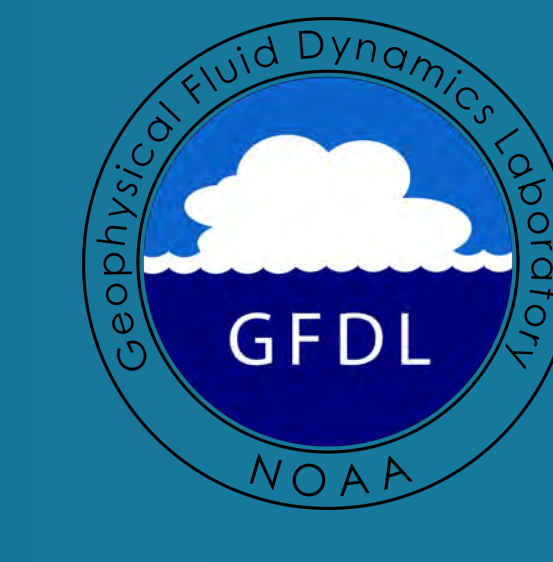
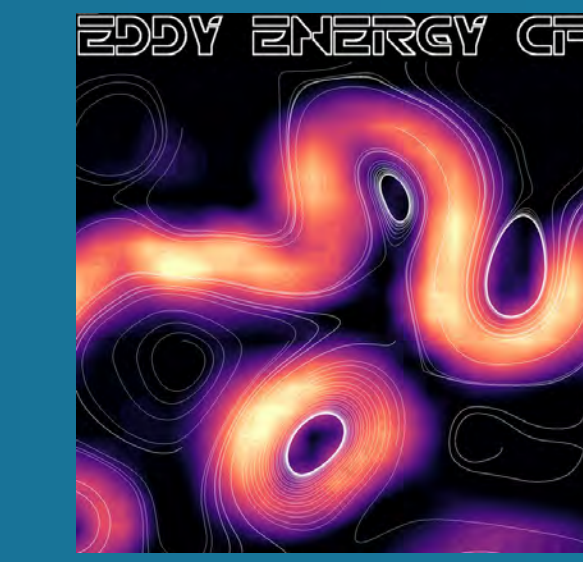


Sensitivity of eddy-permitting ocean simulations to the vertical structure of energy backscatter parameterization

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Motivation

- Eddy-permitting ocean models, with resolution $0.1^\circ - 0.5^\circ$, often exhibit weaker and more surface-intensified eddy kinetic energy than higher-resolution models and observations.
- The kinetic energy backscatter parameterization (e.g., Jansen and Held, 2014, Jansen et al., 2019) was introduced to enhance the energy of mesoscale eddies in eddy-permitting models.
- The vertical structure of backscatter remains unconstrained, which can significantly impact large-scale dynamics (Yankovsky et al., 2024)

Goal of This Study

- Examine the sensitivity of large-scale circulations to the vertical structure of eddy momentum forcing
- Provide guidance on constraining the vertical structure of backscatter parameterization

Approach

- An idealized configuration of **MOM6**, *Neverworld2*, is used to test the backscatter parameterization
- Setup**: Double-hemisphere domain; isopycnal coordinate with 15 layers; forced by zonally uniform zonal wind stress; no buoyancy forcing; adiabatic and hydrostatic

- Momentum equation in vector invariant form:

$$\partial_t \mathbf{u}_n + \frac{f + \zeta_n}{h_n} \hat{\mathbf{x}} \times (h_n \mathbf{u}_n) + \nabla K_n = -\nabla M_n + \mathbf{F}_n^v - \nabla \cdot [v_4 \nabla (\nabla^2 \mathbf{u}_n)] + \nabla \cdot (v_2 \nabla \mathbf{u}_n)$$

- A subgrid EKE (MEKE) equation:

$$\partial_t \text{MEKE} = \dot{e}_{\text{Bhvisc}} - \dot{e}_{\text{backscatter}} - \dot{e}_{\text{diss}} - \dot{e}_{\text{adv}}$$

← Biharmonic damping → Backscatter
 $(v_2 \text{ is negative})$

- Backscatter counteracts the biharmonic viscosity term by injecting the dissipated energy back to the model at larger scales

