Representation of balanced state in models of geophysical flows

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iow

Game of notes

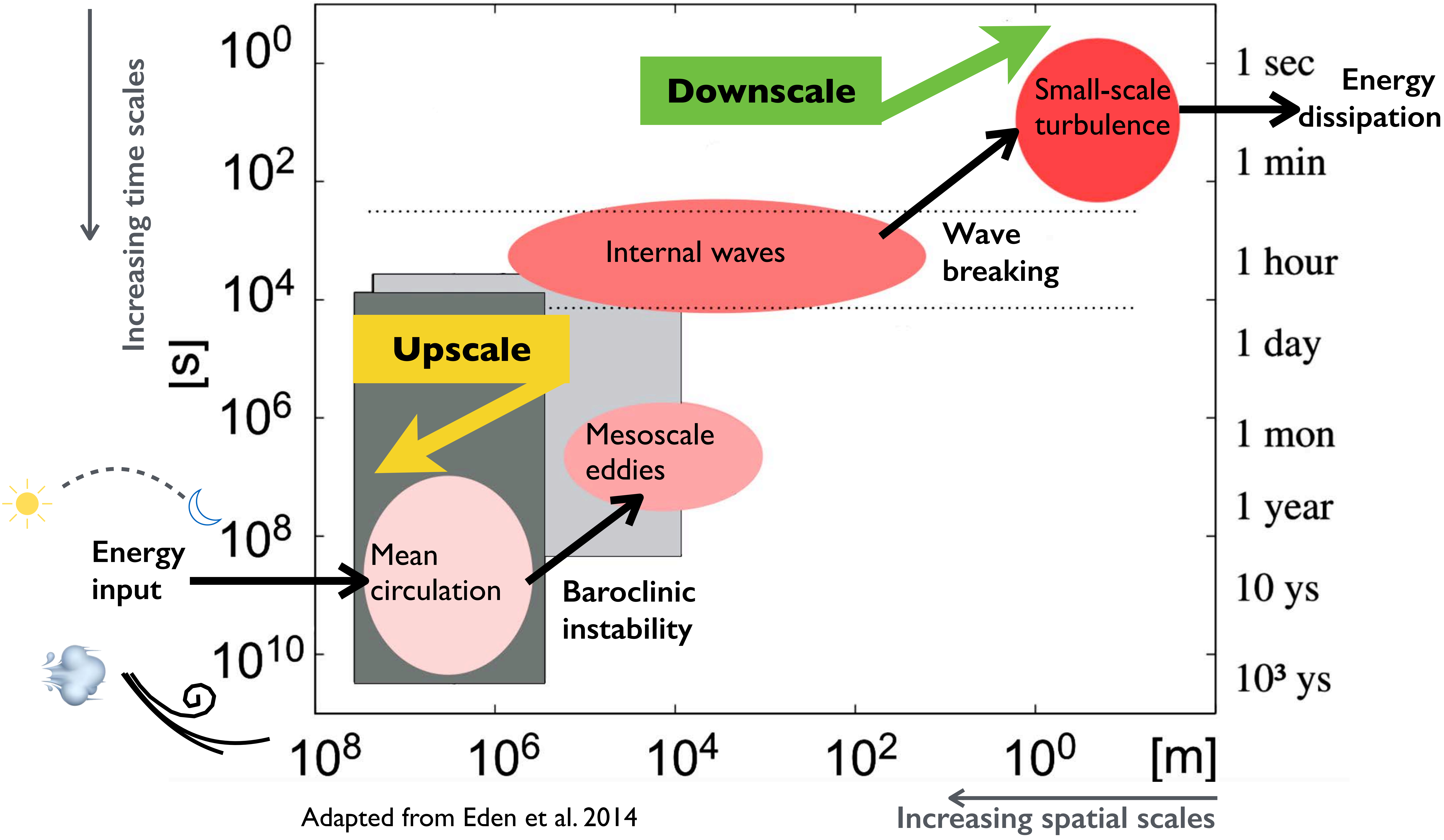
February 12, 1947 : Jule Charney, in a letter to Philip Thompson:

We might say that the atmosphere is a musical instrument on which one can play many tunes. And nature is a musician more of the Beethoven than of the Chopin type.

Low notes refer to the slow rotational motions whereas high notes to the high frequency internal gravity waves.

The bulk of the energy is contained in the slow rotational motions and the amplitude of the high frequency components is small.

Energy pathways- what's missing?



Adapted from Eden et al. 2014

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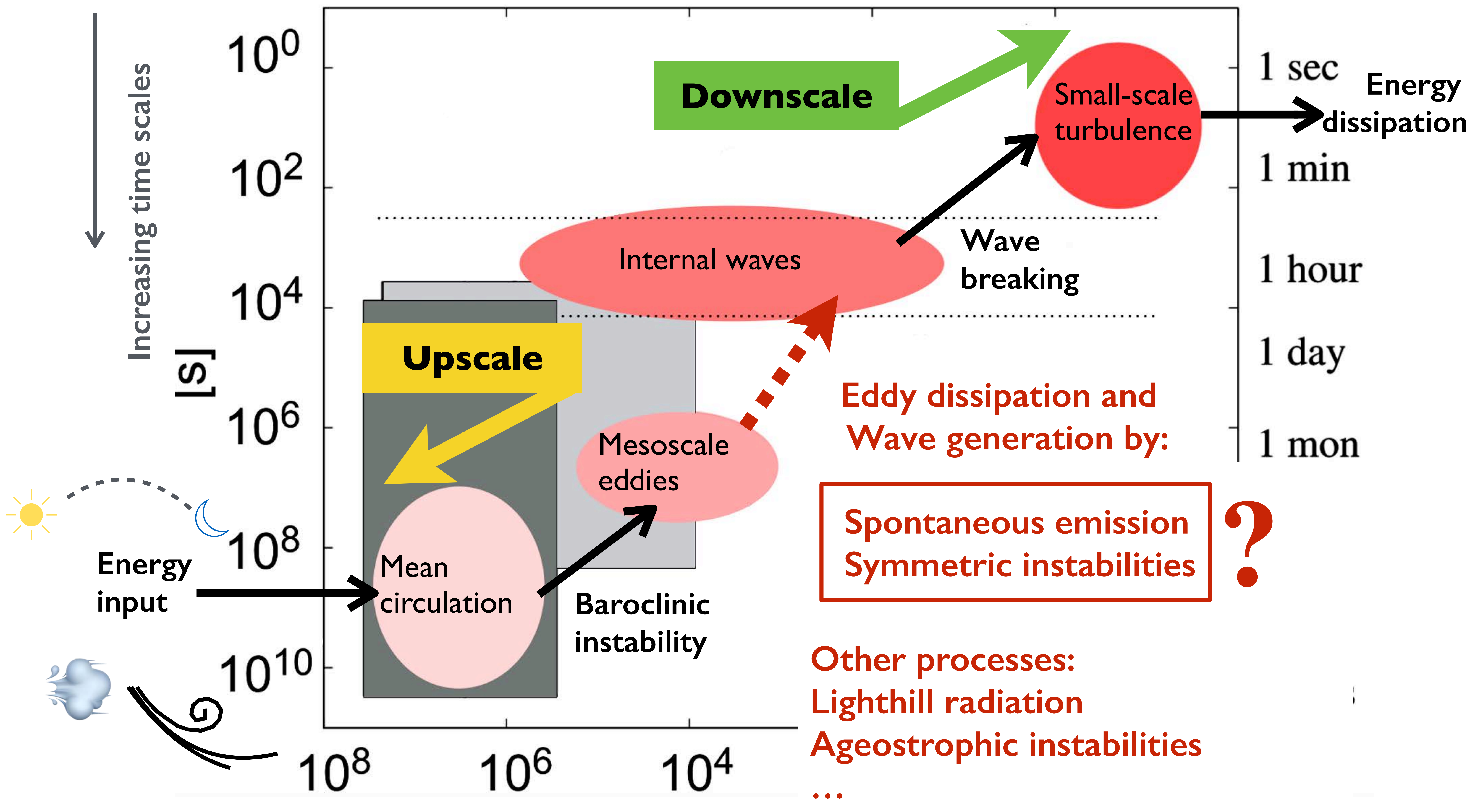

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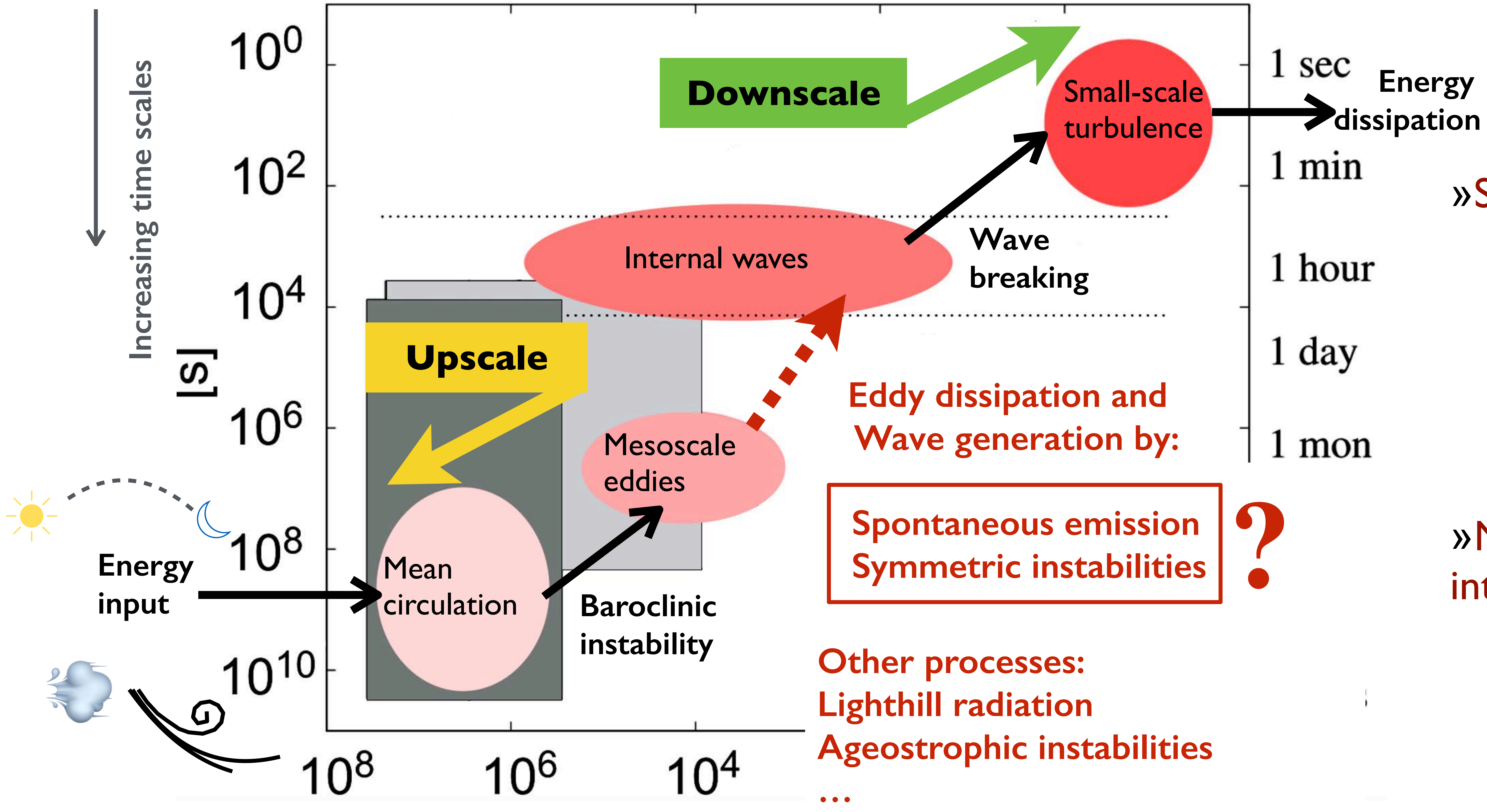


