Multi-scale Ocean simulaTIONS based on mesh refinement strategies with local adaptation of dynamics and physics (MOTIONS)

F. Lemarié[1], F. Auclair[2], L. Bordois[3], N. Ducousso[3], A. Duran[4], Y. Morel[5], L. Roblou[2], and all members of the MOTIONS consortium

[1] Univ. Grenoble Alpes, Inria, CNRS, Grenoble INP, LJK, 38000 Grenoble, France [2] Laboratoire d'Aérologie, Université de Toulouse, CNRS, UPS, France
[3] Service Hydrographie et Océanographie de la Marine (SHOM) [4] Université Claude Bernard Lyon 1, Institut Camille Jordan, France [5] LEGOS, Université de Toulouse, CNRS, IRD, CNES, France

Context and Motivations

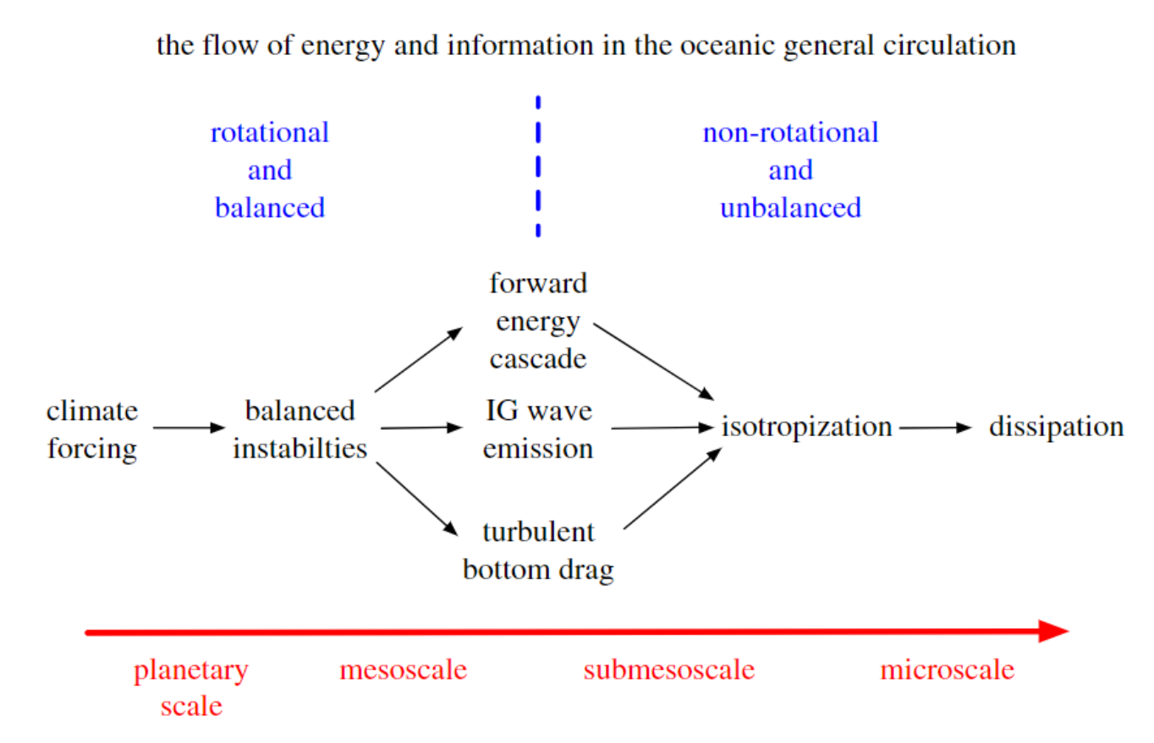
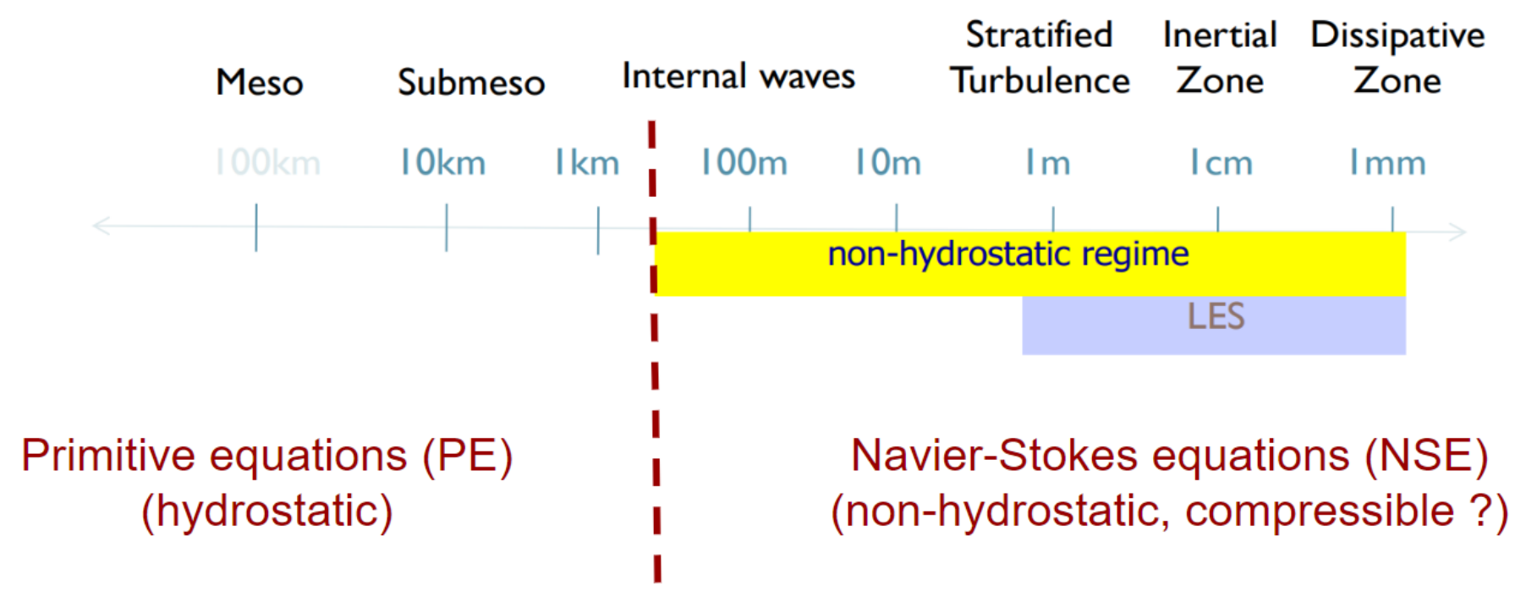


