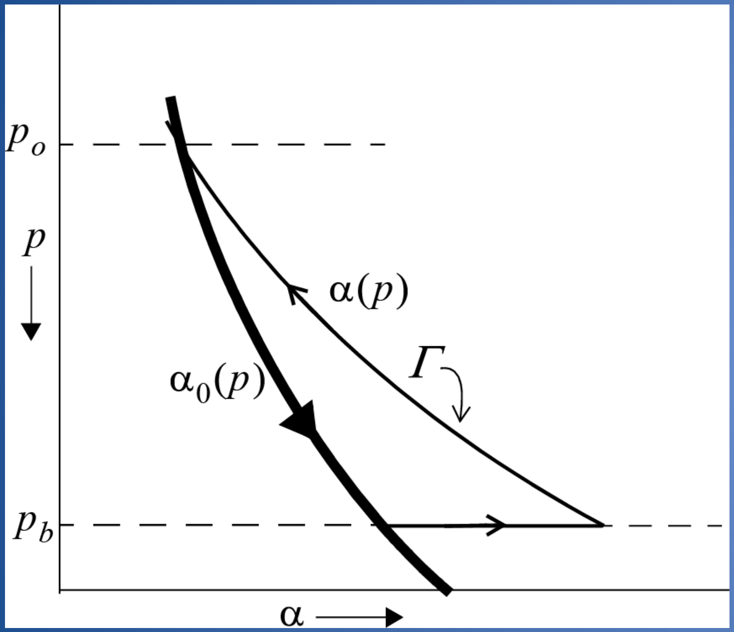
1. Approximations in the CAPE algorithm*
2. Approximations in the closed formulas
3. Updates to the closed formulas