Adapted from Eden et al. 2014

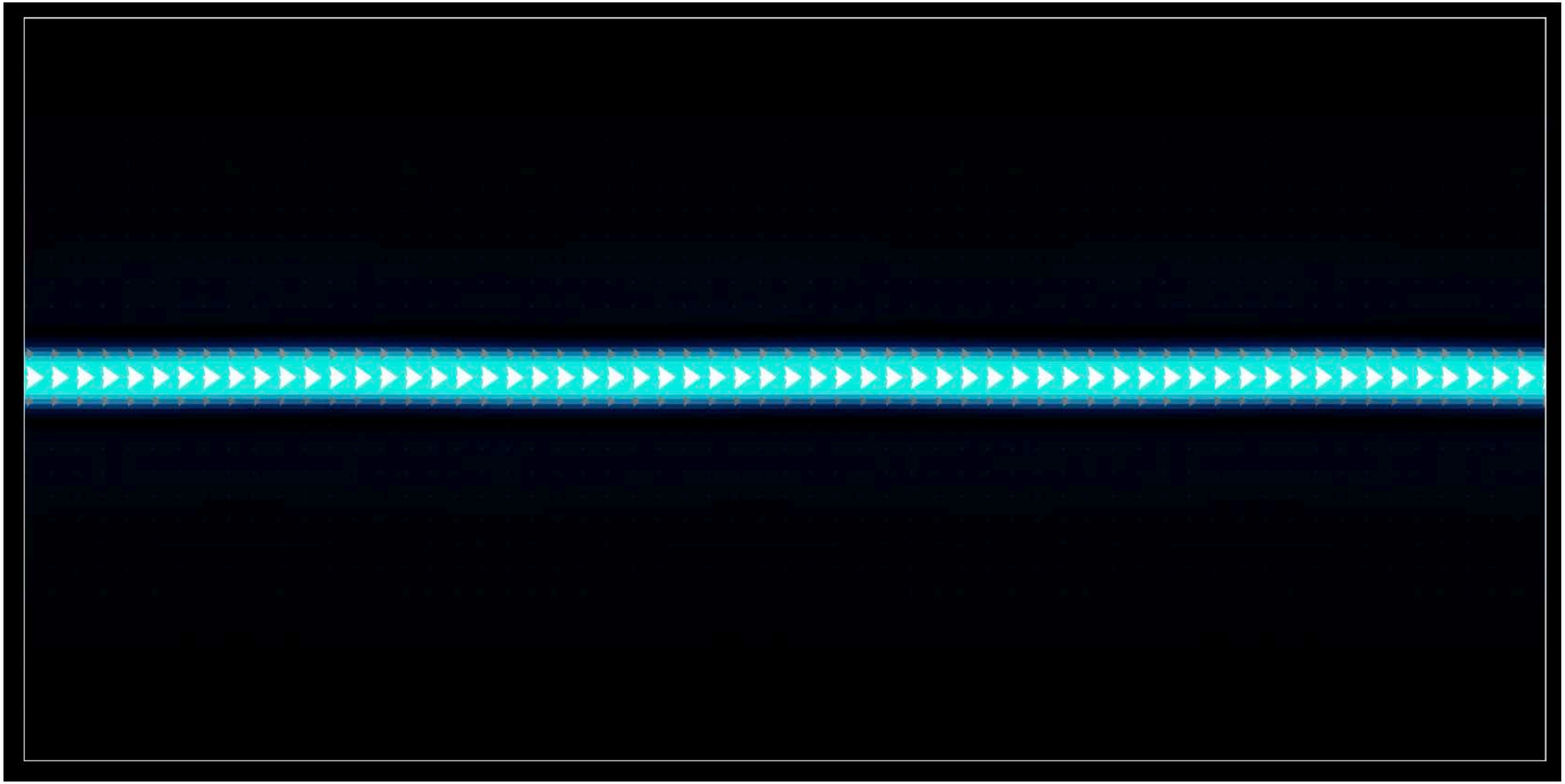
- Scale separation:
  - Timescale separation: fast vs. slow — Ro dependence
- Nonlinear interactions
  - Spontaneous loss of balance
  - Slow manifold

Slow manifold: To be or not to be?

Year	Author(s)	Title	Volume, Pages
1986	Lorenz, E. N.	*On the Existence of a Slow Manifold.*	**43**, 1547–1558.
1987	Lorenz, E. N. and Krishnamurthy, V.	*On the Nonexistence of a Slow Manifold.*	**44**, 2940–2950.
1991	Jacobs, S. J.	*Existence of a Slow Manifold in a Model System of Equations.*	**48**, 893–902.
1992	Lorenz, E. N.	*The Slow Manifold — What Is It?*	**49**, 2449–2451.
1994	Boyd, John P.	*The Slow Manifold of a Five-Mode Model.*	**51**, 1057–1064.
1996	Fowler, A. C. and Kember, G.	*The Lorenz-Krishnamurthy Slow Manifold.*	**53**, 1433–1437.
1996	Camassa, R. and Tin, Siu-Kei	*The Global Geometry of the Slow Manifold in the Lorenz-Krishnamurthy Model.*	**53**, 3251–3264.

From Lynch 2000

Spontaneous wave emission from balanced flow



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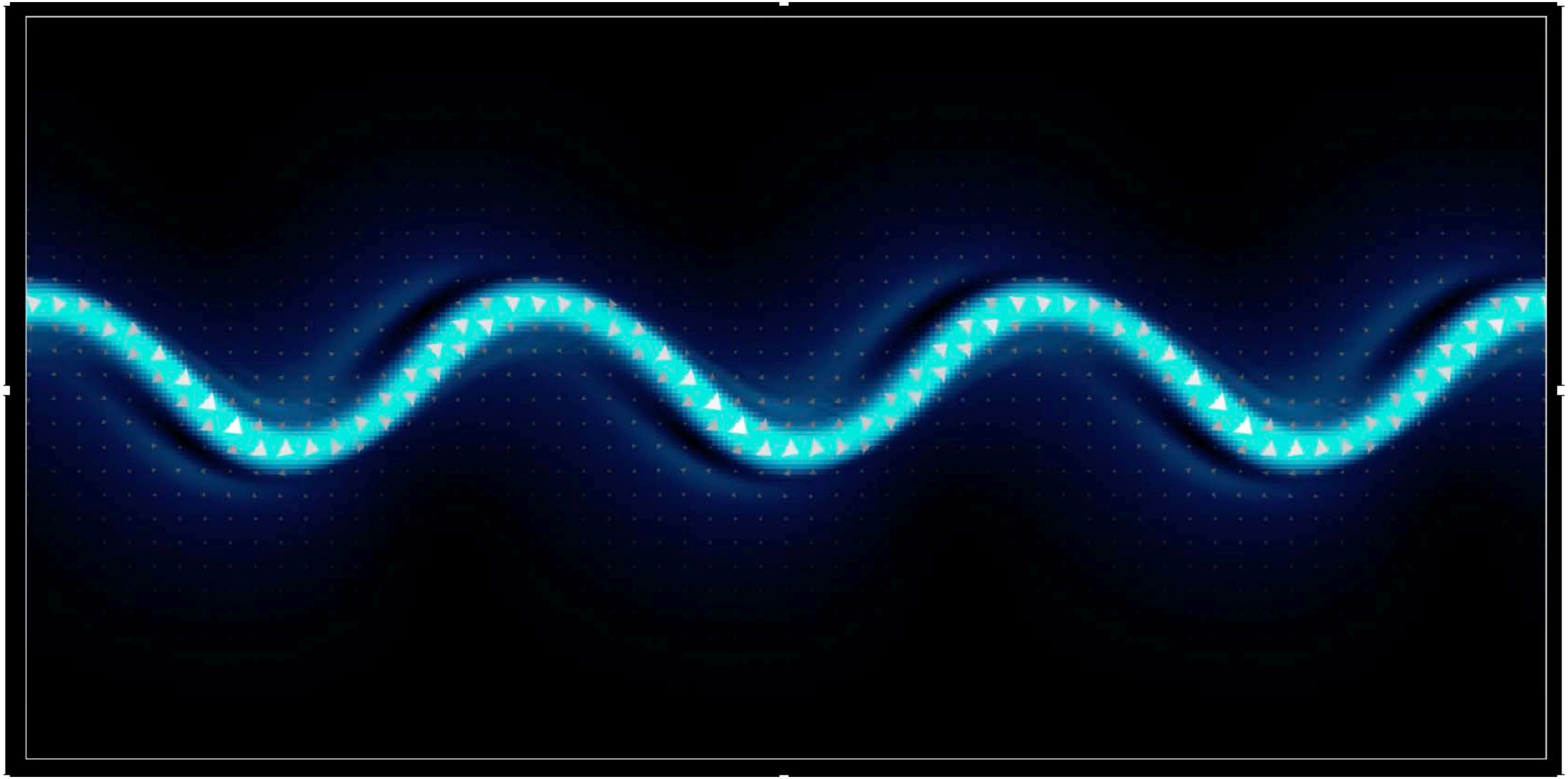

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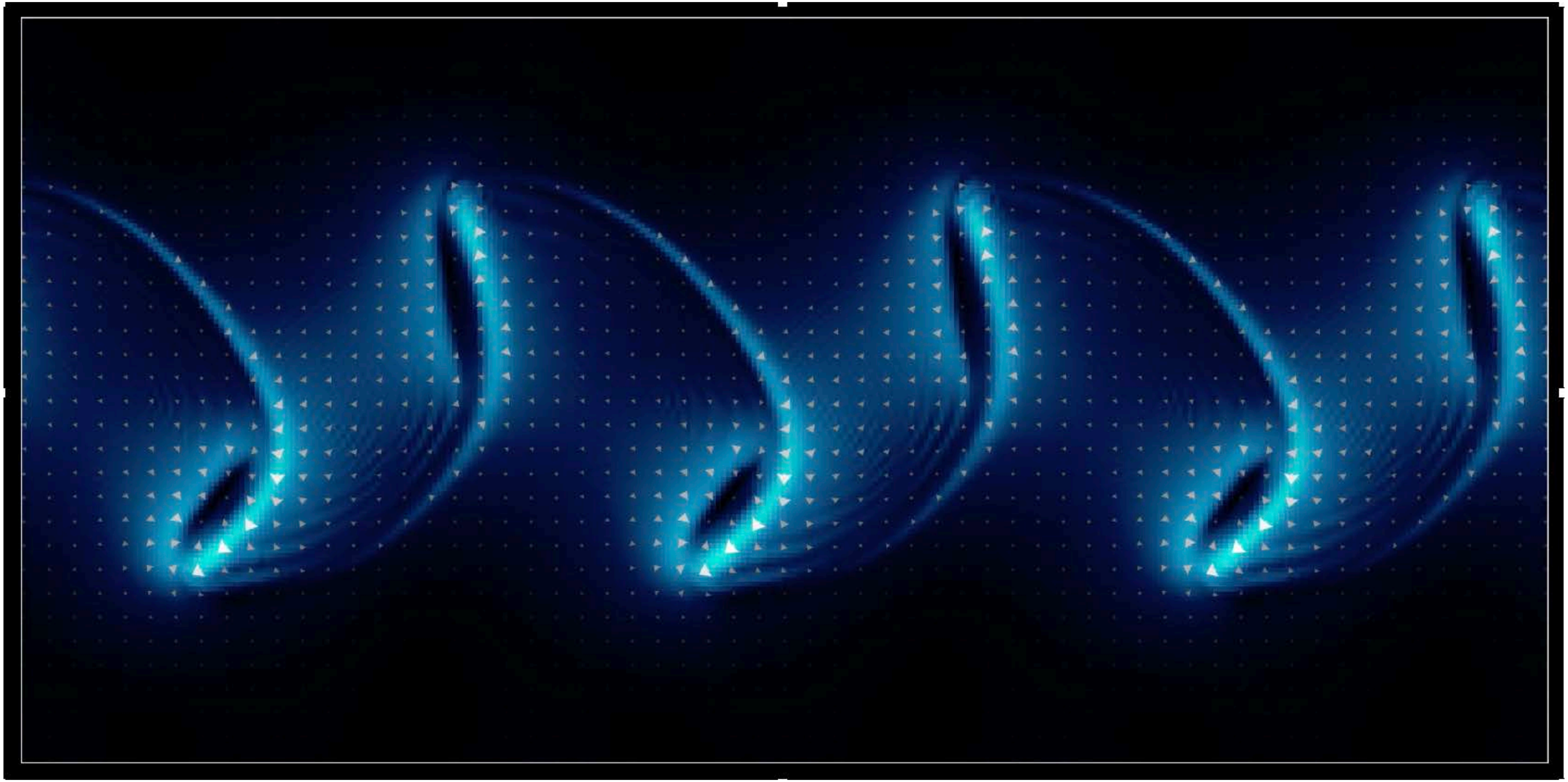

