Towards a generalized vertical coordinate to properly represent vertical modes in an oceanic model

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<u>OMDP/COMMODORE Ocean Model Development Workshop</u>

GENERAL CONTEXT (1)- Vertical structure in ocean model

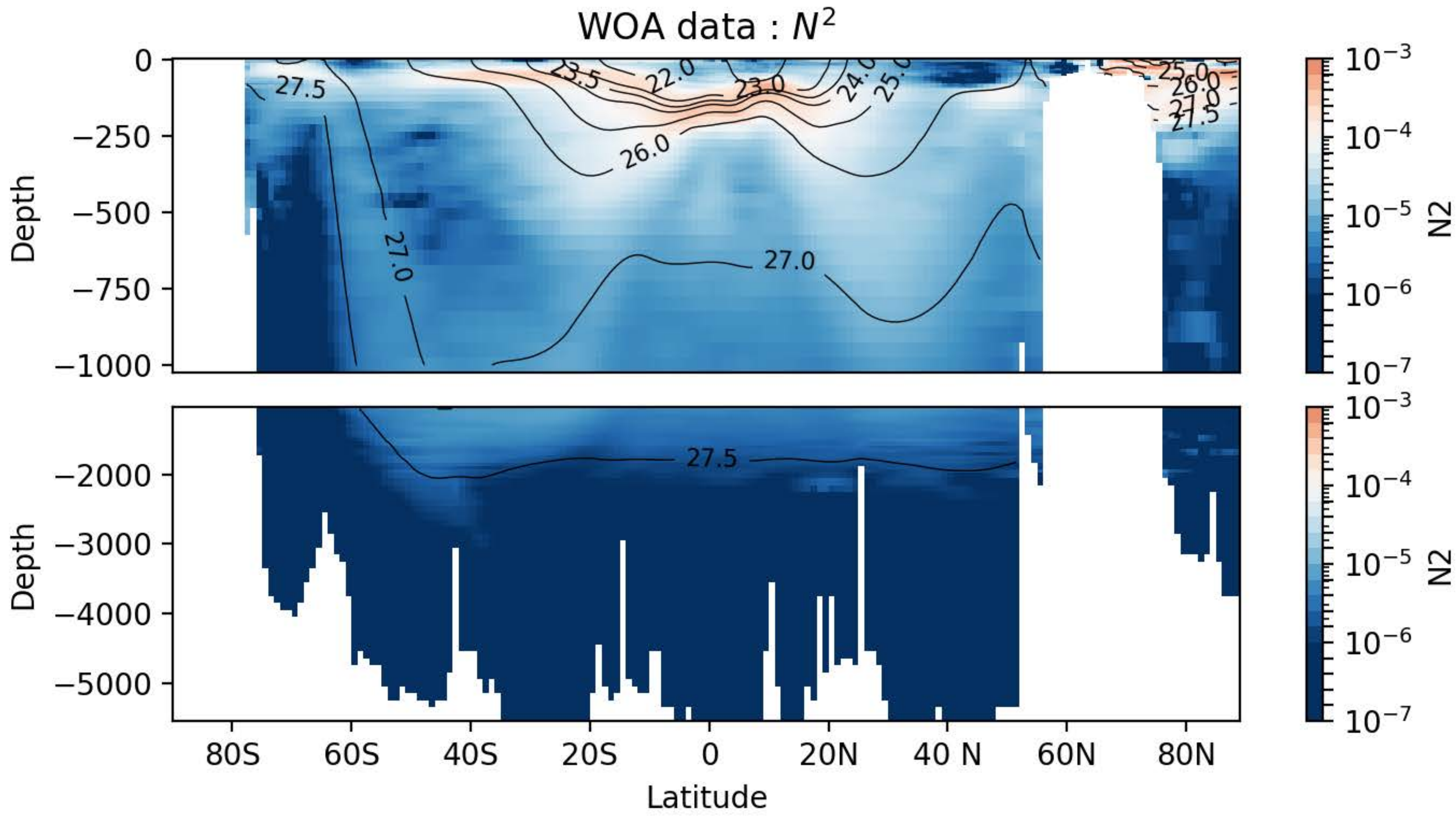
Ocean : an **inhomogeneous** environment with strong vertical variations of salinity S and temperature T

Brunt-Väisälä Frequency (N^2) : a measure of fluid stability in the vertical

$$N^2(z) = g\left(\alpha\frac{\partial T}{\partial z} - \beta\frac{\partial S}{\partial z}\right)$$

Discrete representation of internal waves ?

Stewart et al. 2017 -> Criteria used to define the placement of vertical levels in MOM5



GENERAL CONTEXT (2)- Internal waves, an eigenvalue problem

2D linearized Primitive Equation

$\longrightarrow$

1D eigenvalue problem

GENERAL CONTEXT (2)- Internal waves, an eigenvalue problem

2D linearized Primitive Equations

(Boussinesq, hydrostatic)

