

# Hybrid vertical coordinate in the Bergen Layered Ocean Model (BLOM)

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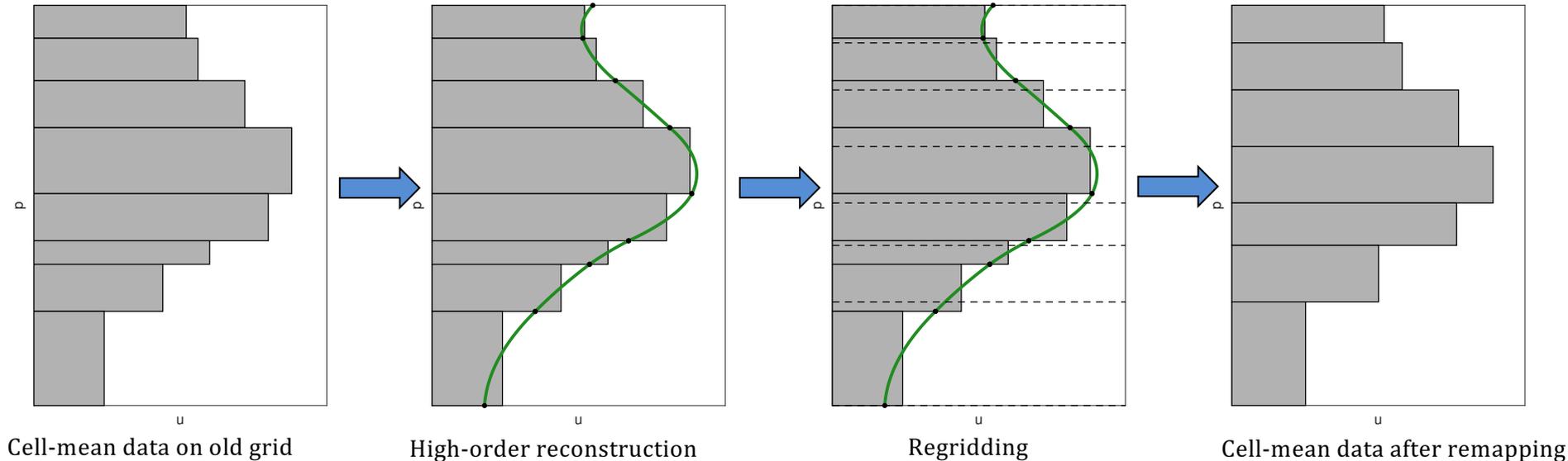
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# Outline

- The ALE method
- Reconstruction: general, robustness, limiting
- Vertical coordinate and regridding
- Physical parameterizations with the ALE method
- Coupled ocean/sea-ice simulations
- Impact of reconstruction accuracy
- Summary

## The ALE method

- The **arbitrary Lagrangian-Eulerian** (ALE) method first integrates the layer conservation equations forward in a truly Lagrangian phase, then remap variables to a desired vertical grid in a second phase.
- A suitable regridding approach and accurate remapping is crucial for the application of the ALE method for ocean climate modelling.



## Reconstruction - general

- BLOM uses the **piecewise polynomial method** and high-order extensions as described by White and Adcroft (2008); White et al. (2009).
- Support for **PCM, PLM, PPM** and **PQM** reconstructions on non-uniform grids.
- **Compact schemes** (Lacor et al. 2004) are used for obtaining edge value and slope estimates. The 4<sup>th</sup> order accurate edge values (**ih<sub>4</sub>**) for PPM are related to cell averages by

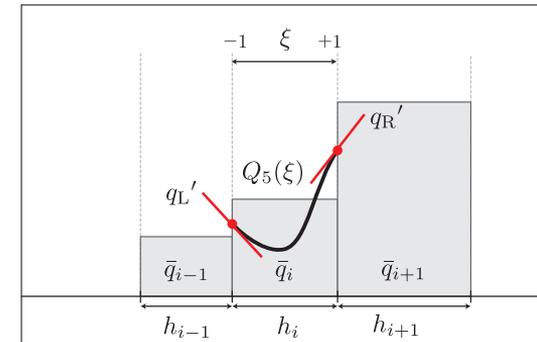
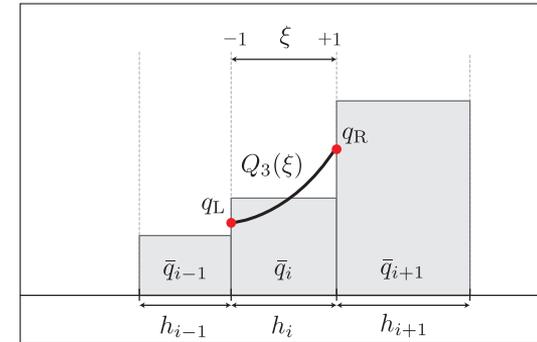
$$\alpha u_{j-\frac{1}{2}} + u_{j+\frac{1}{2}} + \beta u_{j+\frac{3}{2}} = b\bar{u}_j + c\bar{u}_{j+1}$$

while 6<sup>th</sup> order accurate edge values (**ih<sub>6</sub>**) and 5<sup>th</sup> order accurate edge slopes (**ih<sub>5</sub>**) for PQM are found by

$$\alpha u_{j-\frac{1}{2}} + u_{j+\frac{1}{2}} + \beta u_{j+\frac{3}{2}} = a\bar{u}_{j-1} + b\bar{u}_j + c\bar{u}_{j+1} + d\bar{u}_{j+2}$$

$$\tilde{\alpha} u'_{j-\frac{1}{2}} + u'_{j+\frac{1}{2}} + \tilde{\beta} u'_{j+\frac{3}{2}} = \tilde{a}\bar{u}_{j-1} + \tilde{b}\bar{u}_j + \tilde{c}\bar{u}_{j+1} + \tilde{d}\bar{u}_{j+2}$$

- Explicit schemes are used to obtain boundary edge values and slopes, with selectable order of accuracy from 1<sup>st</sup> to 6<sup>th</sup>.

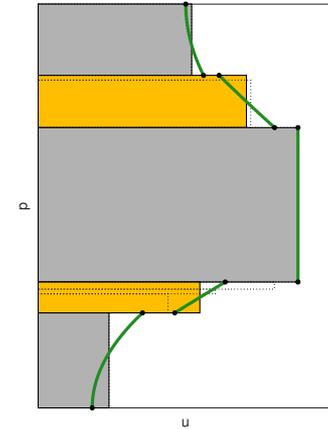
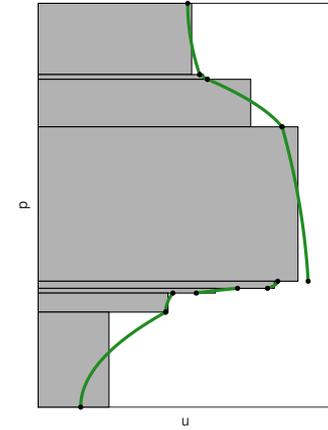


Courtesy: Engwirda and Kelley (2016)

## Reconstruction - robustness

- **Highly variable cell widths** are handled by **merging cells** as necessary to maintain overall high accuracy and well-conditioned linear equation systems. With  $n$  cells in a stencil  $S_n$ , the cell widths  $h_i$  must obey the following criterion for all stencils:

$$\frac{\prod_{i \in S_n} h_i}{[\max_{i \in S_n}(h_i)]^n} > \varepsilon_n$$

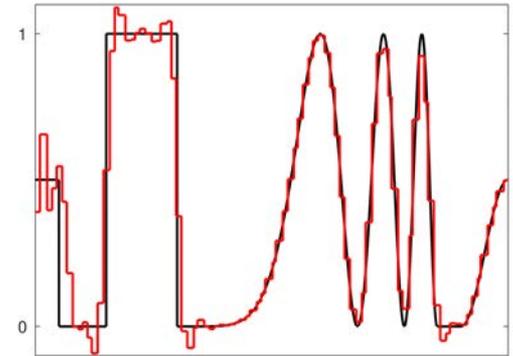
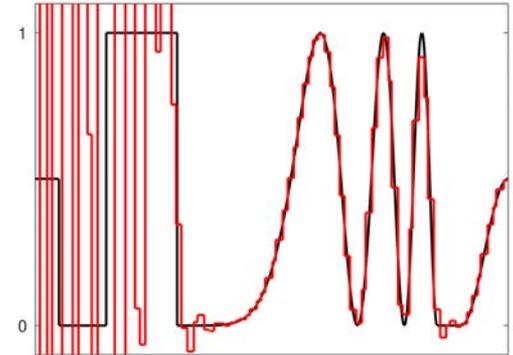


