

Data-driven parameter mesoscale eddies usi Eliassen-Palm flux CLIVAR-OMDP Workshop, September 2020 and on More information and application are at

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X @M2LInES https://m2lines.github.io







Postdoctoral positions available: **Climate Process Team on Ocean Transport and Eddy Energy** Funded by the National Oceanographic and Atmospheric Administration and the National Science Foundation



N Itir א יאר אזי ג' e אמר pos יחי ar available as part of a multi-institution Climate Pro Team (CPT) on Ocean Transport and Eddy Energy. The CPT aims to survey, improve, and unify advances i energy-, flow-, and scale-aware parameterizations of mesoscale eddies, in process st nd g. al to in ... els; constrain parameters and parameterized fluxes through a synthes p-to-d te pse reaction of ocean energetics and transport; and implement and assess schemes w CC-c. s clin ate .odels at NCAR, NOAA-GFDL, and DOE-LANL. The expectation is inodernized, energetically-consistent mesoscale eddy parameterizations will significantly re climate model biases in ocean currents, stratification, and transport.

- New York University (Supervised by Laure Zanna): Unification of buoyancy and tracer closure Assessment and parameterization of vertical energy structure; Parameterization of the grey zone. More information and application at https://apply.interfolio.com/68119.
- University of Colorado, Boulder (Supervised by Ian Grooms): Assessment of 2D eddy energy ation parameterization of eddy energy transport; parameterizing dissipation in the eddy https://jobs.colorado.edu/jobs/JobDetail/?jobId=20799.
- Woods Hole Oceanographic Institution (Supervised by Sylvia Cole): Characterizing \bullet scale dependent EKE from observations; quasi-3D eddy buoyancy and momentum statistic from observations; analysis of vertical eddy structure in observations; synthesis of observat More information and application are at

https://careers.whoi.edu/opportunities/view-all-openings/science-research/ (position 19-08-0

Princeton University (Supervised by Alistair Adcroft): Implement parameterizations of mesoscale eddies in process, idealized an consistent and optimized formulation of closures; development a lienvironnement and unified closures; evaluation of new closures in climate mode application at https://www.princeton.edu/acad-positions/position



















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or any o







KE backscatter / Inverse cascade







KE backscatter / Inverse cascade

Down-scale energy transfer via APE extraction and up-scale energy transfer via KE backscatter are often parameterised separately

e.g., Kraichnan 1976; Frederiksen 1997; Berner 2009; Jansen 2014

e.g., Gent McWilliams 1990; Griffies 1998

But they're two essential aspects of one process







KE backscatter / Inverse cascade

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But they're two essential aspects of one process

Parameterisation of Ocean mesoscale eddies whereby **momentum** and thickness fluxes are considered together





 $\hat{u}_n =$ Thickness-weighted average framework:

$$=rac{h_n u_n}{\overline{h}_n}$$
 Thic

$$\frac{\partial \hat{u}_n}{\partial t} + \hat{u}_n \frac{\partial \hat{u}_n}{\partial x} + \hat{v}_n \frac{\partial \hat{u}_n}{\partial y} - f \hat{v}_n + \frac{\partial \overline{p}_n}{\partial x} - \widehat{F}_n^{(u)} = -N$$

$$\frac{\partial \hat{v}_n}{\partial t} + \hat{u}_n \frac{\partial \hat{v}_n}{\partial x} + \hat{v}_n \frac{\partial \hat{v}_n}{\partial y} + f \hat{u}_n + \frac{\partial \overline{p}_n}{\partial y} - \widehat{F}_n^{(v)} = -N$$