$$\partial_x u + \partial_z w = 0$$

$$\partial_t u + \partial_x p = 0$$

$$\partial_z p + b = 0$$

$$\partial_t b + w\, N^2(z) = 0$$

Rigid lid boundary condition

Separation of variables

$$w(x,z,t) = W(z)\, \exp(i(kx - \omega t))$$

1D eigenvalue problem

$$W''(z) + \lambda N^2(z) W(z) = 0$$

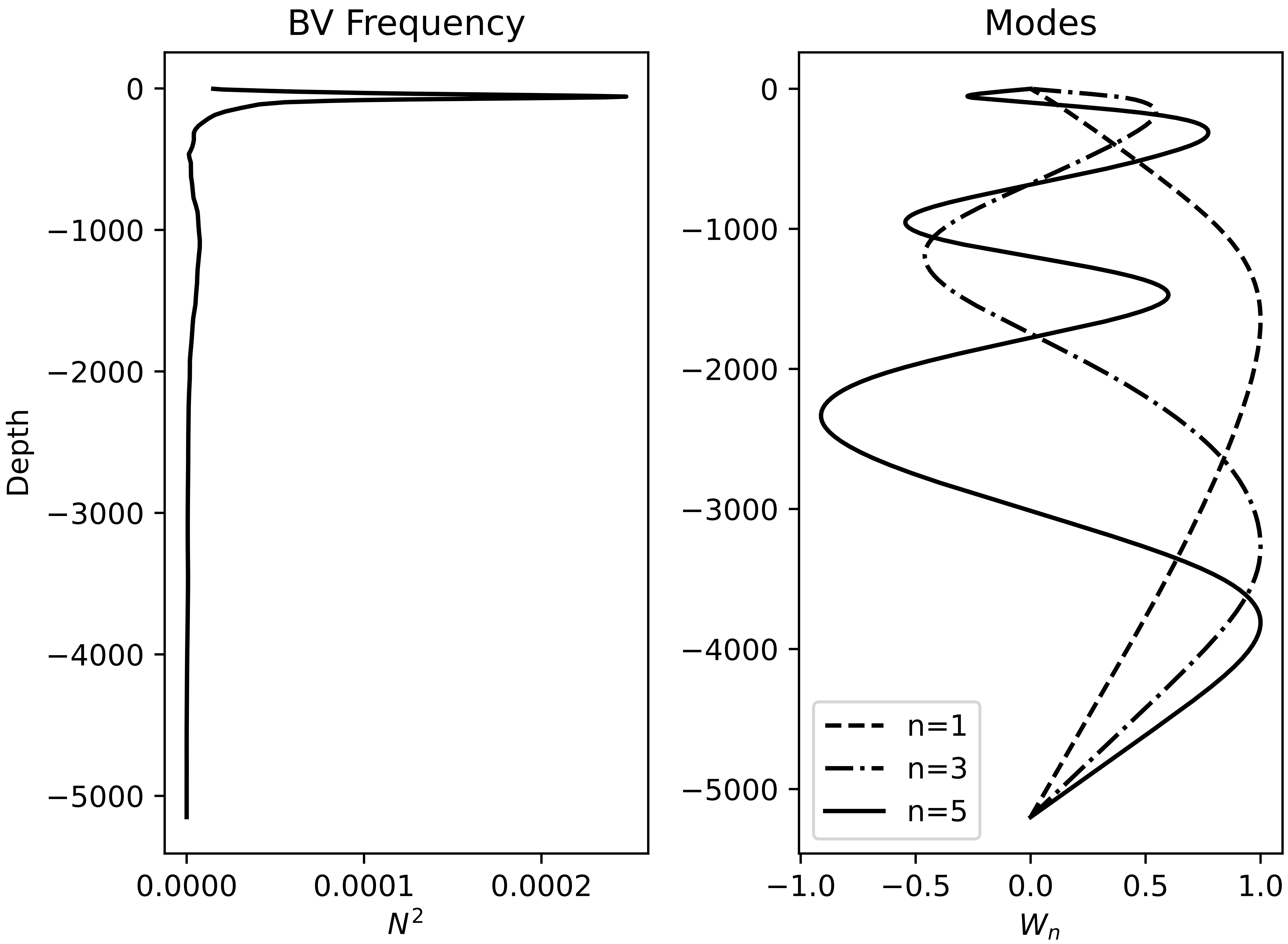
$$W(0) = W(-H) = 0$$

- Linear equation -> modes W_n
- Eigenvalues -> velocities of the modes

$$\lambda_n = \left(\frac{\omega^2}{k^2}\right)_n = \frac{1}{c_n^2}$$

- With minor modifications : free-surface boundary condition and Coriolis force

1D eigenvalue problem



2D linearized Primitive Equations : **the example of constant N^2**

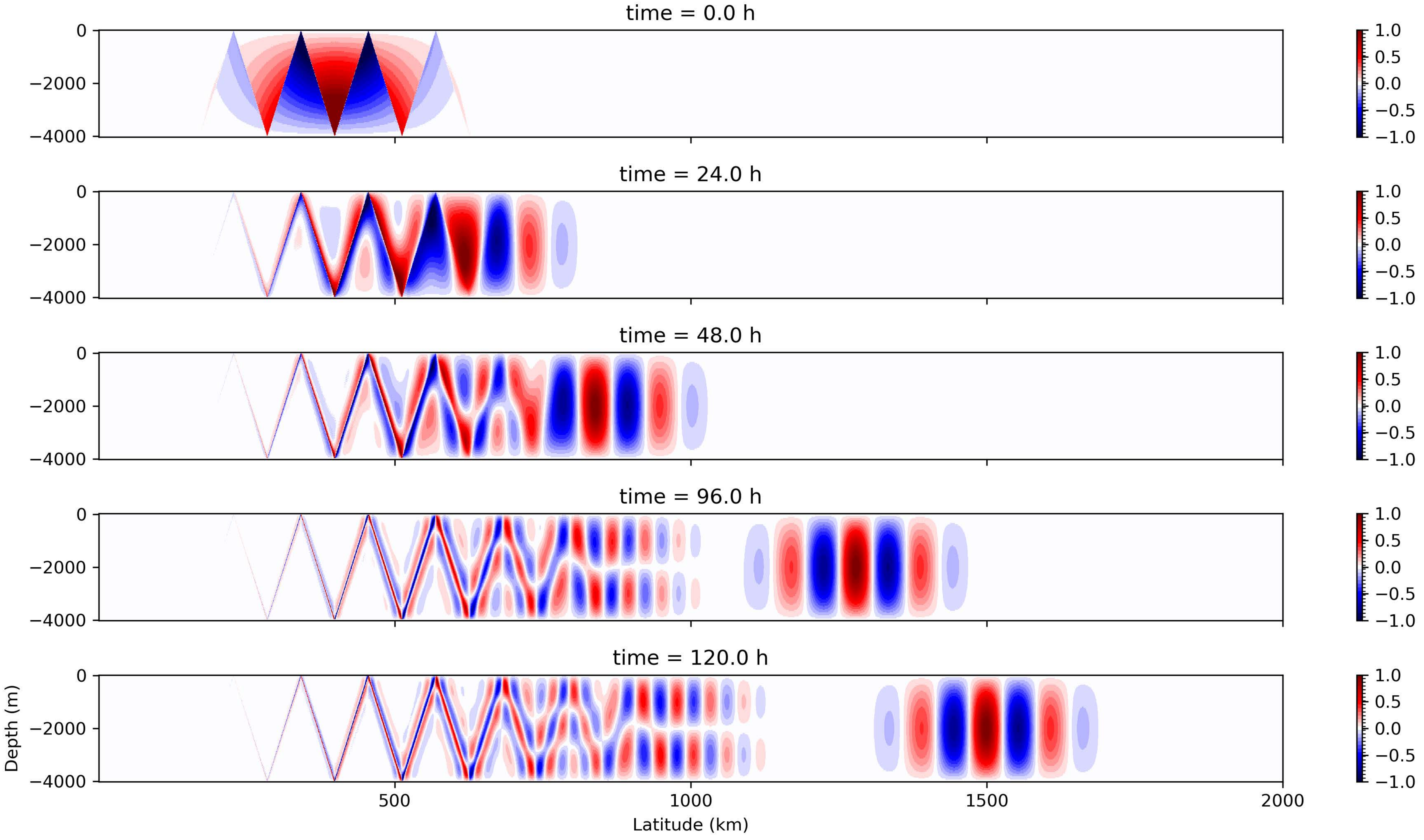
Vertical mode decomposition