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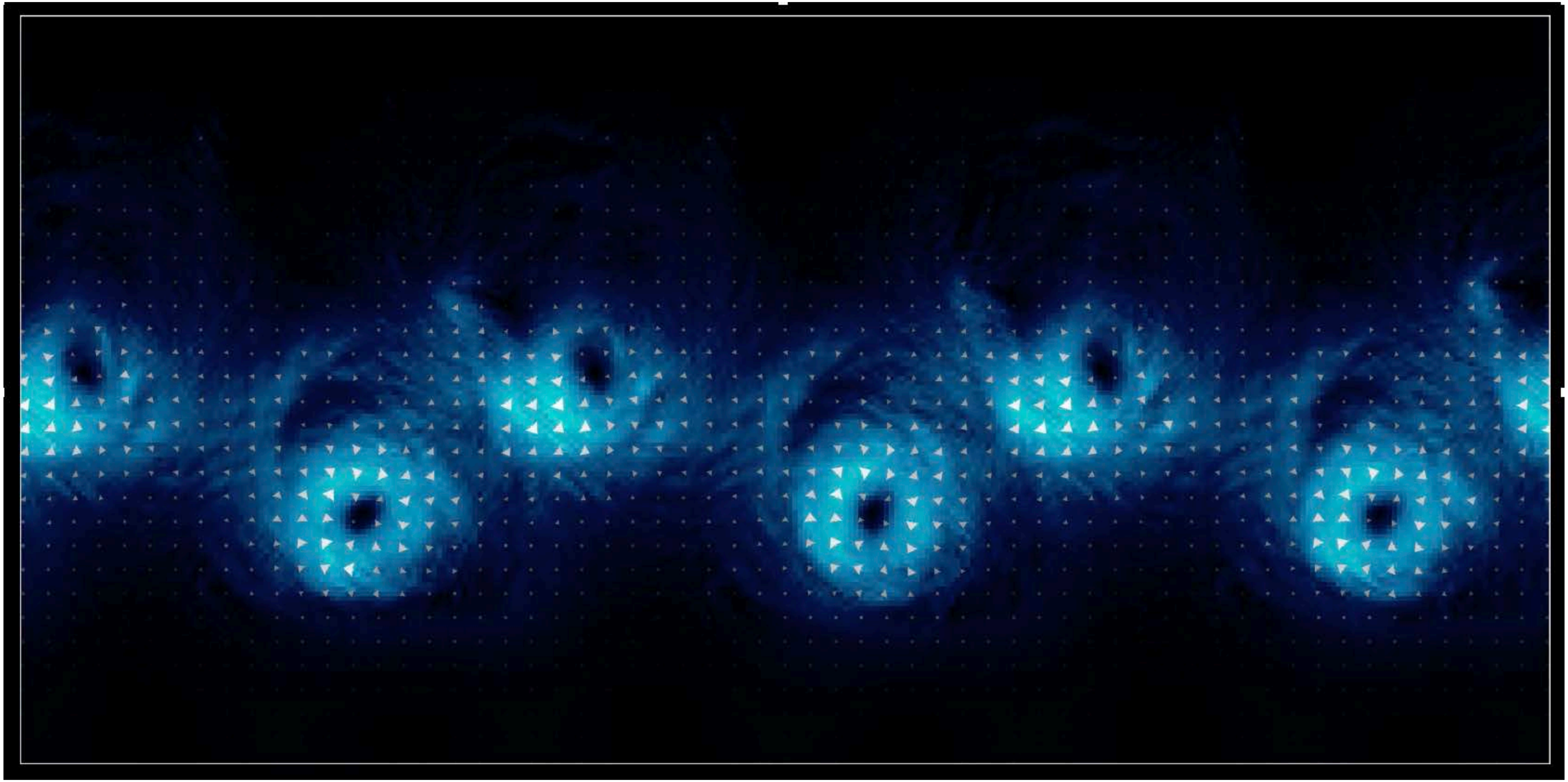
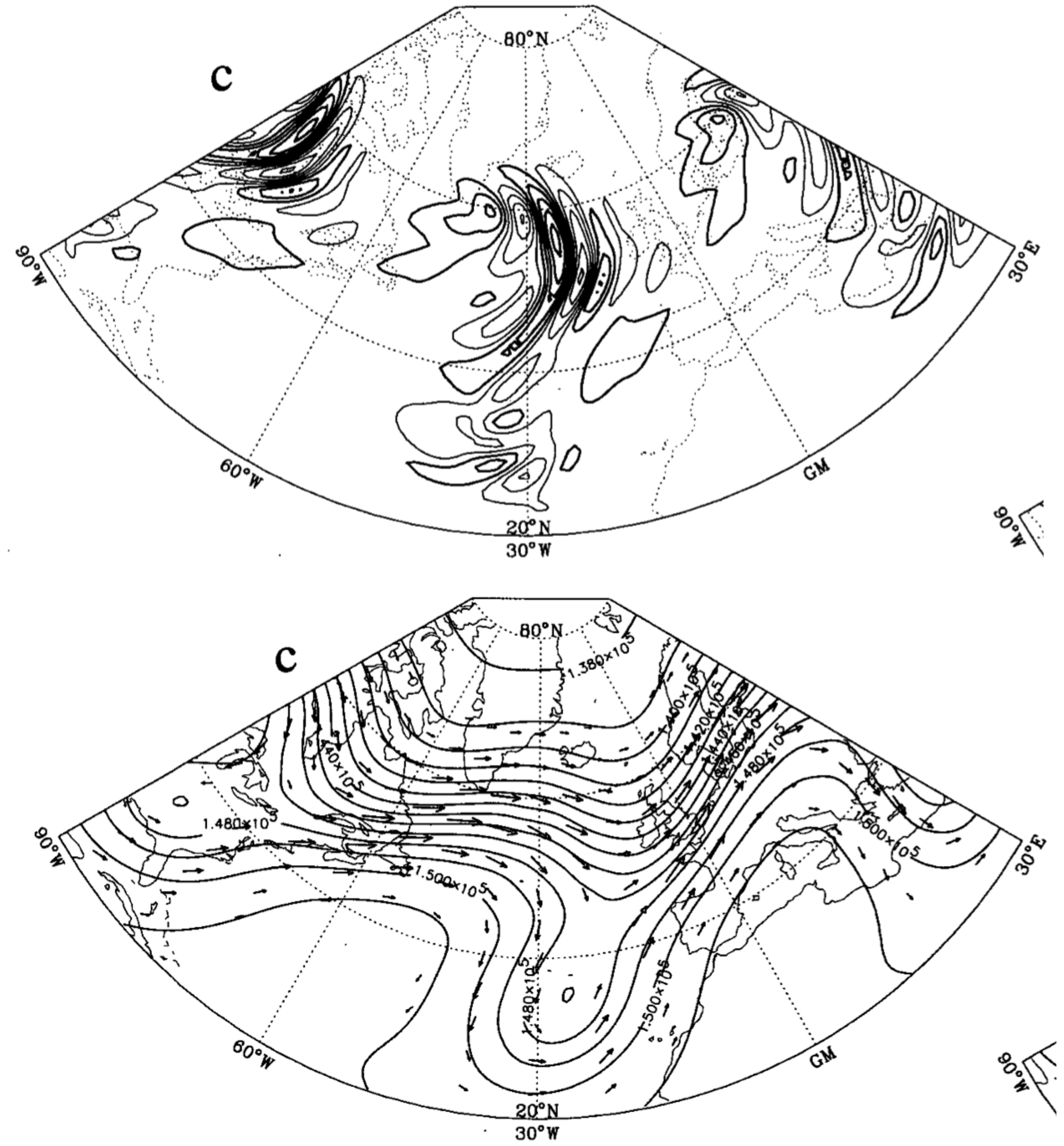


Diagnosing waves in atmosphere and ocean

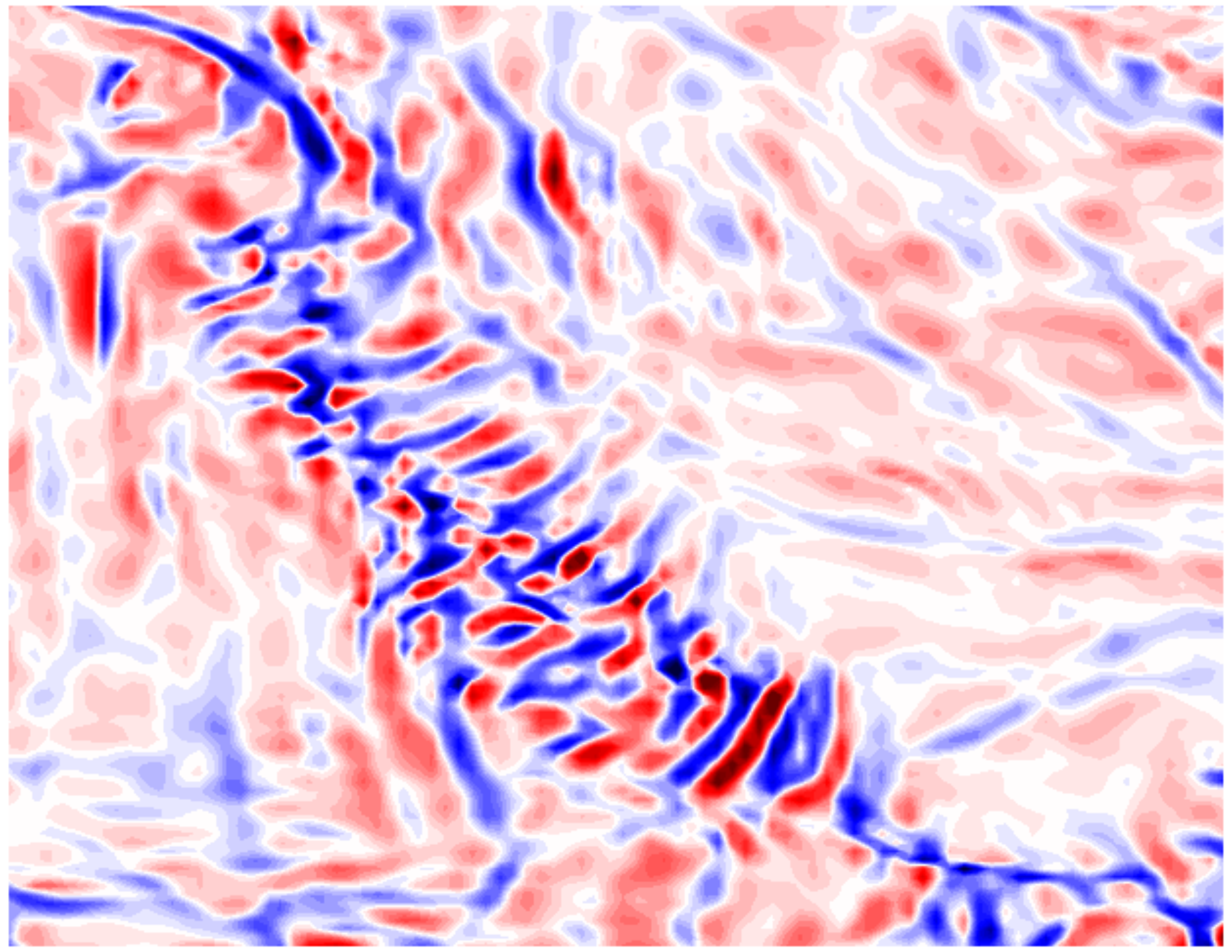
» Are these true wave signals or 'apparent wave signals' (slaved modes)?

Horz. velocity divergence and geopotential (lower) at 130 mb (upper troposphere) in idealized simulation



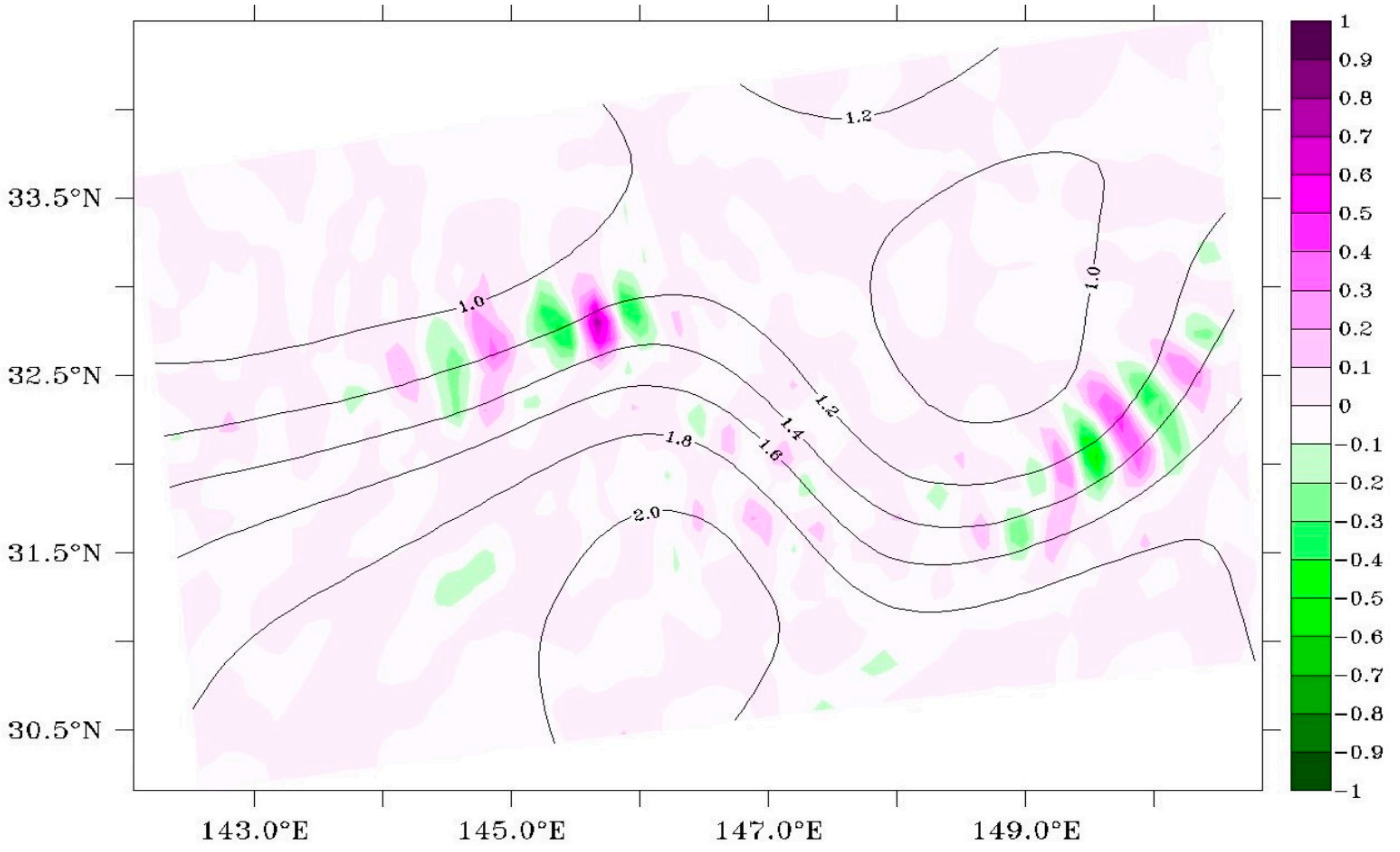
from O'Sullivan and Dunkerton ('95)