- The antiviocosity, v_2 , is formulated following Jansen et al. (2019),

$$v_2(x, y, z) = -c_0 \sqrt{2\text{MEKE}(x, y, z)} L_{\text{mix}}(x, y)$$

where c_0 is a constant, L_{mix} is the mixing length

- MEKE equation is solved for either a **2D** field or a **3D** field (Juricke et al., 2019). For the 2D case, v_2 is prescribed with a vertical mode structure

- Different vertical structure of backscatter is tested:

- 2D MEKE + barotropic (BT) mode
- 2D MEKE + equivalent barotropic (EBT) mode
- 2D MEKE + surface quasigeostrophic (SQG) mode
- 3D MEKE

- The formulation of SQG vertical structure is given by Zhang et al. (2024):

$$\Phi_{\text{SQG}}(z, \Delta) \approx e^{2k_g z_s}$$

where $z_s = \int_z^0 \frac{N}{|f|} dz$, and $k_g = \max(k_{Rh}, \frac{c}{\Delta})$,

Δ is the grid spacing, k_{Rh} is the inverse of Rhines scale, and c is a tuning parameter

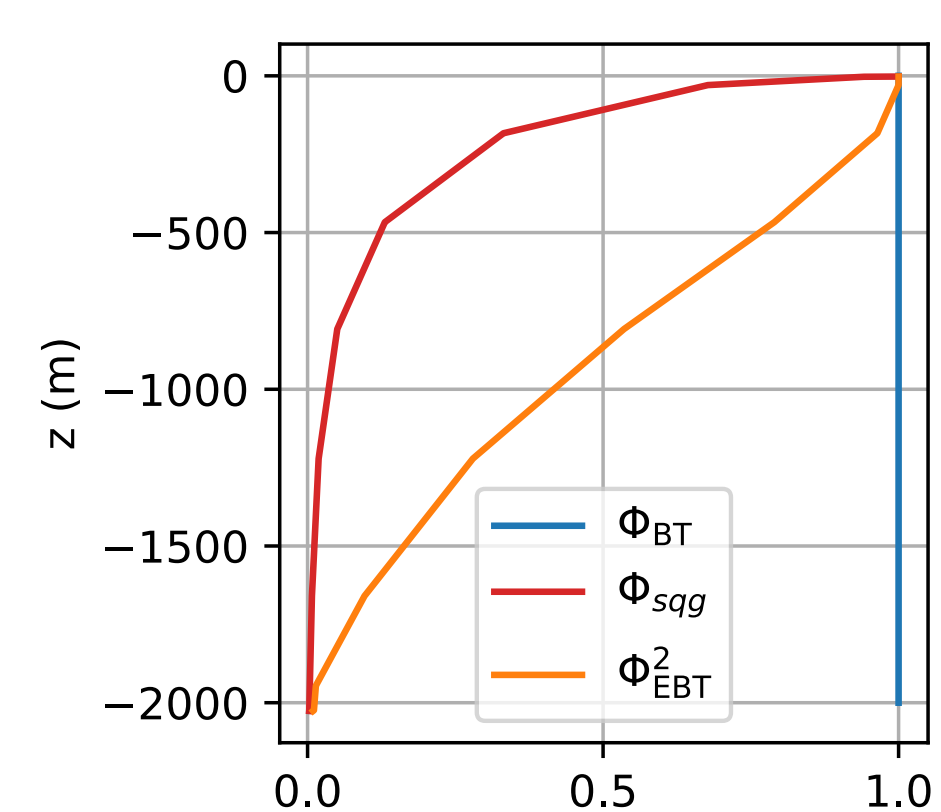


Figure 1. Vertical structure of the barotropic (blue), square of equivalent barotropic (orange), and SQG mode (red) at a location in the Southern Ocean of the model

Reference

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Response to the Backscatter Parameterization