$$b(x,z,t) = \sum_{n=1}^{N} b_n(x,t)\, W_n(z)$$

Here :

- $N = 2.10^{-3}$ (**constant**)
- M2 tide Period : 12.4 h
- $L_x = 2000\ km$
- $H = 4000\ m$

Analytical solution



PROBLEMATIC - Discrete representation of Vertical modes

One needs to discretize the **2D** linearized Primitive Equations

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Discrete **1D** eigenvalue problem in the vertical

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How to reduce the eigenvalue error due to discretization ?

Outline

1. Buoyancy position on a staggered grid

2. Optimal choice of vertical level position

1. Buoyancy position on a staggered grid

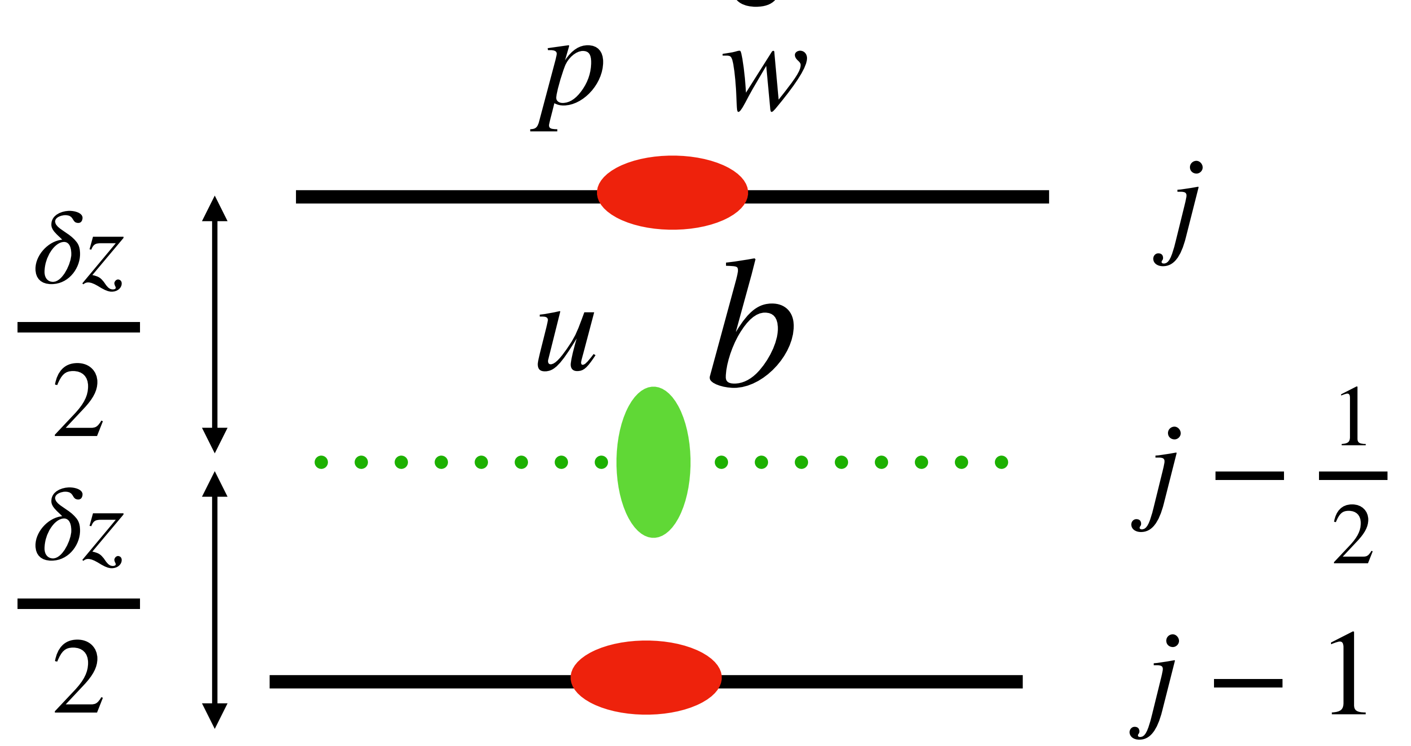
OBJECTIVE 1- Impact of variable positioning on eigenvalue approximation