Vertical velocity in a model of baroclinic instability



From Chouksey et. al ('18)

High-pass filtered w and pressure (contours) in the Gulf Stream

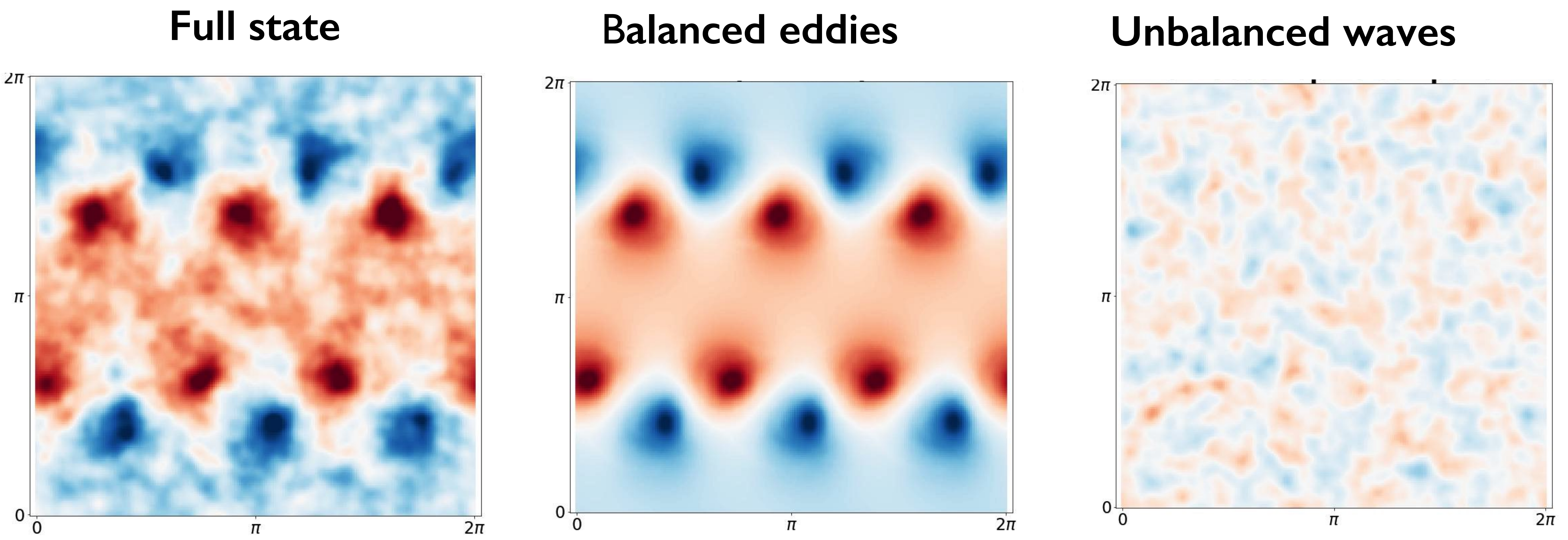


from von Storch, Badin, and Oliver ('19)

Determination of Balance

$$\partial_t \hat{z} - i\boldsymbol{A} \cdot \hat{z} = Ro\, \hat{\boldsymbol{n}}$$

Linear   Nonlinear



- **Higher Order Balance**
  - Chouksey et al. 2022 (JPO)
  - Eden et al. 2019 (JPO)
- Two timescales: slow and fast
- Expansion in Rossby number
- up to 4th order

- **Optimal Balance**
  - Masur and Oliver 2020 (JPO)
- Iterative forward-backward integration
- optimal solution of the balanced state

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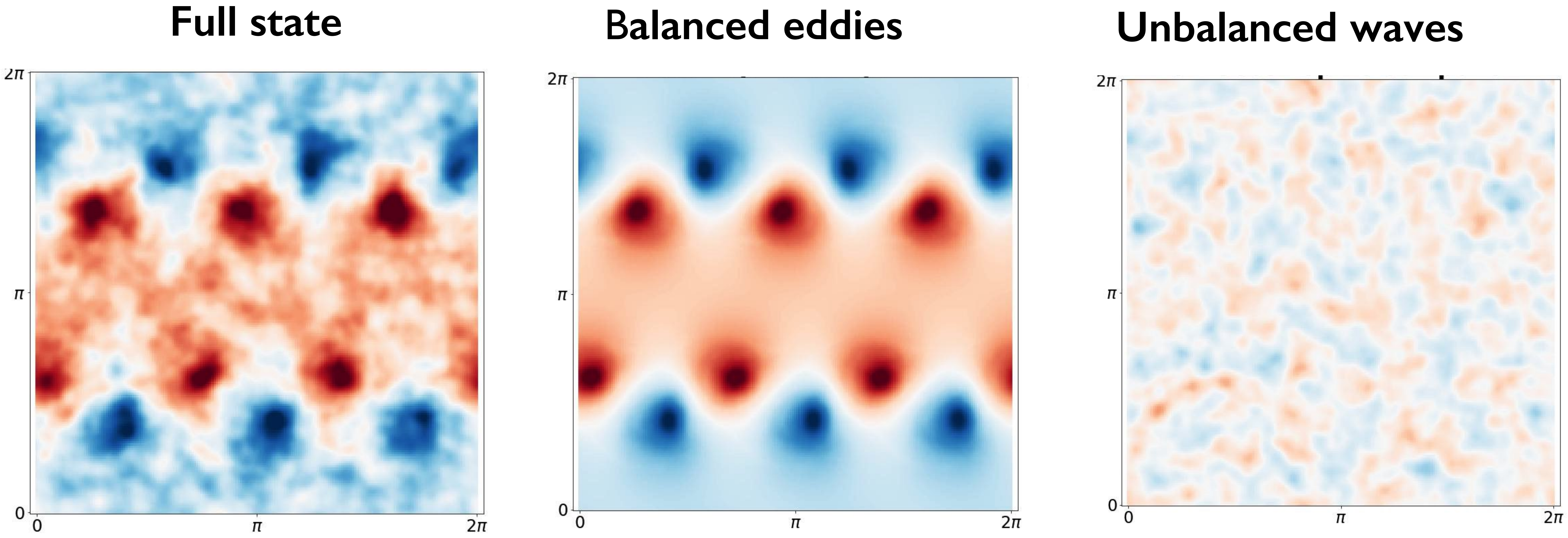

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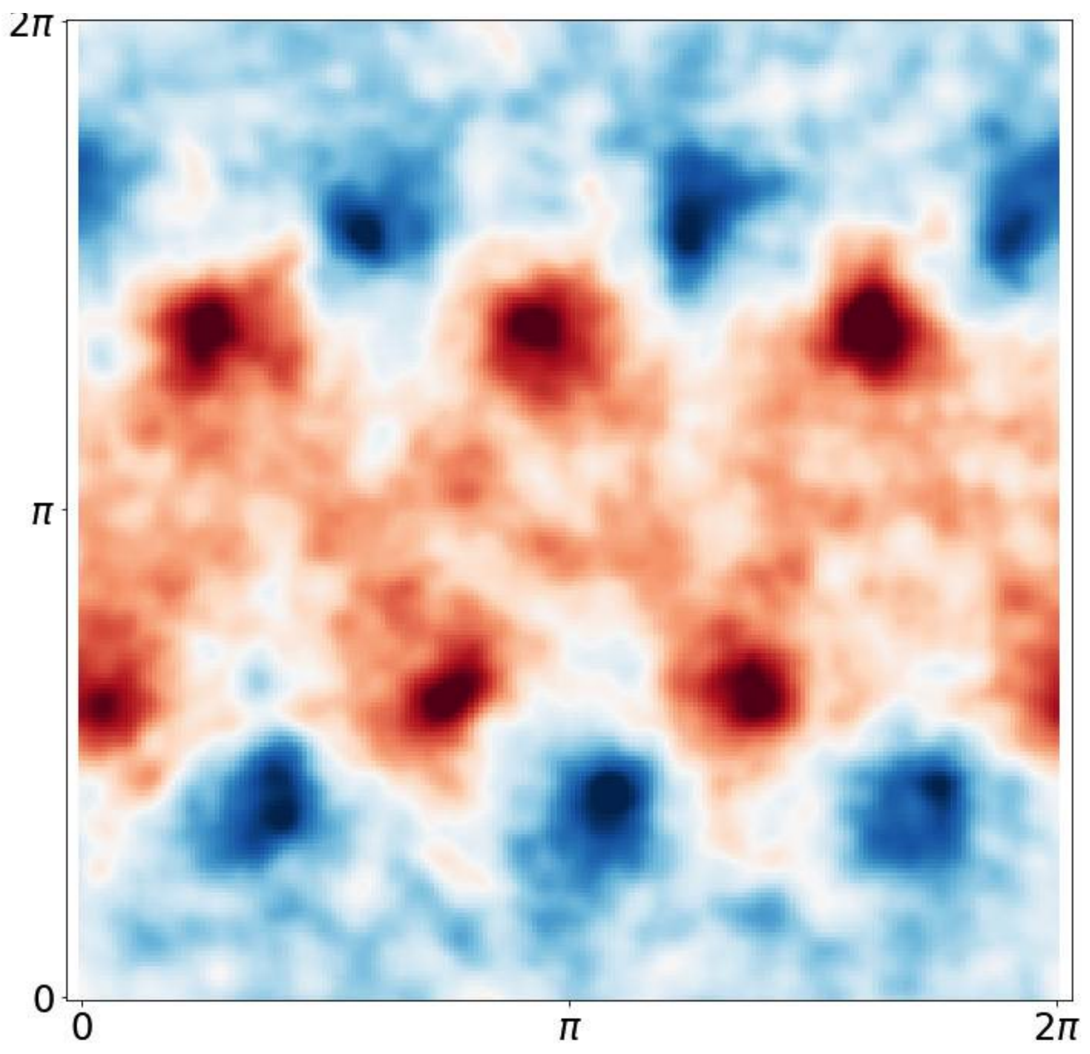
» **Implementation in different**
  » **Models**
  » **Codes**
  » **Configurations**
  » **Discretizations**
  » **Methods**
  » **Regimes**

