



ODEN INSTITUTE

FOR COMPUTATIONAL ENGINEERING & SCIENCES

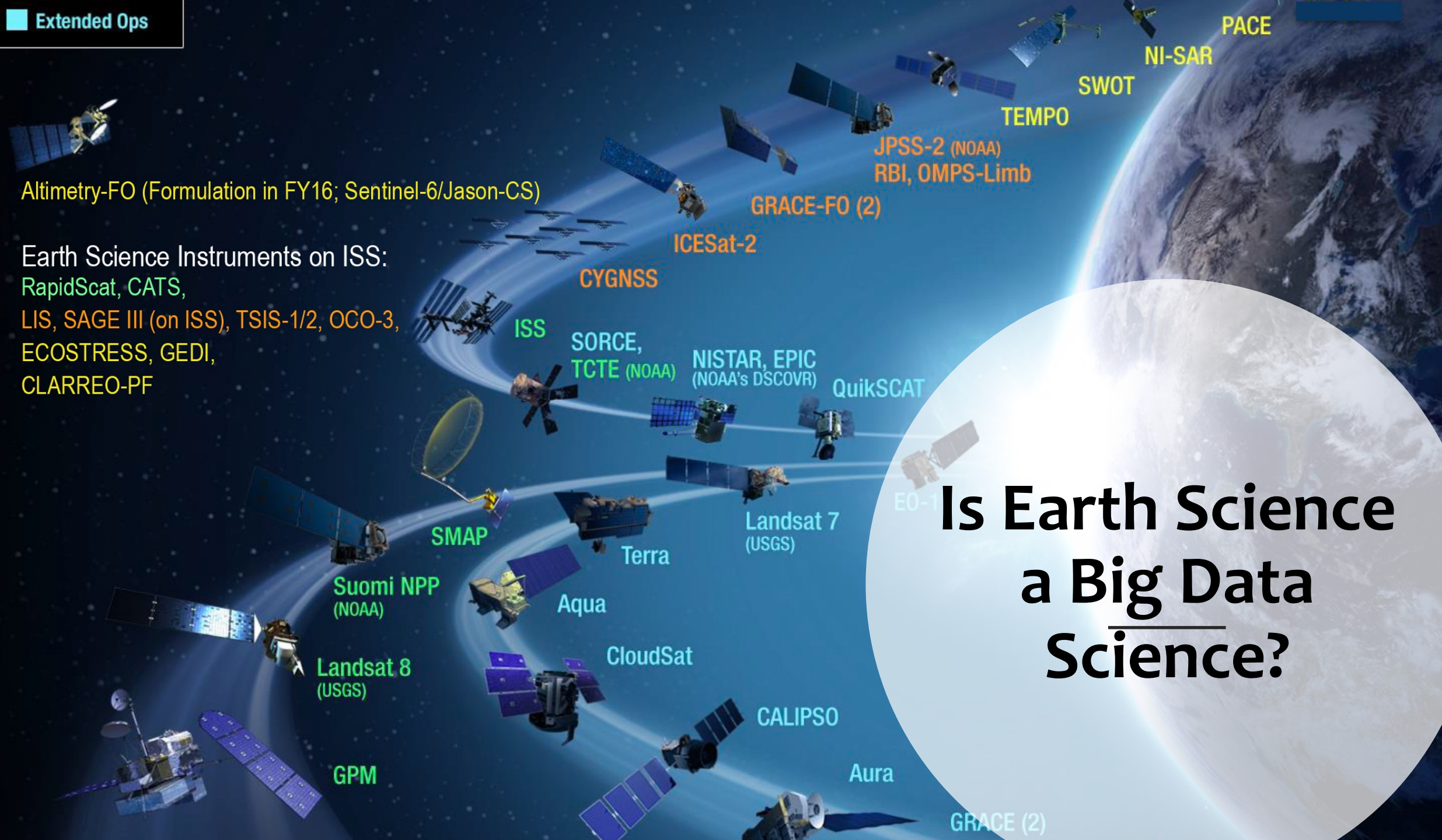
LEARNING FROM (SPARSE) OBSERVATIONS THROUGH THE LENS OF MODELS

Patrick Heimbach, *the DJ4Earth & ECCO groups*
The University of Texas at Austin, TX

<https://ecco-group.org>
<https://crios-ut.github.io>
<https://dj4earth.github.io>

Altimetry-FO (Formulation in FY16; Sentinel-6/Jason-CS)

Earth Science Instruments on ISS:
RapidScat, CATS,
LIS, SAGE III (on ISS), TSIS-1/2, OCO-3,
ECOSTRESS, GEDI,
CLARREO-PF



Is Earth Science
a Big Data
Science?

**Is Oceanography
a “big data”
science?**

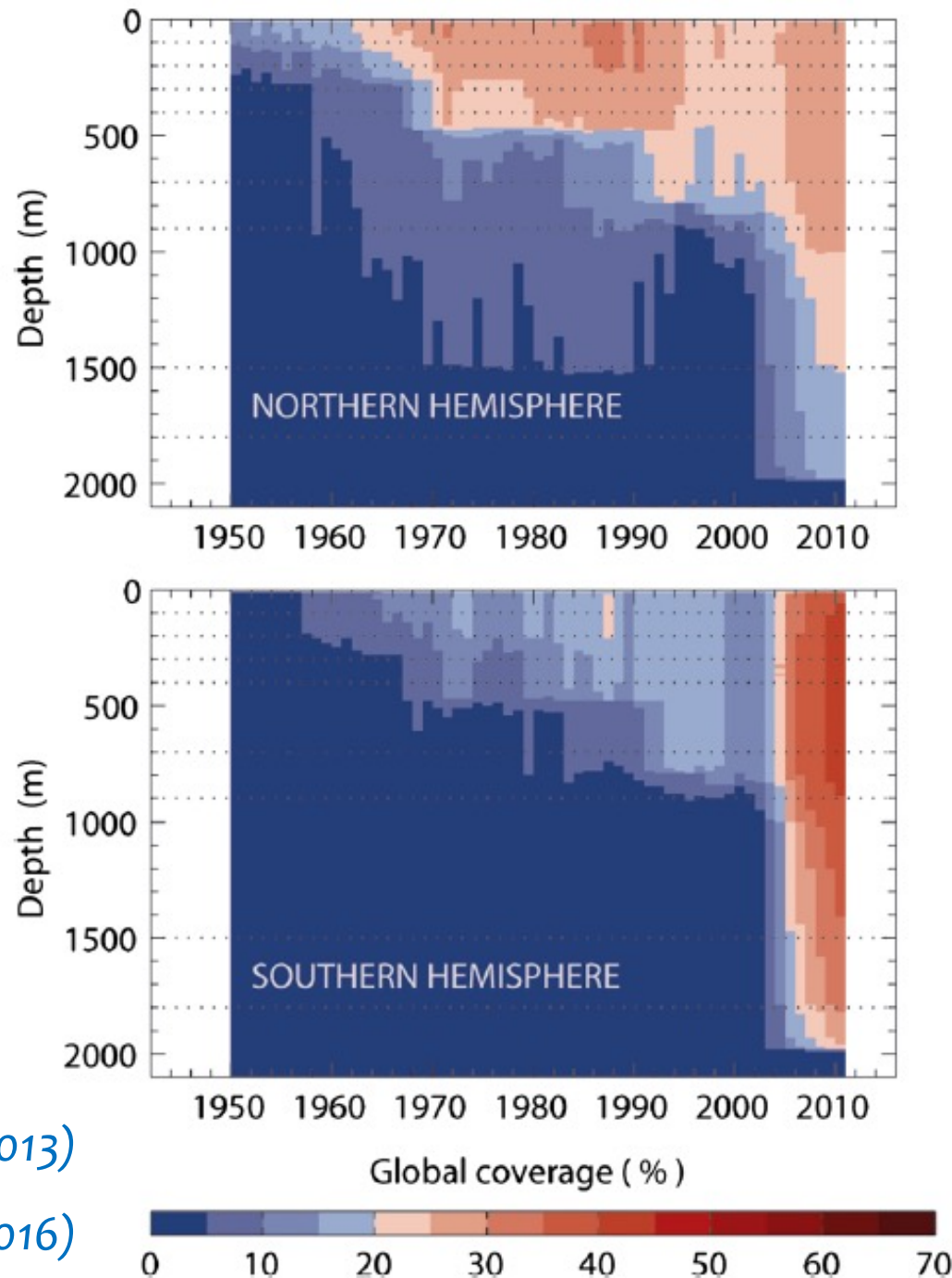
Yes & No ...