2D linearized Primitive Equation

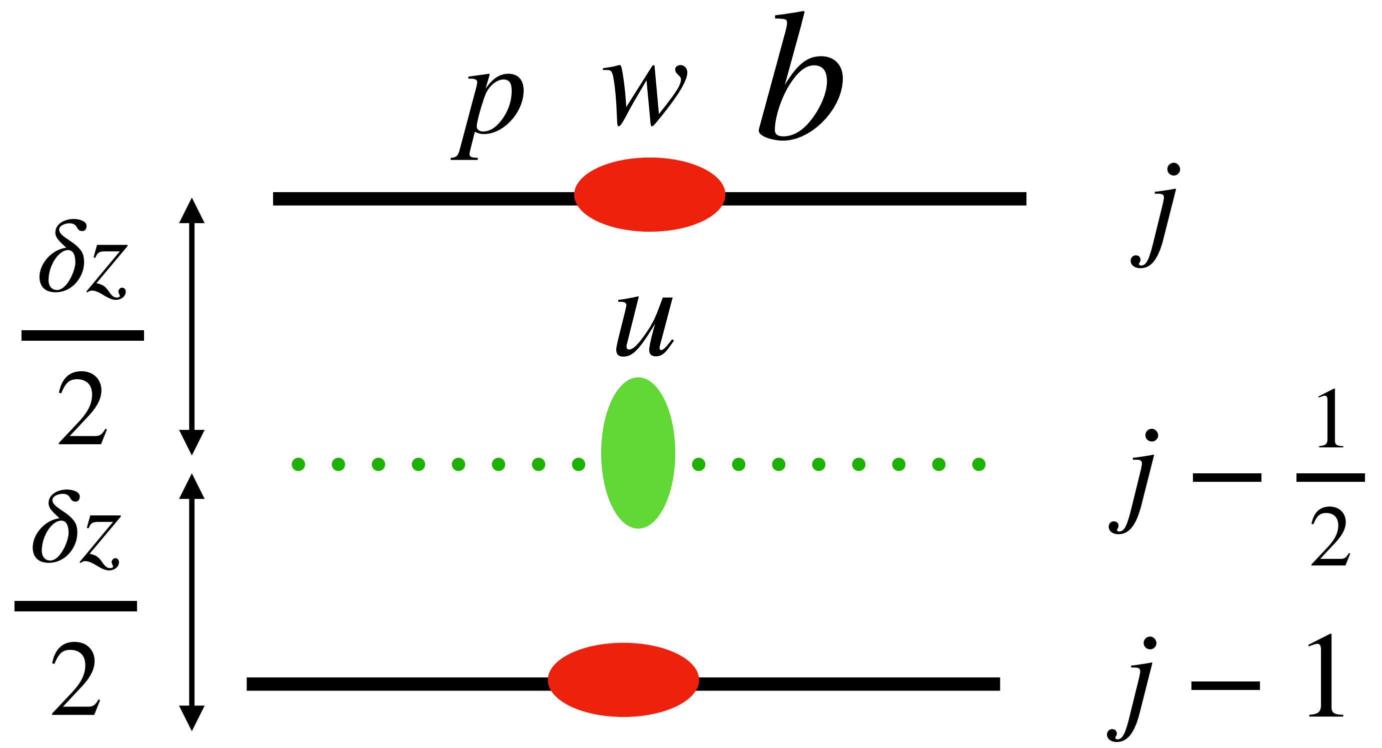
Vertical staggered grid

1D eigenvalue problem

Lorenz grid



Charney-Phillips grid



$$\Delta \mathbf{W} + \lambda_L N_0^2 \, \mathbf{B}_L \mathbf{W} = 0$$

$$\longrightarrow \quad \lambda_L = \lambda^C + \delta z^2 \mu_L + o(\delta z^2)$$

$$\Delta \mathbf{W} + \lambda_{CP} N_0^2 \, \mathbf{B}_{CP} \mathbf{W} = 0$$

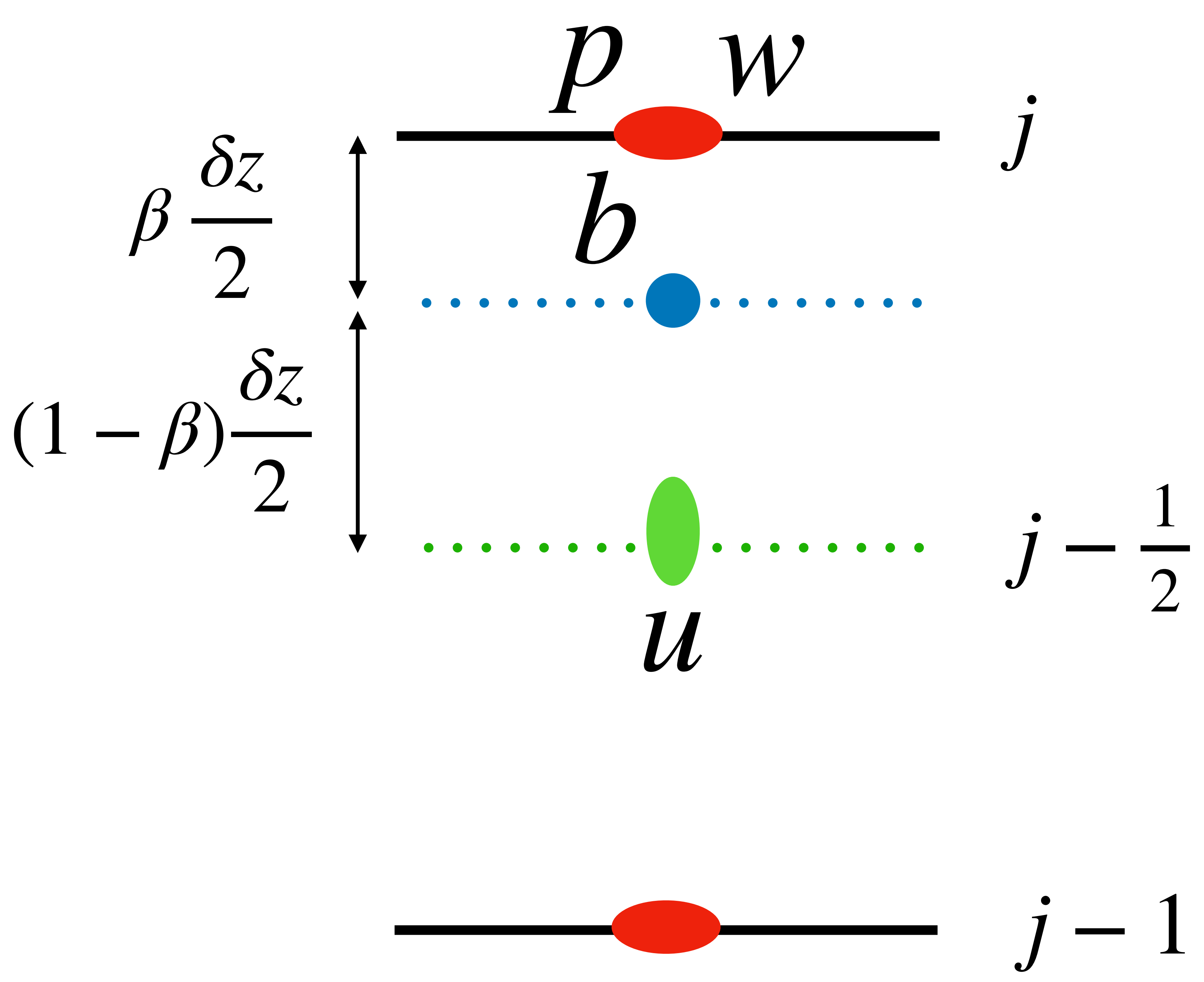
$$\longrightarrow \quad \mu_{CP} \neq \mu_L$$