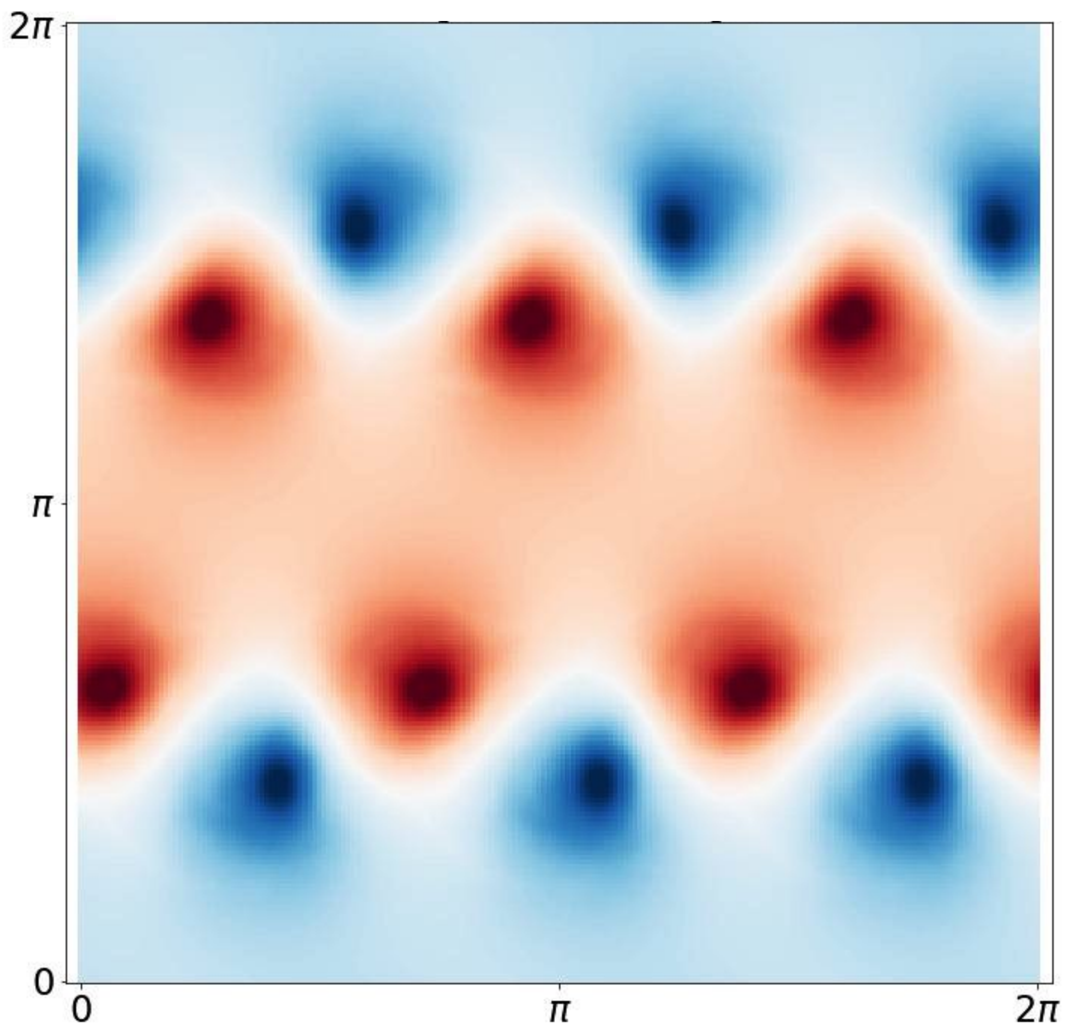
Determination of Balance

$$\partial_t \hat{\boldsymbol{z}} \underbrace{- i\boldsymbol{A}\cdot\hat{\boldsymbol{z}}}_{\text{Linear}} = \underbrace{Ro\,\hat{\boldsymbol{n}}}_{\text{Nonlinear}}$$

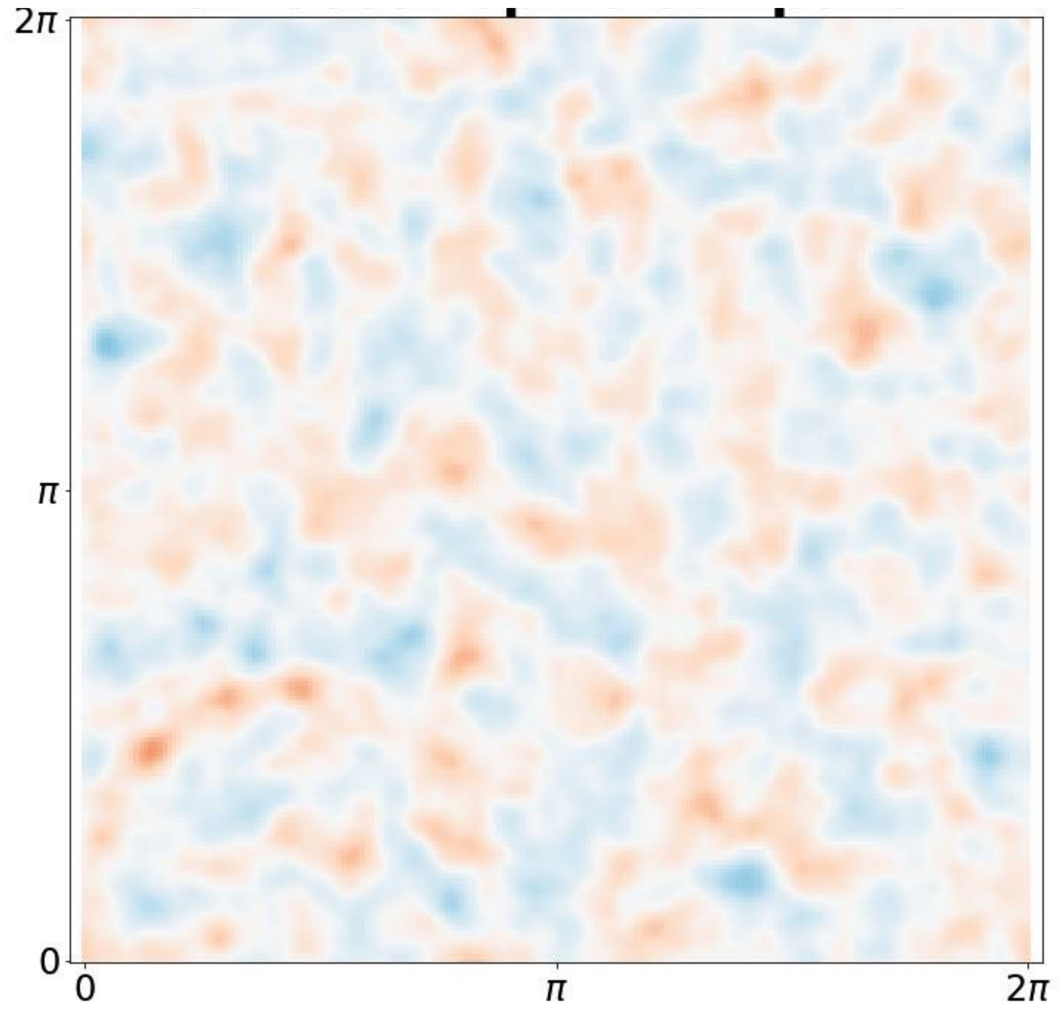
Full state



Balanced eddies



Unbalanced waves



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» **Optimal Balance with Time Averaging**
- »Rosenau 2023 N E W !!

» Higher accuracy, no Fourier transform
» Realistic flows: boundaries, ß-plane

Nonlinear modal decomposition

Non-linear normal mode initialization (NNMI)
Machenhauer (1977), Baer and Tribbia (1977), Warn et. al (1995)

Single layer model (scaled):

$$\partial_t \boldsymbol{u} + \underset{\neg}{\boldsymbol{u}} + \boldsymbol{\nabla} h = -Ro\ \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} \qquad \partial_t h + c^2 \boldsymbol{\nabla} \cdot \boldsymbol{u} = -Ro\ \boldsymbol{\nabla} \cdot h\boldsymbol{u}$$

Fourier space: vector $\hat{z}(\boldsymbol{k}) = (\hat{u}, \hat{v}, \hat{h})^T$

$$\partial_t \hat{z} - \underbrace{i\boldsymbol{A} \cdot \hat{z}}_{\text{Linear}} = \underbrace{Ro\ \hat{\boldsymbol{n}}}_{\text{Non-linear}}$$

$$\boldsymbol{A} = \begin{pmatrix} 0 & -i & -k_x \\ i & 0 & -k_y \\ -c^2 k_x & -c^2 k_y & 0 \end{pmatrix}$$

from C-grid discrete operators

	Balanced mode	**Unbalanced mode**
Eigenvalues:	$\omega^0 = 0$	$\omega^\pm = \pm\sqrt{1 + c^2 k^2}$
Eigenvectors:	$\boldsymbol{q}^0,\ \boldsymbol{p}^0$	$\boldsymbol{q}^\pm,\ \boldsymbol{p}^\pm$

Projection: mode amplitude $g^s = \boldsymbol{p}^s \cdot \hat{z}$ with $s = 0, \pm$

Higher order decomposition

» **Modal representation:** $\partial_t g^s - i\omega^s g^s = Ro\, \mathbf{p}^s \cdot \hat{\mathbf{n}} = -iRo\, I^s(g^0, g^\pm)$

$$(Ro\, \partial_T + \partial_{t*})\, g^s - i\omega^s g^s = -iRo\left(I^s(g^0,0) + I^s(0,g^\pm) + K^s(g^0,g^\pm)\right)$$

» **Weak interaction assumption:** weakly growing waves $g^\pm = Ro\, f_1^\pm + Ro^2\, f_2^\pm + \ldots$

» expansion in Ro as e.g. in Warn (1996)

» introduce fast and slow time scale with T = Ro t* and $\partial_t = Ro\, \partial_T + \partial_{t*}$

» slow mode g^0 varies on T only, while fast mode $g^\pm$ has two time scales t* and T

for increasing order in Ro:

» **SLOW MODE s=0**

$$\partial_T g^0 = -iI^s(g^0,0)$$
$$\partial_T g^0 = -iI^s(g^0,f_1^\pm) + iI^s(0,f_1^\pm)$$
$$\partial_T g^0 = -iI^s(g^0,f_2^\pm) + iI^s(0,f_2^\pm) - iI^0(0,f_1^\pm)$$

» **FAST MODE s=±**

$$\partial_{t*} f_1^\pm - i\omega^\pm f_1^\pm = -iI^\pm(g^0,0)$$
$$\partial_T f_1^\pm + \partial_{t*} f_2^\pm - i\omega^\pm f_2^\pm = -iK^\pm(g^0,f_1^\pm)$$
$$\partial_T f_2^\pm + \partial_{t*} f_3^\pm - i\omega^\pm f_3^\pm = -iI^\pm(0,f_1^\pm) - iK^\pm(g^0,f_2^\pm)$$

» suppress any wave generation by $\partial_{t*} f_n^\pm = 0 \rightarrow$ 'slaved' modes f_n

» **Machenhauer(1977)**

» **QG balanced state**

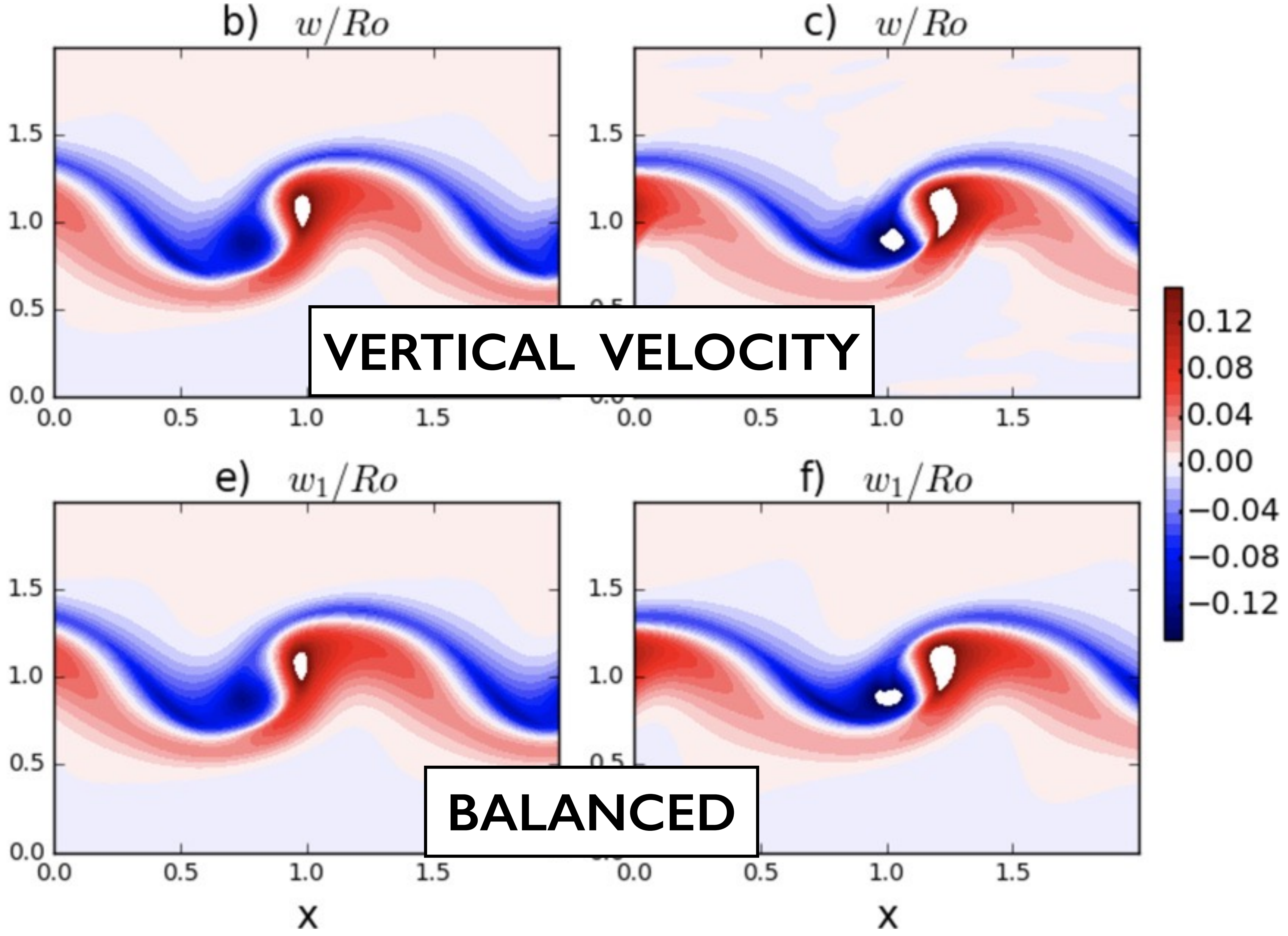
$$\boxed{f_1^\pm = I^\pm(g^0,0)/\omega^\pm}, \quad f_2^\pm = \left(K^\pm(g^0,f_1^\pm) - i\partial_T f_1^\pm\right)/\omega^\pm\ , \ \ldots$$