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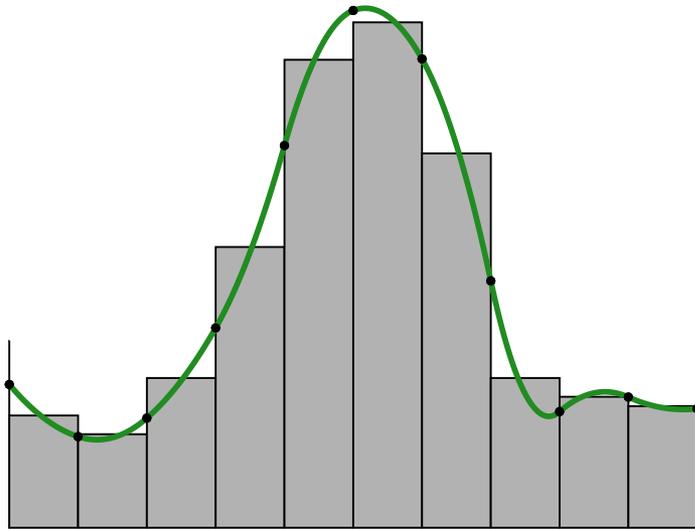
$$\frac{\prod_{i \in S_n} h_i}{[\max_{i \in S_n}(h_i)]^n} > \varepsilon_n$$

- If the  $ih_6/ih_5$  scheme leads to a tridiagonal matrix row that is not **diagonally dominant** ( $|\alpha| + |\beta| > 1$ ), replace that row with coefficients from the reduced order scheme  $ih_4/ih_3$ .
- When using PQM in realistic  $1^\circ$  configuration,  $\sim 10\%$  of the  $ih_6/ih_5$  coefficients had to be replaced.

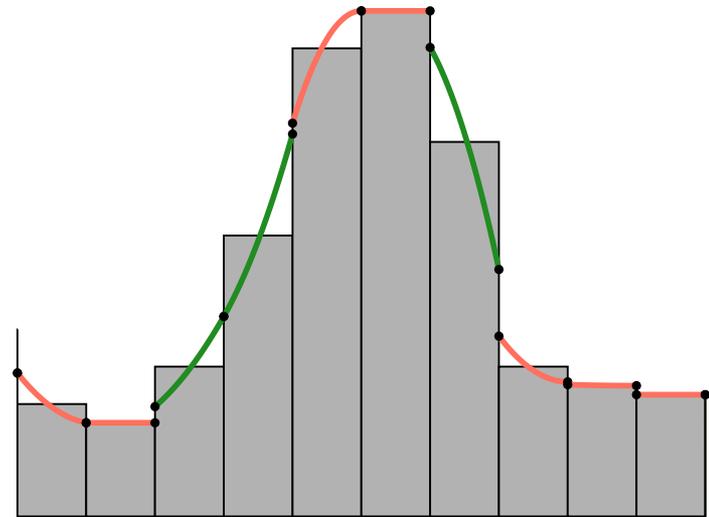


## Reconstruction - limiting

- A **non-oscillatory limiting** for PPM and PQM reconstructions, optionally positive definite, improves accuracy at smooth extrema, while controlling spurious oscillations near sharp features. Inspired by Colella and Sekora (2008), but simplified.



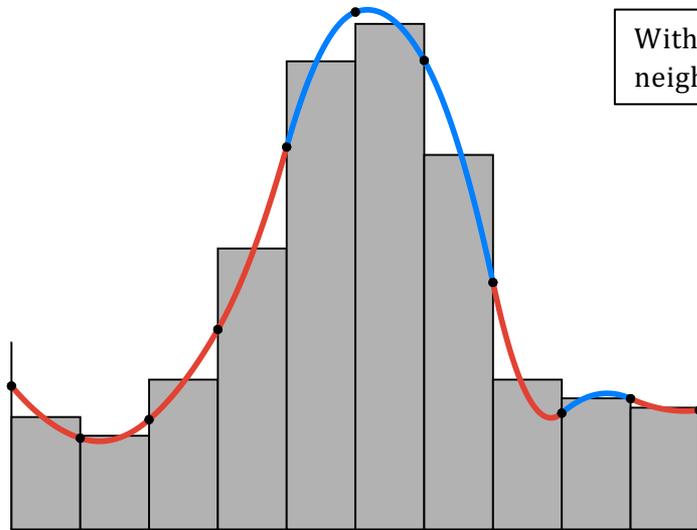
PPM reconstruction, no limiting



PPM reconstruction, monotonic limiting

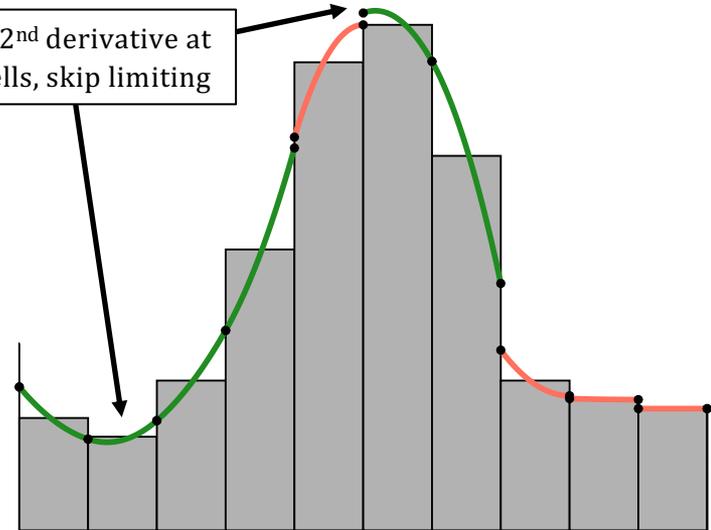
## Reconstruction - limiting

- A **non-oscillatory limiting** for PPM and PCM reconstructions, optionally positive definite, improves accuracy at smooth extrema, while controlling spurious oscillations near sharp features. Inspired by Colella and Sekora (2008), but simplified.



PPM reconstruction, no limiting, colour based on sign of 2<sup>nd</sup> derivative

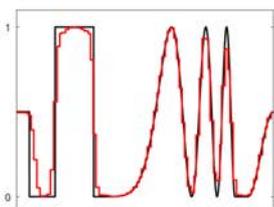
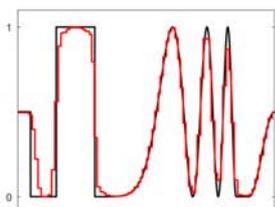
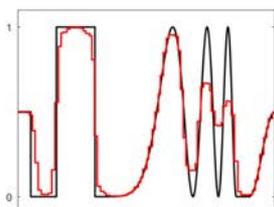
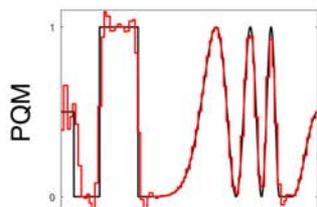
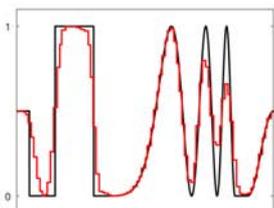
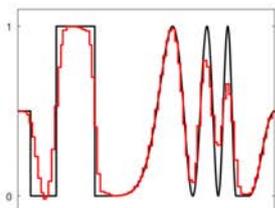
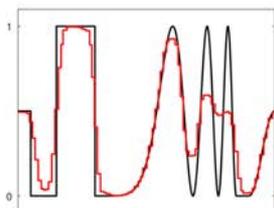
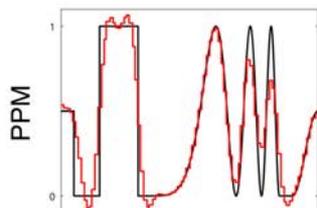
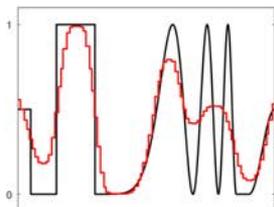
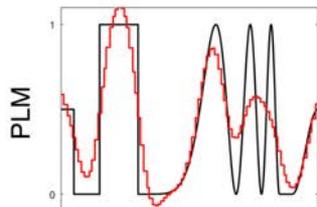
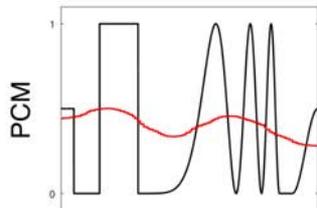
With same sign 2<sup>nd</sup> derivative at neighbouring cells, skip limiting



PPM reconstruction, non-oscillatory limiting

## Reconstruction - limiting

- A **non-oscillatory limiting** for PPM and PCM reconstructions, optionally positive definite, improves accuracy at smooth extrema, while controlling spurious oscillations near sharp features. Inspired by Colella and Sekora (2008), but simplified.
- Options for **limiting at the boundary**:
  - **Piecewise constant** reconstruction at boundary is the only strictly monotonic approach possible, but leads to unrealistic layer structure towards bathymetry in long simulations with hybrid coordinate.
  - Preferred option is a type of **Neumann boundary condition** with constraint on the slope of the edge adjacent to the boundary.