$$\widehat{\mathbf{E}}_{n} = \left(\widehat{E}_{n}^{(u)}, \widehat{E}_{n}^{(v)}\right) = \begin{pmatrix} \widehat{\left(u_{n}^{''2} + \frac{1}{2\bar{h}_{n}}g_{n-1/2}^{r}\overline{\eta_{n-1/2}^{''2}}\right) & \widehat{\left(u_{n}^{''v_{n}^{''}}\right) \\ \widehat{\left(u_{n}^{''v_{n}^{''}}\right)} & \widehat{\left(v_{n}^{''2} + \frac{1}{2\bar{h}_{n}}g_{n-1/2}^{r}\overline{\eta_{n-1/2}^{''2}}\right) \\ \widehat{\left(\eta_{n-1/2}^{d}\overline{\partial_{x}}p_{n-1}^{'}\right)} & \widehat{\left(\eta_{n-1/2}^{d}\overline{\partial_{y}}p_{n}^{''}\right)} \\ \widehat{\left(u_{n}^{''v_{n}^{''}}\right)} & \widehat{\left(u_{n}^{''v_{n}^{''}}\right)} \\ \widehat{\left(u_{n-1/2}^{d}\overline{\partial_{x}}p_{n-1}^{'}\right)} & \widehat{\left(u_{n-1/2}^{d}\overline{\partial_{y}}p_{n}^{''}\right)} \\ \widehat{\left(u_{n-1$$

e.g. Marshall et al. (2012); Young (2012); Madison and Marshall (2013); Loose et al (2023)

Energy + Momentum + Thickness = Eliassen-Palm Flux

To capture the full eddy energy cycle, we target the Eliassen-Palm Flux









Thickness-weighted average framework: $\hat{u}_n = \frac{h_n u_n}{\overline{\tau}}$

$$\frac{\partial \hat{u}_n}{\partial t} + \hat{u}_n \frac{\partial \hat{u}_n}{\partial x} + \hat{v}_n \frac{\partial \hat{u}_n}{\partial y} - f \hat{v}_n + \frac{\partial \overline{p}_n}{\partial x} - \widehat{F}_n^{(u)} = -\nabla \frac{\partial \hat{v}_n}{\partial t} + \hat{u}_n \frac{\partial \hat{v}_n}{\partial x} + \hat{v}_n \frac{\partial \hat{v}_n}{\partial y} + f \hat{u}_n + \frac{\partial \overline{p}_n}{\partial y} - \widehat{F}_n^{(v)} = -\nabla \frac{\partial \hat{v}_n}{\partial y} + f \hat{u}_n + \frac{\partial \overline{p}_n}{\partial y} - \hat{F}_n^{(v)} = -\nabla \frac{\partial \hat{v}_n}{\partial y} + f \hat{u}_n + \frac{\partial \overline{p}_n}{\partial y} - \hat{F}_n^{(v)} = -\nabla \frac{\partial \hat{v}_n}{\partial y} + f \hat{u}_n + \frac{\partial \overline{p}_n}{\partial y} - \hat{F}_n^{(v)} = -\nabla \frac{\partial \hat{v}_n}{\partial y} + f \hat{u}_n + \frac{\partial \overline{p}_n}{\partial y} - \hat{F}_n^{(v)} = -\nabla \frac{\partial \hat{v}_n}{\partial y} + f \hat{u}_n + \frac{\partial \overline{p}_n}{\partial y} - \hat{F}_n^{(v)} = -\nabla \frac{\partial \hat{v}_n}{\partial y} + f \hat{u}_n + \frac{\partial \overline{p}_n}{\partial y} - \hat{F}_n^{(v)} = -\nabla \frac{\partial \hat{v}_n}{\partial y} + f \hat{v}_n + \frac{\partial \overline{p}_n}{\partial y} + f \hat{v$$