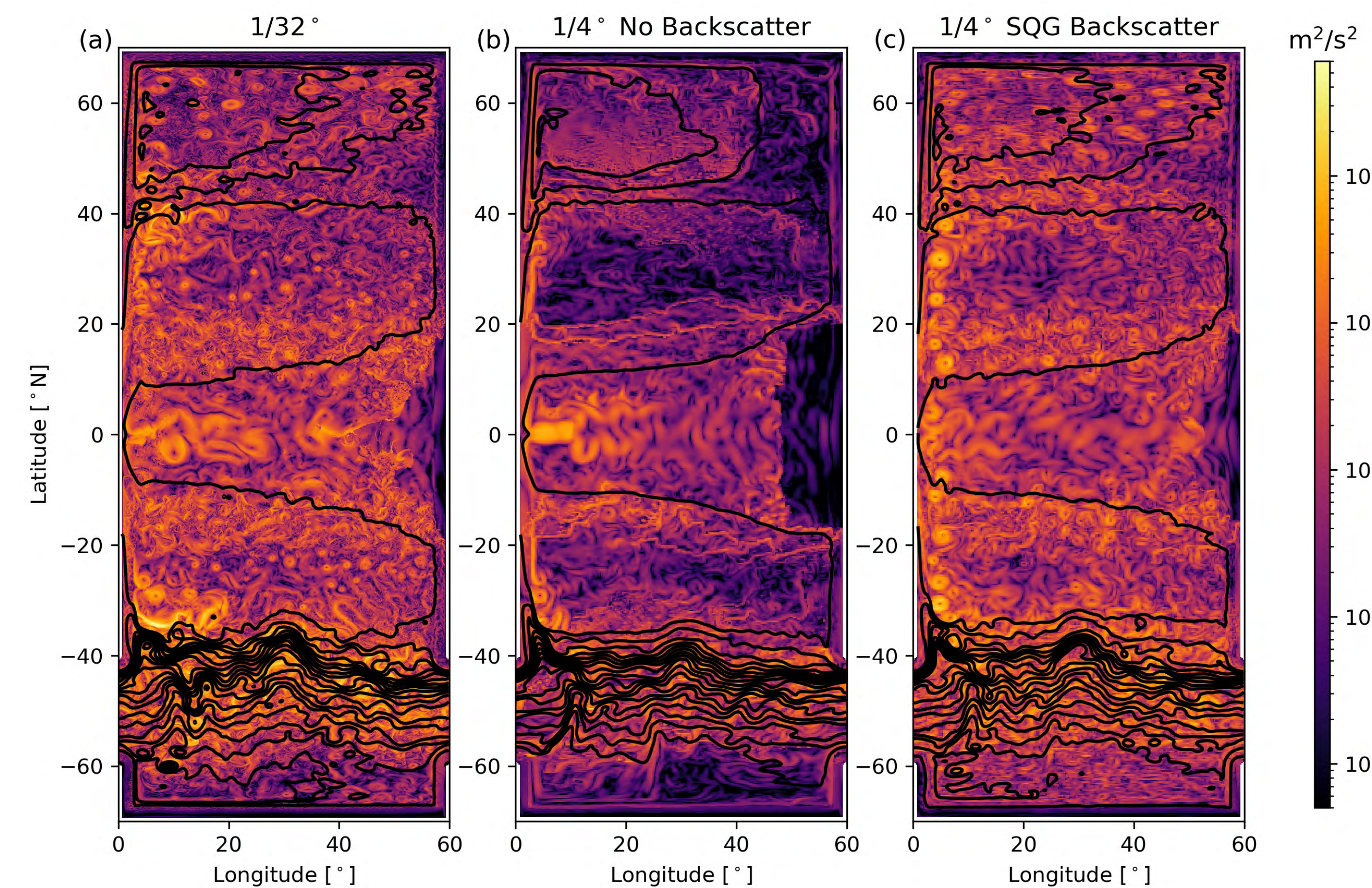


Figure 2. Surface kinetic energy snapshot in (a) $1/32^\circ$ simulation (b) $1/4^\circ$ simulation without the backscatter, and (c) $1/4^\circ$ simulation with the backscatter using an SQG structure. Black lines are 500-day mean sea surface height contours.