Unlimited

Monotonic

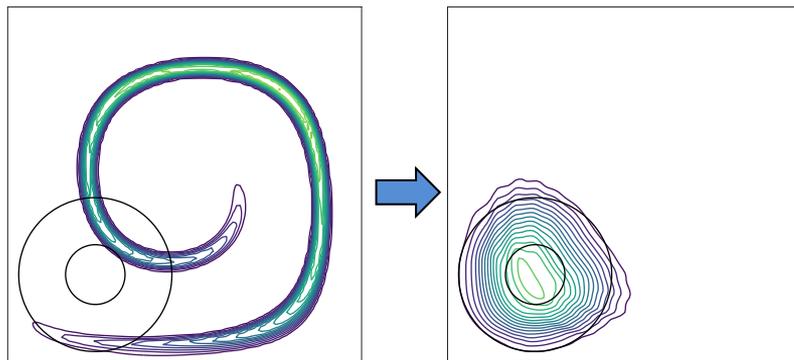
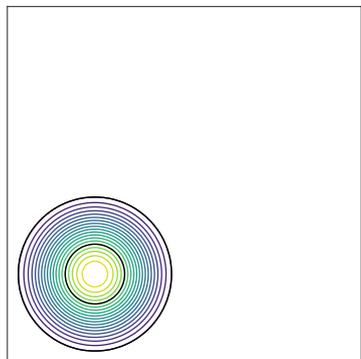
Non-oscillatory

Non-oscillatory, pos. def.

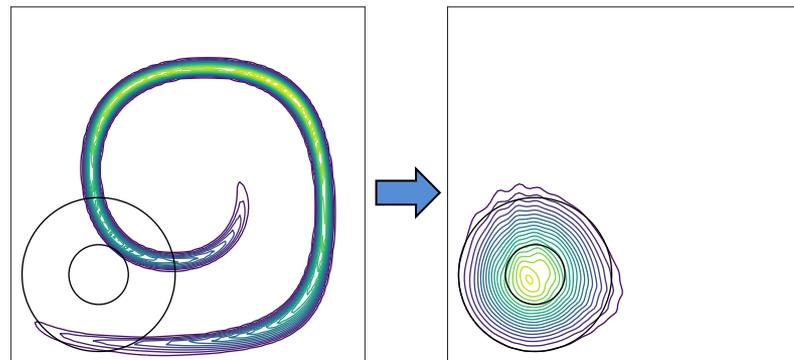
Available polynomial reconstruction methods and options for limiting, tested with 1000 remappings between a uniform and randomly perturbed non-uniform grid.

# Non-oscillatory limiting applied to 2D advection

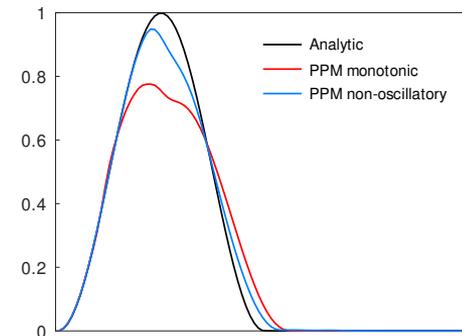
Nondivergent deformational  
flow test by Durran (1999)



PPM with monotonic limiter

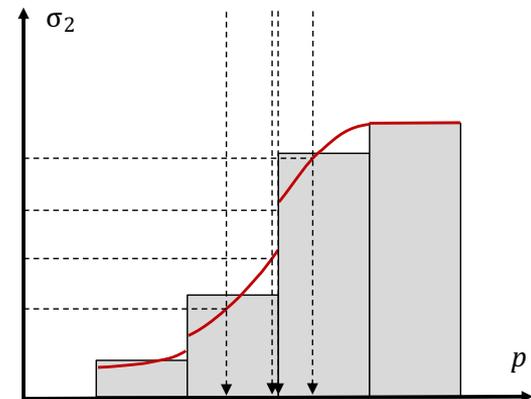


PPM with non-oscillatory limiter

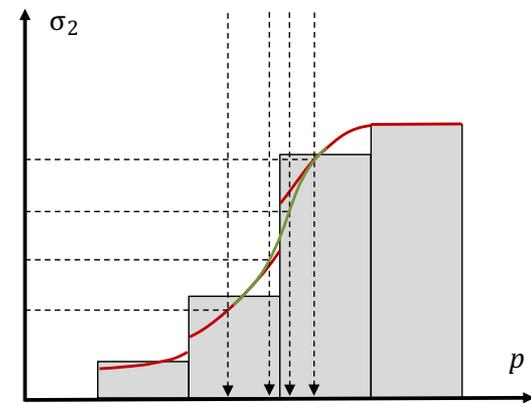


## Vertical coordinate and regridding

- Using the ALE method, BLOM currently supports regridding to the vertical coordinates: ***p*-level** and **hybrid**.
- Hybrid vertical coordinate:
  - Hybrid between **continuous isopycnals** (prescribed  $\sigma_2$  at layer interfaces) and *p*-levels (prescribed pressure at layer interfaces).
  - For continuous isopycnals, regridded interfaces can be found either by **direct** intersection with reconstructed  $\sigma_2$  or by **nudging** interfaces to gradually reduce deviation from prescribed  $\sigma_2$ , making use of reconstructed  $\theta$  and  $S$ .
  - In general, the maximum of regridded continuous isopycnals and *p*-levels is chosen to achieve desired vertical resolution in the PBL.
  - A **transition zone** between continuous isopycnals and *p*-levels are beneficial for a smoother change in vertical resolution.