*Bister and Emanuel *JGR* 2002

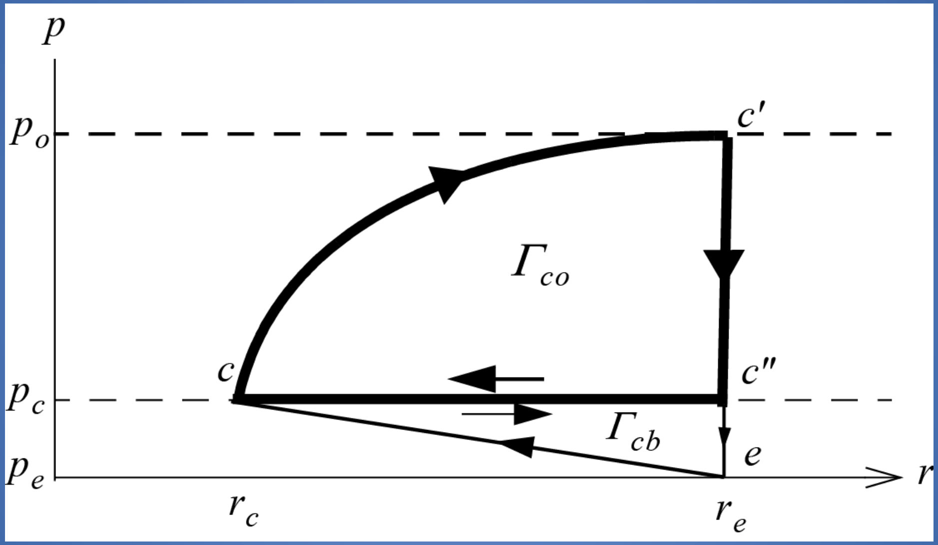
CAPE defined as a contour integral



$$CAPE = \int_{p_o}^{p_b} (\alpha - \alpha_0) dp = -\oint_{\Gamma} \alpha dp$$

“Hurricane CAPE” may have $\alpha \neq \alpha_0$ at bottom

Vertical cross-section in pressure coordinates

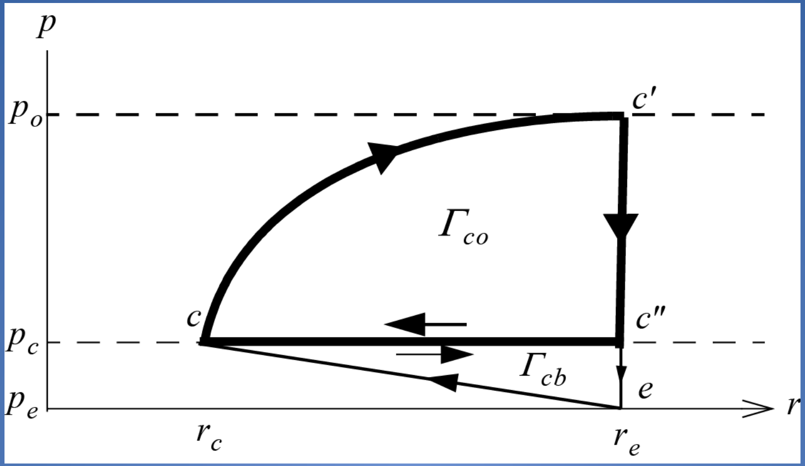


Dynamical constraint:

$$-\alpha dp = d(v^2/2 + \Phi)$$

Γ

$$-\oint_{\Gamma} \alpha dp = -\frac{1}{2}(v^2 - v'^2) + R\overline{T_{vb}}\log(\frac{p_e}{p})$$



Γ_o

$$-\oint_{\Gamma_o} \alpha dp = CAPE$$

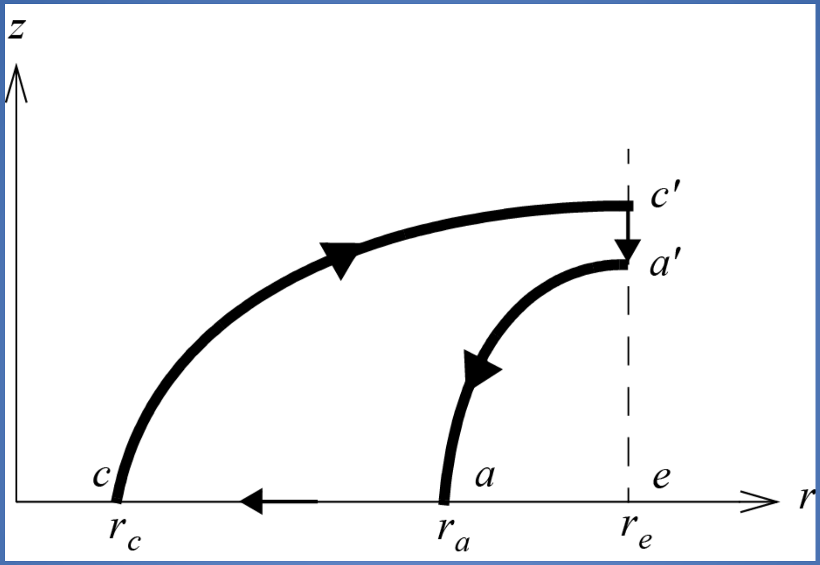
Γ_b

$$-\oint_{\Gamma_b} \alpha dp = R(\overline{T_{vb}} - \overline{T_{vo}})\log(\frac{p_e}{p})$$

$$R\overline{T_{v0}}\log\left(\frac{p_e}{p}\right) = CAPE + \frac{1}{2}(v^2 - v'^2)$$

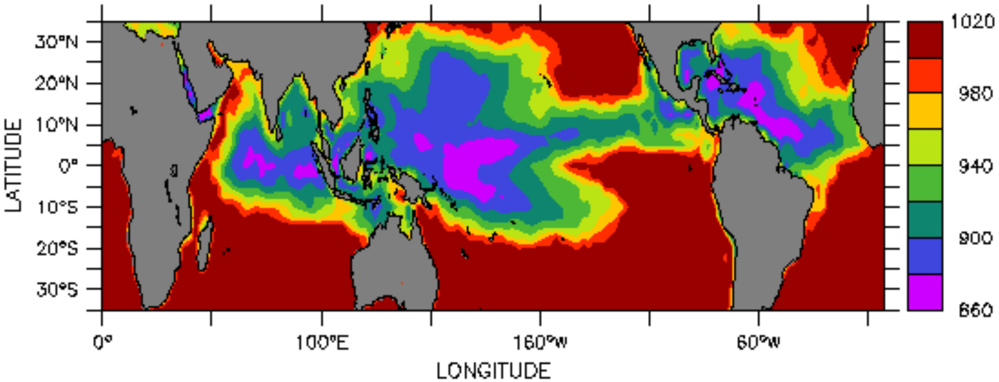
Holland 97
Bister-Emanuel 02

If SLP is known at a second overturning radius (a)

