Accurate calculation of horizontal pressure forces on cells defined by steeply sloping coordinates

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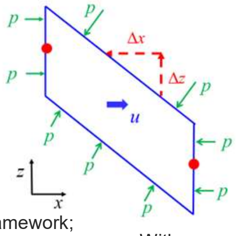
1. Summary

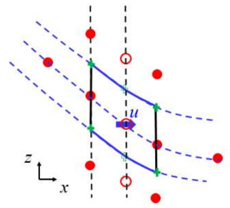
- We want NEMO to be able to use more generic vertical coordinates that do not represent the bathymetry as a series of steps
- We focus on two methods, **djc** and **ff**, that conserve momentum by integrating line integrals around the cell faces
- We use cubic interpolating polynomials. For very rough bathymetry the interpolation is constrained to avoid overshoots
- The two methods give quite similar results that are better than second order methods for the standard SEAMOUNT test-case
- The final velocities are almost the same whether cells are intialised as point values or grid-cell mean values

2. Motivation

- This is a step towards more generic vertical coordinates for NEMO
- With stepped bathymetry there are well known issues with flow over sills (Bruciaferri et al 2024), unphysical side-walls (Adcroft & Marshall 1998) and the representation of bottom torques (Styles et al 2021)
- Shchepetkin & McWilliams (2003) showed that higher-order schemes with limiters can perform well

3. Methods





- The net horizontal pressure force on a velocity cell can be calculated as the sum of the forces on the faces (ff) of the cell (figure on left - Lin 1997)
- This is a good "conservative" framework; action equals reaction; there is a good analogue of the bottom pressure torque
- The horizontal force on the upper face segment Δx is - pΔz
- The total horizontal force on the cell is

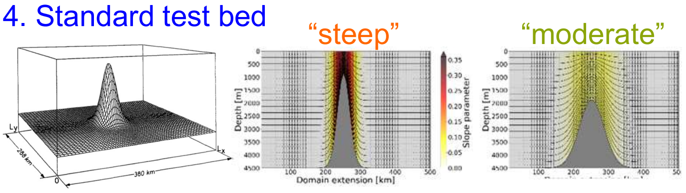
$$F_x = -\oint_C \left(p\frac{\partial z}{\partial s}\right) ds$$

- The density Jacobian (djc) method of Shchepetkin & McWilliams (2003) involves a similar line integral of the density

$$G_x = -\oint_C \left(\rho\frac{\partial z}{\partial s}\right) ds$$

- Second order methods to interpolate ρ, p and calculate the line integrals are not accurate enough. Need to use higher-order methods (figure on right)
- With very rough bathymetry it is necessary to constrain the interpolations to avoid overshoots
- The **ffq_ccs** and **djc** schemes use constrained cubic splines; **ffq_cub** uses a simple unconstrained cubic interpolation
- The density, rhd, in NEMO has the main effect of compressibility removed (Roquet et al 2015)
- The **prj** scheme interpolates the front & back pressures to the same height and differences them. Action ≠ reaction and there are problems near the bottom
- We have also tried a pre-conditioner that vertically interpolates then subtracts the density profile at the deepest point from all other points in the stencil. This scheme also does not guarantee action = re-action

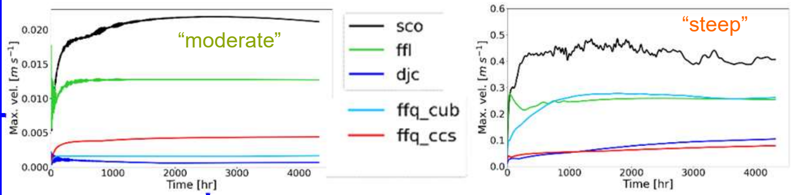
4. Standard test bed



- Classic HPG test case of Beckmann and Haidvogel (1993)
- Isolated 2D Gaussian seamount in an E-W periodic channel (above left)
- Ocean initialised at rest with exponentially decaying density $\rho(z)$
- Velocities should be identically zero
- Details follow the two configurations of Ezer et al. (2002) closely
- No thermal diffusivity, EOS80; Centred advection; A_M = 500 m²/s
- Seamounts have different steepness (figures above on centre & right)
- Maximum of slope parameter $r_{max} = \Delta H/2H$ is 0.36 for "steep" and 0.07 for "moderate" configurations

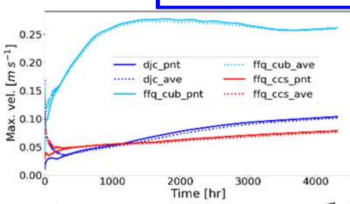
5. Main results

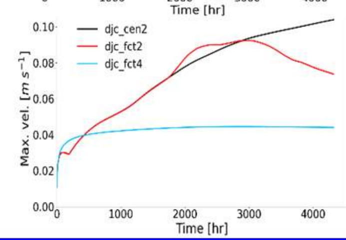
- Integrations for 180 days
- **sco** is the standard NEMO scheme
- **ffl** is forces on faces with simple linear interpolation (Lin 1997)
- **djc** and **ffq_cub** give the smallest maximum currents after spin-up for a "moderate" slope
- They are stable even though they do not conserve an analogue of energy
- For the "steep" case the **ffq_ccs** and **djc** schemes give the smallest maximum currents. The **ffq_cub** scheme is no better than **ffl**
- Even for the moderate configuration the prj scheme quickly generates much larger currents (not shown). The pre-conditioner gives small initial currents but large currents emerge (not shown)



6. Effects of cell mean initialisation and advection schemes

- Our **djc, ffq_ccs** and ffq_cub schemes treat the cell densities as point values. We could treat them as cell mean values, but this would roughen the density field and the limiters would need revision
- Marsaleix et al 2009 note that this issue affects schemes differently
- The upper figure to the right shows results for the steep configuration initialising cells using point values (pnt; solid lines) and cell average values (ave; dotted lines)
- The initial velocity errors are larger for ave (as expected) but the velocity errors after spin-up are very similar
- Mellor et al (1998) argue that after spin-up the velocity errors depend on the curl of the error in the horizontal pressure forces
- The lower figure to the right shows that the tracer scheme has a significant impact on the velocity errors after spin-up
- The fct schemes reduce the velocity errors; the fct4 scheme reduces them more than the fct2 scheme!





7. Questions for further work

- Why does cell mean initialisation have little impact?
- Can a competitive forces on faces scheme be derived that treats tracers as cell mean values (cf Engwirda et al 2017)?
- How good are results for hybrid (e.g., vanishing quasi-sigma or multi-envelope) coordinates?
- What smoothing of envelope bathymetries (with multi-enveloping or Brinkman penalisation methods) gives most reliable results?
- Can the subtraction of a reference profile be stabilised (e.g. by ensuring that action equals reaction – it seems not)?
- What are the computational costs? How can they be kept low?

A first version of these results was published as IMMERSE Deliverable 3.3: Accurate calculation of pressure forces Dec 2021. This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 821926

8. References

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