OBJECTIVE 1- Impact of variable positioning on eigenvalue approximation

<u>Interpolation between CP and Lorenz grid</u>

2D linearized Primitive Equation

New grid



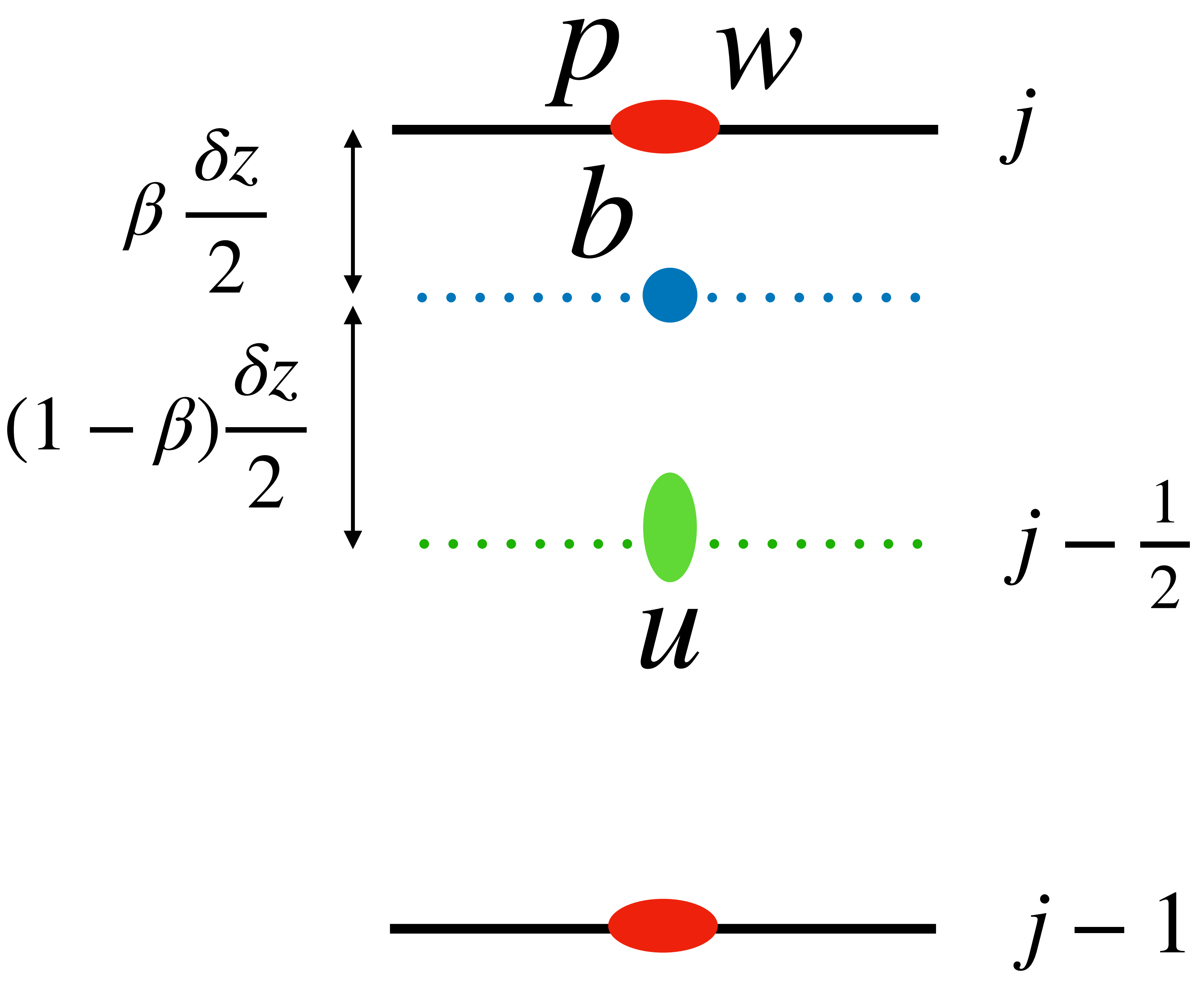
1D eigenvalue problem

OBJECTIVE 1- Impact of variable positioning on eigenvalue approximation

Interpolation between CP and Lorenz grid

2D linearized Primitive Equation

New grid



1D eigenvalue problem

$$\Delta \mathbf{W} + \lambda N_0^2 \, \mathbf{B}(\beta)\mathbf{W} = 0$$

$\beta = 0 \longrightarrow$ Charney-Phillips grid

$\beta = 1 \longrightarrow$ Lorenz grid

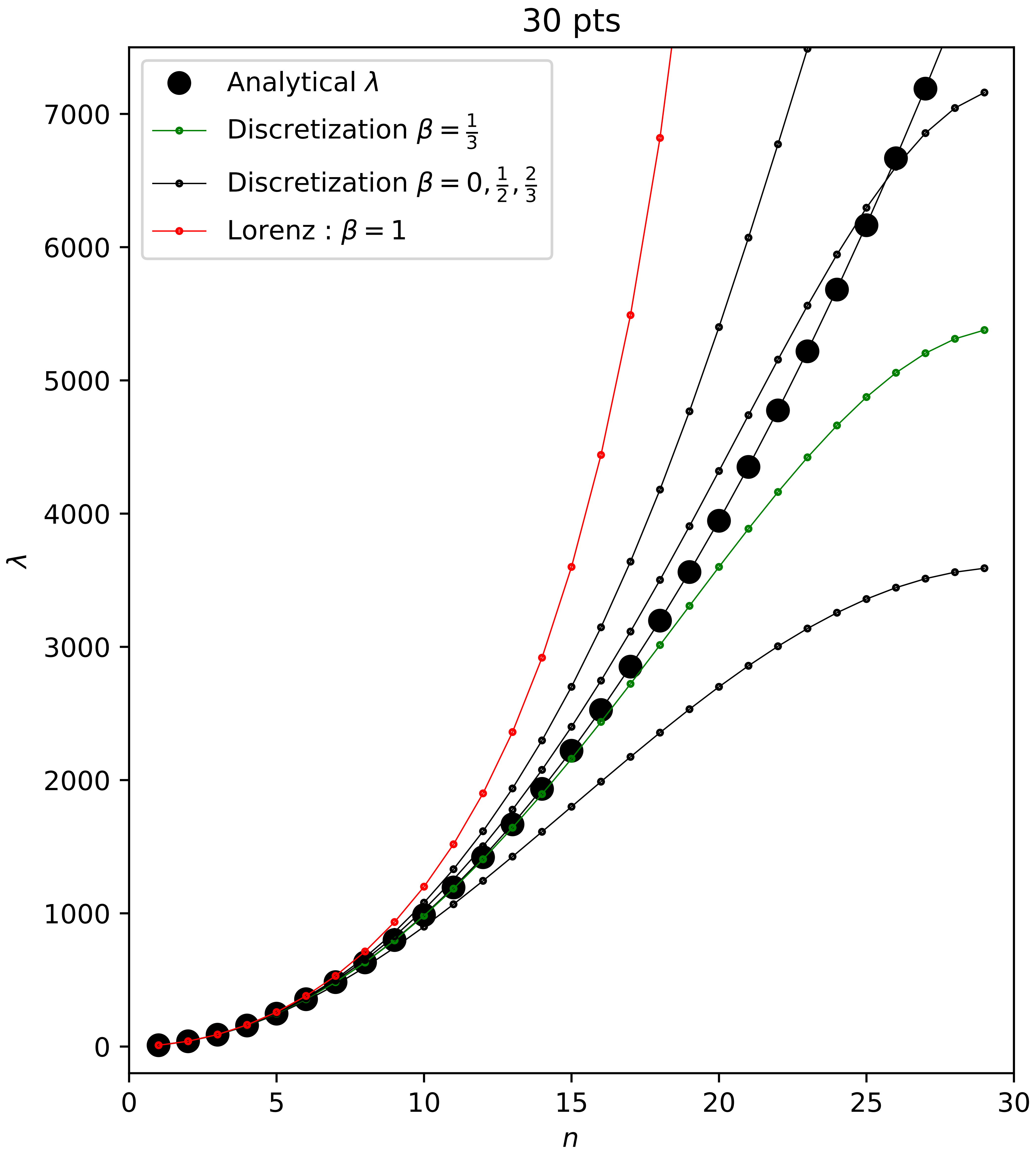
Optimal value of β

$$\lambda = \lambda^C + \delta z^2 \cancel{\mu(\beta)} + o(\delta z^2)$$

RESULT - Eigenvalues

Optimal $\beta = \frac{1}{3}$

$N = cst$

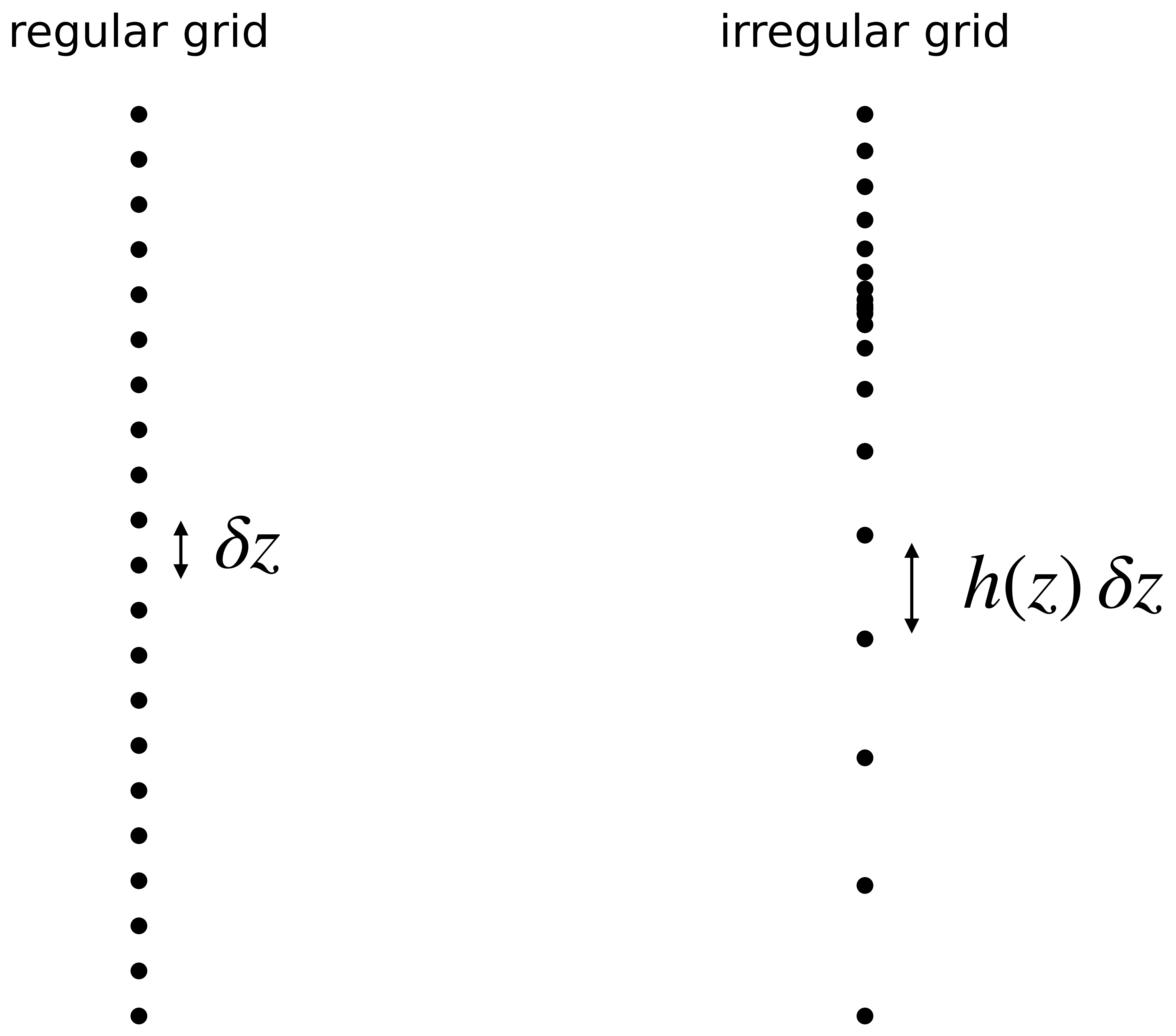


By interpolation between two vertical grid, one can **reduce the errors**

2. Optimal choice of vertical level position

OBJECTIVE 2 - Move grid points to minimize eigenvalue errors

Goal : find the optimal h which minimizes $|\mu|$