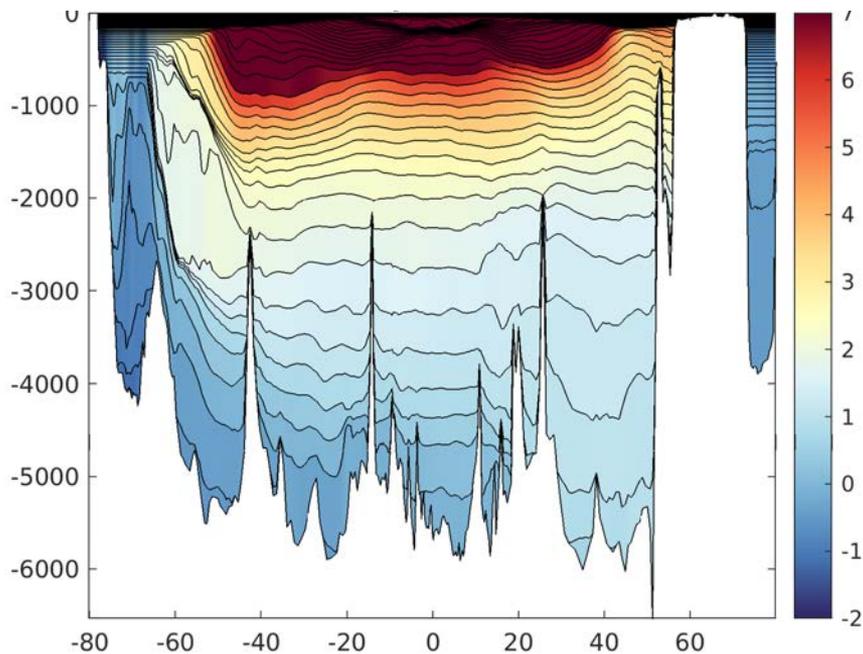
Intersection with PPM reconstruction



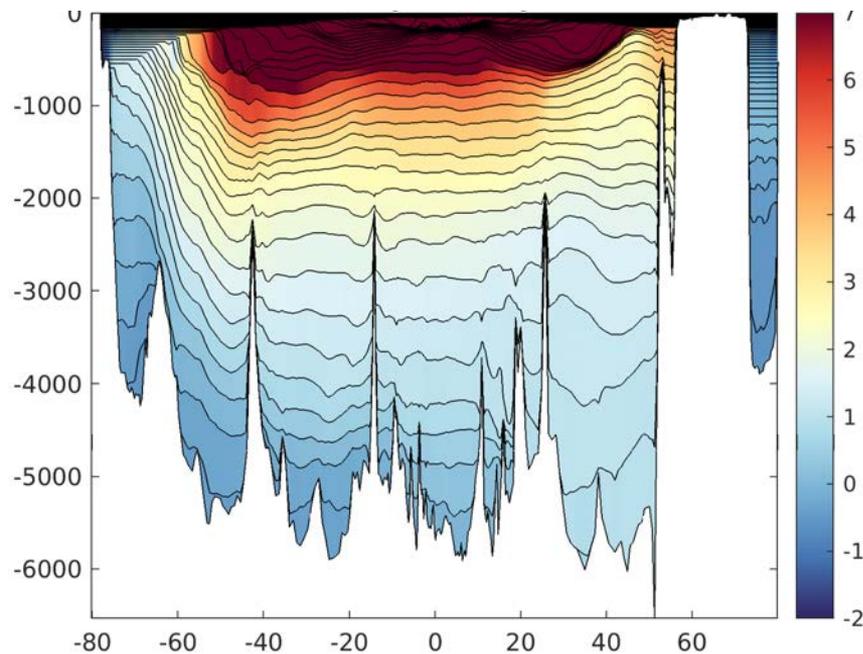
Intersection with continuous reconstruction

## Vertical coordinate and regridding

Impact of regridding approaches for hybrid coordinate.



Direct intersection with reconstructed  $\sigma_2$

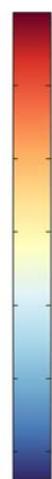
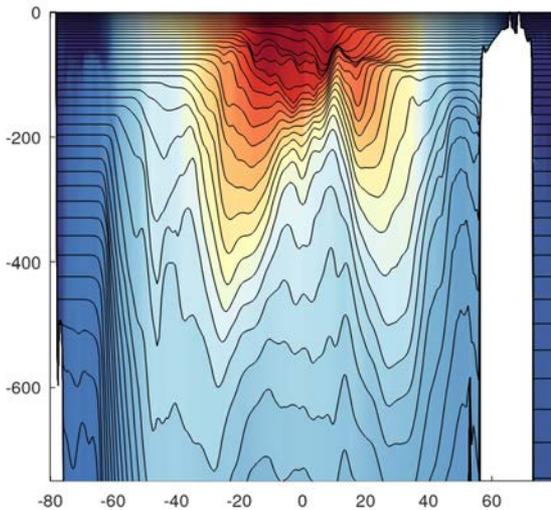
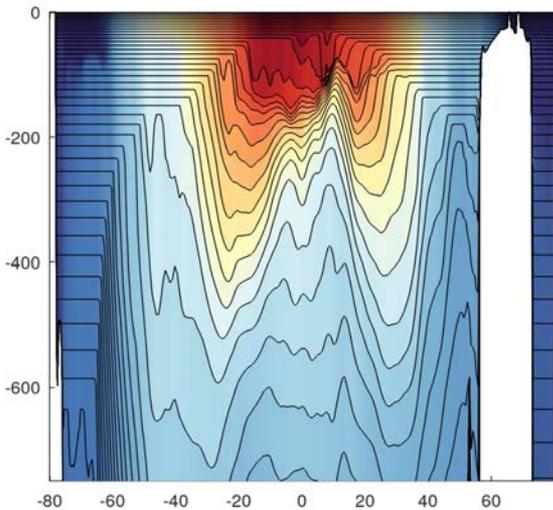


Regridding by nudging towards prescribed  $\sigma_2$

# Vertical coordinate and regridding

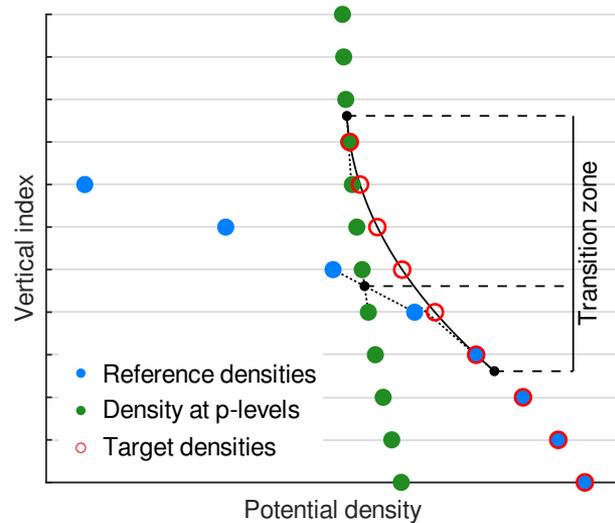
Pacific, No transition zone

Pacific, Transition zone



No transition zone

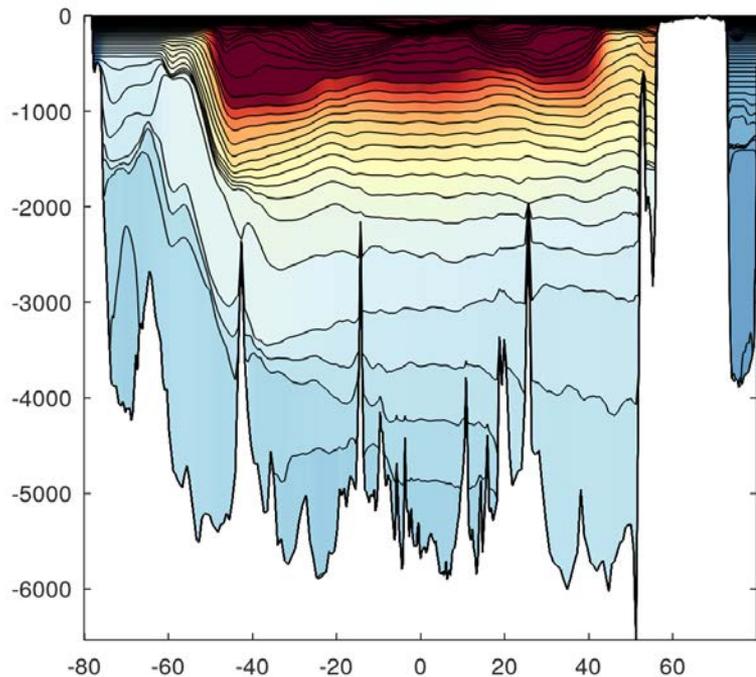
Transition zone between continuous isopycnals and  $p$ -levels



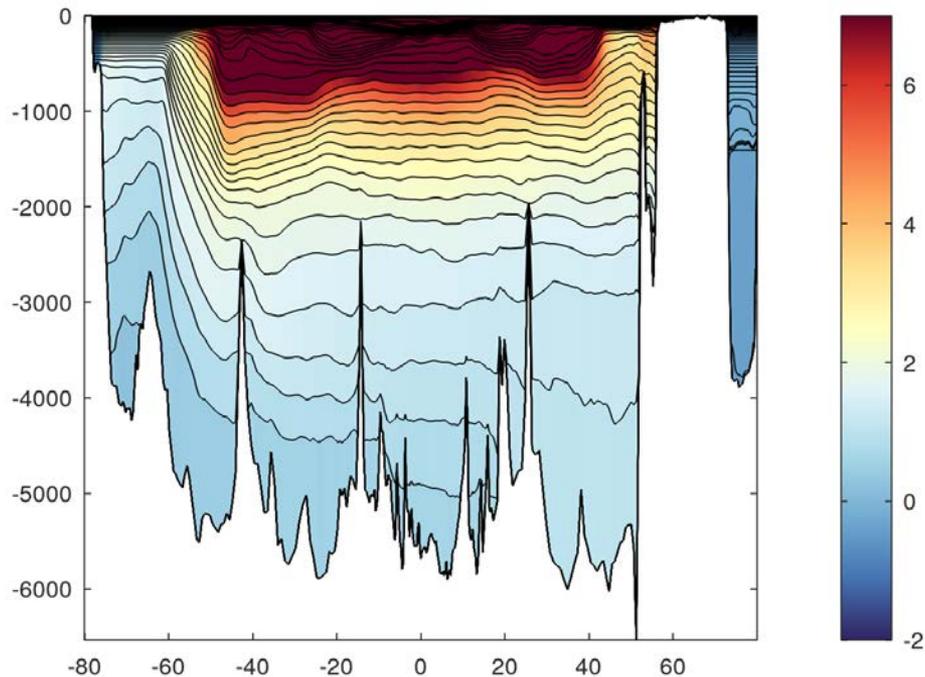
Schematic of the construction of target densities in the transition zone

## Vertical coordinate and regridding

Impact of limiter choice in long simulations.