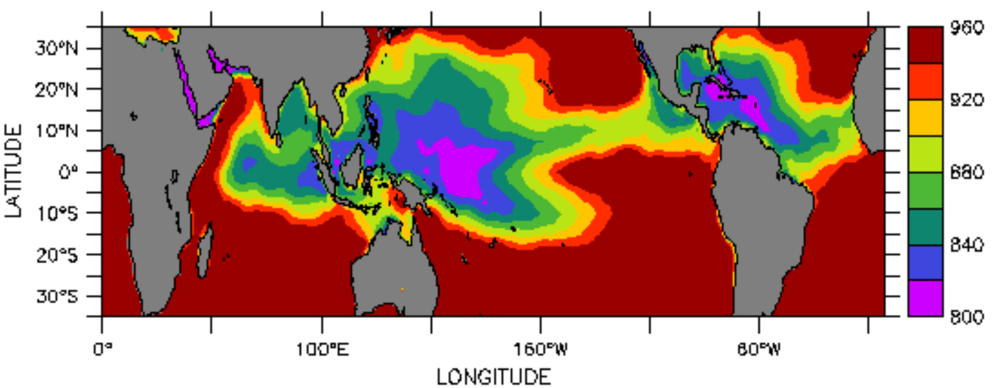


$$R\overline{T_{v0}}\log\left(\frac{p_a}{p_c}\right) = CAPE_c - CAPE_a + \frac{1}{2}\left({v_c}^2 - {v_a}^2\right)$$

Emanuel 86,95
Emanuel code



IEW pressure PI (hPa, Emanuel−pseudo)



IEW pressure PI (hPa, Holland−pseudo)

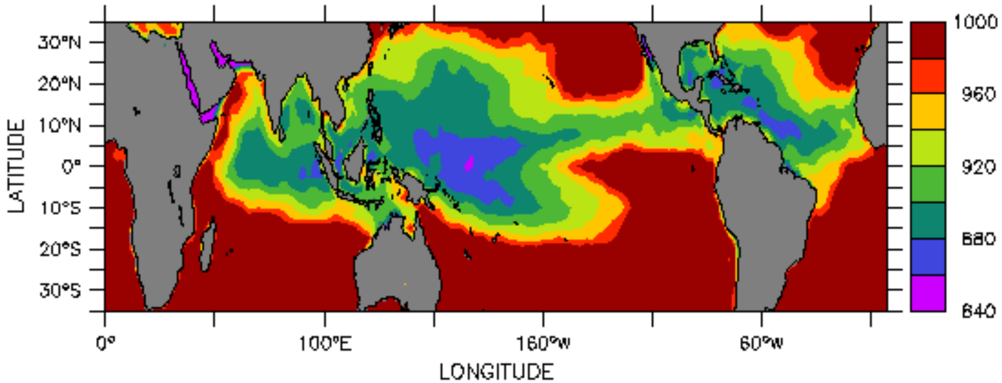
$$R\overline{T_{v0}}\log\left(\frac{p_a}{p_c}\right) = CAPE_c - CAPE_a$$

Take $p_a = p_e$; solve for p_c

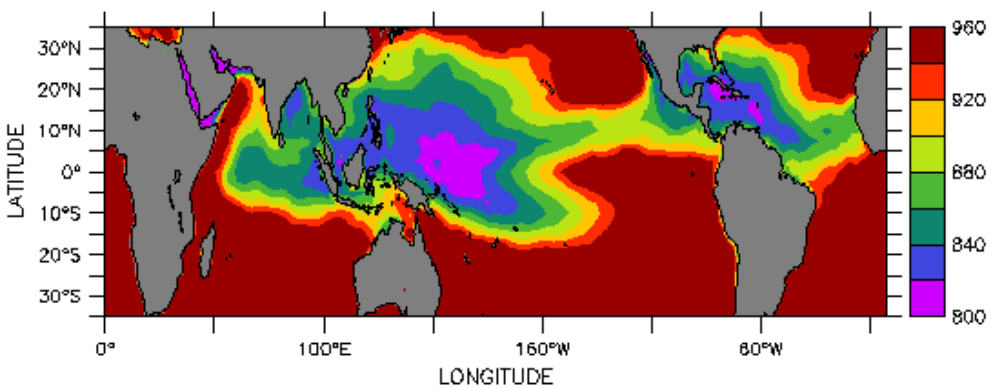
Pressure PI with $\boldsymbol{v^2 = 0}$

$$R\overline{T_{v0}}\log\left(\frac{p_e}{p}\right) = CAPE$$

Solve for p



OEW pressure PI (hPa, E95−pseudo)