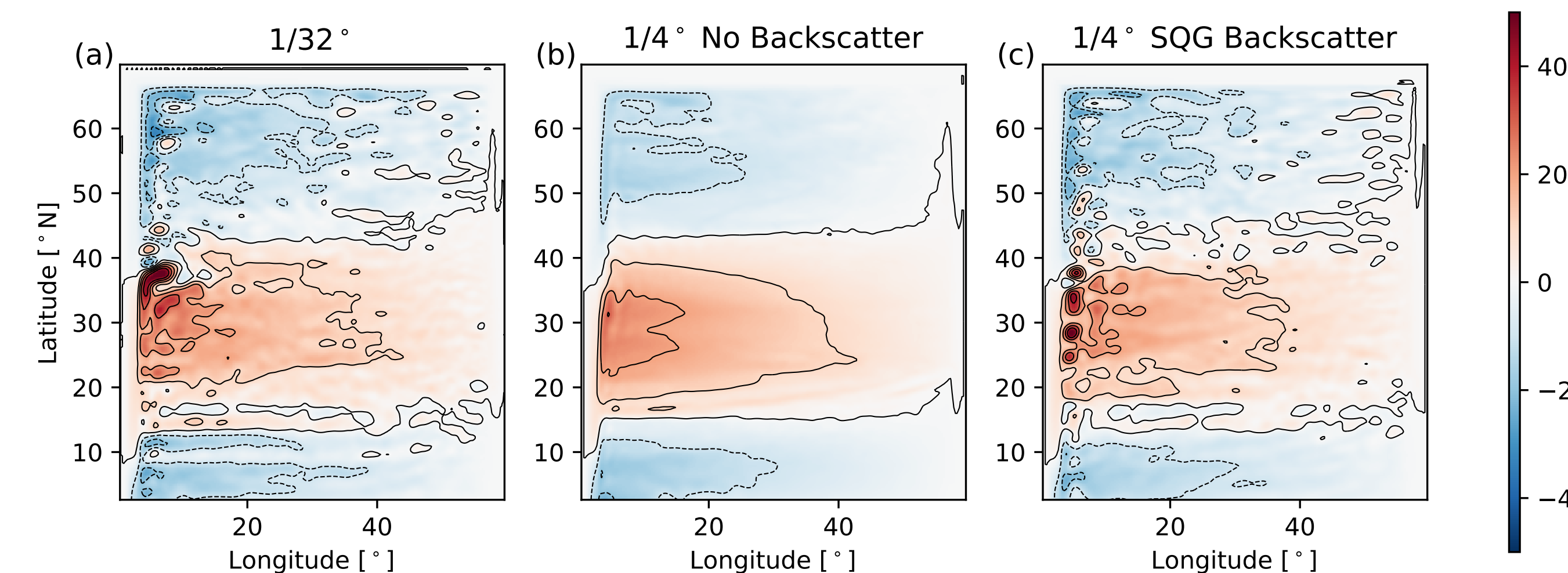


Figure 3. As figure 2 but for the 500-day mean barotropic streamfunction of gyres in the North Hemisphere.

- Kinetic energy and gyre transport are improved by the backscatter parameterization for all vertical structures

- Mean available potential energy (APE) is computed as

$$\text{APE} = \frac{1}{2} g' (\bar{\eta} - \eta_{\text{ref}})^2$$

where η is the isopycnal interface height and η_{ref} is the initialized interface height

- APE measures the **baroclinicity** of mean flow

- BT and EBT backscatters lead to too weak mean APE in the **Southern Ocean**

- SQG and 3D backscatters** improves the mean APE that matches the high-resolution simulation

