Modeling noctilucent cloud dust charging with a quantum algorithm

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Objectives

- Model the dust charging process of ice grains in noctilucent clouds using a quantum algorithm
- Assess the benefits of this model relative to the classical simulation and analytic approaches
- Assess the downsides of this model relative to the classical simulation and analytic approaches

Background

Noctilucent clouds (NLCs) are located in the mesosphere and are only visible under the following conditions:

- at latitudes closer to 50-65°,
- during the summer,
- and shortly after sunset or shortly before sunrise.

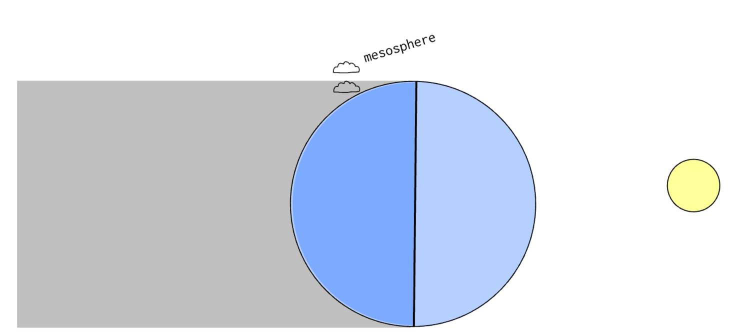

Figure 1: The conditions required for visible NLCs.

During NLC dust charging, we model the following process until steady-state of electron-dust collision frequency ν_e and ion-dust collision frequency ν_i, as well as plasma density are achieved:

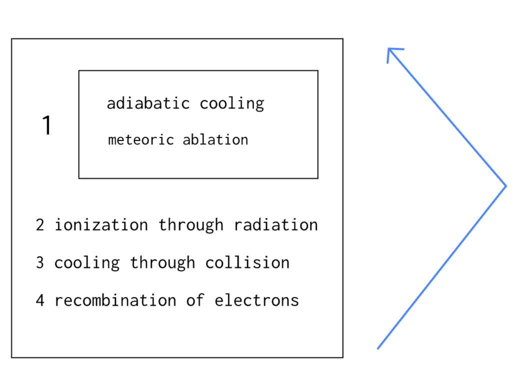

Figure 2: NLC dust charging process (loops until $\nu_e = \nu_i$ and $Q_e = \alpha n_e n_i + D_e n_e n_d$).

Quantum algorithm

Algorithm 1 of Montanaro [1] takes an algorithm A (which outputs a value between $[0,1]$) and returns an estimate of A's output's mean.

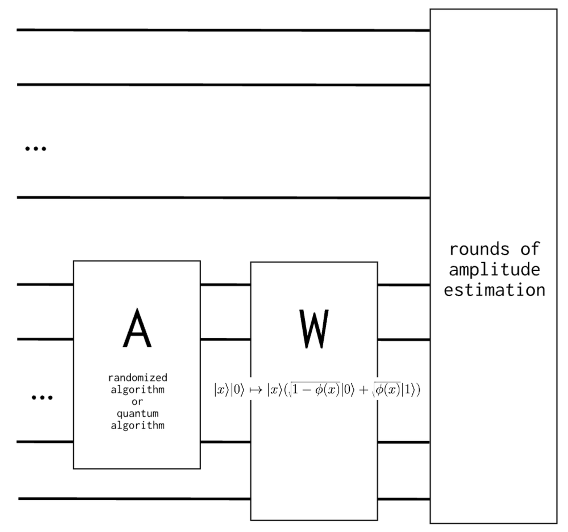

Figure 3: The quantum algorithm outputs the mean Havnes parameter $\mathbb{E}[H]$, $H = 1 - n_e/n_i$, which conveniently lies between $[0,1]$ whenever ice grains charge negative.

Constructing the algorithm

Constructing A so that it maps input states bijectively:

1. every timestep is a permutation of computational basis states
2. randomness is reproducible, never traced out
3. arithmetic is fixed point, since IEEE rounding is not injective

W is constructed simply. There are then t iterations of amplitude estimation at the end of the circuit. The runs of this algorithm are compared against rounds of sampling the distribution created by running classical Monte Carlo in Nicolov et al. [2].

Key takeaway

Demanding extreme accuracy makes quantum advantageous. This is unnecessary when modeling NLC dust charging.