Figure 1: Dynamical route to dissipation (figure from McWilliams, 2016)



Focus of the project (funded from Feb. 2024 to Jan. 2028)

▷ **Transition from hydrostatic to NH dynamics** (occurs around a horizontal grid size of tens of meters)

▷ Representing both **downscaling** and **upscaling across the submesoscale currents (SMCs)** (feedback of NH processes on water-mass mixing and ocean circulation)

Range of scales of interest: submesoscale currents (SMCs)

$\mathrm{O}(0.1-10\ \mathrm{km}/0.01$ - $1\ \mathrm{km})$ in the horizontal/vertical; $\mathrm{O}(\mathrm{days}/\mathrm{hours})$ in time

"Large scales" : largest scales of the SMC range ≡ hydrostatically balanced dynamics
→ represented by most regional ocean forecasting systems

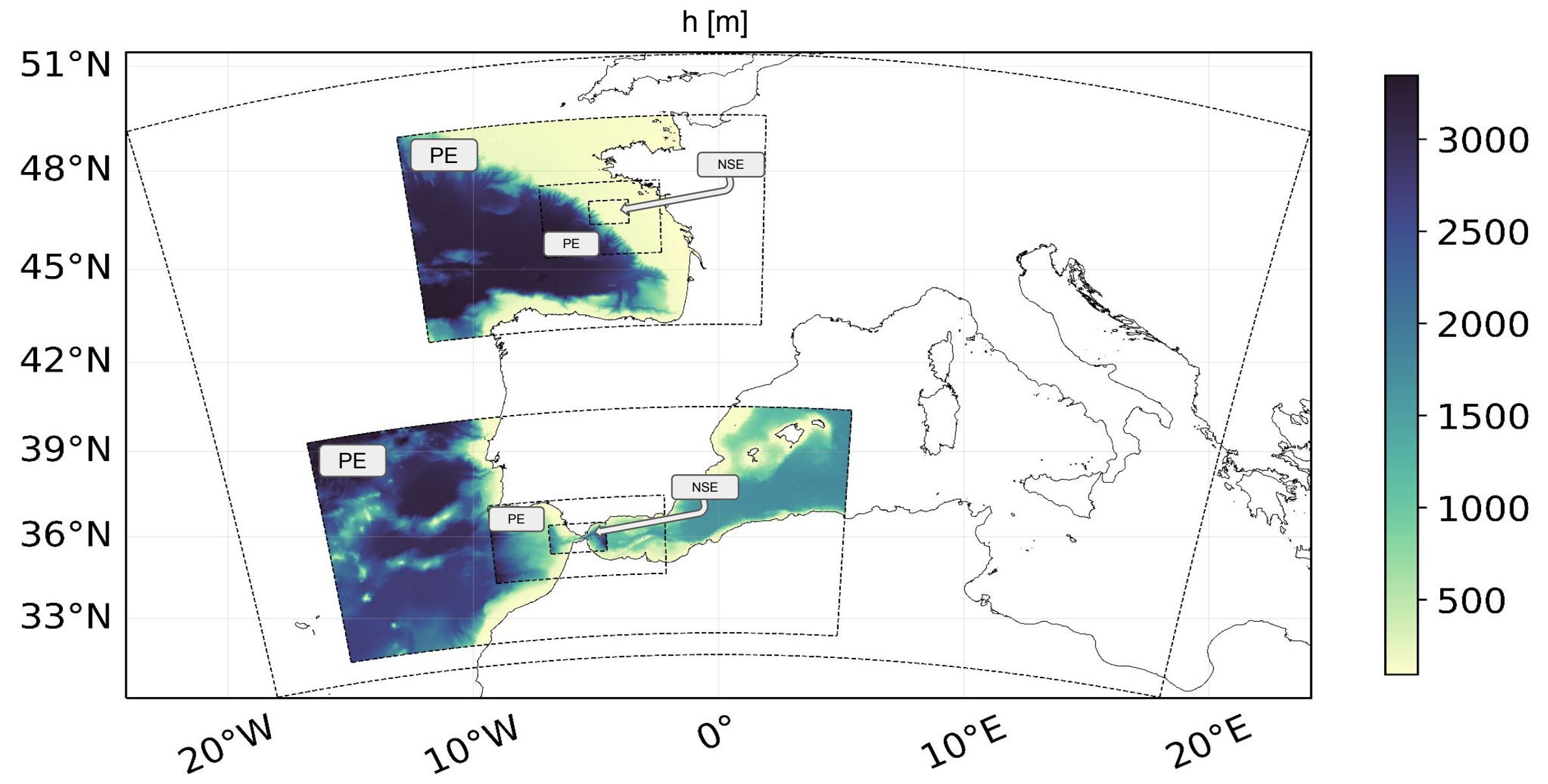
"Fine scales" : finest scales of the SMC range ≡ non-hydrostatic (NH) dynamics
→ not accessible for PE models

Objectives

I) Provide robust and efficient numerical algorithms allowing a **multiscale modeling strategy based on block-structured mesh refinement with local adaptation of model equations, numerics and physics** in selected areas of interest.

II) Assessment of developments through simulation of **fine-scale NH processes and their feedback to larger scales** within the Mediterranean / NE Atlantic dynamical continuum

III) Comparing simulations with **in-situ observations** at fine scales available from Ifremer & SHOM in the Bay of Biscay and Gibraltar strait (towed Moving Vessel Profiler (MVP) CTD casts, moorings, drifters, Argo floats, eOdyn, etc)



⇒ Acquire new numerical perspectives for fine-scale dynamics and its feedback on the large scale.

Existing building blocks in the CROCO modeling system

CROCO: free-surface, structured grid oceanic model jointly developed by CNRS, Ifremer, Inria, IRD, SHOM and UT3

→ Primitive Equation solver (expanded on the basis of ROMS-AGRIF/UCLA)

Finite diff./vol. approach : **C-grid** staggering in the horizontal and **Lorenz-grid** in the vertical

Time-stepping : Leap-Frog-Adams-Mouton predictor corrector with split-explicit treatment of nonlinear free-surface

Momentum advection (conservative form) Linear schemes of order 2 to 6, WENO5z, TVD

Tracer advection
Horizontal : Linear schemes of order 2 to 6, WENO5z
Vertical : WENO5z, Compact4 (a.k.a. splines reconstruction)

Vertical coordinate : Quasi-Eulerian (terrain-following)

