Detecting instantaneous tidal signals in ocean models utilizing streaming band-pass filters

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Introduction

Input of tidal energy into the world's oceans is about 3.5 TW.

- 70% is dissipated in the shallow water due to bottom friction.
- 30% is dissipated in the deep ocean due to internal wave drag.

Parameterization of internal wave drag is needed in global ocean models.

$$\frac{D\mathbf{u}}{Dt} + f\mathbf{k}\times\mathbf{u} = -g\nabla(\eta-\eta_{eq}-\eta_{SAL}) - \underbrace{\frac{C_d|\mathbf{u}|\mathbf{u}}{H}}_{\text{Bottom friction}} - \underbrace{\frac{\mathbb{C}\cdot\mathbf{u}}{H}}_{\text{Internal wave drag}}$$

- η_{eq} = Equilibrium tide
- η_{SAL} = Self-attraction and loading
- C_d = Bottom drag coefficient
- $\mathbb{C}$ = Internal wave drag tensor (units = m/s)

Internal wave drag is subject to the frequency-dependent scaling factor,

$$\sqrt{1-\frac{f^2}{\omega^2}},$$

which is difficult to implement in the time domain and is usually neglected.

Goal: To develop a computationally efficient algorithm for implementing the frequency-dependent internal wave drag in Modular Ocean Model version 6 (MOM6), in order to improve the realism of tides in MOM6 simulations.

Filter design

To apply frequency-dependent drag, we need to determine the instantaneous tidal velocities while the model is running. This can be done by constructing a streaming band-pass filter in the form of a second-order ODE,

$$\frac{d^2u_1}{dt^2} + \alpha\omega_1\left(\frac{du_1}{dt}-\frac{du}{dt}\right) + \omega_1^2 u_1 = 0, \quad \alpha > 0,$$

- u_1 is the output signal, representing the tidal velocity in the model,
- u is the input signal, which is the broadband model output,
- ω_1 is the resonant or target frequency of the filter,
- α is the damping coefficient, determining the bandwidth of the filter.

In Fourier space, u_1 and u are related through the filter transfer function,

$$\mathscr{A}(\omega;\alpha) = \frac{i\alpha\omega}{(1-\omega^2)+i\alpha\omega}, \quad \omega \geq 0, \quad \alpha > 0,$$

where ω is the frequency of the input signal, normalized by ω_1.