$$\widehat{\mathbf{E}}_{n} = \left(\widehat{E}_{n}^{(u)}, \widehat{E}_{n}^{(v)}\right) = \begin{pmatrix} \widehat{\left(u_{n}^{''2} + \frac{1}{2\overline{h}_{n}}g_{n-1/2}^{r}\overline{\eta_{n-1/2}}\right) & (\widehat{\left(u_{n}^{''v_{n}^{''}}\right) \\ (\widehat{\left(u_{n}^{''v_{n}^{''}}\right)} & (\widehat{\left(v_{n}^{''2} + \frac{1}{2\overline{h}_{n}}g_{n-1/2}^{r}\overline{\eta_{n-1/2}}) \\ (\overline{\eta_{n-1/2}^{\frac{\partial}{\partial x}}p_{n-1}^{'}}) & (\overline{\eta_{n-1/2}^{\frac{\partial}{\partial y}}p_{n-1}^{''}} \end{pmatrix}$$

Zonal

Meridional

e.g. Marshall et al. (2012); Young (2012); Madison and Marshall (2013); Loose et al (2023)

Energy + Momentum + Thickness = Eliassen-Palm Flux

To capture the full eddy energy cycle, we target the Eliassen-Palm Flux







MOM6 Double Gyre Configuration



Perezhogin et al (2024)

Commonly used configuration as stepping stone e.g.,

- Dhruv's presentation;
- Perezhogin et al (2024);
- Zhang et al (2023)

GFDL MOM6 ocean model in Double Gyre configuration

- Two layers
- Initialized from rest
- Driven by wind and equilibrated by bottom friction
- Biharmonic Smagorinsky model





Double Gyre simulation data at $1/32^{\circ}$ resolution





Parameterisation Development: 2) Filter and coarsen



Filter and coarsen to $1/4^{\circ}$







Filter and coarsen to 1/4°





Parameterisation Development: 3) Coarse flow-field representations

Calculate Spatial gradients of flow field variables





Relative vorticity







Parameterisation Development: 4) Diagnose Eliassen-Palm Fluxes

Calculate Spatial gradients of flow field variables



AND

Calculate Sub-filter EPF tensor components



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2 hidden layers 8 neurons each

Coarse-grain derivatives of filtered flow variables

Trained on $\sim 4 \times 10^6$ **samples** (288 snapshots)

- $\sim 5 \times 10^5$ samples for testing (36 snapshots)

Parameterisation Development: 5) Artificial Neural Network



OUTPUTS:

• Reynolds' stresses and dual form stresses

 $\sim 5 \times 10^5$ samples for validation (36 snapshots)











2 hidden layers 8 neurons each

Coarse-grain derivatives of • filtered flow variables

$$\widehat{\mathbf{E}}_{n} = \left(\widehat{E}_{n}^{(u)}, \widehat{E}_{n}^{(v)}\right) = \begin{pmatrix} \left(\widehat{u_{n}^{''2}} + \frac{1}{2\overline{h}_{n}}g_{n-1/2}^{r}\overline{\eta_{n-1/2}^{''}}\right) & \left(\widehat{u_{n}^{''}v_{n}^{''}}\right) \\ \left(\widehat{u_{n}^{''}v_{n}^{''}}\right) & \left(\widehat{v_{n}^{''2}} + \frac{1}{2\overline{h}_{n}}g_{n-1/2}^{r}\overline{\eta_{n-1/2}^{''}}\right) \\ \left(\overline{\eta_{n-1/2}^{\frac{\partial}{\partial x}}p_{n-1}^{'}}\right) & \left(\overline{\eta_{n-1/2}^{\frac{\partial}{\partial y}}p_{n-1}^{'}}\right) \end{pmatrix}$$