OEW pressure PI (hPa, BE02−pseudo)

$p_a = p_e$ (code)

Pressure PI with $\boldsymbol{v^2 \neq 0}$

$p_a < p_e$ (BE 2002)

Definitions of PI

	IEW $(v^2 = 0)$	OEW $(v^2 > 0)$
$p_a = p_e$	E86, E88	E95, code
$p_a < p_e$	H97	BE02

Velocity closure

$$-\alpha dp = Tds - dk$$

$$-\frac{1}{2}\left(v^2 - v'^2\right) + R\overline{T_{vb}}\log\left(\frac{p_e}{p}\right) = \left(\tilde{T}_b - \tilde{T}_0\right)(s - s_e)$$

Differentiate once wrt inflow radius:

$$-\left(\frac{1}{r^2} - \frac{1}{{r_e}^2}\right)M\frac{\partial M}{\partial r} = (T_b - T_o)\frac{\partial s}{\partial r}$$

Then use aerodynamic law:

$$v^2 = \frac{c_k}{c_d}(T_b - T_o)(s^* - s)$$

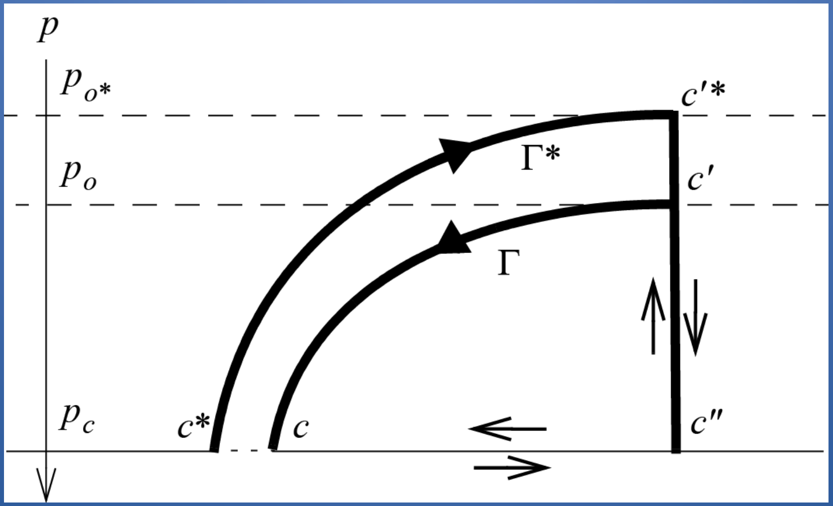
PI algorithm for velocity closure

$$CAPE = -\oint_{\Gamma_o} \alpha dp = (\tilde{T}_B - \tilde{T}_0)([s - s_0(p')]$$

$$\tilde{T}_B = [k - k_0(p')]/[s - s_0(p')]$$

$$CAPE^* = -\oint_{\Gamma_{o*}} \alpha dp = (\tilde{T}_B - \tilde{T}_0)([s^* - s_0(p')]$$