» first order slaved mode

Wave generation at higher orders

1st ORDER

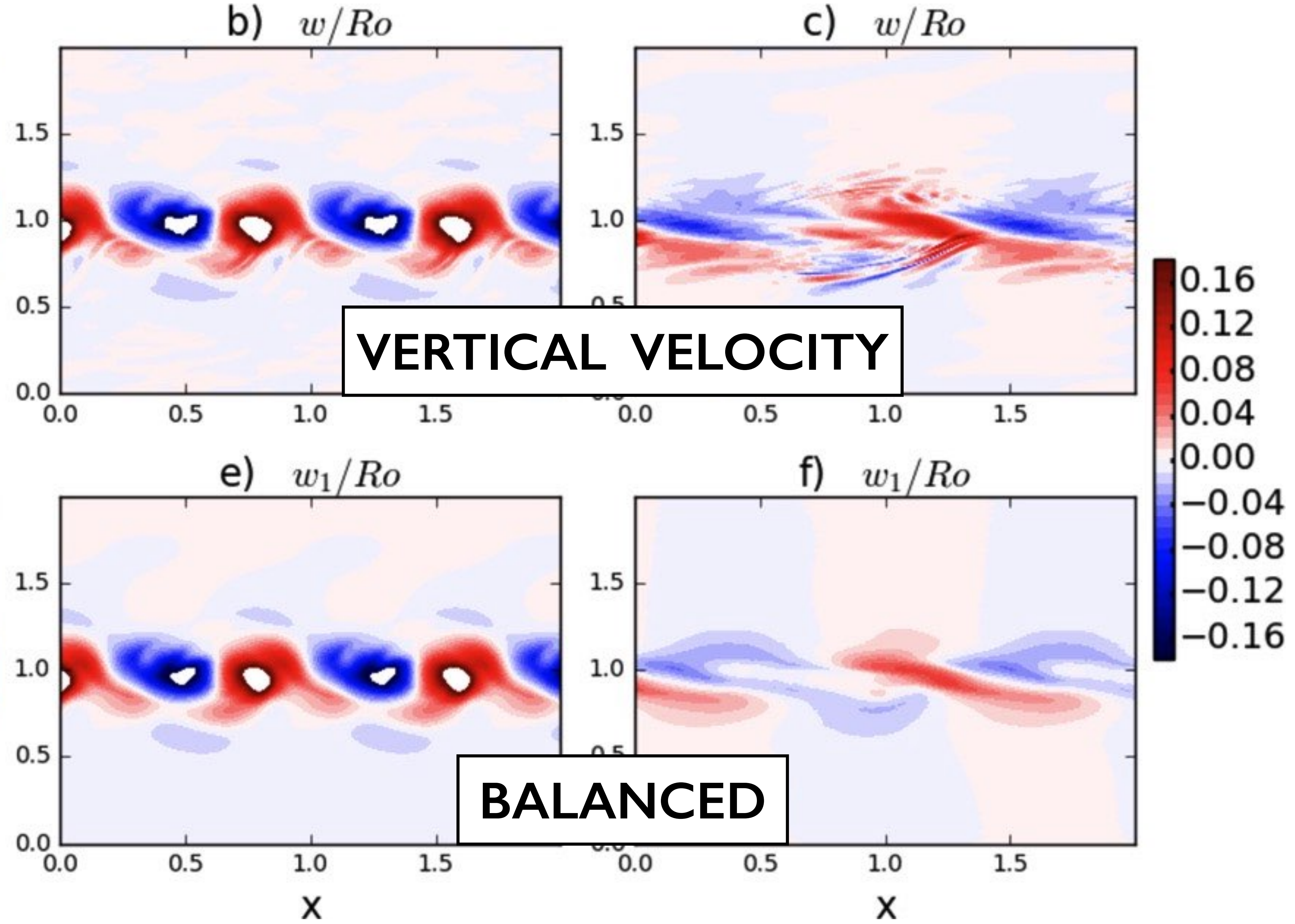
Ro=0.2 **Ro=0.3**



» SPONTANEOUS EMISSION
  » Waves only at higher orders
  » Dominated by slaved modes

1st ORDER

Ro=0.2 **Ro=0.3**



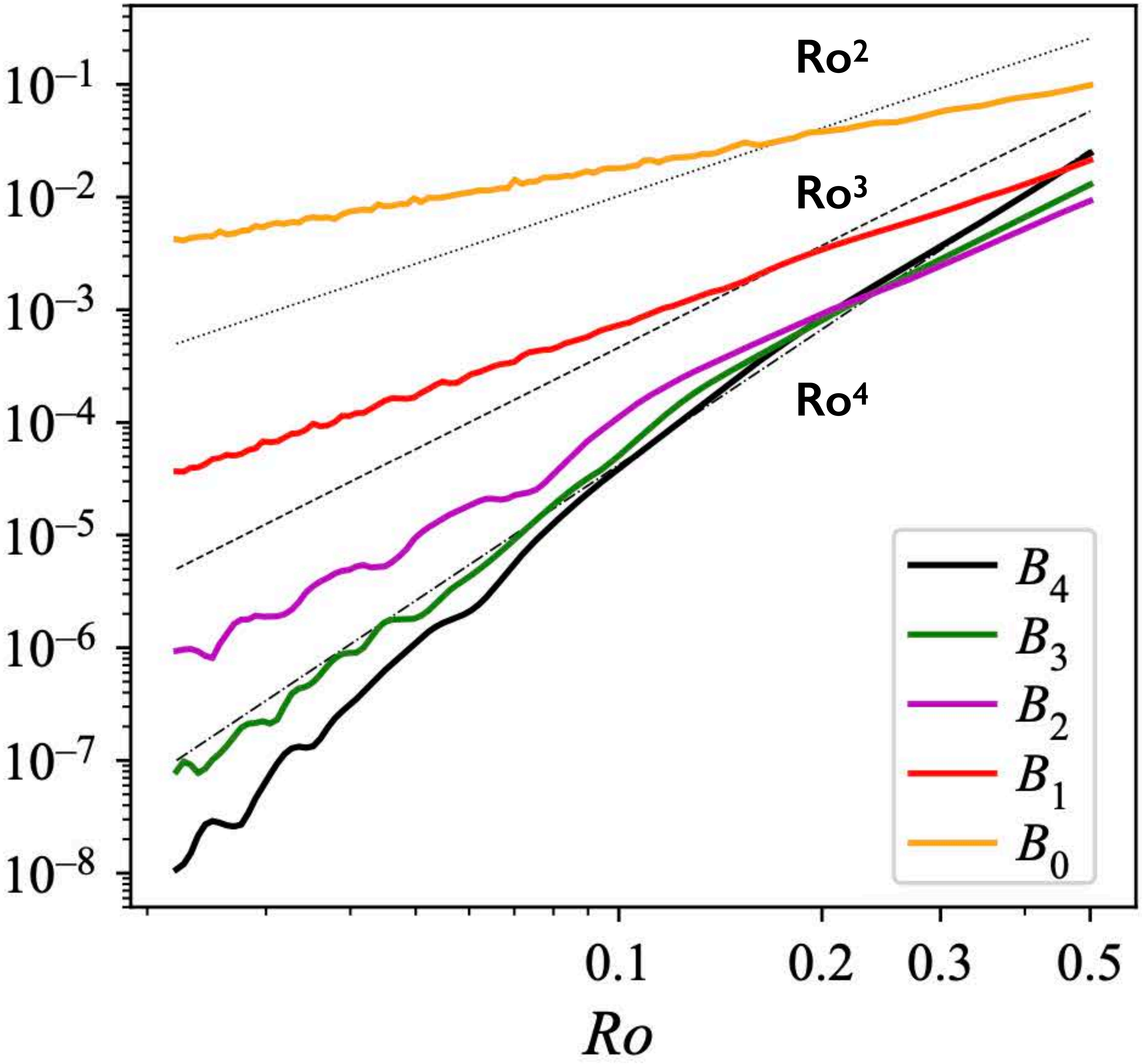
» SYMMETRIC INSTABILITIES
  » Waves already at lower orders
  » More prominent

Diagnosed imbalance: higher orders

Diagnosed imbalance

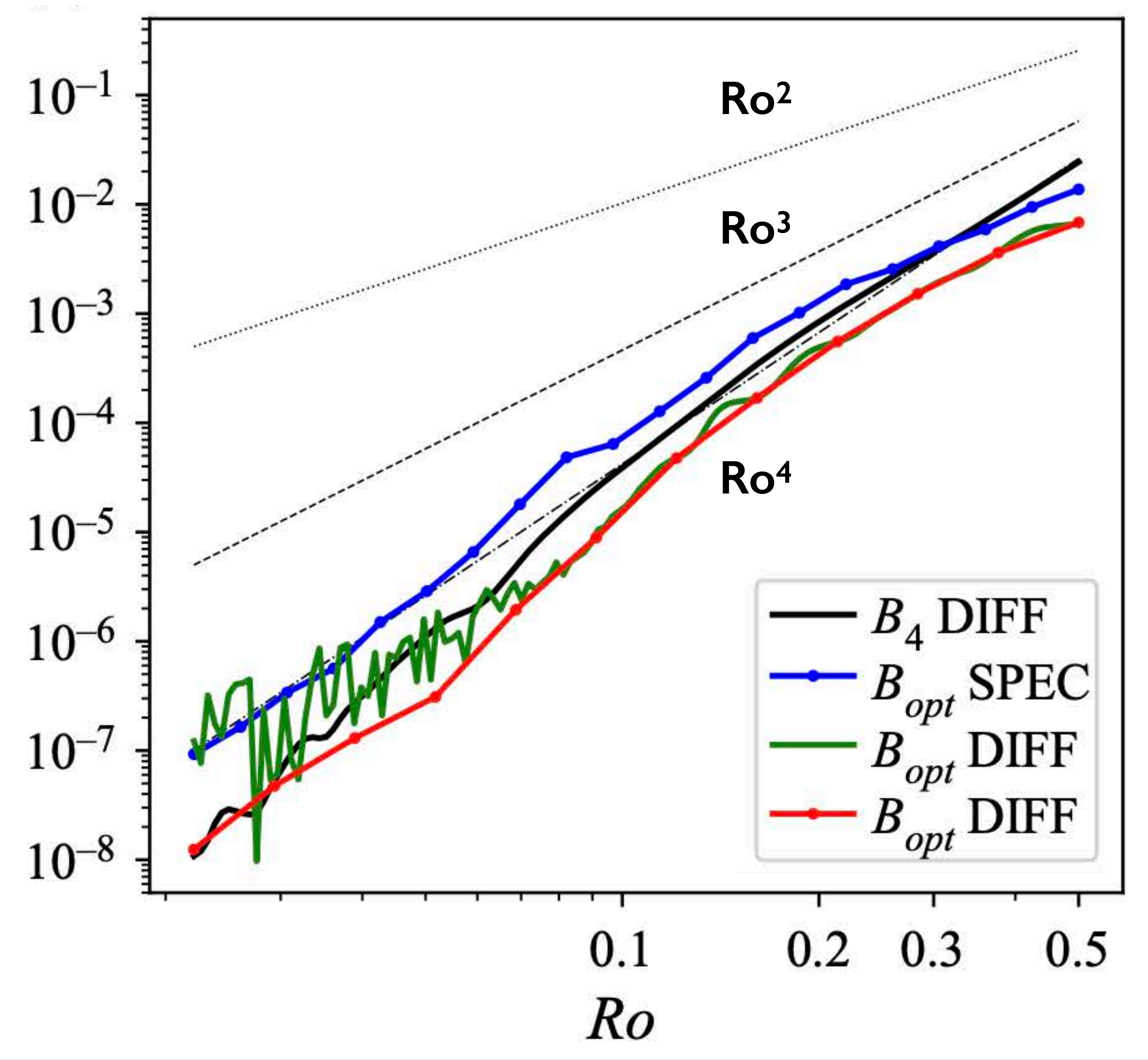
$$I(\boldsymbol{u}) = \frac{\|\boldsymbol{u}' - \boldsymbol{u}''\|}{\frac{1}{2}\left(\|\boldsymbol{u}'\| + \|\boldsymbol{u}''\|\right)}$$

evolved state $\boldsymbol{u}'$
rebalanced state $\boldsymbol{u}''$
norm (rms) $\|..\|$