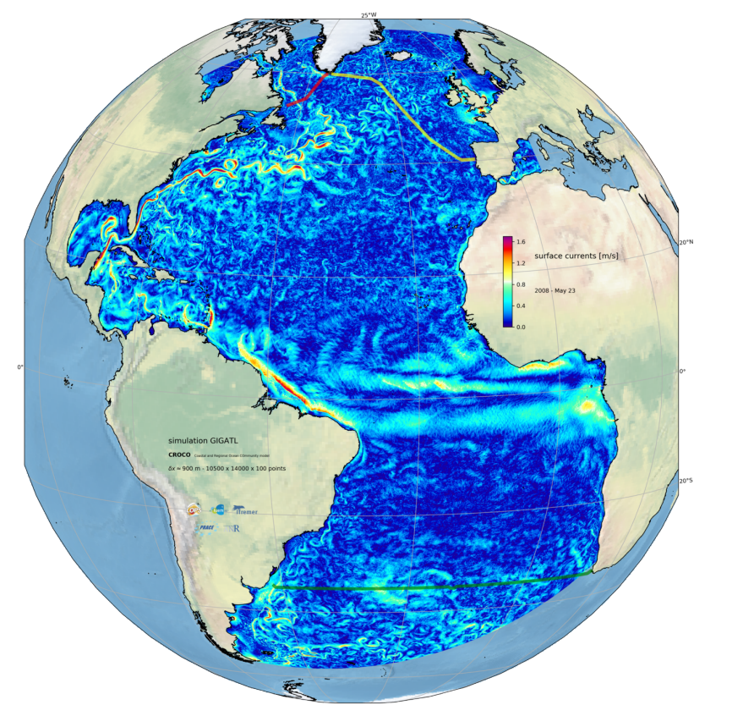


Figure 2: GIGATL Atlantic Ocean numerical simulations performed with CROCO (Gula et al., 2021)

→ Navier-Stokes Equation solver

Governing equations: compressible non-hydrostatic free-surface equations
→ retain fast acoustic waves and associated numerical stiffness