$$\tilde{T}_B = [k^* - k_0(p')]/[s^* - s_0(p')]$$



$$CAPE^* - CAPE = (\tilde{T}_B - \tilde{T}_o)(s^* - s)$$

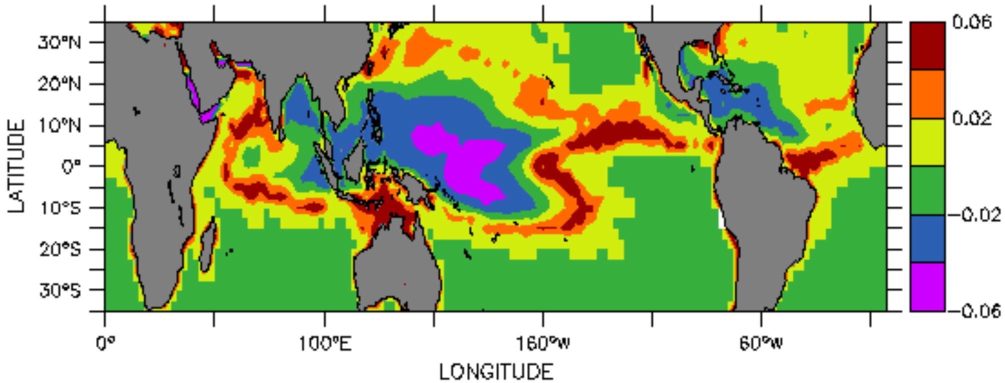
$$\tilde{T}_B = (k^* - k)/(s^* - s)$$

$$\tilde{T}_o = \int_s^{s^*} T_0 ds\,/(s^* - s)$$

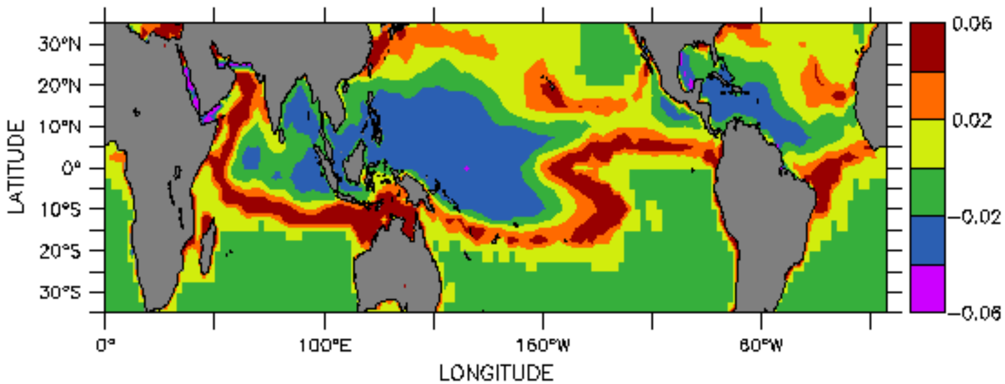
Compare:

$$v^2 = \frac{c_k}{c_d}(T_b - T_o)(s^* - s)$$

$$\Longrightarrow \boxed{v^2 \approx \frac{c_k}{c_d}(CAPE^* - CAPE)}$$ Bister-Emanuel 02



relative error due to averaging (revers)



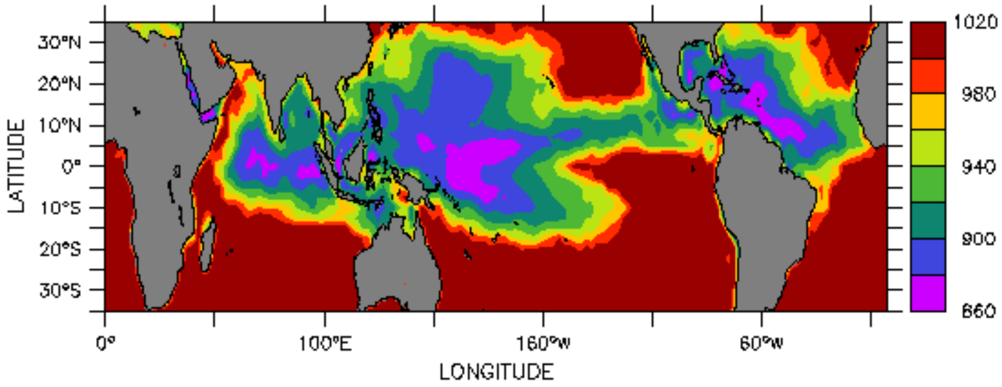
relative error due to averaging (pseudo)

Error from *T* averaging (%)