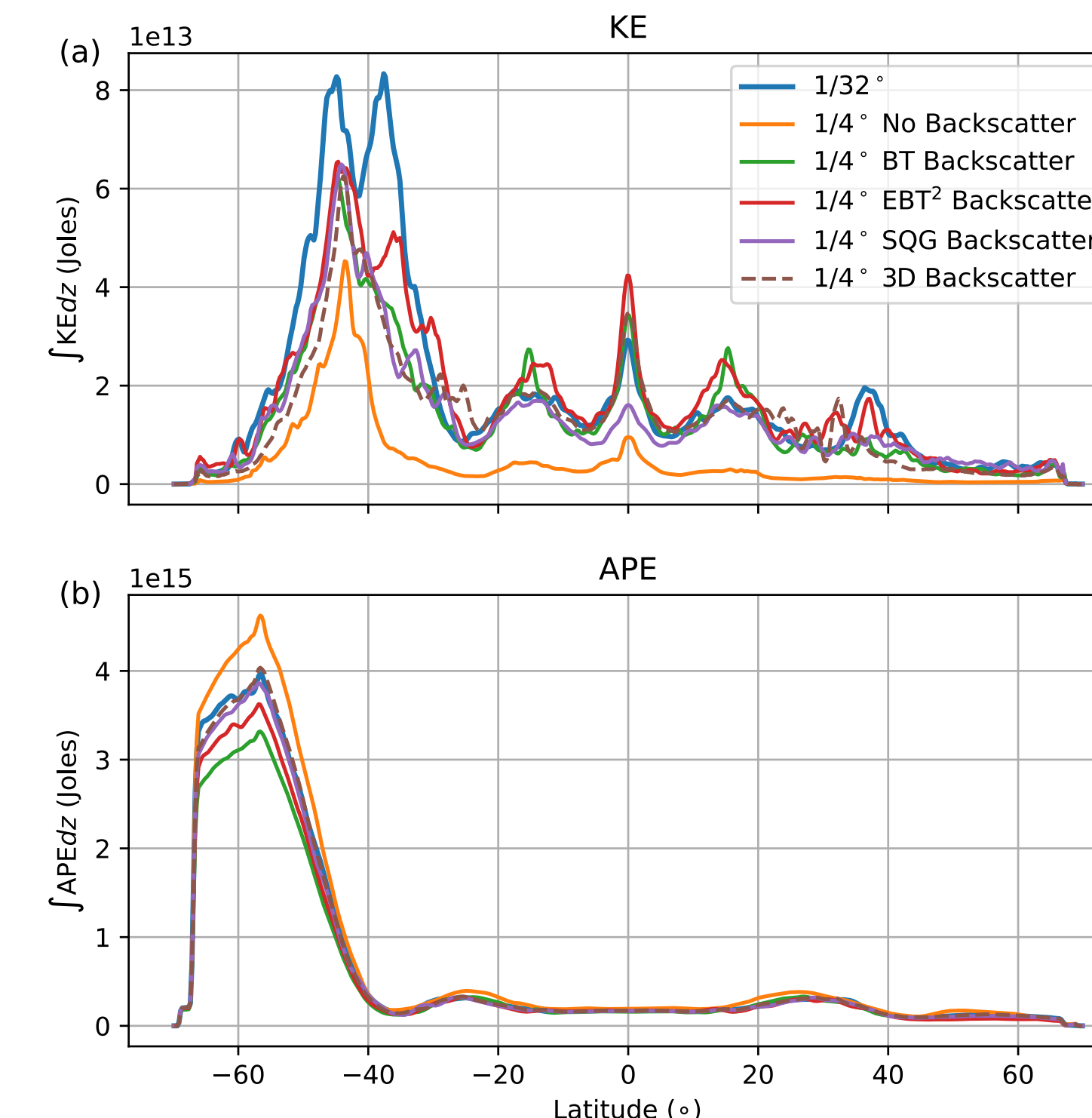


Figure 4. Zonally averaged vertical integral of (a) kinetic energy and (b) mean available potential energy. Blue, orange, green, red and purple lines are for the $1/32^\circ$ simulation, $1/4^\circ$ simulation without backscatter, $1/4^\circ$ simulation with the BT, EBT, and SQG backscatters, respectively. Grey dashed line indicates the $1/4^\circ$ simulation with backscatter informed by 3D MEKE.