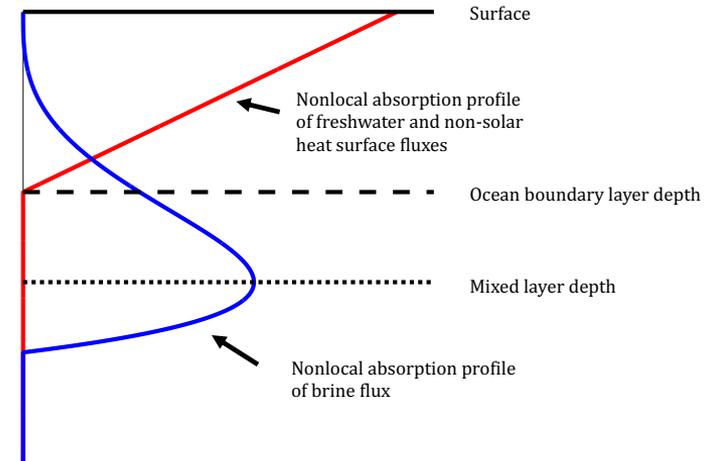
PPM with monotonic limiter



PPM with non-oscillatory limiter

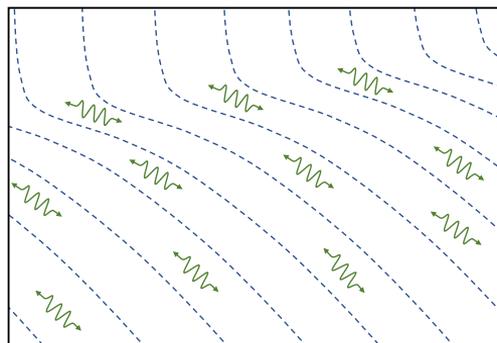
## Physical parameterizations with the ALE method

- Vertical diffusivities are obtained using the nonlocal K Profile Parameterization (**KPP**; Large et al. 1994), as implemented in the **CVMix community package**.
- A deviation from the standard KPP is nonlocal distribution of surface momentum flux and nonlocal distribution of brine from sea-ice freezing.
- The KPP parameterization gives insufficient mixing in gravity currents and work remains to handle this in a satisfactory manner.
- Currently exploring surface wave effects and double diffusion.

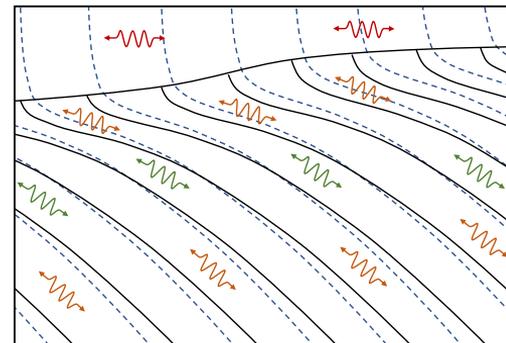


## Physical parameterizations with the ALE method

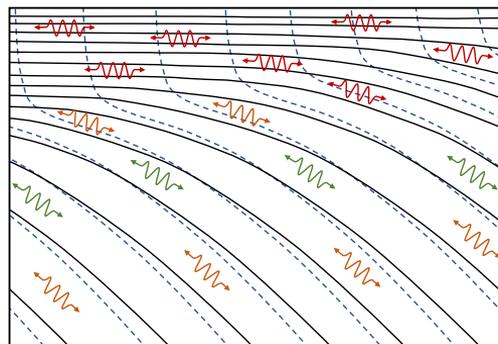
For eddy-induced diffusion with the ALE method, BLOM makes use of neutral diffusion along nonlocal neutral sublayers, as recently proposed by Shao et al. (2020).



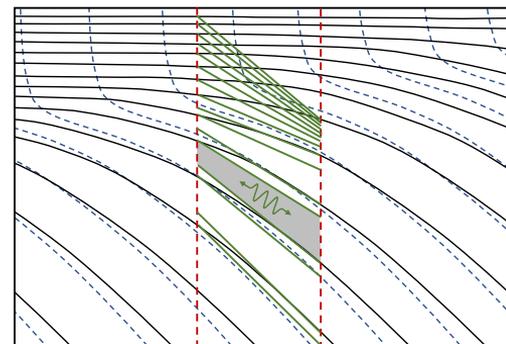
Neutral diffusion



Isopycnal layer diffusion



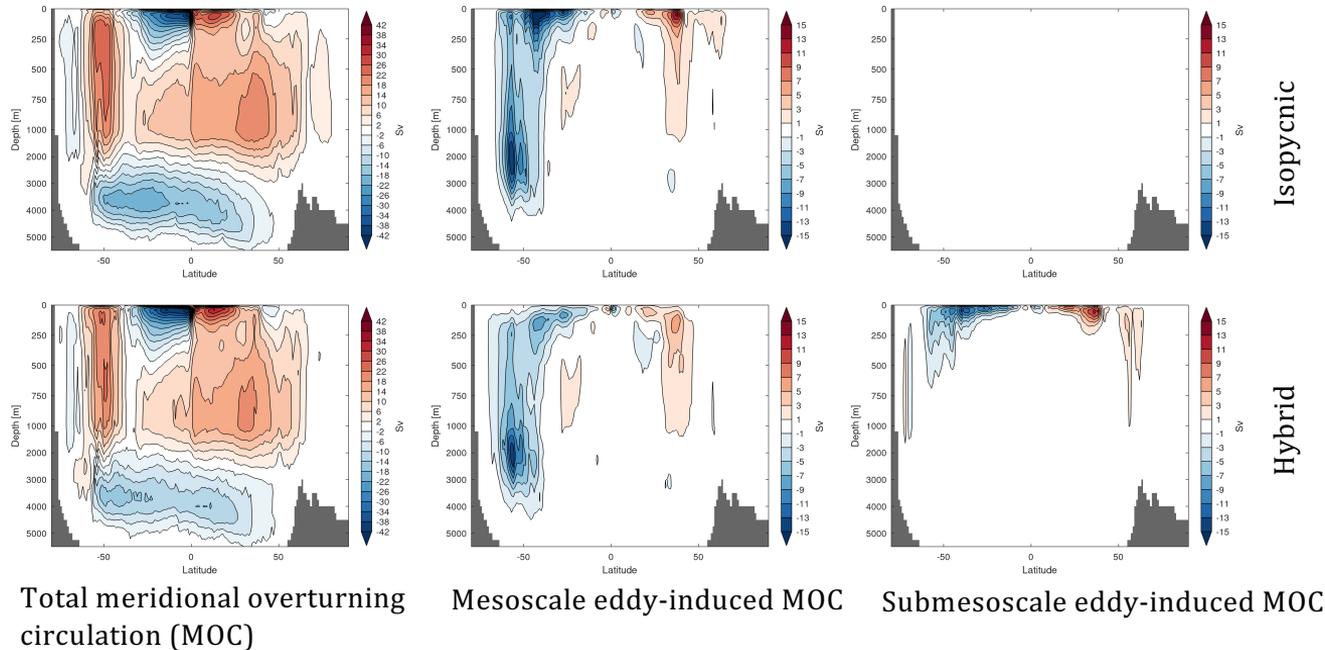
Hybrid layer diffusion



Neutral sublayers

# Physical parameterizations with the ALE method

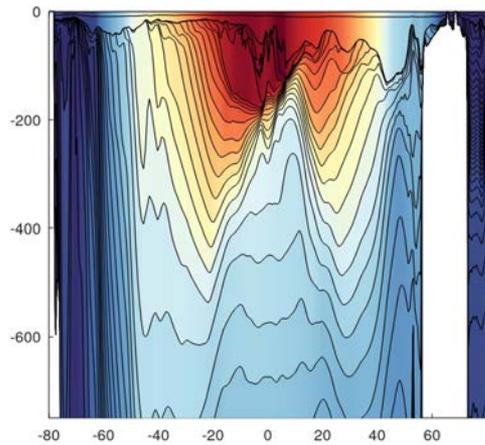
- The parameterization of Gent-McWilliams (GM) eddy-induced transport makes use of the neutral slope estimation of the neutral diffusion algorithm.
- With vertical resolution in the PBL with hybrid vertical coordinate, **submesoscale eddy-induced transport** (Fox-Kemper et al., 2008) has now been fully implemented.