$\tilde{T}_B - \tilde{T}_o$ vs $T_b - T_o$

2. Approximations in the thermodynamics: Fully moist vs. "moist-like"

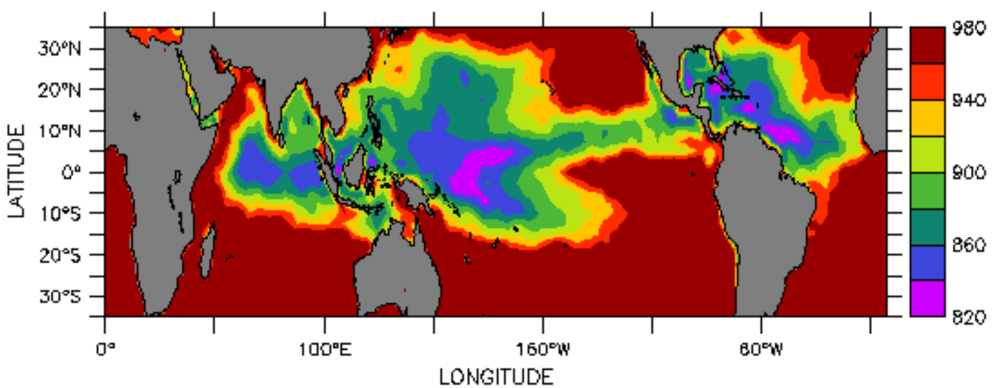
"Moist-like" neglects condensate heat capacity, loading



IEW pressure PI (hPa, pseudo)

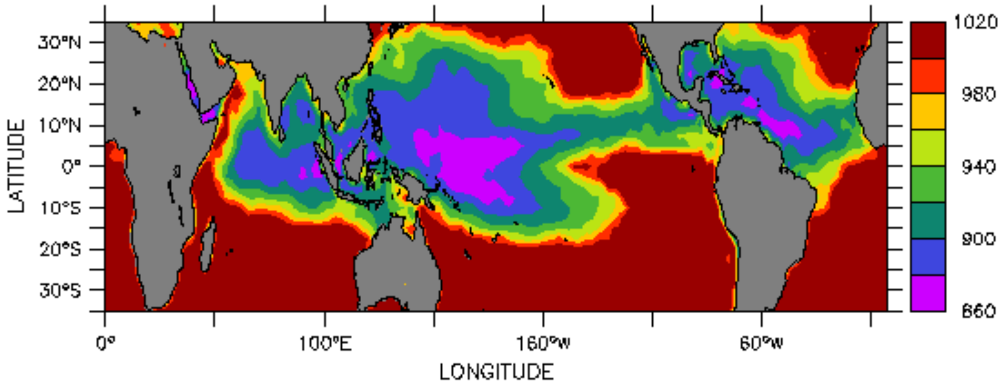
Full moisture (code)

Pressure PI with $\boldsymbol{v^2 = 0}$

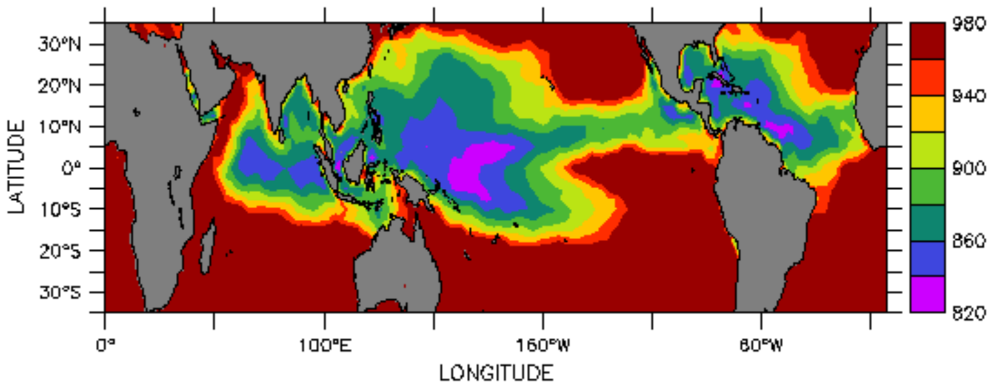


IEW pressure PI (hPa, dry−pseudo)

Moist-like



OEW pressure PI (hPa, pseudo)