⇒ Design of a 2-mode, split-explicit time scheme for compressible NH ocean models (Auclair et al., 2024).

⇒ The fast *kernel* is 3d and no longer just barotropic.

⇒ Implemented as an overlay of the standard PE solver

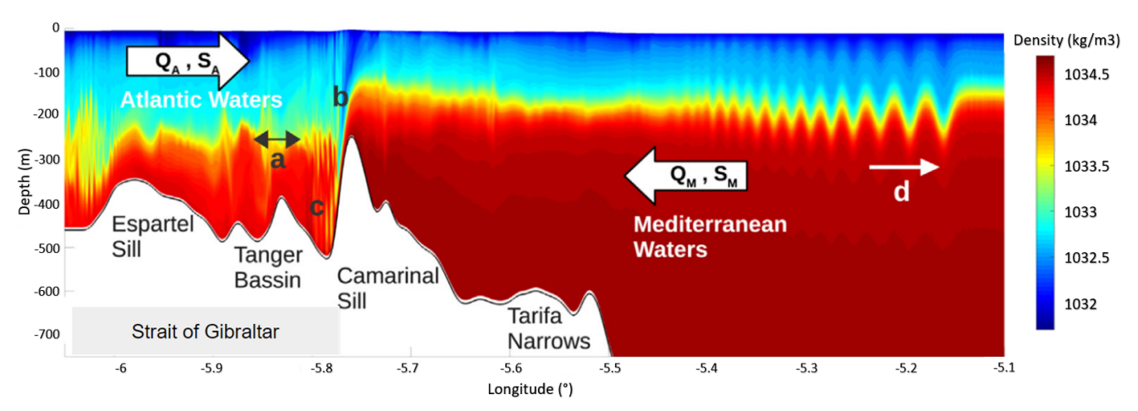
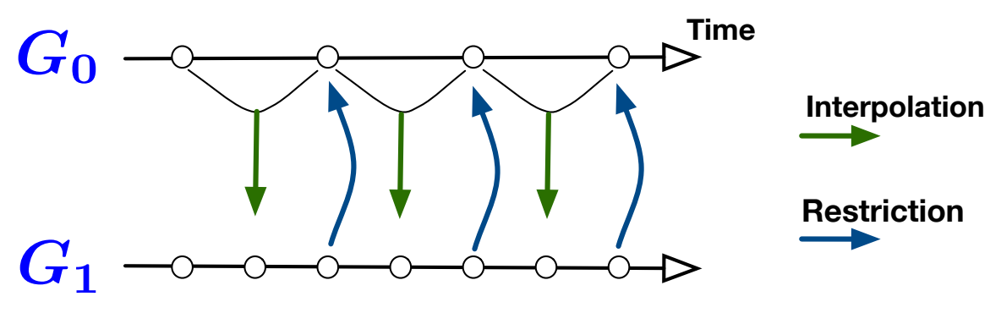
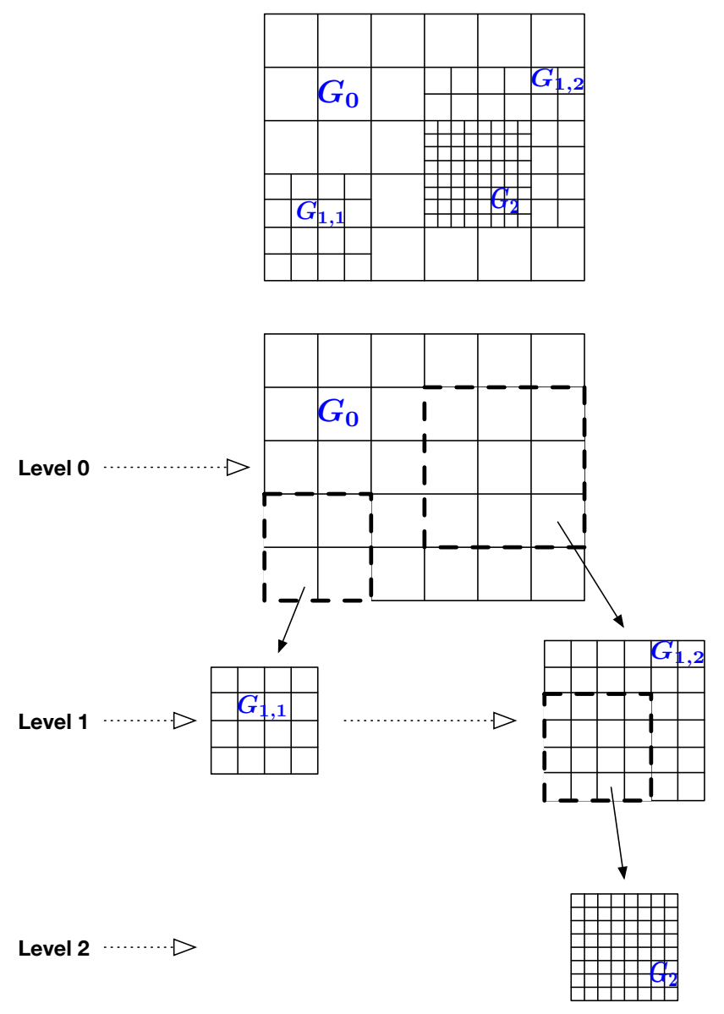