Energetics in the Southern Ocean

Isopycnal Structure

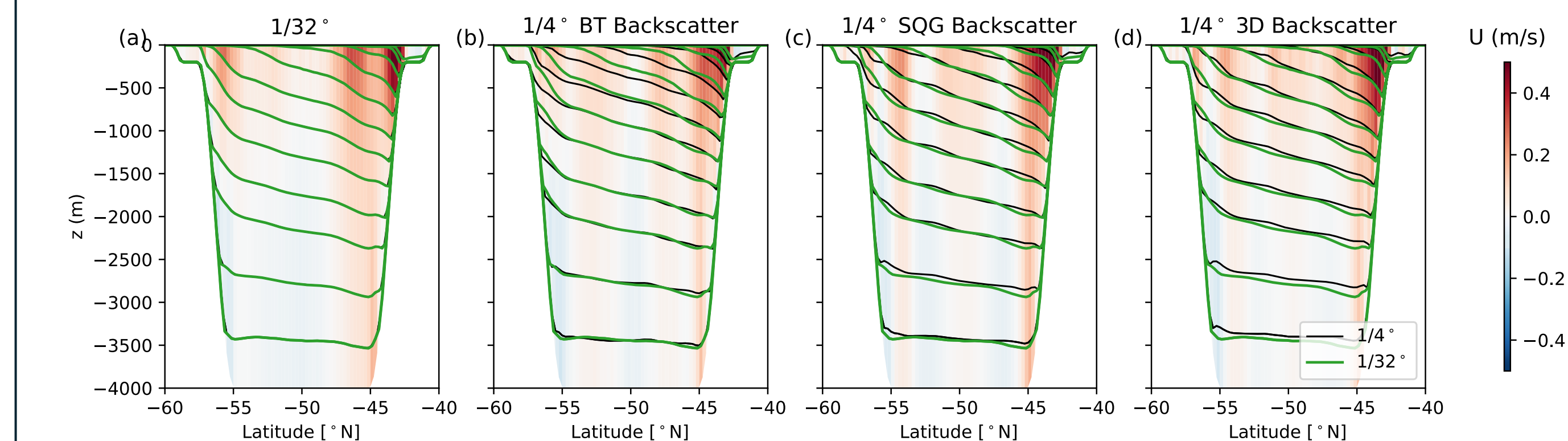


Figure 5. Isopycnal structure in a meridional section of Southern Ocean in (a) $1/32^\circ$ simulation, (b)–(d) $1/4^\circ$ simulations with the BT, SQG, and 3D backscatter, respectively. Green and black lines are isopycnals in the $1/32^\circ$ and $1/4^\circ$ simulations, respectively. Color shading indicates the mean zonal velocity.

- Backscatter with deep vertical structure (BT and EBT²) leads to flatter isopycnals (i.e., reduced baroclinicity) in the upper ocean
- A more surface-intensified vertical structure (SQG and 3D) leads to better isopycnal structure

Energy Contribution by Backscatter

Energy tendency due to horizontal viscous stress in spectral space:

$$\mathbf{F}_n^h = -\nabla \cdot [v_4 \nabla (\nabla^2 \mathbf{u}_n)] + \nabla \cdot (v_2 \nabla \mathbf{u}_n)$$