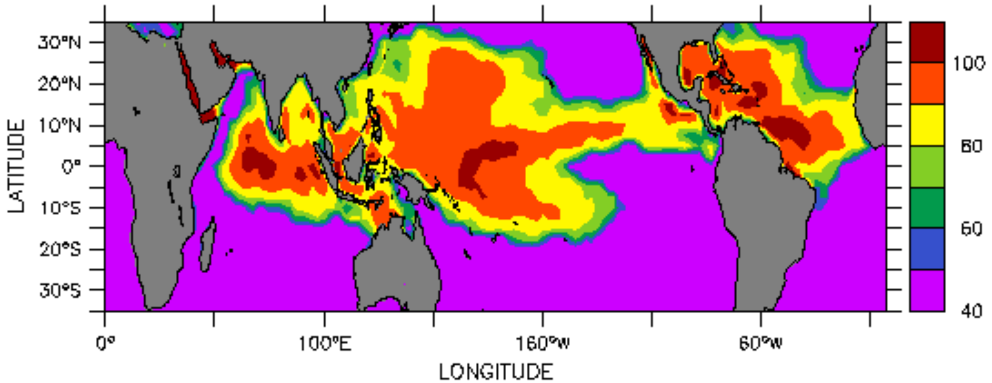


OEW pressure PI (hPa, dry−pseudo)

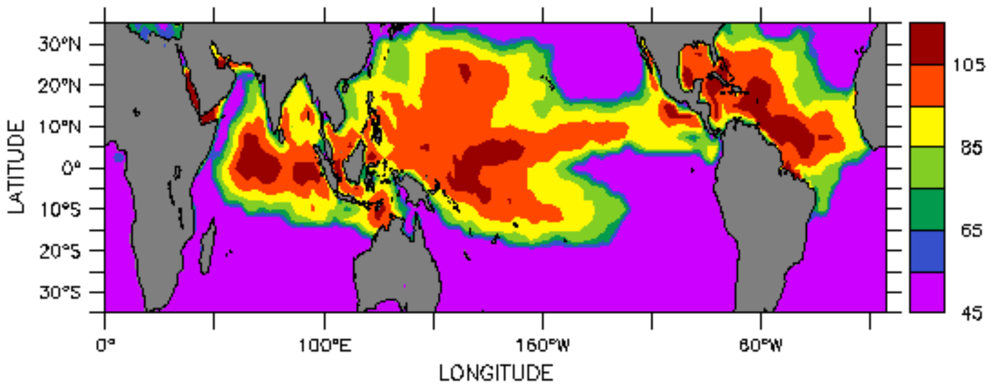
Full moisture (code)

Pressure PI with $\boldsymbol{v}^2 \neq \mathbf{0}$

Moist-like



velocity PI (m/s, pseudo)



velocity PI (m/s, dry−pseudo)

Full moisture (code)

Velocity PI

Moist-like

3. Closed formulas for full moisture

Thermodynamics:

$$\left(c_p + wc_{pv} + lc_l\right)dT - (1+Q)\alpha dp = -Ldw$$ 1st law

$$-\alpha dp = Tds - dk + Q\alpha dp + c_l T dQ$$ 1st & 2nd laws

Dynamics:

$$-\alpha dp = d(v^2/2 + \Phi)$$

$$k = \left(c_p + c_l Q\right)T + Lw$$

$$Q = w + l$$

$$\Phi = gz$$

Irreversible case (full moisture)

Define:

$$\int_0^{\Phi_o} Q d\Phi = w_b \boldsymbol{\Phi_q} \qquad \int_{T_b}^{T_o} Q dT = w_b(\boldsymbol{T_q} - T_b) \qquad \int_{0T_b}^{T_o} Q dlogT = w_b(\boldsymbol{\Lambda_q} - \Lambda_b)$$

$$\overline{(1 + w_b)RT_{vb}}\log\left(\frac{p_a}{p_c}\right) = D_{a,c} + (1 + \breve{w}_c)\frac{{v_c}^2}{2} - (1 + \breve{w}_a)\frac{{v_a}^2}{2}$$

$$D_{a,c} = (\tilde{T}_b - \tilde{T}_o)(s_c - s_a) + [(G_b - \boldsymbol{G_q})w_b]_a^c$$

$$G = \Phi + c_l(T - \tilde{T}_o \log T)$$

Velocity PI for irreversible full moisture case

$$v^2 = \frac{c_k}{c_d}(1+w)^{-1}[(T_b - T_o)(s * -s) + \left(G_b - \boldsymbol{G_q}\right)(w^* - w)]$$

Effective outflow geopotential Φ_q (pseudoadiabatic)

Environment

Eyewall

Saturated Eyewall