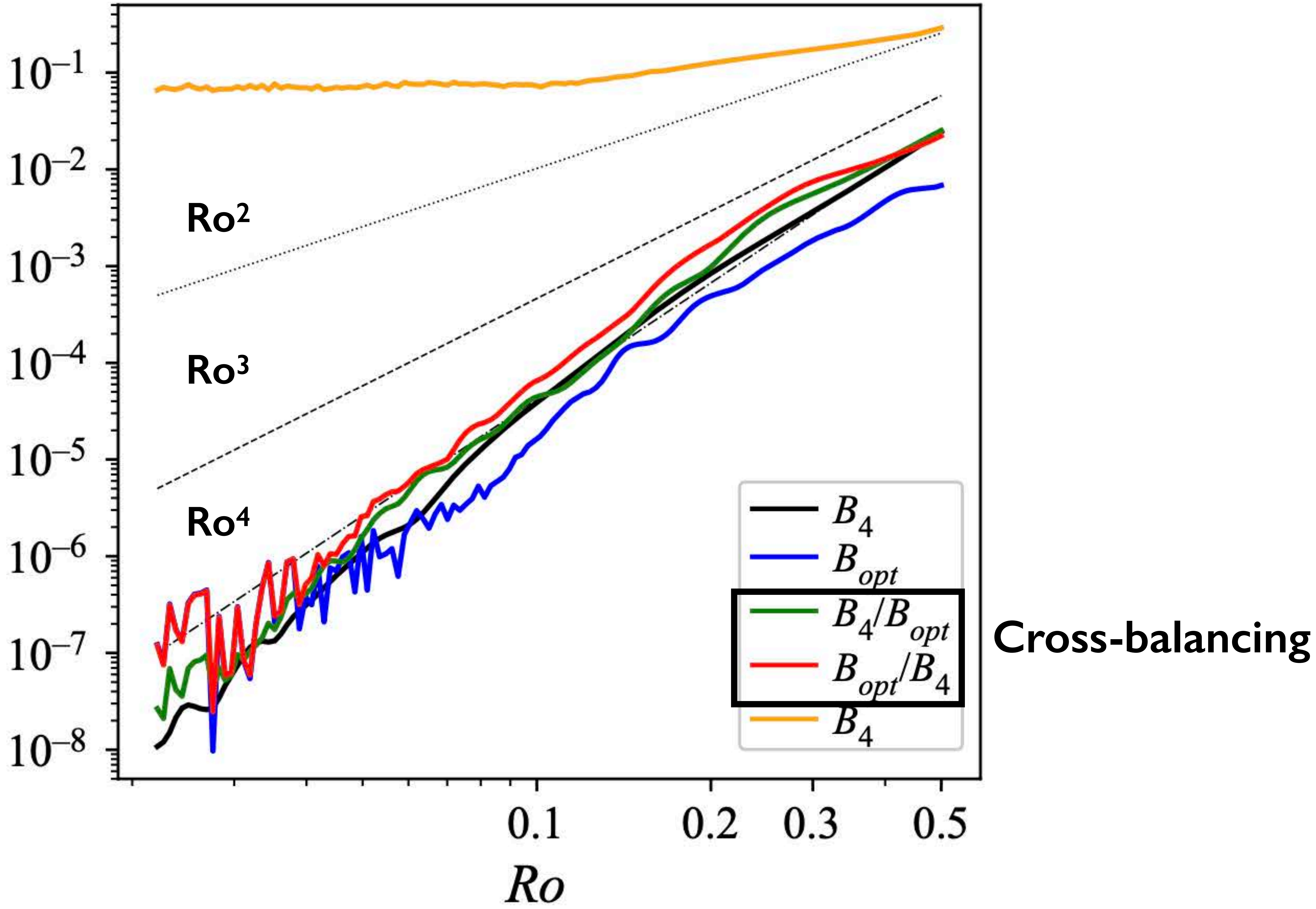
Preservation of Balance

- The quality of preservation of balance might depend on the numerical scheme (e.g. Mohebalhojeh & Dritschel 2000).
- Here, we show that adapting the notion of balance when changing between the finite difference and the spectral scheme yields comparably very good preservation of balance.
- Mixing notions of balance across numerical schemes—> quality of preservation of balance drops.



Diagnosed imbalance: Cross-balancing

- Balancing the model with one method and diagnosing the imbalance with the other one
  - both methods find approximately the same balanced states

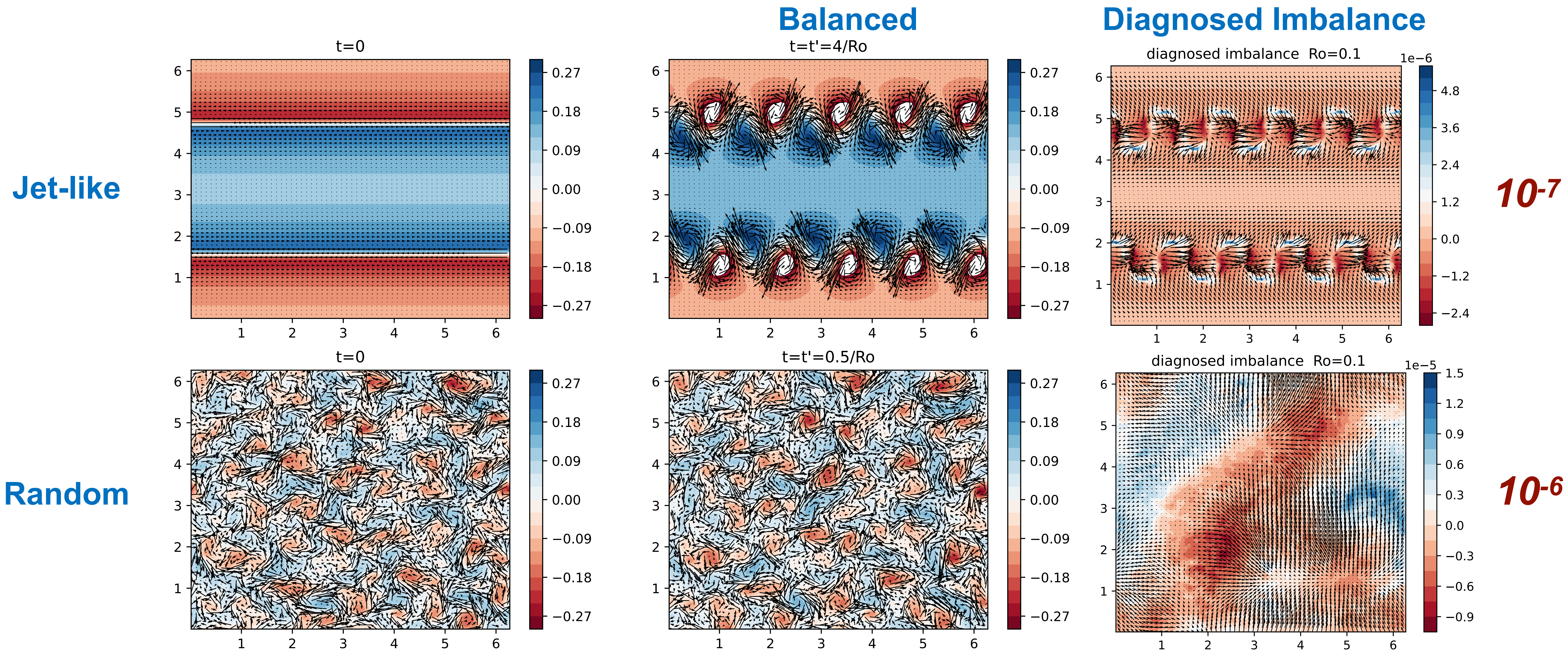


Nearly invariant slow manifold

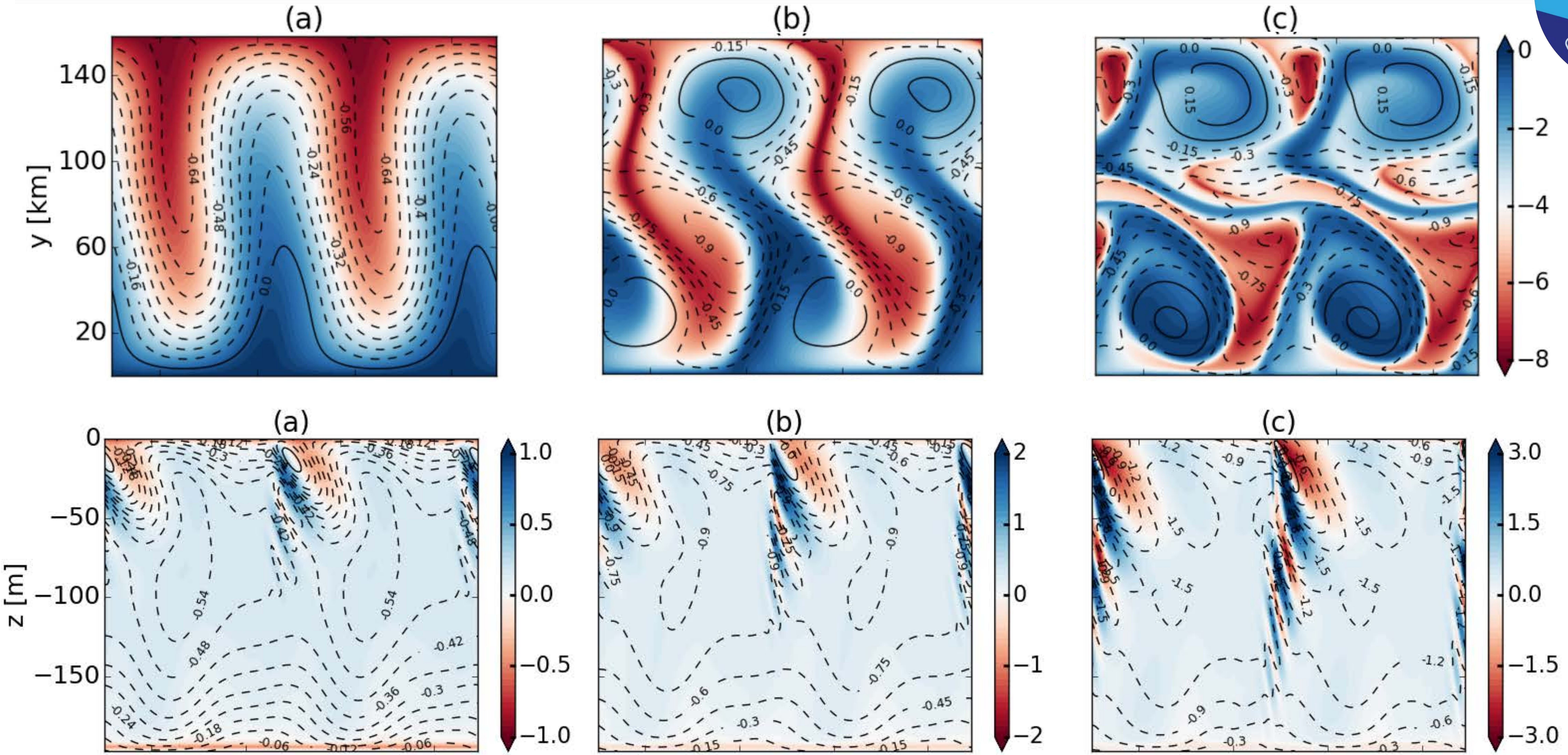
- Very small diagnosed imbalance
  - negligible wave emission
  - <u>*nearly* invariant slow manifold</u>