Oceanography: A sparse data problem ...

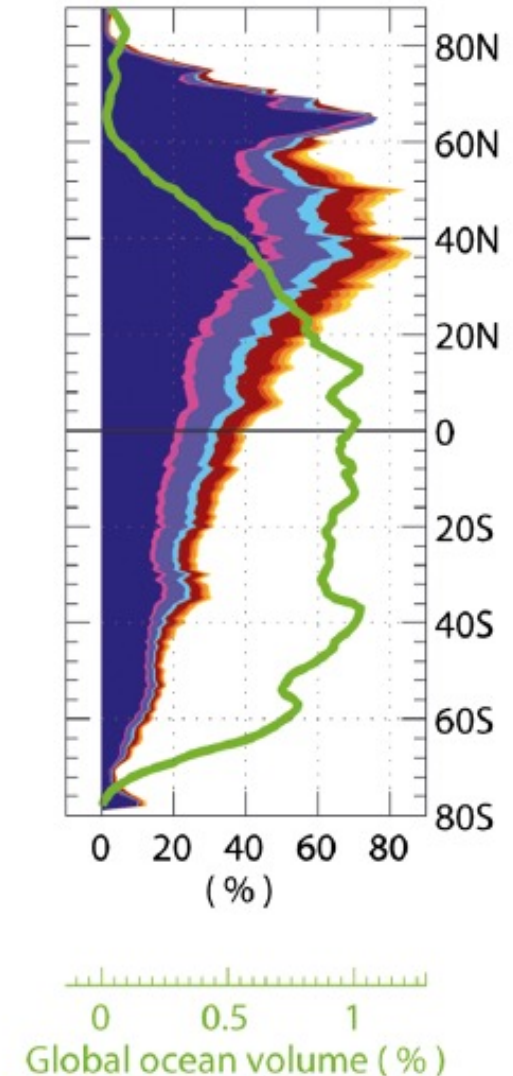
Observational sampling
coverage for ocean
temperature in the
upper 2000 m
1950 – 2010
(mean ocean depth:
~ 3900 m)

Abraham et al., Rev. Geophys. (2013)

Wunsch (2016)



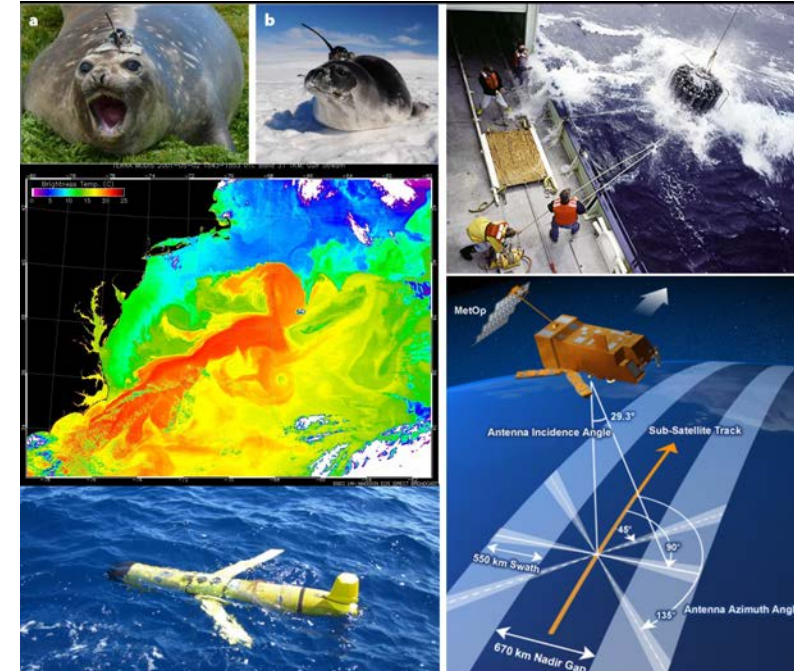
Mean zonal coverage
(1950–2011)



(colors refer to
depth ranges)

Two incomplete
knowledge
reservoirs

an eclectic, patchy, heterogeneous
observing system



numerical models

that require
uncertain
inputs



Session 1: Ocean Dynamics and Circulation

1. How have the recent advances in ocean modeling helped our understanding of the ocean's dynamics and the role of the ocean in the climate system? How does understanding dynamics feed into improving OGCMs?
2. How can we better use observations to evaluate and advance ocean models? Do we have the observations needed, including for evaluating high-resolution models?
3. Is the ocean modeling community tackling the relevant problems, including model development efforts?
4. What are your thoughts on a hierarchical modeling approach from coarse to ultra-high resolution modeling?

What is Data Assimilation / Inverse Modeling?

Kaminski et al., *The Cryosphere* (2015):

“Ideally, ...

- ... all observational data streams are interpreted simultaneously,
- ... with the process information provided by the model,
- ... [which leads to] a consistent picture of the state of the system,
- ... that balances all the observational constraints,
- ... taking into account all the respective uncertainty ranges.”

Penny et al., *Front. Mar. Sci.* (2019):

“DA allows information provided from observations to be propagated in time and space to unobserved areas using the dynamical and physical constraints imposed by numerical models.”

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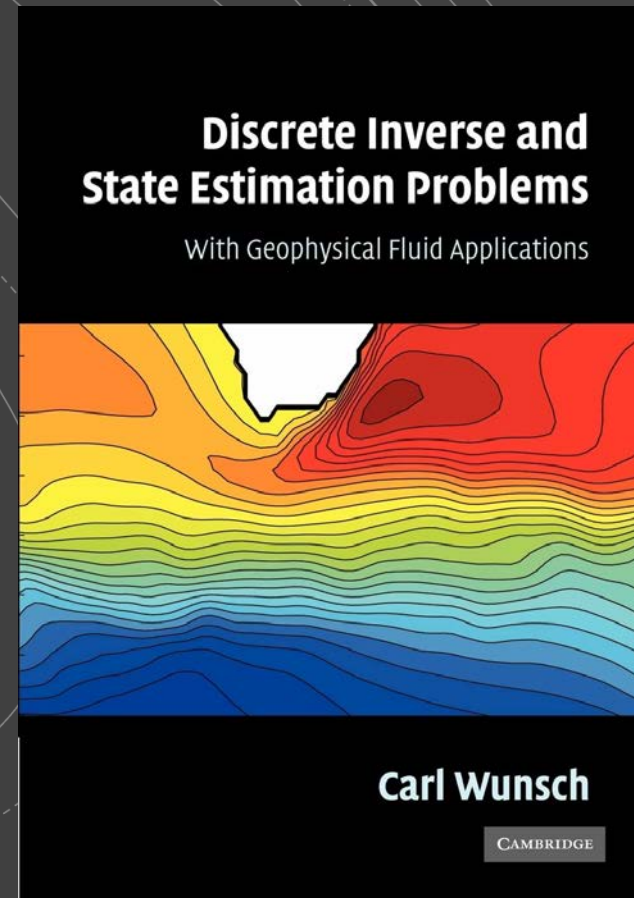
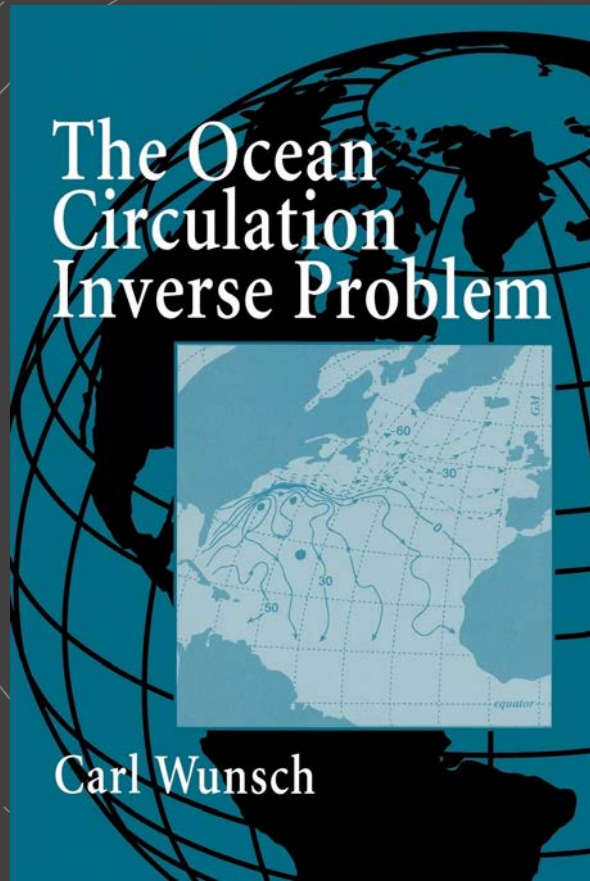
“DA allows information provided from observations to be propagated in time and space to unobserved areas using the dynamical and physical constraints imposed by numerical models.”