Figure 3: Illustration of fine-scale processes in the Strait of Gibraltar simulated using the CROCO NSE kernel in a 2DV configuration with a 50 m resolution. The visible processes include : (a) Small amplitude internal wave, (b) Hydraulic Jump, (c) KH instabilities. (d) Internal solitary waves. (Source : Hilt et al. (2020))

→ 2-way nesting capability

- Full two-way coupling [Debreu et al., 2012]
  → solution unaffected by nesting when the refinement coefficient is one.
- Local space and time refinement (unlimited number of grids)
- Fully conservative (volume and tracer via refluxing)
- Numerical schemes and SGS parameterizations can differ from one level of resolution to another
- Implemented via the AGRIF library `http://agrif.imag.fr/`





Features to be consolidated and/or developed for MOTIONS purposes

I) Nesting of a high-resolution NSE zoom in a low-resolution PE grid with 2-way interactions

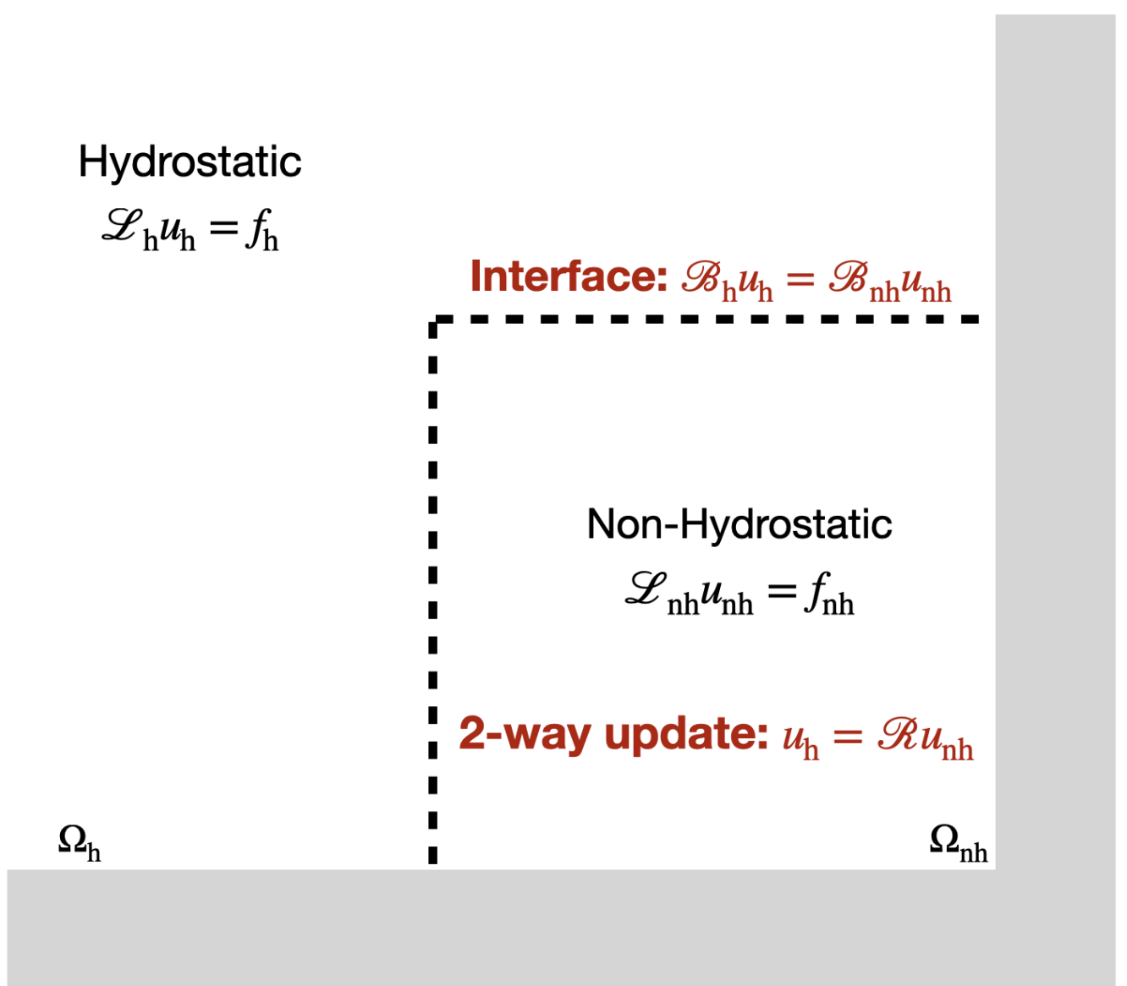
II) Efficient numerical treatment of the fast acoustic/gravity waves for the NSE kernel (e.g. by considering only hydrostatic surface gravity waves)