Eigenvalue

$$\lambda = \lambda^C + \delta z^2\,\mu(h) + o(\delta z^2)$$

OBJECTIVE 2 - Move grid points to minimize eigenvalue errors

Goal : find the optimal h which minimizes $|\mu|$

$$\mu = -\frac{\int h^2(z)\left(W_C''(z)^2 + \lambda^C\left(N^2(z)W_C(z)^2\right)''\right)dz}{12\int N^2(z)W_C(z)^2\,dz}$$

$W_C \longrightarrow$ Eigenfunction of the continuous problem

With the constraint $\displaystyle\int \frac{1}{h(z)}dz = H$ to keep the total length constant.

For one eigenvalue :

In some cases, a possible analytical approach of vertical grids

For more than one eigenvalue :

Complicated optimization problem

OBJECTIVE 2 - Move grid points to minimize eigenvalue errors

Method : Resolution of an **optimization** problem

Distances between successive grid points

$$h_j = \frac{z_{j+1} - z_j}{\Delta z}$$

← Regular grid

- Regular grid : $h_j = 1$
- Finer grid : $h_j < 1$
- Coarser grid : $h_j > 1$

Cost function

$$J = \sum_{n=1}^{\mathcal{N}} \frac{1}{n^2} \left(\frac{\lambda_n(h) - \lambda_n^C}{\lambda_n^C} \right)^2$$