Data Assimilation and Inverse Modeling

The DA / inverse problem is learning from ...

- a set of available (usually sparse, heterogeneous) observations
- ... AND known physics/dynamics,
- ... by solving a gigantic least-squares model-data misfit minimization

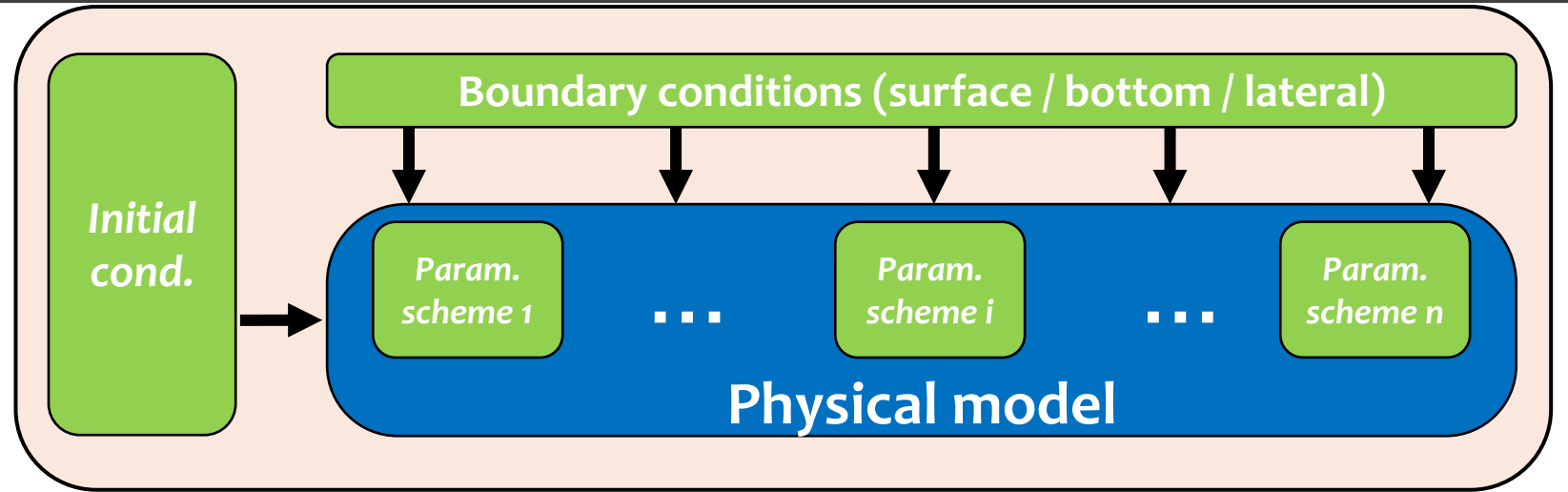
“Data assimilation” is much more than its use in numerical weather prediction



The background is a dark gray color with several concentric, overlapping circles of varying shades of gray. A single dashed white line also curves across the scene, intersecting the solid circles.

What do we mean by
“Learning”?

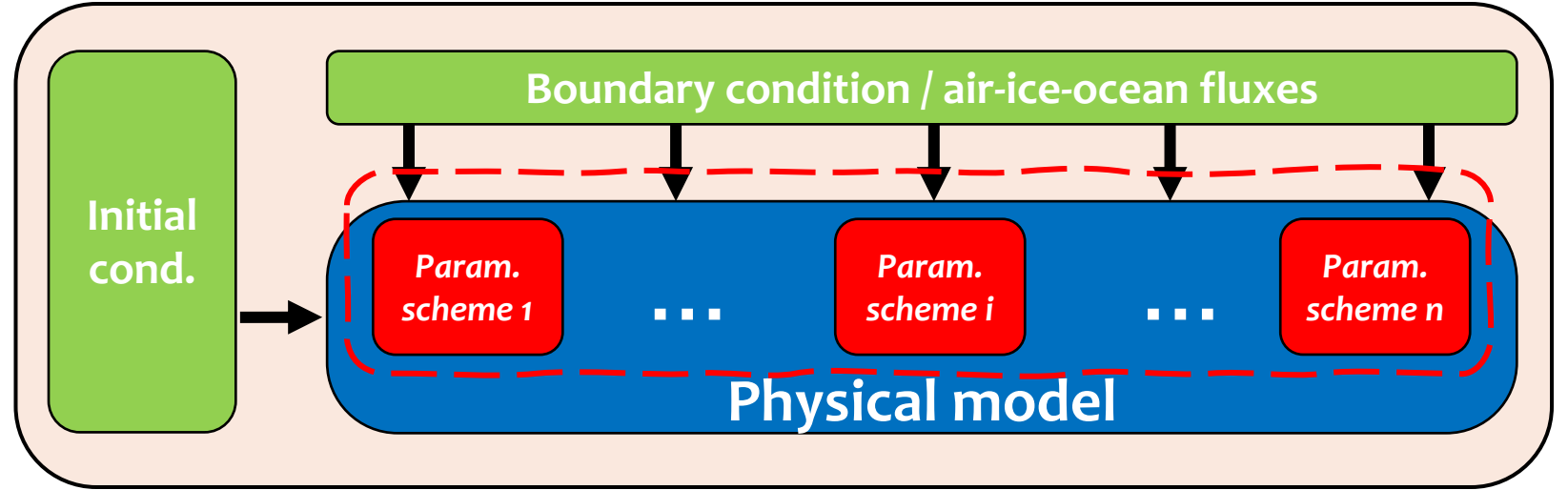
Learn ...



Learn model parameters

Physical model has many empirical parameters:

- constitutive laws
- subgrid-scale parameterization schemes

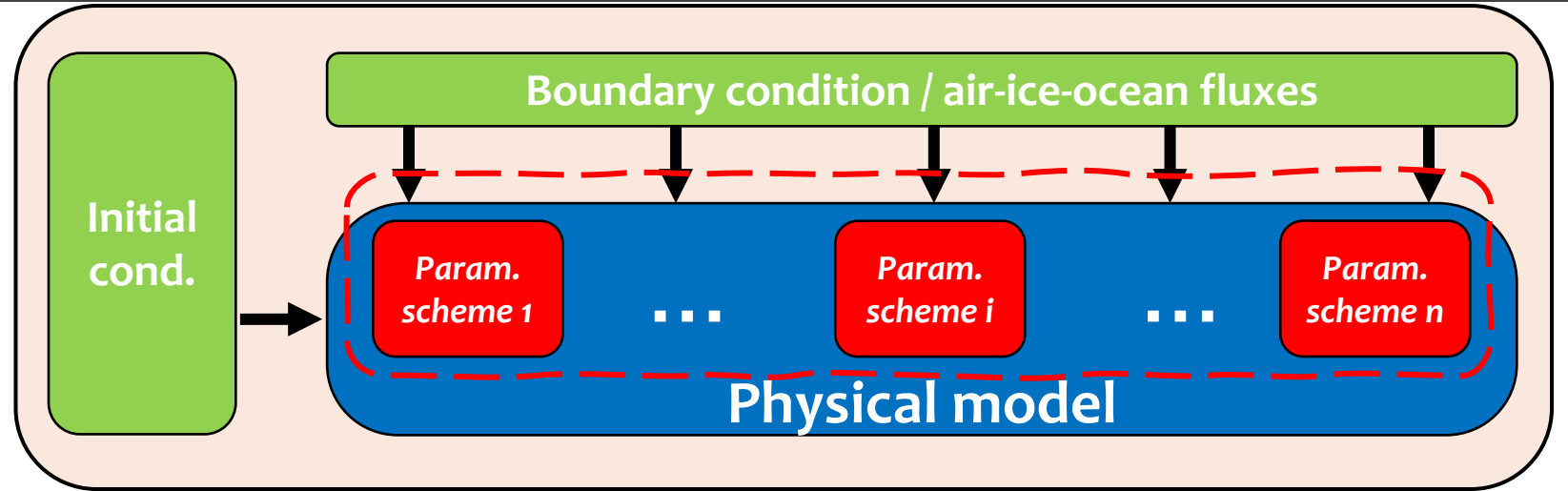


Learn model parameters

Physical model has many empirical parameters:

- constitutive laws
- subgrid-scale parameterization schemes

parameter estimation using observations is essential



THE ART AND SCIENCE OF CLIMATE MODEL TUNING

BAMS

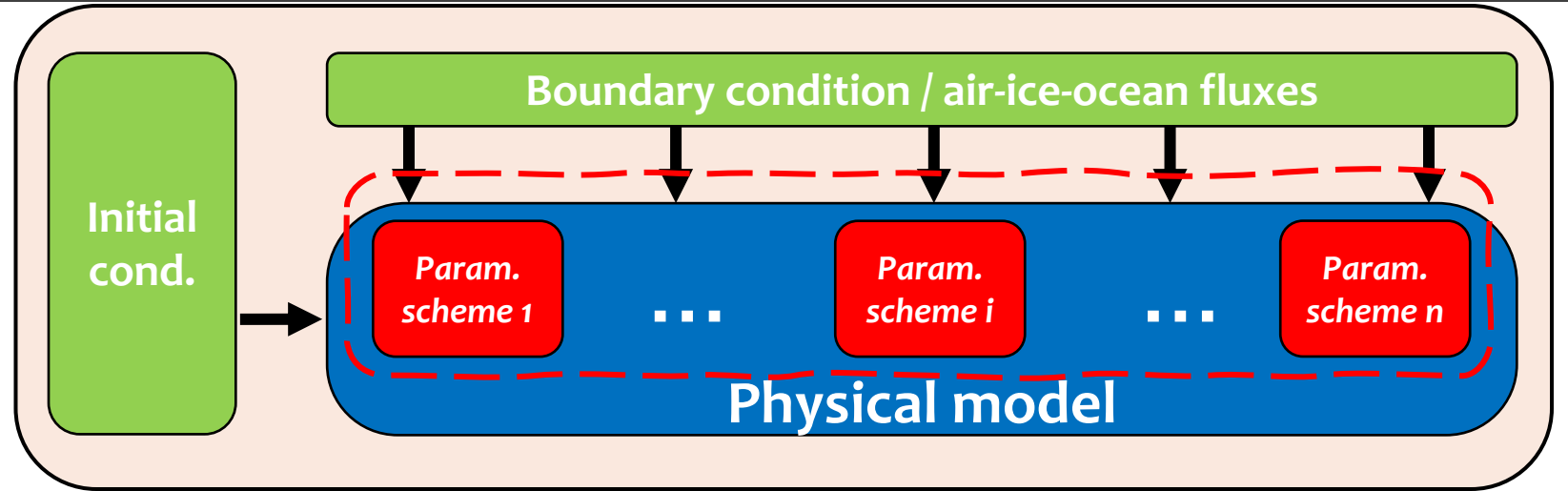
FRÉDÉRIC HOURDIN, THORSTEN MAURITSEN, ANDREW GETTELMAN, JEAN-CHRISTOPHE GOLAZ, VENKATRAMANI BALAJI, QINGYUN DUAN, DORIS FOLINI, DUOYING JI, DANIEL KLOCKE, YUN QIAN, FLORIAN RAUSER, CATHERINE RIO, LORENZO TOMASSINI, MASAHIRO WATANABE, AND DANIEL WILLIAMSON

We survey the rationale and diversity of approaches for tuning, a fundamental aspect of climate modeling, which should be more systematically documented and taken into account in multimodel analysis.

Learn model parameters

Physical model has many empirical parameters:

- constitutive laws
- subgrid-scale parameterization schemes



***parameter estimation
to calibrate model
parameters***



Ocean Sci., 11, 839–853, 2015
www.ocean-sci.net/11/839/2015/
doi:10.5194/os-11-839-2015
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Ocean Science

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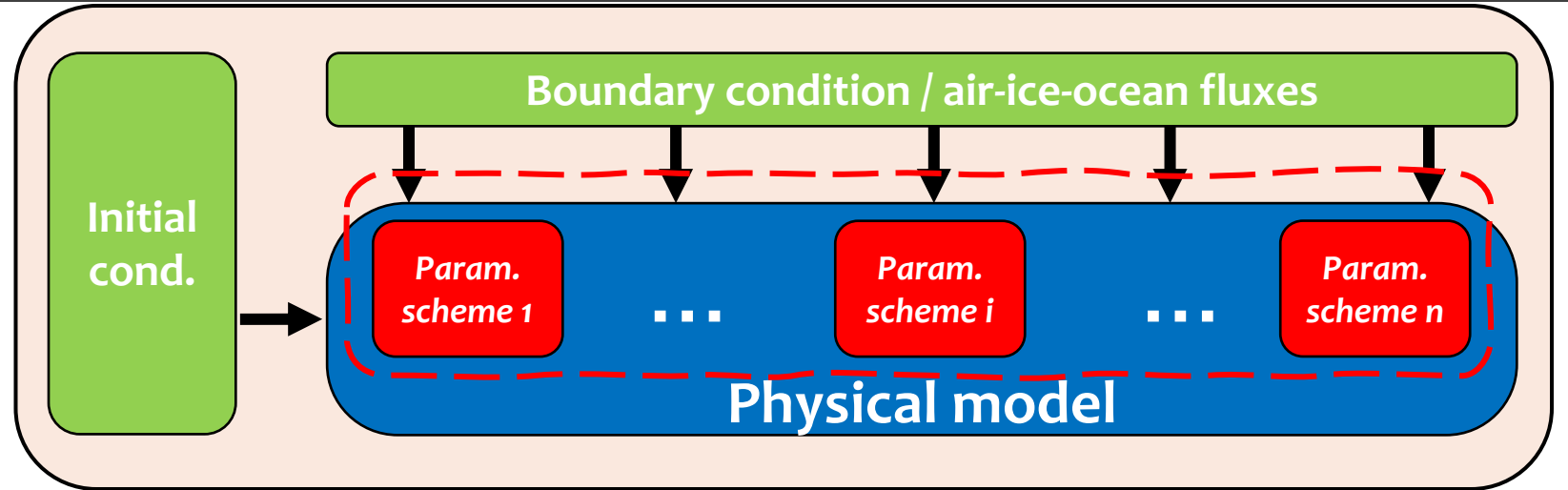
**On the observability of turbulent transport rates by Argo:
supporting evidence from an inversion experiment**

G. Forget¹, D. Ferreira², and X. Liang¹

Learn model parameters

Physical model has many empirical parameters:

- constitutive laws
- subgrid-scale parameterization schemes



parameter estimation to calibrate model parameters



Ocean Sci., 18, 729–759, 2022
<https://doi.org/10.5194/os-18-729-2022>
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Ocean Science