III) Deployment of large realistic simulations and associated HPC aspects (I/Os, pre/post processing, take advantage of the GPU version of PE/NSE solvers, ...)

IV) Subgrid-scale modeling for the O(10 m) resolution NSE zooms (Implicit vs explicit vs mixed explicit/implicit subgrid-scale-modeling)

Coupling hydrostatic and NH models

Problem : Find the interface operators $\mathscr{B}_{\mathrm{h}}$ and $\mathscr{B}_{\mathrm{nh}}$ (1-way) and the update operator $\mathscr{R}$ (2-way)



Objective : coupling methodology aiming at **matching vertical modes** combined with a **Perfectly Matched Layer (PML)** (Hu et al., 2008) (*P. Lozano's thesis work*)

Non-hydrostatic model

$$\begin{aligned} \rho_0\partial_t u &= -\partial_x p' \\ \rho_0\partial_t w &= -\partial_z p' - g\rho' \\ \partial_x u + \partial_z w &= 0 \\ \partial_t \rho' &= \frac{N^2}{g}\rho_0 w \end{aligned}$$

Boundary conditions

$$w(x, z=-H, t) = 0$$
$$\partial_x p'(x, z=0, t) = \rho_0 g w(x, z=0, t)$$

→ IW dispersion relation :

$$\Omega^2 = \frac{N^2 k_x^2}{k_x^2 + k_z^2} < N^2$$

Associated Sturm-Liouville eigenvalue problem

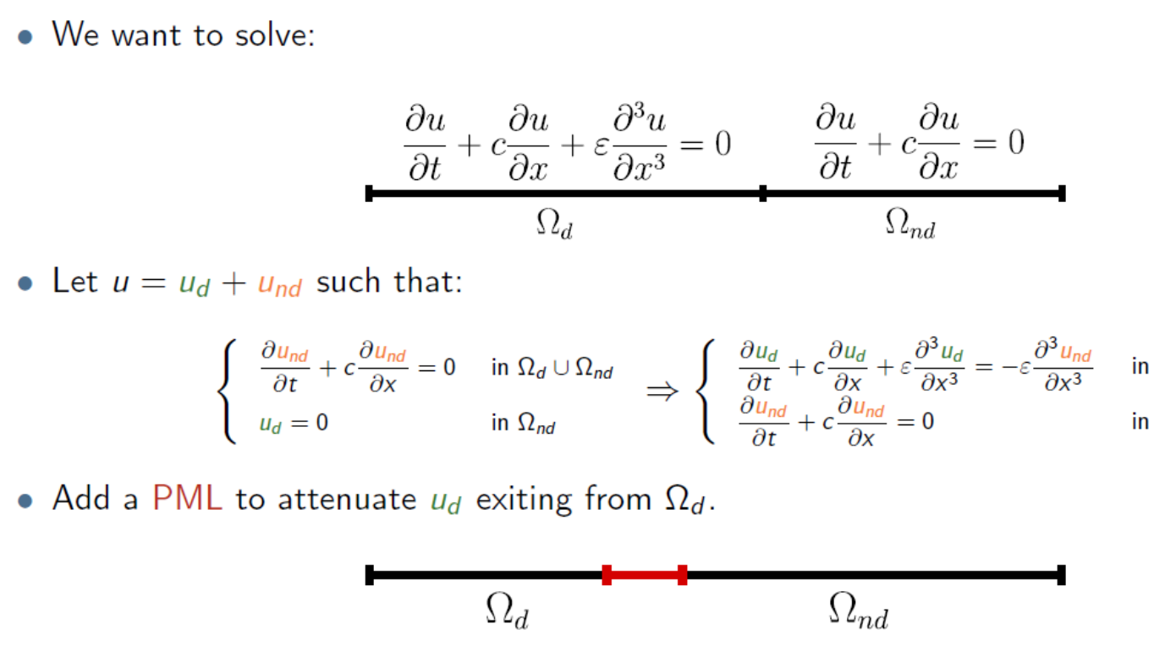
$$\begin{aligned} W''(z) + \lambda\left(N^2(z) - \Omega^2\right) W(z) &= 0 \\ W(-H) &= 0 \\ W'(0) - g\lambda W(0) &= 0 \end{aligned}$$