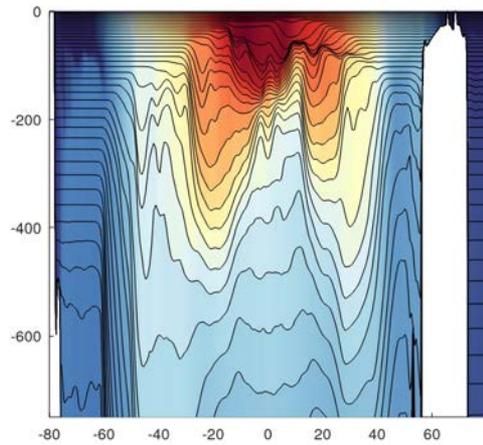
## Coupled ocean/sea-ice simulations

- Run in the NorESM framework with CICE5 as sea-ice component.
- OMIP2 forcing (JRA55 based atmospheric forcing).
- Tripolar ocean grid with  $1^\circ$  resolution along the Equator.
- 56 vertical levels. Minimum thickness of the top layer is 2.5 m.
- Primarily showing time averages over the last 20 years of the second cycle of OMIP2 forcing.
- These simulations also include iHAMOCC for ocean biogeochemistry

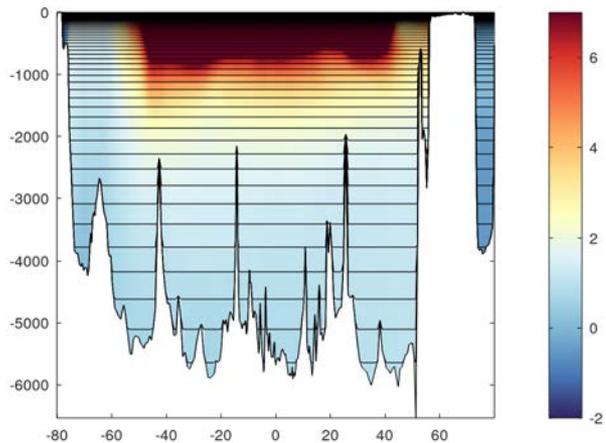
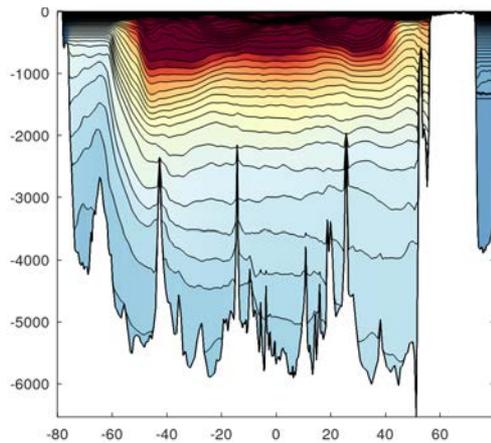
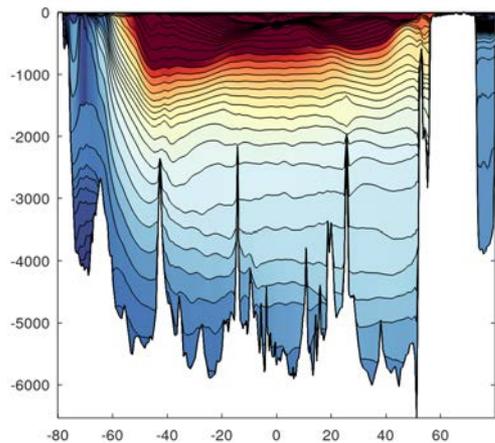
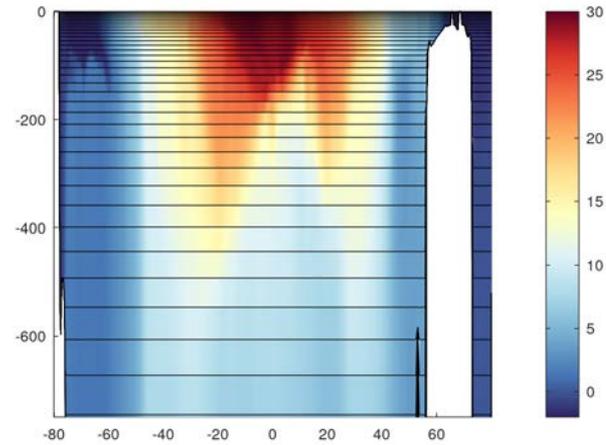
Pacific, Isopycnic