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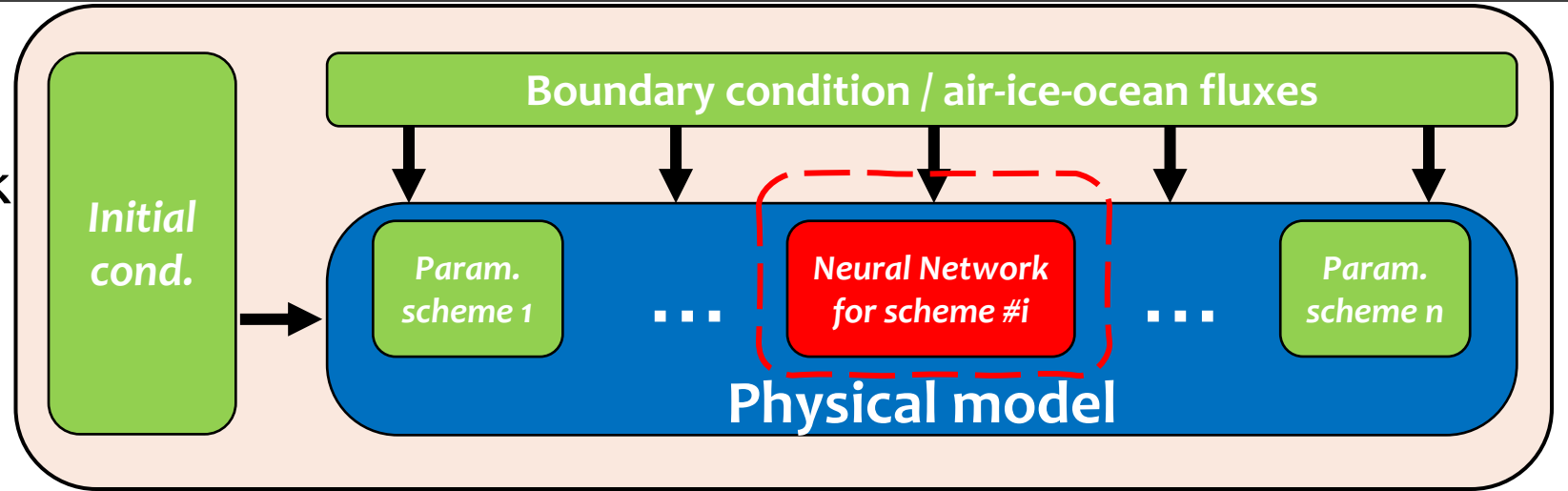
Tracer and observationally derived constraints on diapycnal diffusivities in an ocean state estimate

David S. Trossman^{1,2}, Caitlin B. Whalen³, Thomas W. N. Haine⁴, Amy F. Waterhouse⁵, An T. Nguyen⁶, Arash Bigdeli⁷, Matthew Mazloff⁵, and Patrick Heimbach^{6,8}

Learn surrogate (e.g., NN) of model's parameterization scheme

Parameterization scheme(s)
is replaced by neural network

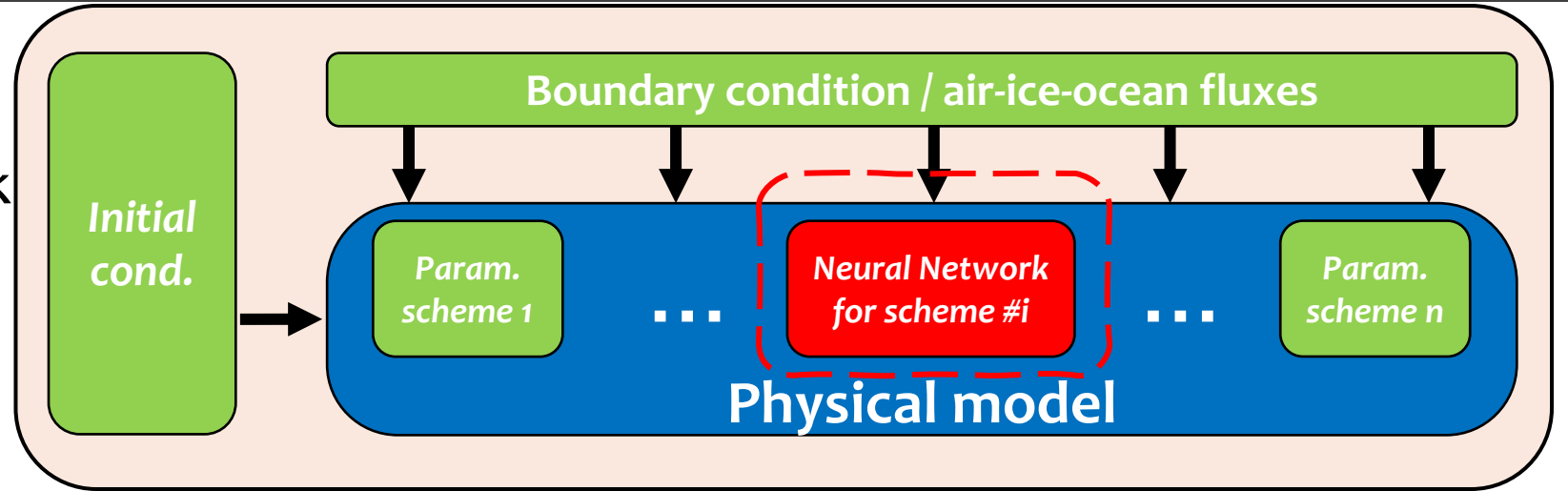
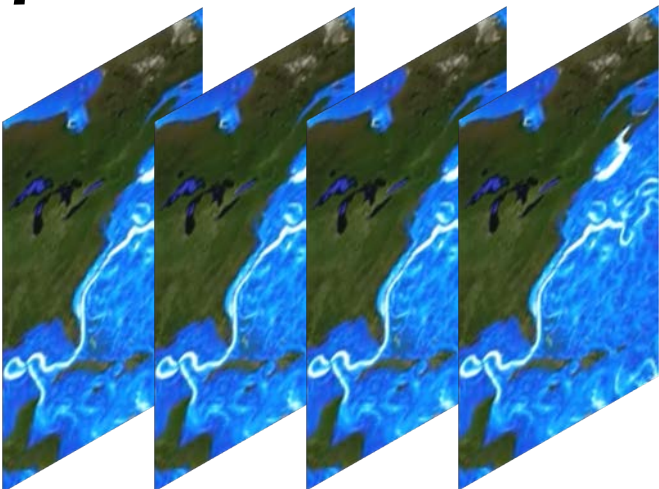
NN is trained on high-fidelity simulation data which resolve scales to be parameterized



Learn surrogate (e.g., NN) of model's parameterization scheme

Parameterization scheme(s) is replaced by neural network

NN is trained on high-fidelity simulation data which resolve scales to be parameterized

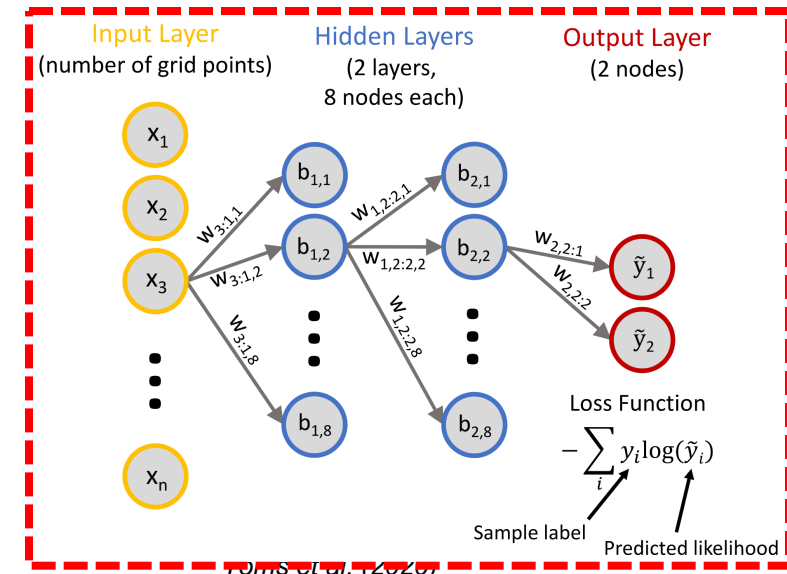


$$\frac{\partial \bar{\Phi}}{\partial t} + (\bar{\mathbf{u}} \cdot \nabla) \bar{\Phi} = \nabla \cdot (\kappa \nabla \bar{\Phi}) + \bar{F}_\Phi + \nabla \cdot \mathbf{S}$$

$$\nabla \cdot \mathbf{S} = \nabla \cdot (\bar{\mathbf{u}} \bar{\Phi} - \bar{\mathbf{u}} \bar{\Phi}) =$$