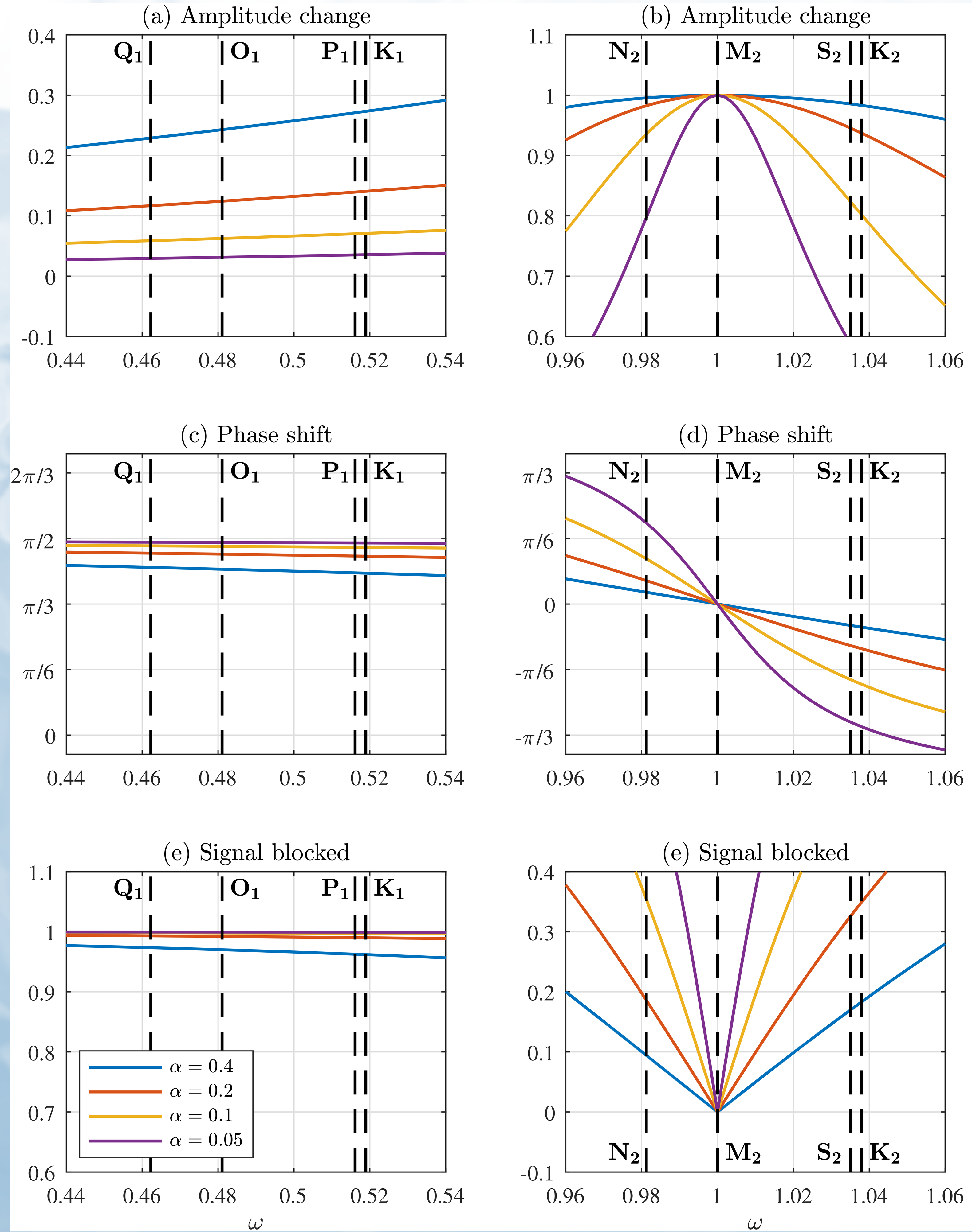

Figure 1: Effects of the M2 filter (target frequency: $\omega_1 = 1.4\times10^{-4}$ s-1) on the diurnal and semi-diurnal frequency bands as functions of ω for different α.

- The filter response is unity at the target frequency.
- The filter response vanishes in both low and high frequency limits.
- The filter's sharpness (rate of decay of the filter response) is determined by α.
- The filter response is symmetric about the target frequency in log-linear scale.

Numerical simulations

The algorithm was implemented in a 1/12° global barotropic model, in which tides were forced by the astronomical tidal potential at the M2 and K1 frequencies. The barotropic velocity field was filtered at the M2 and K1 frequencies, so that internal wave drag could be applied to the M2 and K1 velocities separately. This was done by replacing the wave drag term in the momentum equation by

$$\frac{\mathbb{C}\cdot\mathbf{u}}{H} = \frac{C_{\mathrm{JSL}}}{H}(\kappa_{M2}\mathbf{u}_{M2} + \kappa_{K1}\mathbf{u}_{K1}),$$

where C_{JSL} is the scalar drag coefficient obtained following Jayne & St. Laurent (2001), and κ_{M2} and κ_{K1} are the dimensionless scaling factors. Simulation results were compared against the TPXO9 data by calculating the global mean root mean squared errors (RMSE) of M2 and K1 water elevations for all depths and latitudes.

M2 scaling factor	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
M2 elevation (cm)	11.39	8.54	6.64	5.73	5.77	6.47	7.48	8.58	9.67
K1 elevation (cm)	3.42	3.39	3.36	3.35	3.34	3.33	3.33	3.33	3.32

Table 1: Global mean RMSE of M2 and K1 elevations with $\kappa_{K1} = 0$ and $0 \leq \kappa_{M2} \leq 2$.

K1 scaling factor	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
M2 elevation (cm)	11.39	11.42	11.45	11.49	11.52	11.56	11.60	11.64	11.68
K1 elevation (cm)	3.42	2.95	2.60	2.37	2.27	2.28	2.36	2.51	2.70

Table 2: Global mean RMSE of M2 and K1 elevations with $\kappa_{M2} = 0$ and $0 \leq \kappa_{K1} \leq 2$.

Tables 1 and 2 show that the performance of the filters was as expected during the simulations, since we were able to control the wave drag applied to the diurnal and semi-diurnal frequency bands separately by varying the respective scaling factors. The implementation of the filters and the frequency-dependent drag in MOM6 can be found at https://github.com/c2xu/MOM6/releases/tag/v1.0.0. The algorithm can also be used to de-tide model outputs. Given their ability to capture the temporal variation in tidal signals, the filters are particularly suitable at locations where tides exhibit strong seasonal variations. More detail of the filtering algorithm will soon be available in Xu & Zaron (2024).

References

Jayne, S. R., & St. Laurent, L. C. (2001). Parameterizing tidal dissipation over rough topography. *Geophysical Research Letters*, *28*, 811–814. https://doi.org/10.1029/2000GL012044

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