Pacific, Hybrid



Pacific, p-level

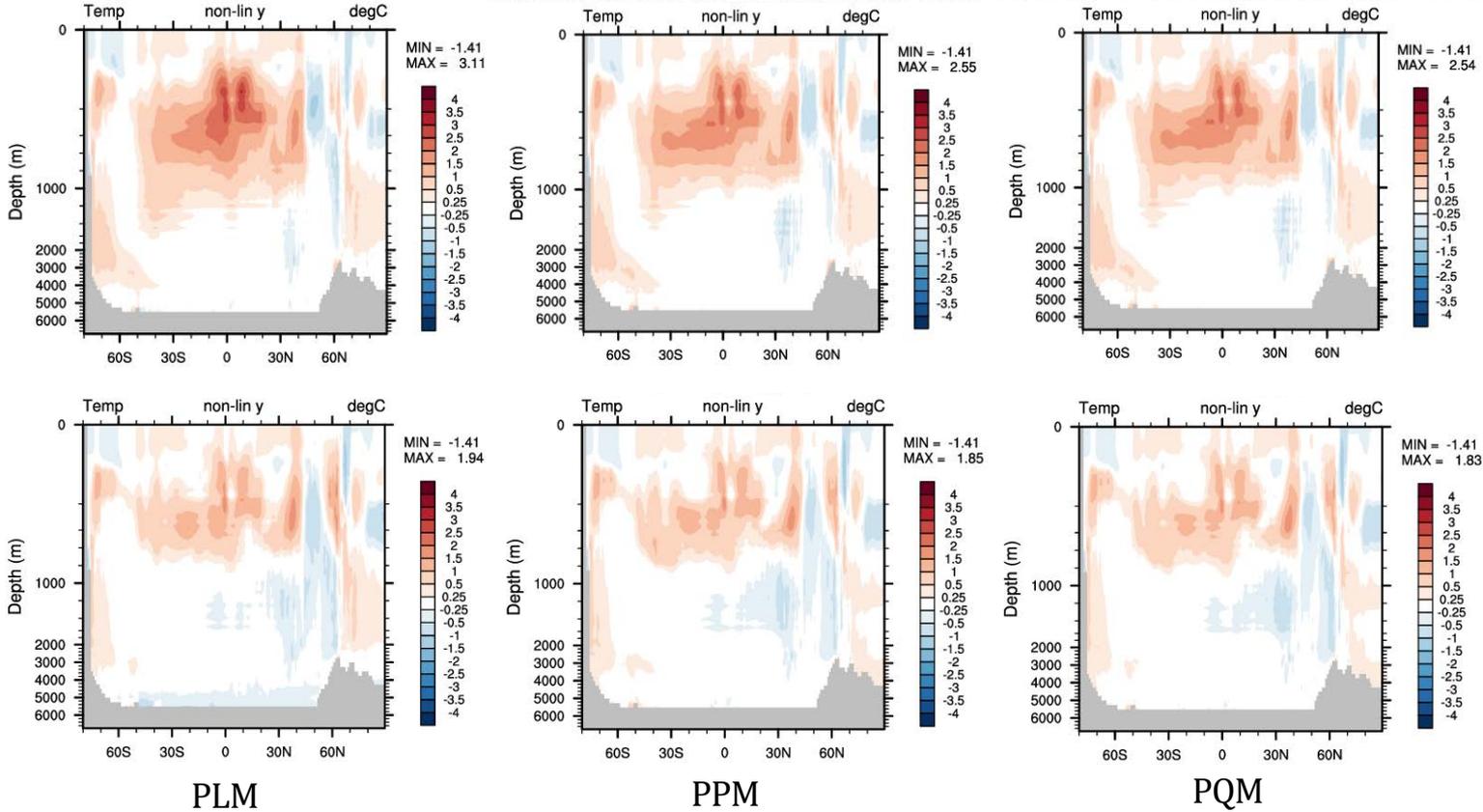


Isopycnic

Hybrid

$p$ -level

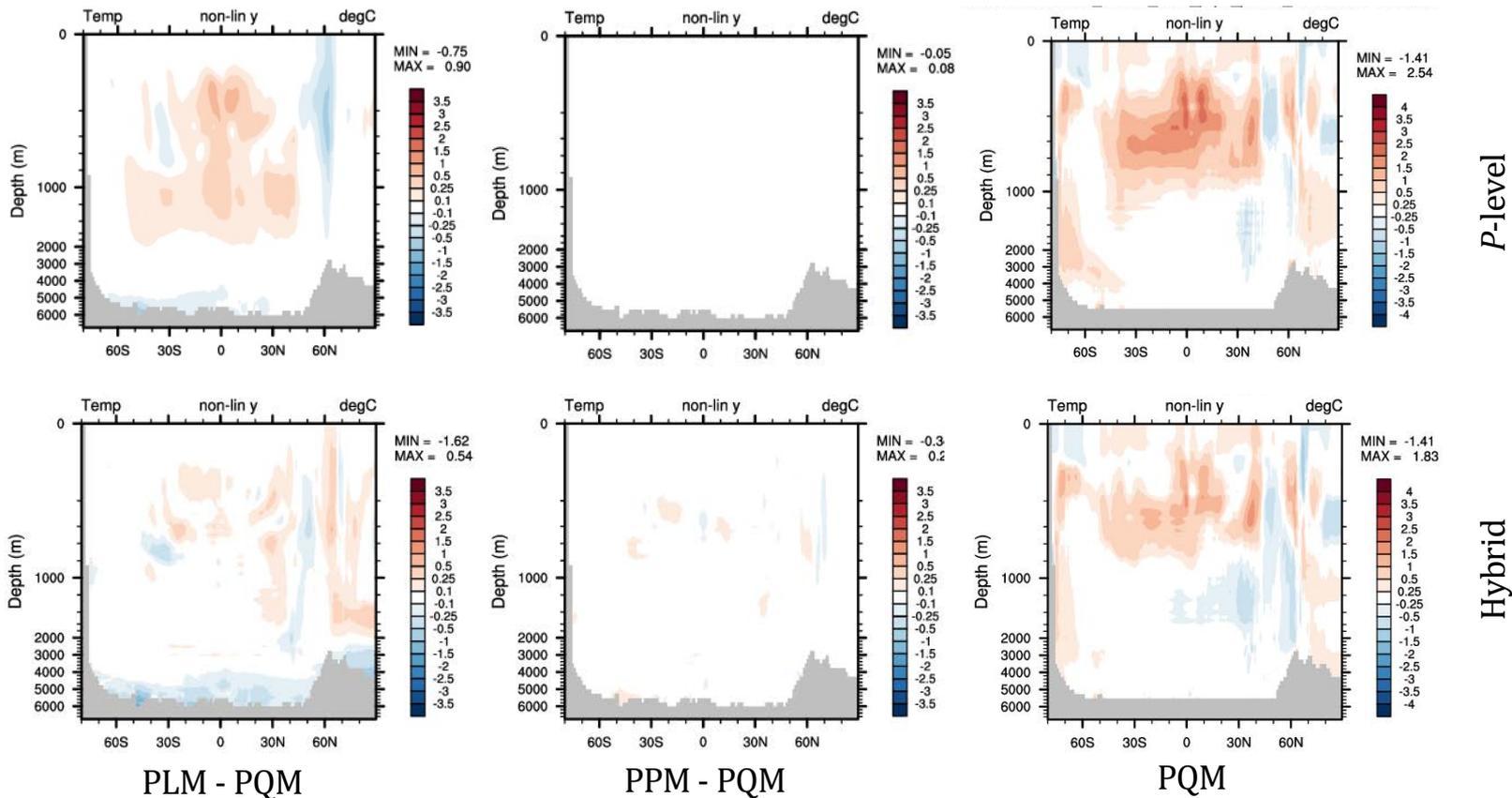
# Impact of reconstruction accuracy



P-level

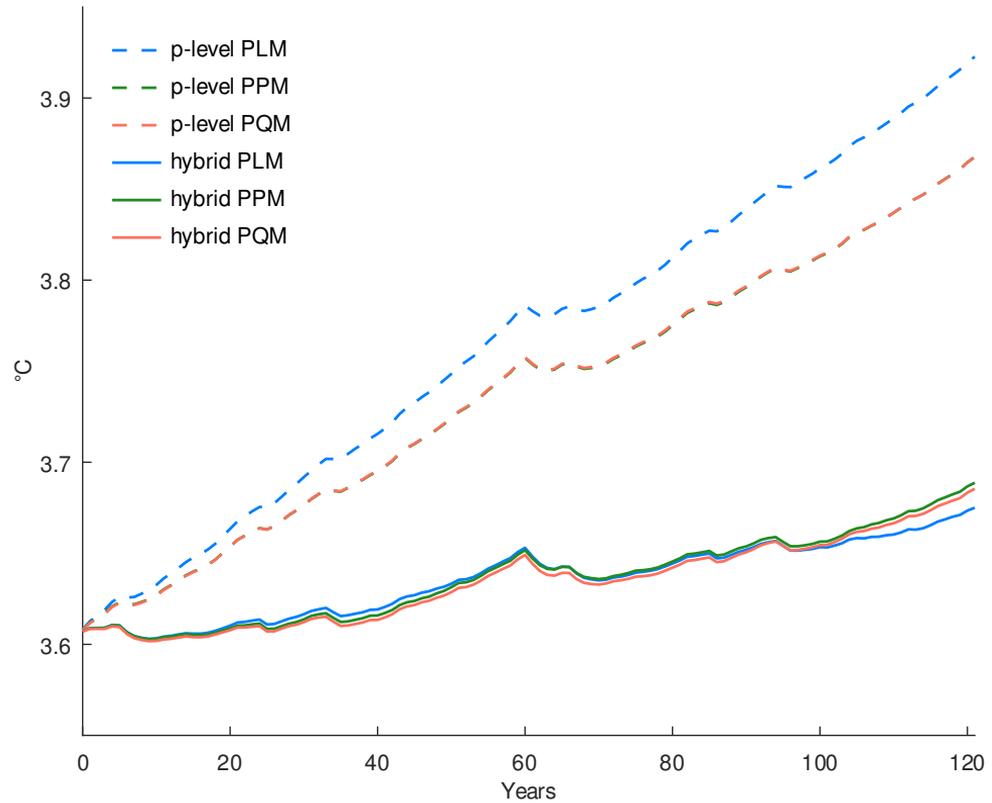
Hybrid

# Impact of reconstruction accuracy



# Impact of reconstruction accuracy

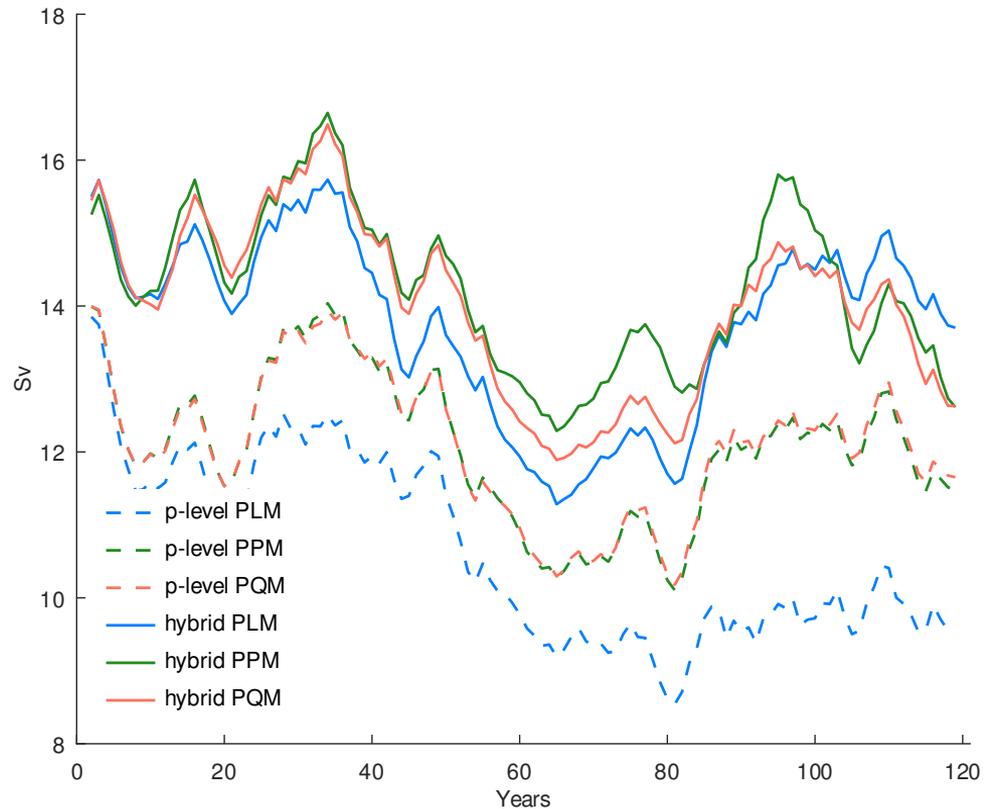
Global mean potential temperature evolution during two OMIP2 cycles.





# Impact of reconstruction accuracy

Evolution of AMOC at 26.5° during two OMIP2 cycles (5 year running mean).

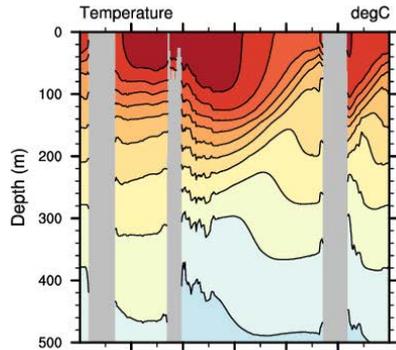


## Summary

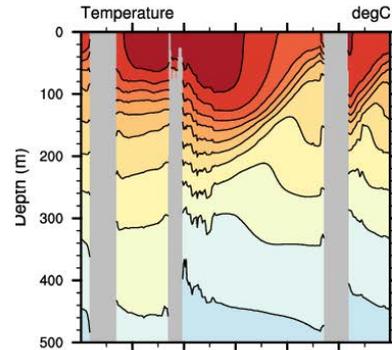