Analytic approach

Nicolov et al. [2] sets up the Havnes parameter as a function of normalized dust potential.

$$H(\psi_d) = 1 - \frac{\sqrt{m_e/M}\,(1+\psi_d)}{\sqrt{\frac{\pi}{2m_e kT_i}}\,\frac{M}{8\tau_R \sigma_n n_n}\left(\psi_d \ln\frac{w(0)}{kT_i\psi_d} + (1-\psi_d)\ln\frac{w(0)/kT_i - 1}{\psi_d - 1}\right)}.$$

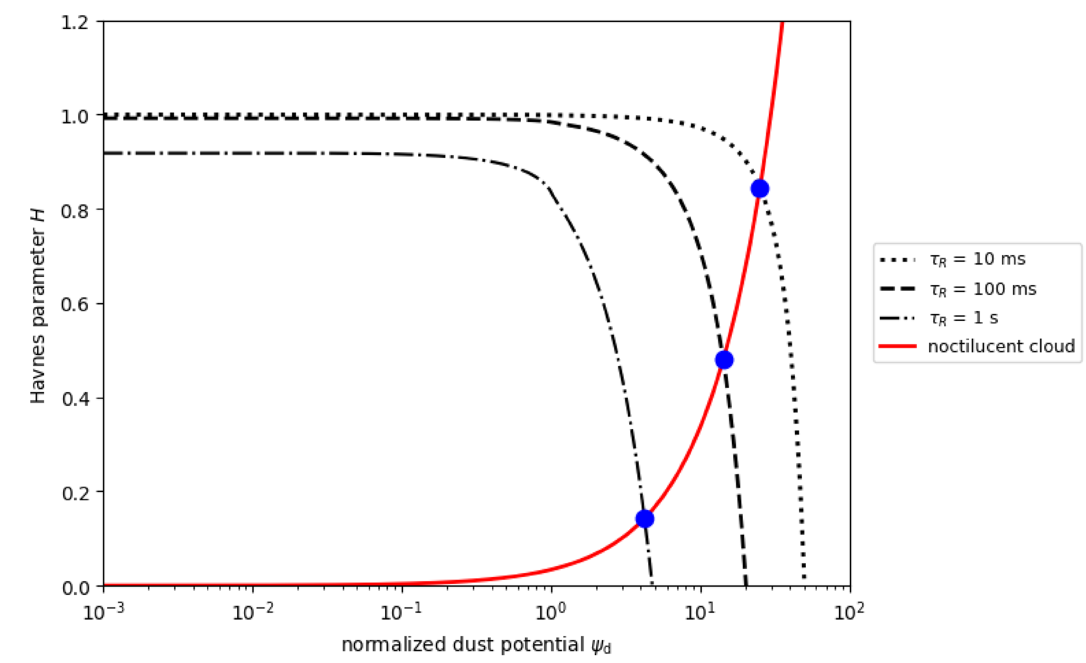

Figure 4: Charge equilibrium is the dust-to-ion charge ratio (H) at which there is an intersection with the electron lifetime curve.

Results

Sampling efficiency. The two methods differ only in how many times the simulation must be run. Counting simulation runs gives:

RMSE on H	MC [2]	Alg. 1
2.5×10^{-3}	2	4 141
2.5×10^{-4}	155	41 385
2.7×10^{-5}	13 475	413 665
2.9×10^{-6}	1.1×10^{6}	4.1×10^{6}
8.8×10^{-7}	1.2×10^{7}	1.2×10^{7}

Table 1: Simulation runs needed for a given accuracy on the Havnes parameter, H.

For cases when error for dust-ion charge ratio must be less than 8.8×10^{-7}, quantum becomes advantageous.

Implications

NLCs might indicate the quantities of molecular gases in the atmosphere.

- Rising CO_2 cools the mesosphere.
- CH_4 oxidation results in mesospheric H_2O.
- Increasing levels of CO_2 and CH_4 can cause more frequent and brighter noctilucent clouds.

Cross-referencing NLC observations from the Earth's surface with impacts of electron bite-out in the stratosphere and thermosphere can give insight into the composition and dynamics of the entire atmosphere.

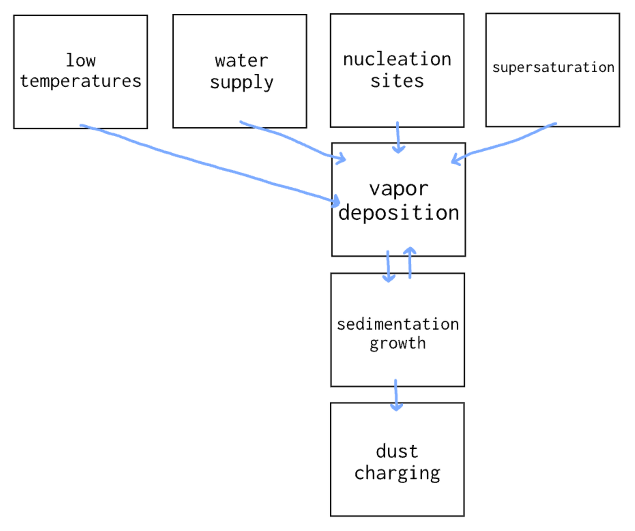

Figure 5: The complete NLC formation process.

References

[1] A. Montanaro. Quantum speedup of Monte Carlo methods. *Proc. R. Soc. A*, 471(2181), September 2015.

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[4] J. M. C. Plane, W Feng, and E. M. C. Dawkins. The mesosphere and metals: Chemistry and changes. *Chem. Rev.*, 115(10), May 2015.