a priori / offline learning

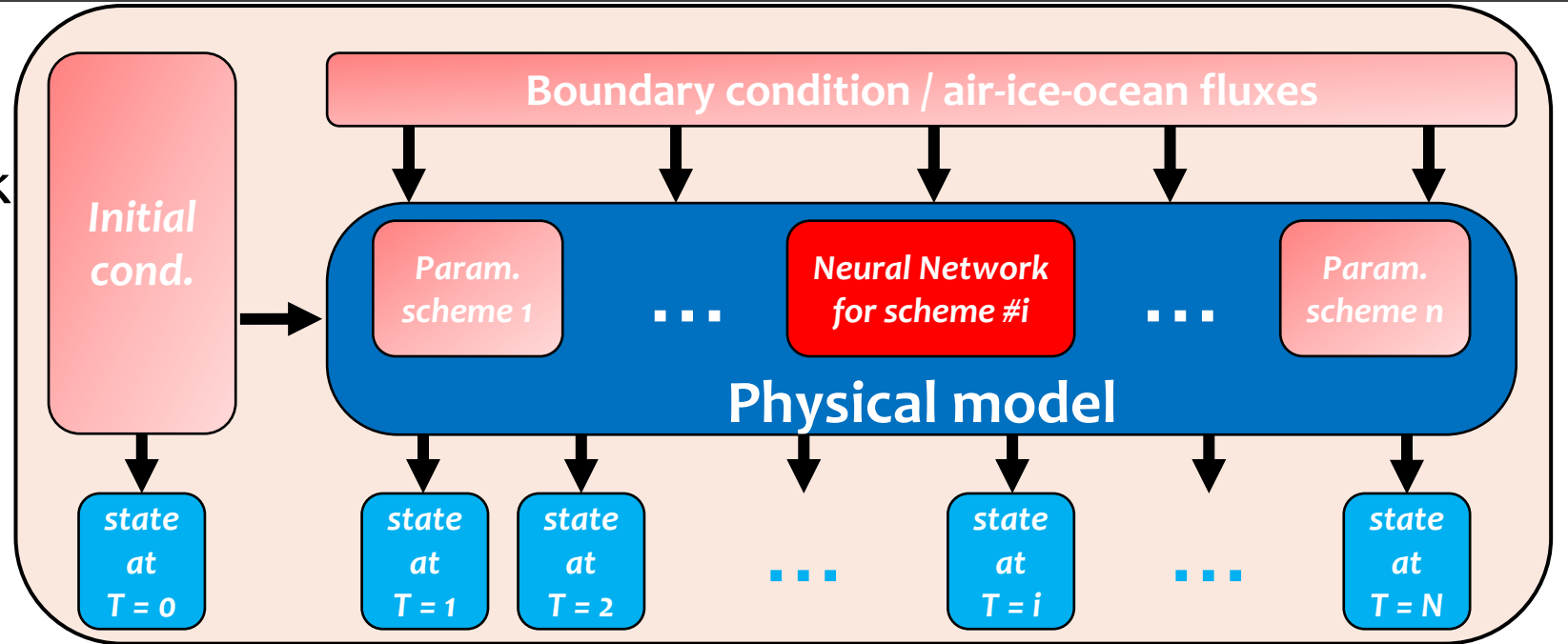
Zanna & Bolton (2021)



Learn hybrid physical/surrogate (NN) model

Parameterization scheme(s)
is replaced by neural network

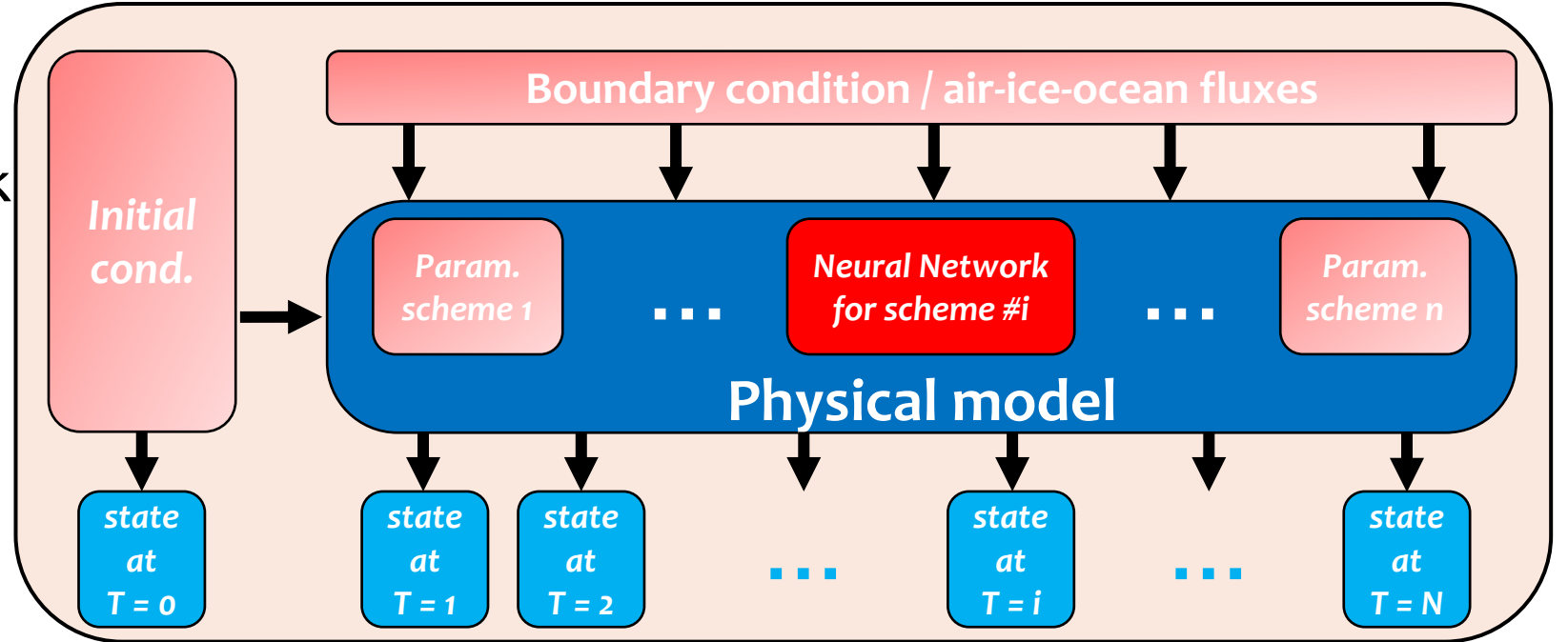
**Training of the NN is
part of “training” of
the physical model
on state variables**



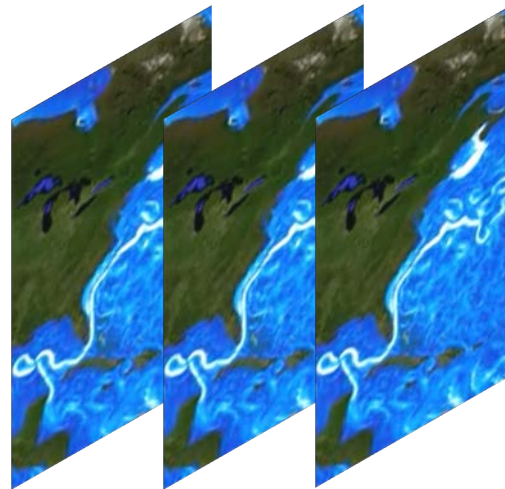
Learn *hybrid* physical/surrogate (NN) model

Parameterization scheme(s)
is replaced by neural network

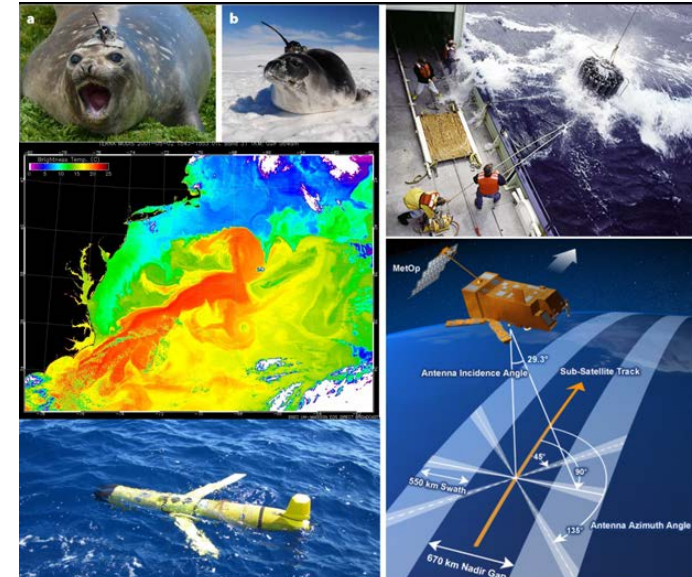
**Training of the NN is
part of “training” of
the physical model
on state variables**