$$I(u) = \frac{\|u' - u''\|}{\frac{1}{2}\left(\|u'\| + \|u''\|\right)}$$

evolved state u'
rebalanced state u''
norm (rms) $\|..\|$



Wave generation at boundaries

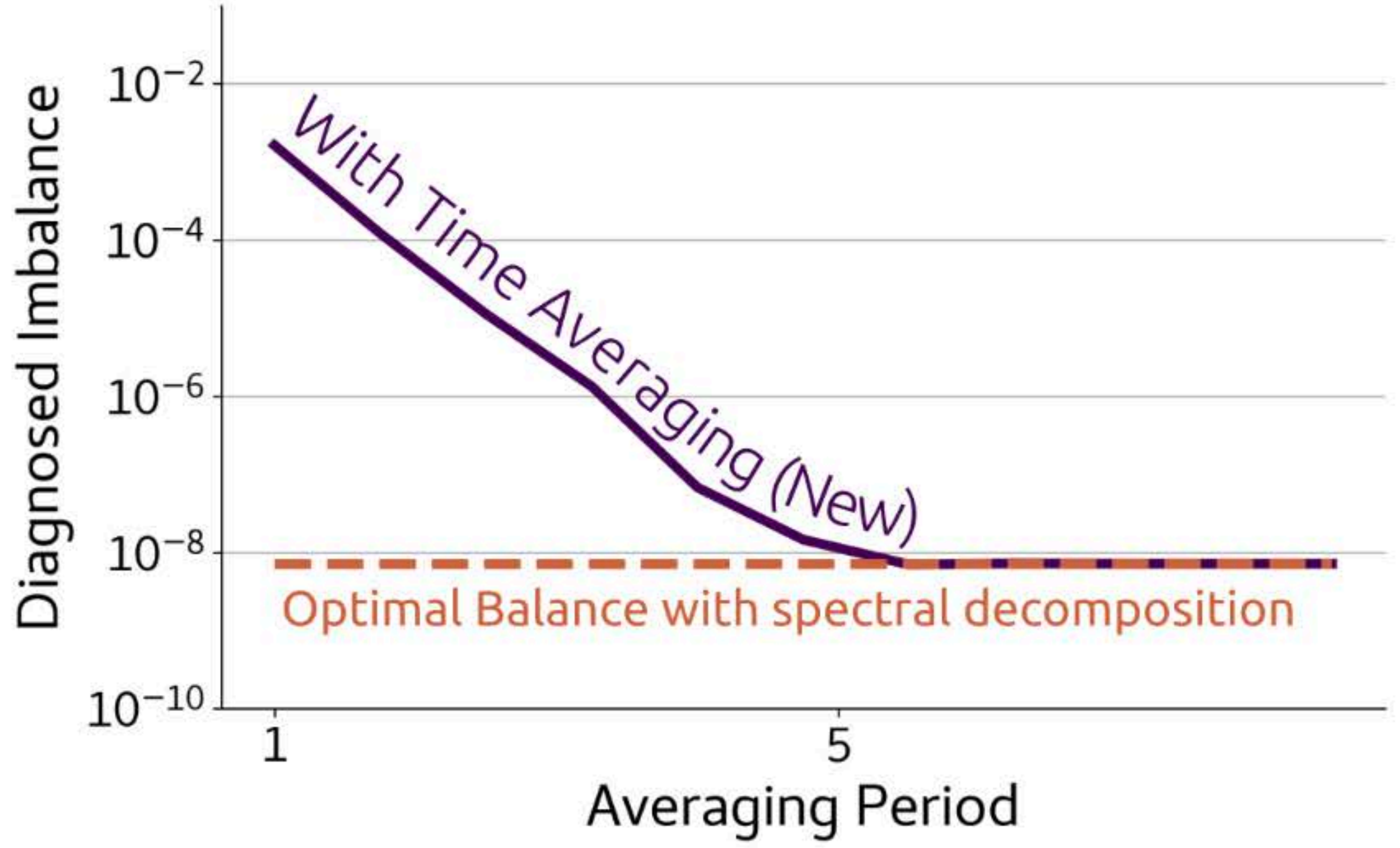


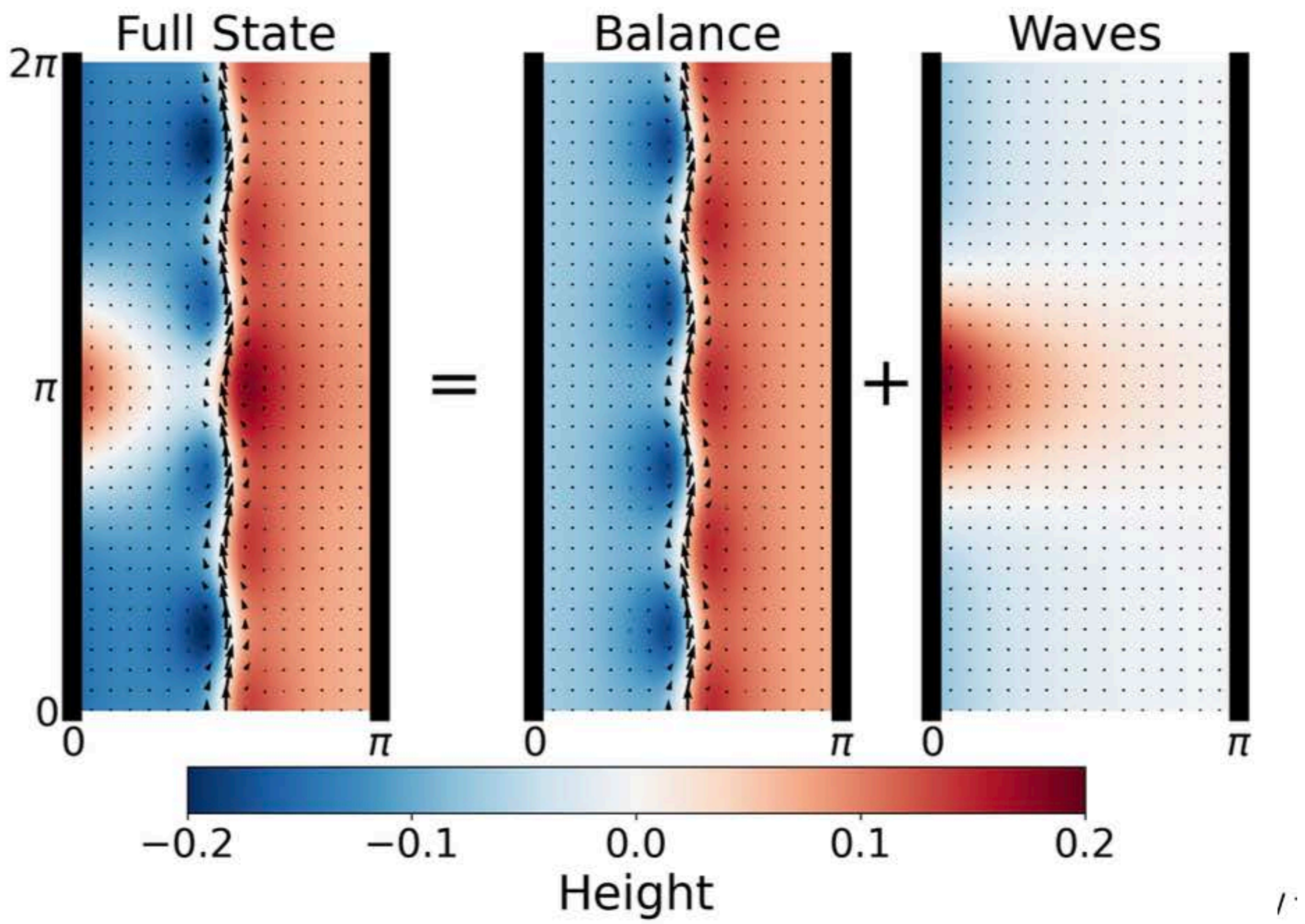
Optimal Balance with Time Averaging

Decomposition in more realistic cases:

» Modification of Optimal Balance —> OBTA

» Replace spectral decomposition with the new time averaging procedure





⇒ **Convergence to exact solution!**

New model: FRIDOM

Framework for Idealized Ocean Models

Implications for:

» Python + JAX => Fast! Runs on GPUs.

» Modular => New diagnostic modules, even parameterizations, can be appended to existing model without touching the source code.

» Highly generalized => Modules for one model can be used for a different one without changes.

» User friendly => Easy to install and run!

» Documentation at https://fridom.readthedocs.io/en/latest/

Courtesy: Silvano Rosenau

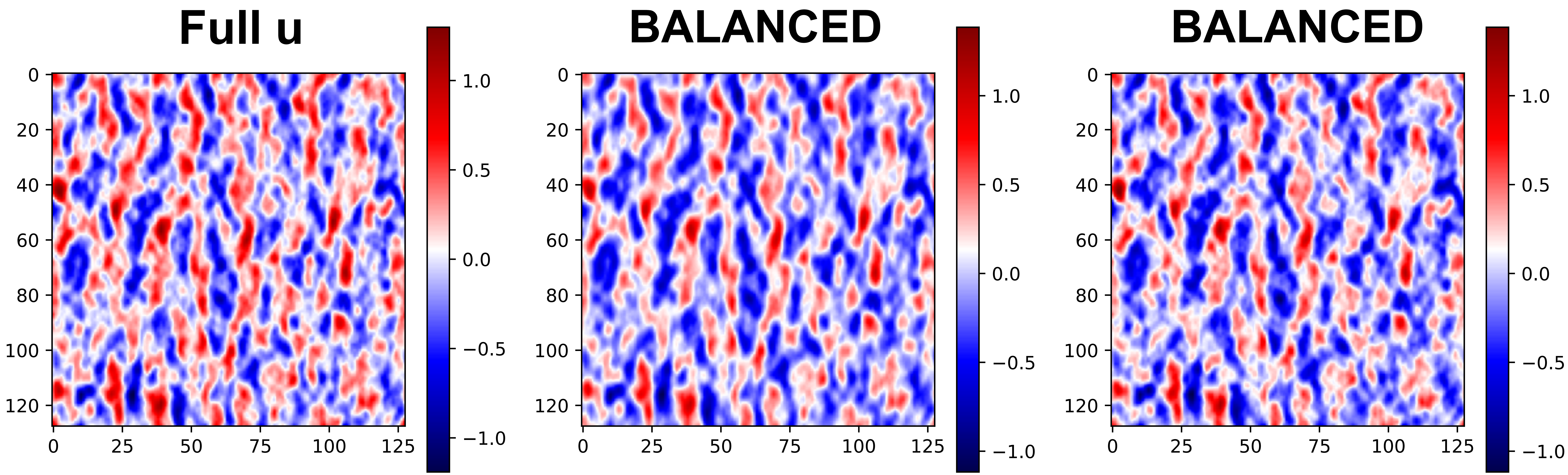
Teaser: Machine learning for SWOT

- » Rigorously Trained neural network from nonlinear flow decomposition
- » Application to ocean observations, e.g. SWOT
- » 2D snapshots
- » Ro (1) regime: no scale separation
- » First results from shallow water model

Courtesy: Nils Brüggemann (high-resolution ICON-O simulations)

Teaser: Machine learning for SWOT

- » Generative adversarial network
  - » generative network
  - » discriminative network
- » Results from shallow water model
  - » 1500 samples: snapshots from initially balanced random field —> Normalize
  - » Training NN for *h*, *u* and *v* for 50 iterations
  - » Evaluating the trained NN on a new unknown sample
  - » Denormalized outputs



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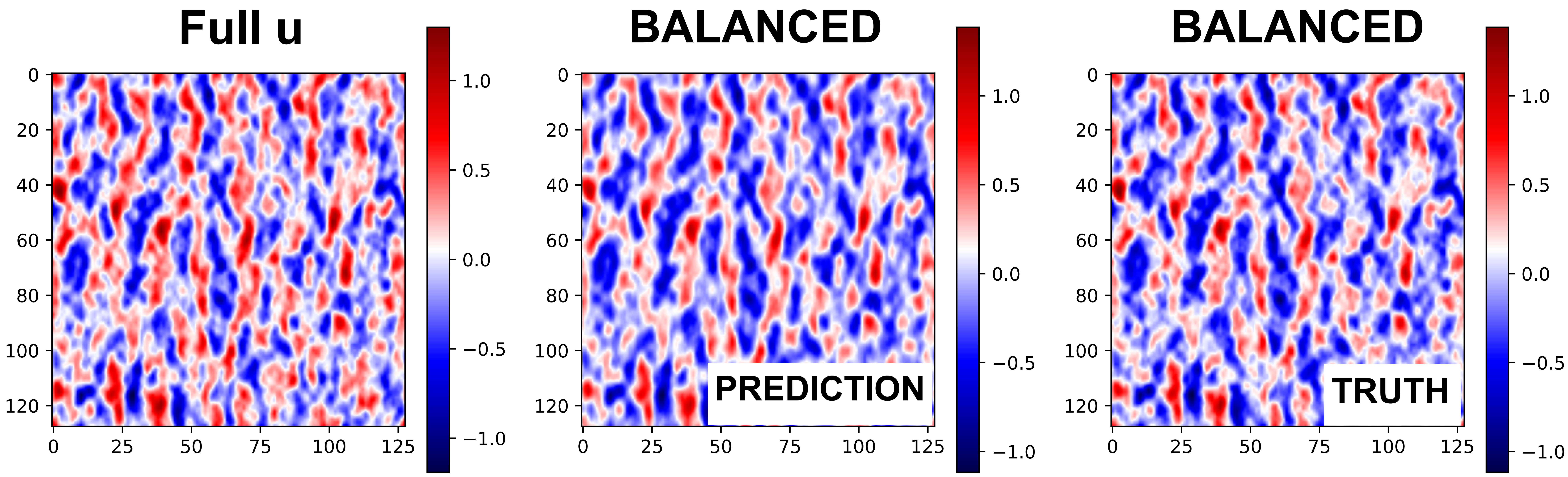

Summary

- Balance
  - Nonlinear flow decomposition
  - Higher order and Optimal balance
    - can be considered equivalent for practical purposes
    - nearly invariant balanced state
- Imbalance
  - Slaved modes dominate the wave signal
  - Spontaneous emission— WEAK
  - **Symmetric instabilities**
- New insights into the representation of balance in geophysical flows
  - Implications for eddy and wave parameterizations

- Ongoing:
  - Balance at boundaries
  - Balance at equator

Summary

» **Revisit balance representation and eddy dissipation!**

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