Expand the variables in the eigenfunctions to obtain a set of uncoupled systems

$$\partial_t u_q + \frac{g}{\lambda_q}\partial_x u_q = 0$$
$$\partial_t \rho_q + \frac{1}{g}\left(\frac{N^2 + \partial_t^2}{N^2 - \Omega^2}\right)\partial_x u_q = 0$$

→ achieve coupling by matching the coefficients of each mode

Case study : coupling KdV/transport



- We want to solve:

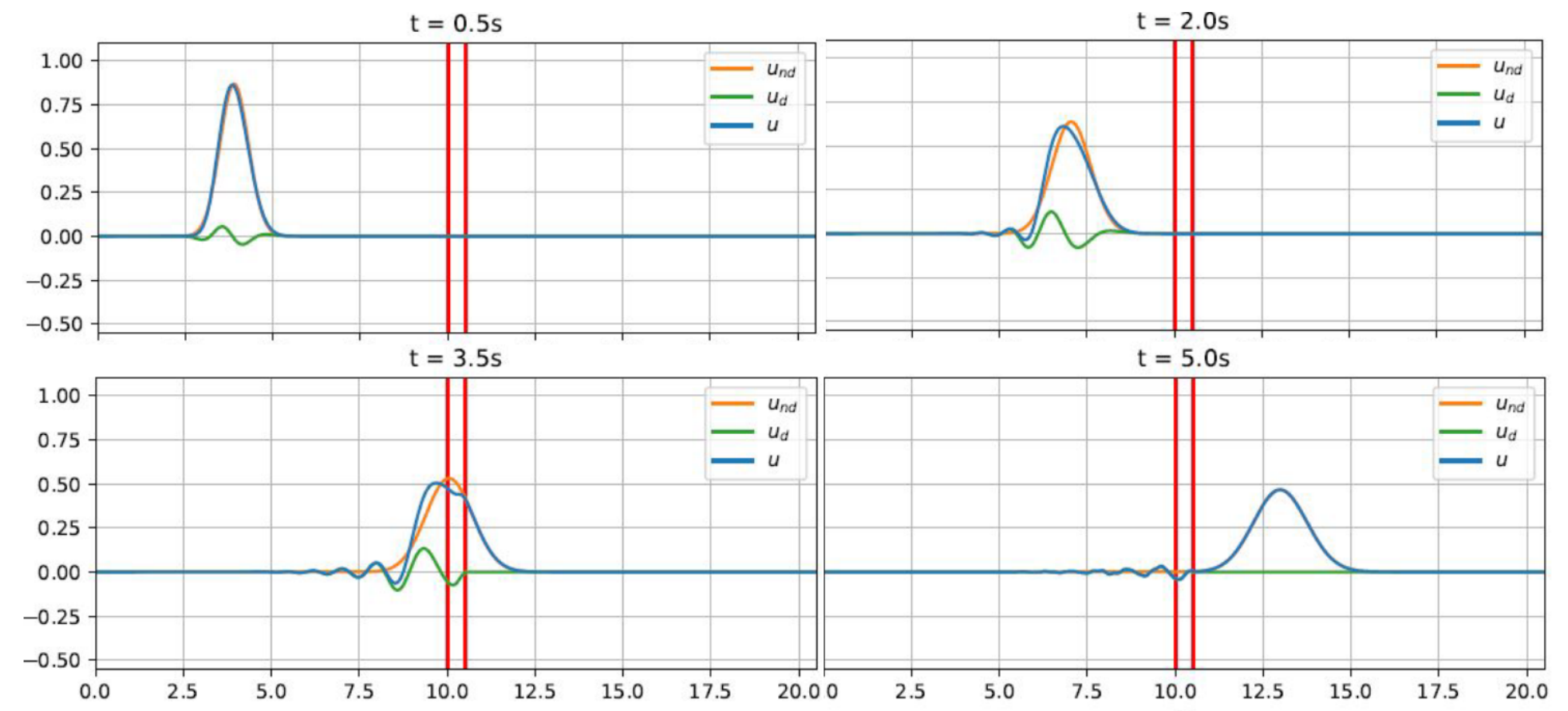
$$\underbrace{\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + \varepsilon\frac{\partial^3 u}{\partial x^3} = 0}_{\Omega_d} \qquad \underbrace{\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0}_{\Omega_{nd}}$$