$\mathcal{N}$ Number of eigenvalues

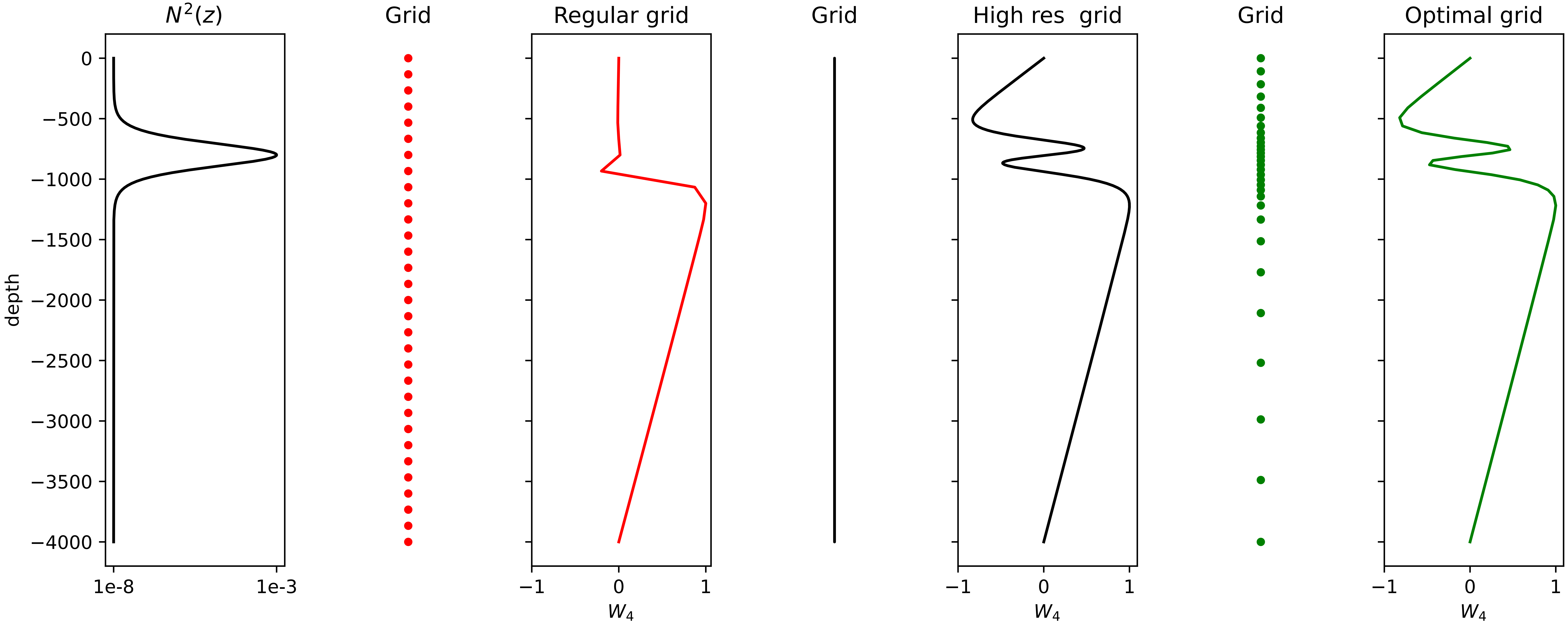
λ^C High resolution eigenvalues

$\lambda(h)$ Eigenvalues of the grid

No analytical solution when $N^2 \neq cst$ => use of a high-resolution reference solution to provide λ^C

1D RESULT - a vertical mode representation

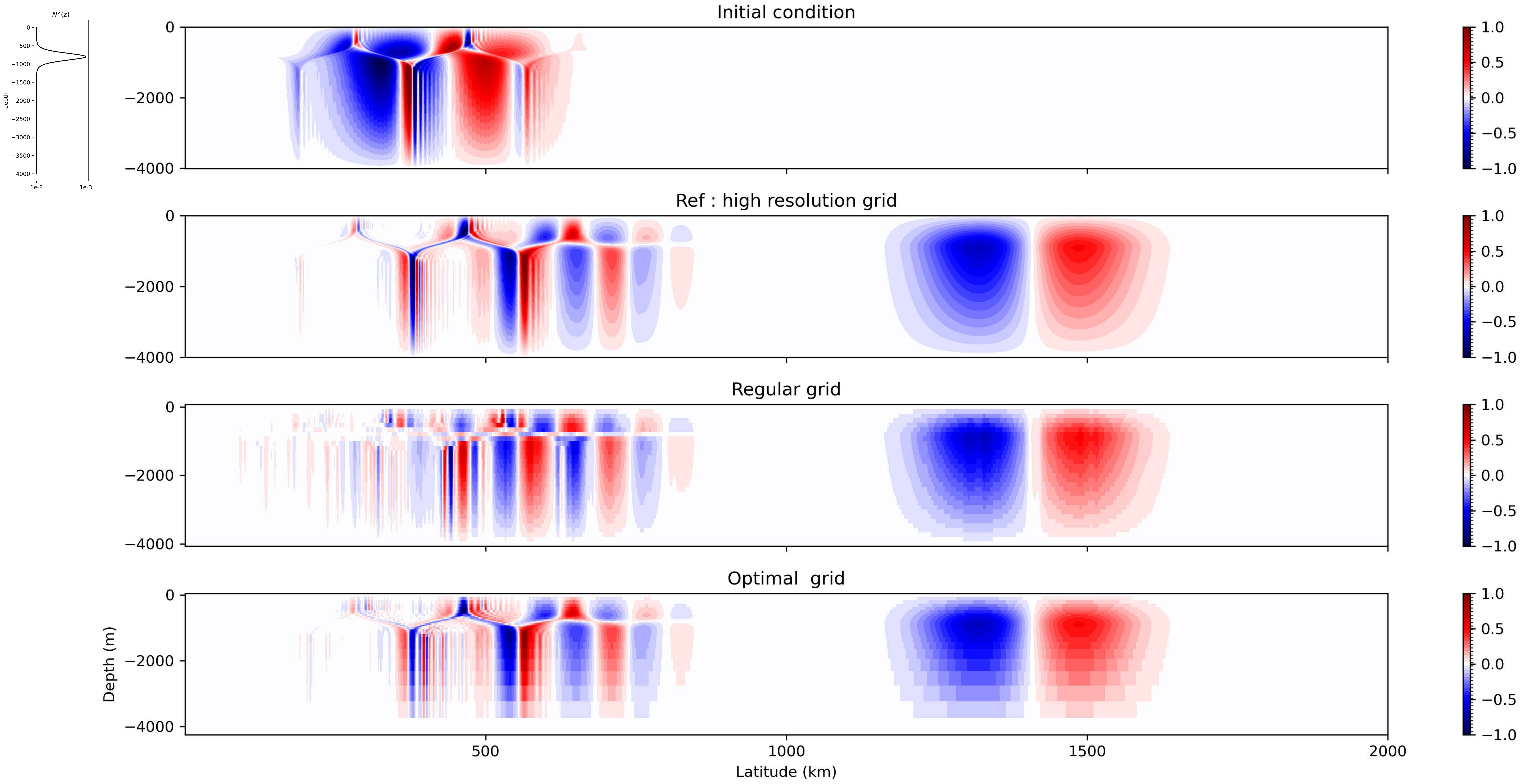
<u>Mode 4 : 30 pts</u>



Number of grid points : 30 pts 900 pts 30 pts

2D RESULT - Internal waves propagation

30 grid points



CONCLUSION AND FUTURE WORK

Objective: improve the **representation of internal wave** propagation by focusing on the role of **vertical grid** staggering and vertical level placement.

Conclusion

- A hybrid between the CP and the Lorenz grids provides a very accurate representation of eigenvalues
  → Is this feasible in a PE model, and what does it involve?

- Optimal placement of vertical levels to represent vertical modes
  → Basis for the definition of a target grid in a V-ALE coordinate (provided we add constraints for refining in boundary layers).
  → Investigate ways to get a cheap estimate of the solution of the optimisation problem.

CONCLUSION AND FUTURE WORK

Objective: improve the **representation of internal wave** propagation by focusing on the role of **vertical grid** staggering and vertical level placement.

<u>Future work</u>

- Comparison with conventional stretching functions used in σ and $z^\star$ coordinates.
- Semi-realistic simulations with NEMO model in QE and V-ALE coordinates (in collaboration with Mercator International Ocean).

Thanks for your attention