**a posteriori / full-model
/ online / end-to-end
learning**



+

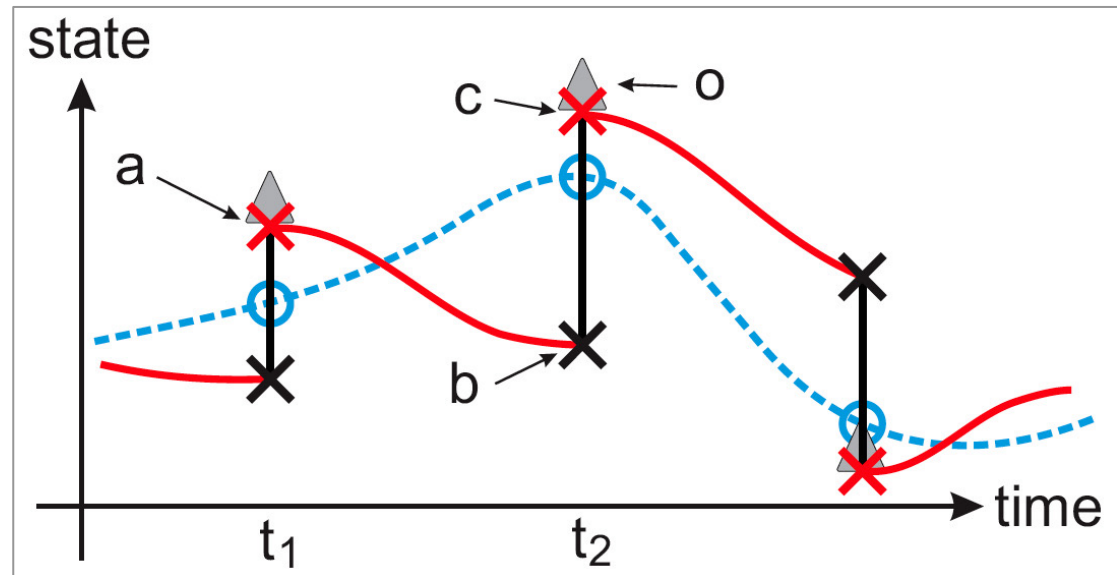
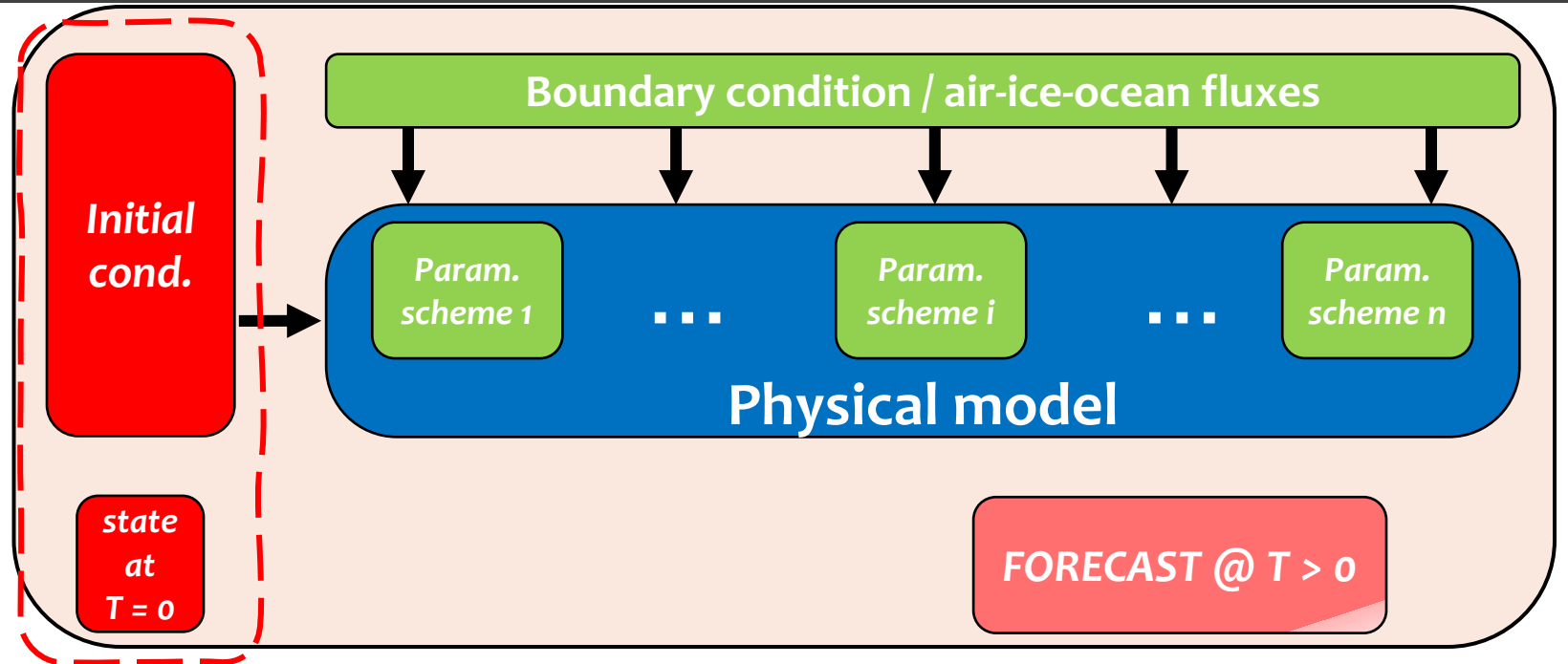


Learn model initial conditions

Find best initial conditions that will produce optimal forecast ...

The **filtering** problem of optimal estimation & control

Initialization for prediction/extrapolation as practiced in **numerical weather prediction**