- Let $u = u_d + u_{nd}$ such that:

$$\begin{cases} \frac{\partial u_{nd}}{\partial t} + c\frac{\partial u_{nd}}{\partial x} = 0 & \text{in } \Omega_d \cup \Omega_{nd} \\ u_d = 0 & \text{in } \Omega_{nd} \end{cases} \Rightarrow \begin{cases} \frac{\partial u_d}{\partial t} + c\frac{\partial u_d}{\partial x} + \varepsilon\frac{\partial^3 u_d}{\partial x^3} = -\varepsilon\frac{\partial^3 u_{nd}}{\partial x^3} & \text{in } \Omega_d \\ \frac{\partial u_{nd}}{\partial t} + c\frac{\partial u_{nd}}{\partial x} = 0 & \text{in } \Omega_{nd} \end{cases}$$

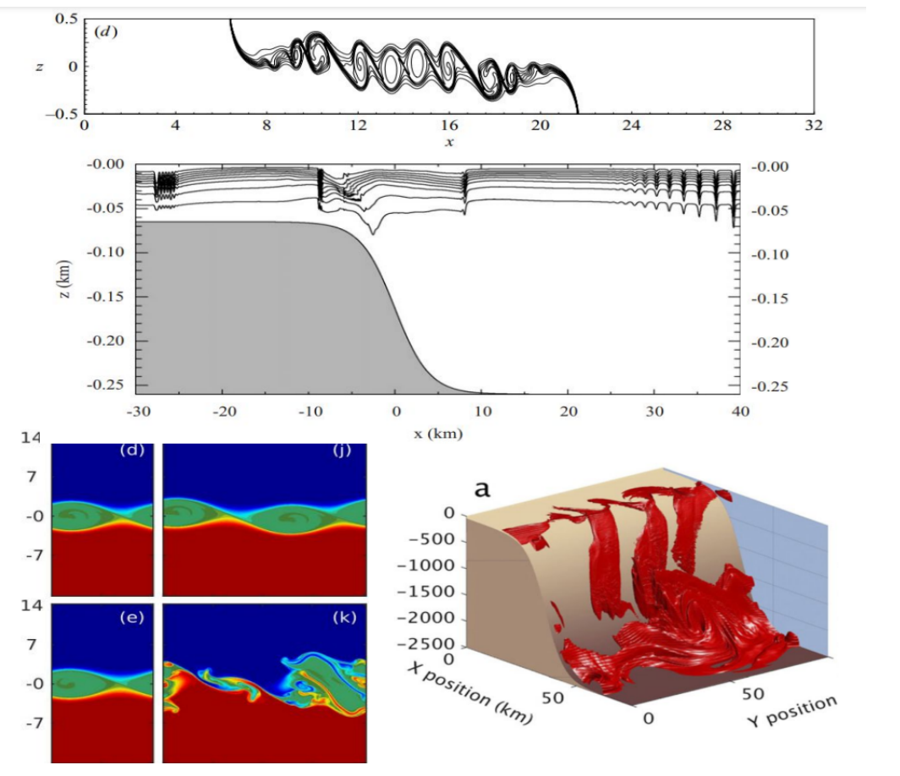
- Add a PML to attenuate u_d exiting from Ω_d.

Numerical experiment



Academic Benchmark

	Description	Reference
TC1	Linear acoustic-gravity waves	(Auclair et al. 2021; Harris & Durran 2010)
TC2	Internal tides	(Lamb 2007; Lamb & Kim, 2012)
TC3	Nonlinear internal solitary waves	(Carr et al. 2011; Dunphy et al. 2011)
TC4	Lock exchange (Bq vs Non-Bq)	(Birman et al. 2005)
TC5	Kelvin-Helmholtz instabilities	(Penney et al., 2020)
TC6	Dense shelf overflow	(Yankovsky & Legg 2019)



TC1, TC2, TC3 : propagative processes over long distances
TC4, TC5, TC6 : local transition to turbulence mechanisms

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