Upscale energy transfer via KE backscatter











Coarse-grain derivatives of • filtered flow variables

$$\widehat{\mathbf{E}}_{n} = \left(\widehat{E}_{n}^{(u)}, \widehat{E}_{n}^{(v)}\right) = \begin{pmatrix} \widehat{\left(u_{n}^{''2} + \frac{1}{2\overline{h}_{n}}g_{n-1/2}^{r}\overline{\eta_{n-1/2}^{''}}\right) & \left(\widehat{u_{n}^{''}v_{n}^{''}}\right) \\ \left(\widehat{\left(u_{n}^{''}v_{n}^{''}\right)} & \left(\widehat{\left(v_{n}^{''2} + \frac{1}{2\overline{h}_{n}}g_{n-1/2}^{r}\overline{\eta_{n-1/2}^{''}}\right) \\ \left(\overline{\eta_{n-1/2}^{\partial}\overline{\partial x}}p_{n-1}^{'}\right) & \left(\overline{\eta_{n-1/2}^{\partial}\overline{\partial y}}p_{n-1}^{'}\right) \end{pmatrix}$$

Parameterisation Development: 5) Artificial Neural Network

Neglecting sub-filter potential energy

e.g., Marshall et al (2012)











Coarse-grain derivatives of • filtered flow variables

$$\widehat{\mathbf{E}}_{n} = \left(\widehat{E}_{n}^{(u)}, \widehat{E}_{n}^{(v)}\right) = \begin{pmatrix} \widehat{\left(u_{n}^{''}\right)} \\ \widehat{\left(u_{n}^{''}v_{n}^{''}\right)} \\ \overline{\left(\eta_{n-1/2}^{-\frac{\partial}{\partial x}}p_{n-1}^{'}\right)} \end{pmatrix}$$

Parameterisation Development: 5) Artificial Neural Network

2 hidden layers 8 neurons each



Upscale energy transfer via KE backscatter



Downscale energy transfer via APE extraction

$$\left(\widehat{u_n''v_n''}\right)$$
$$\left(\widehat{v_n''^2}\right)$$
$$\left(\overline{\eta_{n-1/2}'\frac{\partial}{\partial y}}p_{n-1}'\right)$$











Momentum flux components $\Delta x^{2} ||\nabla u||^{2} \operatorname{ANN}\left(\frac{\nabla u}{||\nabla u||}, \frac{S}{||S||}\right) \approx \widehat{E}_{mom}$ Normalized inputs **Thickness flux components** $\left(\frac{\nabla u}{||\nabla u||}, \frac{S}{||S||}\right) \approx \widehat{E}_{thick}$

$$\Delta x^2 f || \nabla u || || S || \mathbf{ANN} \left(- \frac{1}{10} \right)$$

$$||\nabla u|| = \left(\left(\partial_x \widehat{v} - \partial_y \widehat{u} \right)^2 + \left(\partial_x \widehat{u} - \partial_y \widehat{v} \right)^2 + \left(\partial_x \widehat{v} + \partial_y \widehat{u} \right)^2 \right)^{1/2}$$
$$||S|| = \left(\left(\partial_x \overline{\eta_{top}} \right)^2 + \left(\partial_y \overline{\eta_{top}} \right)^2 + \left(\partial_x \overline{\eta_{bot}} \right)^2 + \left(\partial_y \overline{\eta_{bot}} \right)^2 \right)^{1/2}$$











Momentum flux components $\Delta x^{2} ||\nabla u||^{2} \operatorname{ANN}\left(\frac{\nabla u}{||\nabla u||}, \frac{S}{||S||}\right) \approx \widehat{E}_{mom}$ Normalized inputs **Thickness flux components** $\Delta x^2 f||\nabla u|||S||\operatorname{ANN}\left(\frac{\nabla u}{||\nabla u||},\frac{S}{||S||}\right) \approx \widehat{E}_{thick}$















Momentum flux components $\Delta x^{2} ||\nabla u||^{2} \operatorname{ANN} \left(\frac{\nabla u}{||\nabla u||}, \frac{S}{||S||} \right) \approx \widehat{E}_{mom}$ Normalized inputs **Thickness flux components** $\Delta x^2 f||\nabla u|||S|| \operatorname{ANN}\left(\frac{\nabla u}{||\nabla u||}, \frac{S}{||S||}\right) \approx \widehat{E}_{thick}$