- Several reconstruction methods of different accuracies have been implemented in a robust manner, with multiple limiting options, including a novel non-oscillatory limiter.
- Realism of simulations are sensitive to choice of limiter and regridding approach.
- Physical parameterizations have been adapted to use with the ALE method.
- For reconstruction, small gains were found by going higher in accuracy than PPM with non-oscillatory limiter.
- The vertical reconstruction, (direct) regridding and remapping functionality are implemented in a standalone Fortran module.



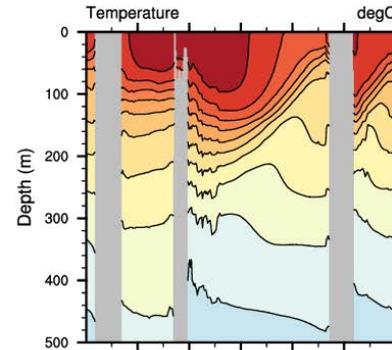
# Impact of reconstruction accuracy



PLM



PPM

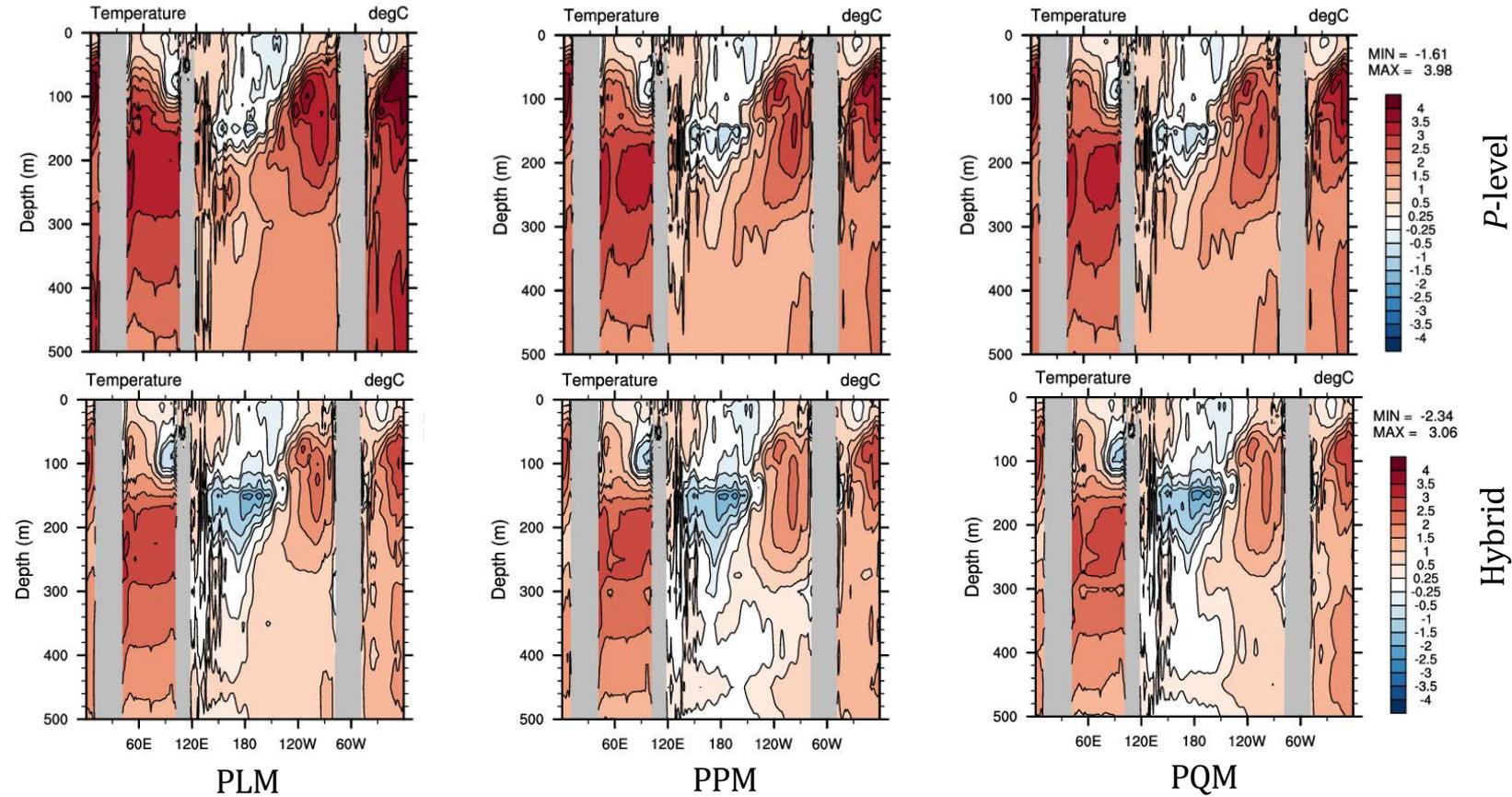


PQM

P-level

Hybrid

# Impact of reconstruction accuracy



# Impact of reconstruction accuracy