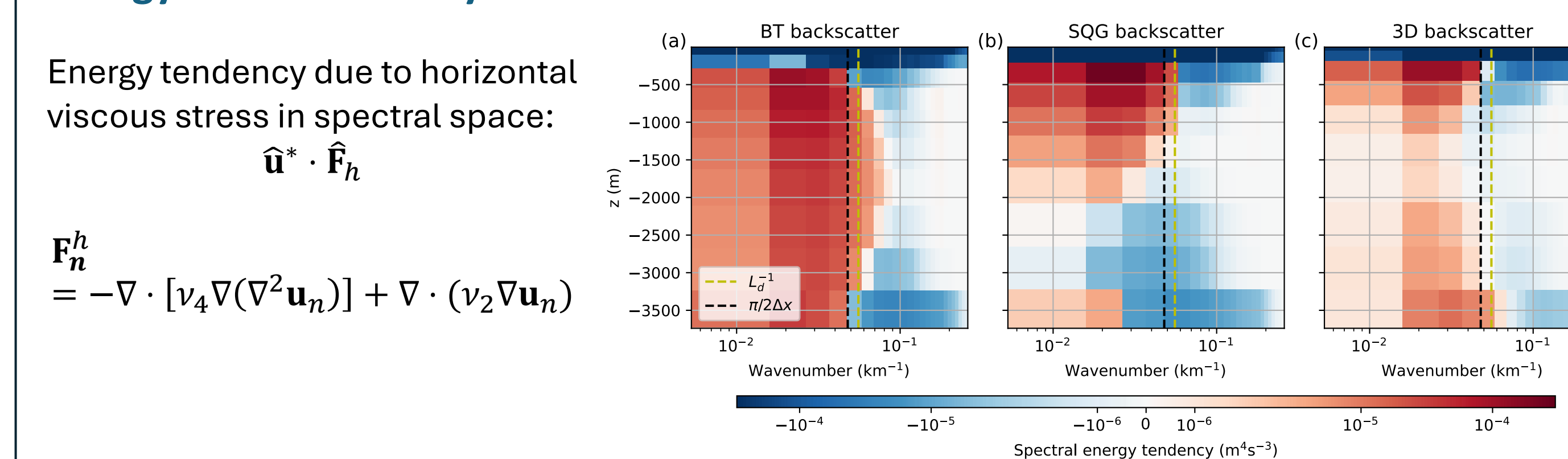


Figure 6. Energy tendency due to horizontal viscous stress as a function of horizontal wavenumber and depth in Southern Ocean. Red and blue indicate energy source and sink, respectively. (a)–(c) $1/4^\circ$ simulation with the BT, SQG, and 3D backscatter, respectively. Black dashed line indicates the wavenumber of $4\Delta x$ wave. Yellow dashed line indicates the inverse of Rossby deformation radius.

- Biharmonic viscosity dissipates energy at scale smaller than $4\Delta x$ (~ 70 km); backscatter injects energy at scales around 200 km
- Forcing is vertically uniform in the BT backscatter case, leading to more barotropic eddies
- SQG and 3D backscatters enhance surface-intensified eddies at about 200 km

Eddy Rectification Effect

KE spectral flux:

$$\Pi = \int_k^\infty \text{Re}[\hat{\mathbf{u}}^* \cdot \mathbf{u} \cdot \nabla \mathbf{u}] dk$$

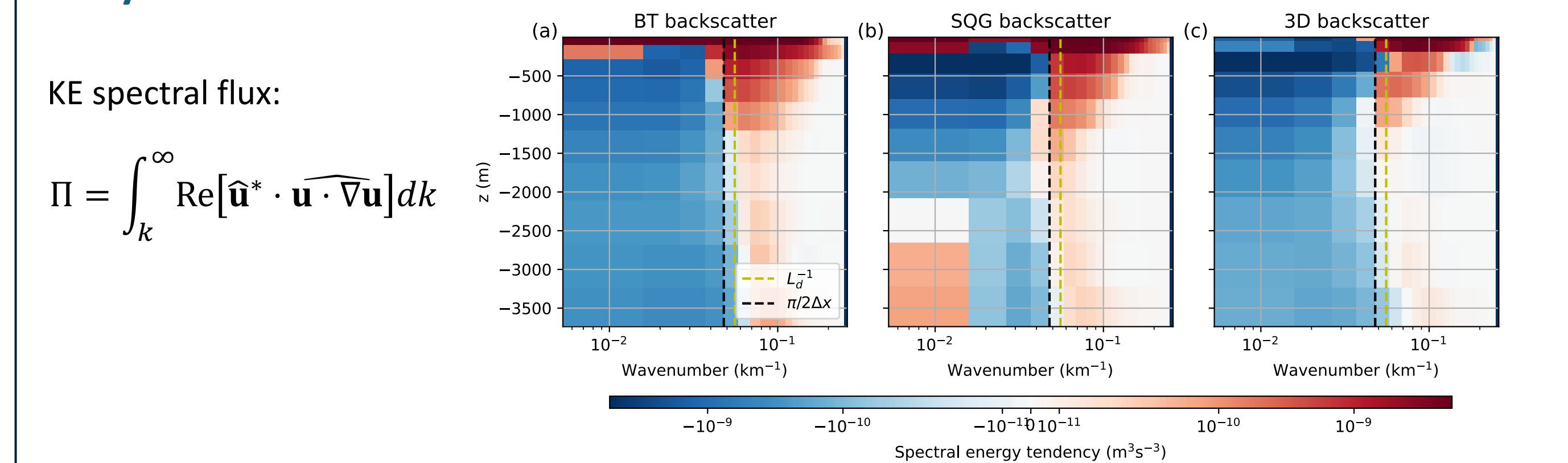


Figure 7. As figure 6 but for the kinetic energy spectral flux. Blue and red indicate inverse and forward energy cascades, respectively.

- Eddies enhance by backscatter rectifies the large-scale circulations through inverse energy cascade
- Spurious barotropic eddies reduce the baroclinicity of mean flow

Conclusion

- Energy backscatter can shape the vertical structure of resolved eddies, which further modulate the large-scale circulation structure
- A more surface-intensified vertical structure like SQG mode leads to better large-scale isopycnal structure by reasonably representing the momentum fluxes of surface-intensified eddies