Momentum flux components $\Delta x^{2} ||\nabla u||^{2} \operatorname{ANN}\left(\frac{\nabla u}{||\nabla u||}, \frac{S}{||S||}\right) \approx \widehat{E}_{mom}$ Normalized inputs **Thickness flux components** $\Delta x^2 f ||\nabla u|||S|| \operatorname{ANN}\left(\frac{\nabla u}{||\nabla u||}, \frac{S}{||S||}\right) \approx \widehat{E}_{thick}$







Offline performance on test data





High R^2 for Reynolds' stresses in top layer



Offline performance on test data



*The R^2 is set to 0.0 when the spatial average $R^2 < 0$



High R^2 for Reynolds' stresses in top layer

Worse performance in bottom layer

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Moderate R^2 for dual form stresses in top layer



*The R^2 is set to 0.0 when the spatial average $R^2 < 0$









Moderate R^2 for dual form stresses in top layer

Poor R^2 for dual form stresses in bottom layer



*The R^2 is set to 0.0 when the spatial average $R^2 < 0$





Offline performance on test data



*The R^2 is set to 0.0 when the spatial average $R^2 < 0$



Online Implementation: Top Layer Mean Flow



 $1/32^{\circ}$ flow field filtered and coarsened to $1/4^{\circ}$ resolution.



Online Implementation: Top Layer Mean Flow



Pesky persistent eddy

1/32° flow field filtered and coarsened to 1/4° resolution.

1/4° resolution run with **no parameterisation** (only Smagorinsky)



Online Implementation: Top Layer Mean Flow



Averages over daily snapshots of ~ 10 simulation years

 $1/32^{\circ}$ flow field filtered and coarsened to $1/4^{\circ}$ resolution.

 $1/4^{\circ}$ resolution run with **no parameterisation** (only Smagorinsky)

1/4° resolution run with EPF ANN parameterisation (in addition to Smagorinsky)

Pesky persistent eddy is weakened by parameterisation!



Isotropic KE transfer spectra



Filtered data



Isotropic Kinetic Energy transfer spectra







Isotropic Kinetic Energy transfer spectra







Isotropic Kinetic Energy transfer spectra



Not much improvement in bottom layer



Concluding remarks

Initial data-driven parametrization to capture the mesoscale energy cycle using the Eliassen-Palm Flux tensor components:

Idealised runs, double gyre

- Offline performance:

 - ... however, improvements offline do not guarantee improvements online

Online performance:

- Doesn't blow up!
- Weakens pesky persistent eddy in mean flow
- Better representation of energy spectral transfer
- Still room for improvement

Upcoming:

- How can we improve representation of thickness fluxes? lacksquare
 - Improve ANN to be implemented

Use of eddy energy system to produce a single parameterisation that captures both momentum and thickness fluxes seems promising 🗠

• Room for improvement in training (more epochs, more data, regularization techniques, etc)





Supplementary Slides



Supplementary Slide: Weighting layer influence during training



Alternate ANN trained, where back-propagated loss depends only on the top layer

'Original' ANN trained, where back-propagated loss depends equally on top and bottom layer

Alternate ANN trained, where back-propagated loss depends only on the bottom layer





Supplementary Slide: EPF parameterization contributions for top layer



1/4° resolution DG run with **full EPF ANN** parameterisation (in addition to Smagorinsky)

1/4° resolution DG run with **horizontal** divergence of EPF (in addition to Smagorinsky)

1/4° resolution DG run with **vertical** divergence of EPF (in addition to Smagorinsky)





Supplementary Slide: ANN with bottom topography ignored











Large-scale flow field: Bottom layer



 $1/32^{\circ}$ flow field filtered and coarsened to $1/4^{\circ}$ resolution.

 $1/4^{\circ}$ resolution run with **no** parameterisation

 $1/4^{\circ}$ resolution run with EPF ANN parameterisation







Strength of Parameterisation