Learn model time-evolving state

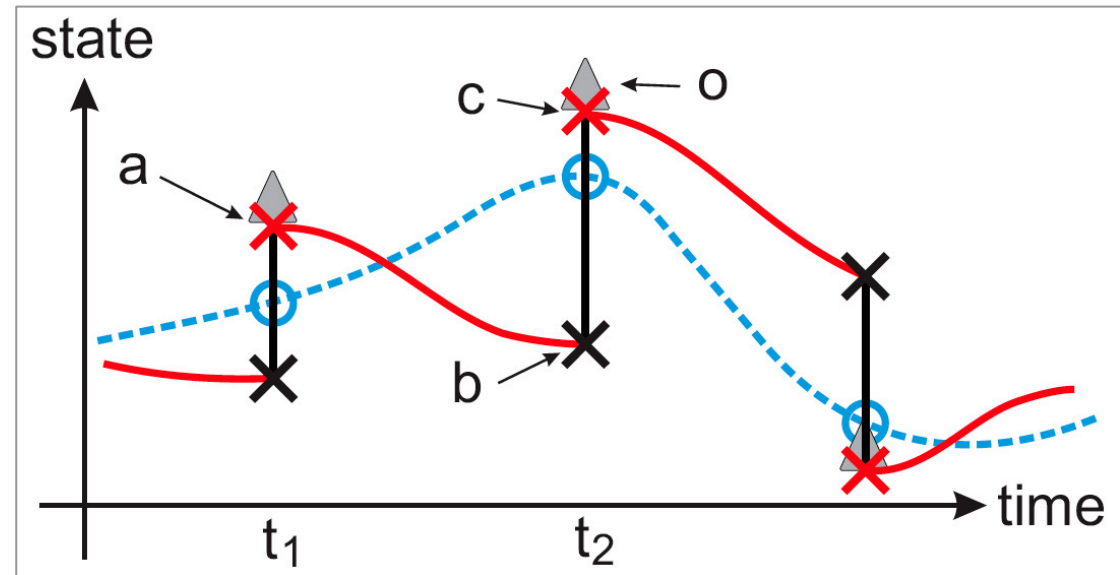
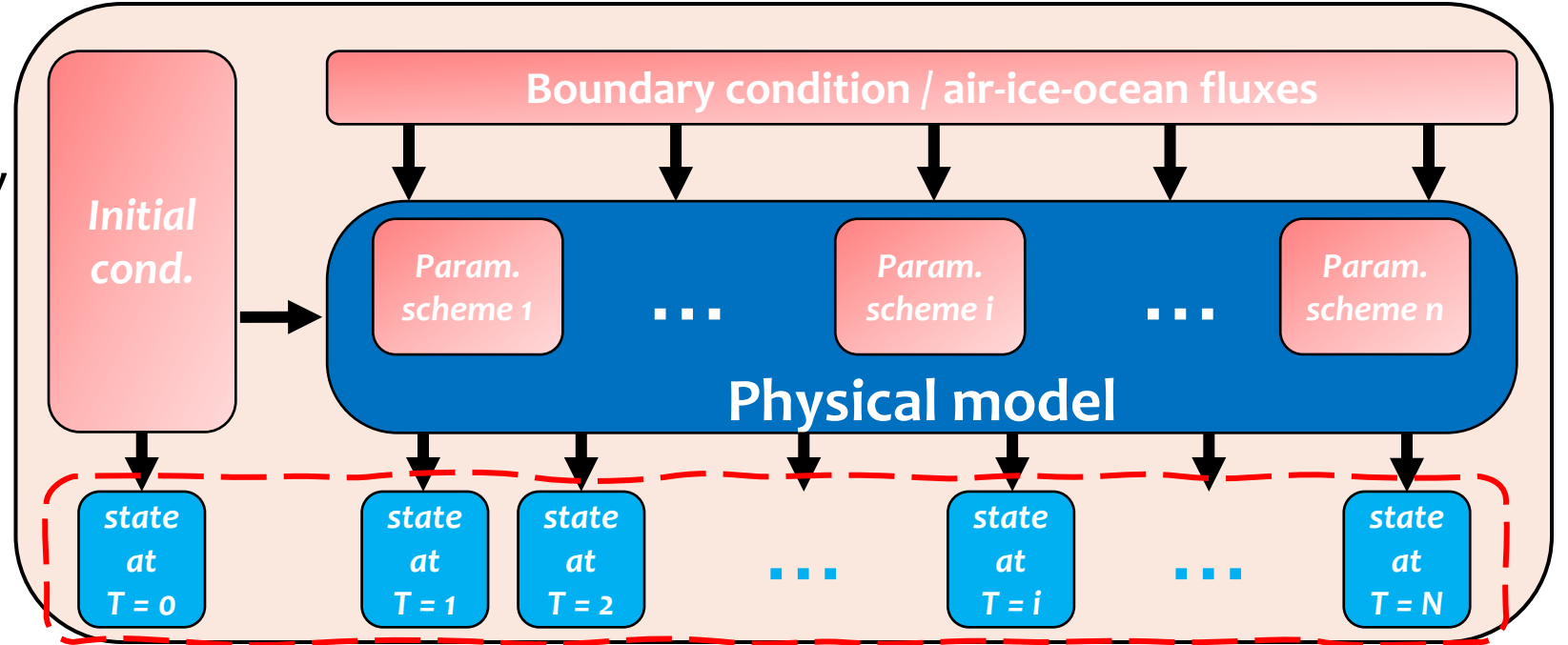
Find model inputs that produce the best dynamically consistent state

The smoothing problem of optimal estimation & control

State & parameter estimation

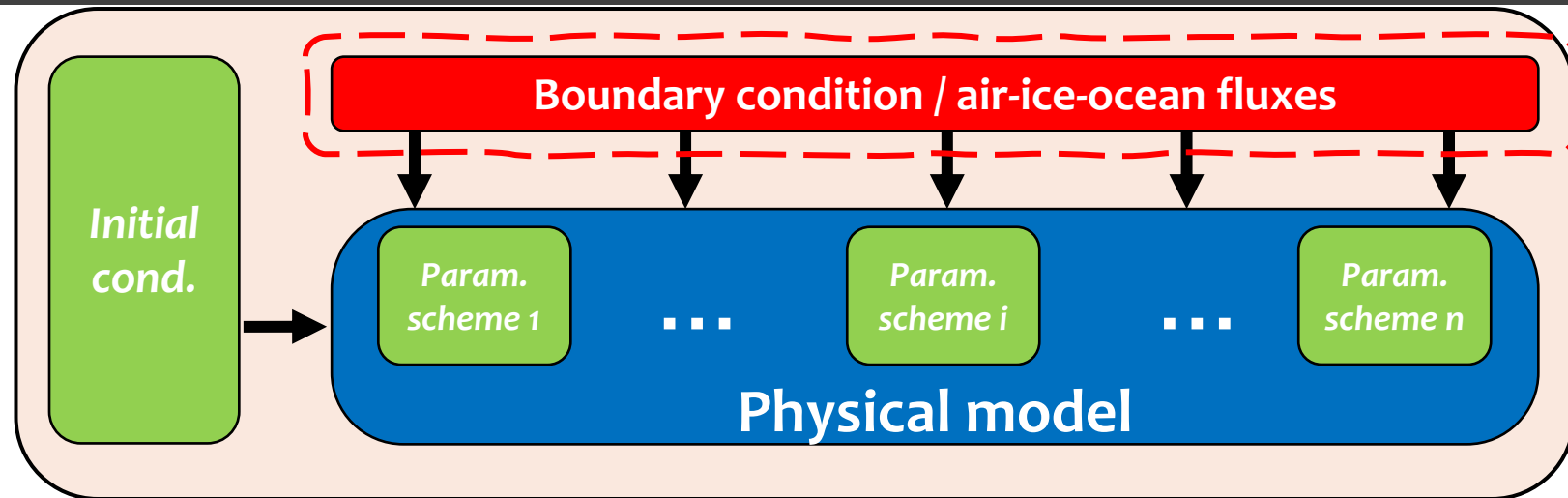
for:

- **Interpolation/reconstruction**
- **transient calibration**



Learn model boundary conditions

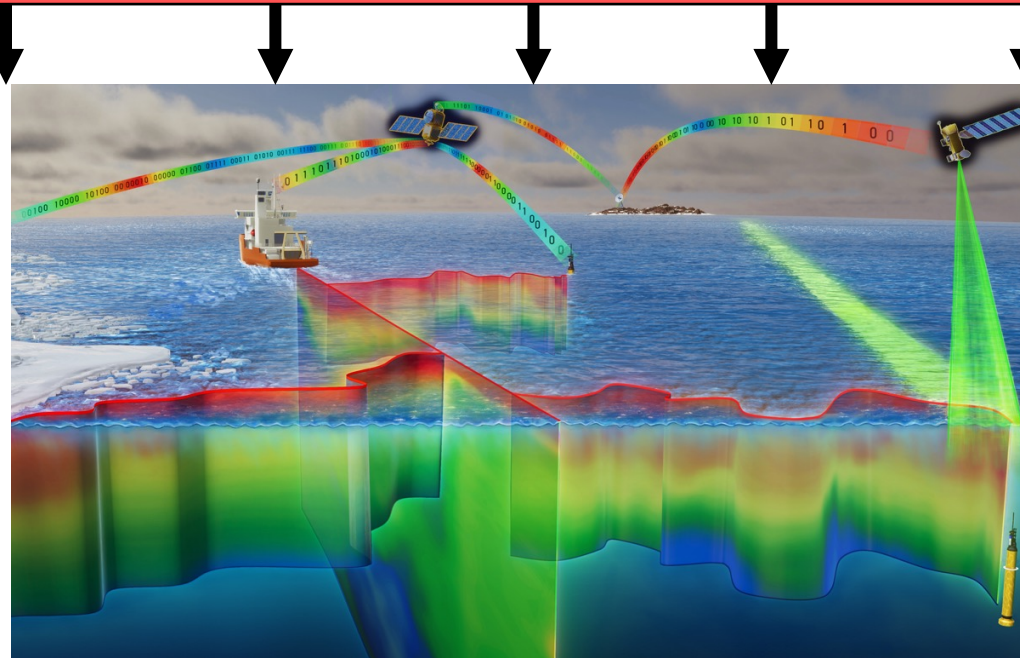
Observations of **ocean** interior, combined with global & local mass/tracer conservation enables ...



... **inversion for surface fluxes** that are required to match interior observations

Example for CO₂ air-sea fluxes (similar for heat fluxes)

air-sea fluxes of CO₂ inferred from interior measurements



SOCCOM

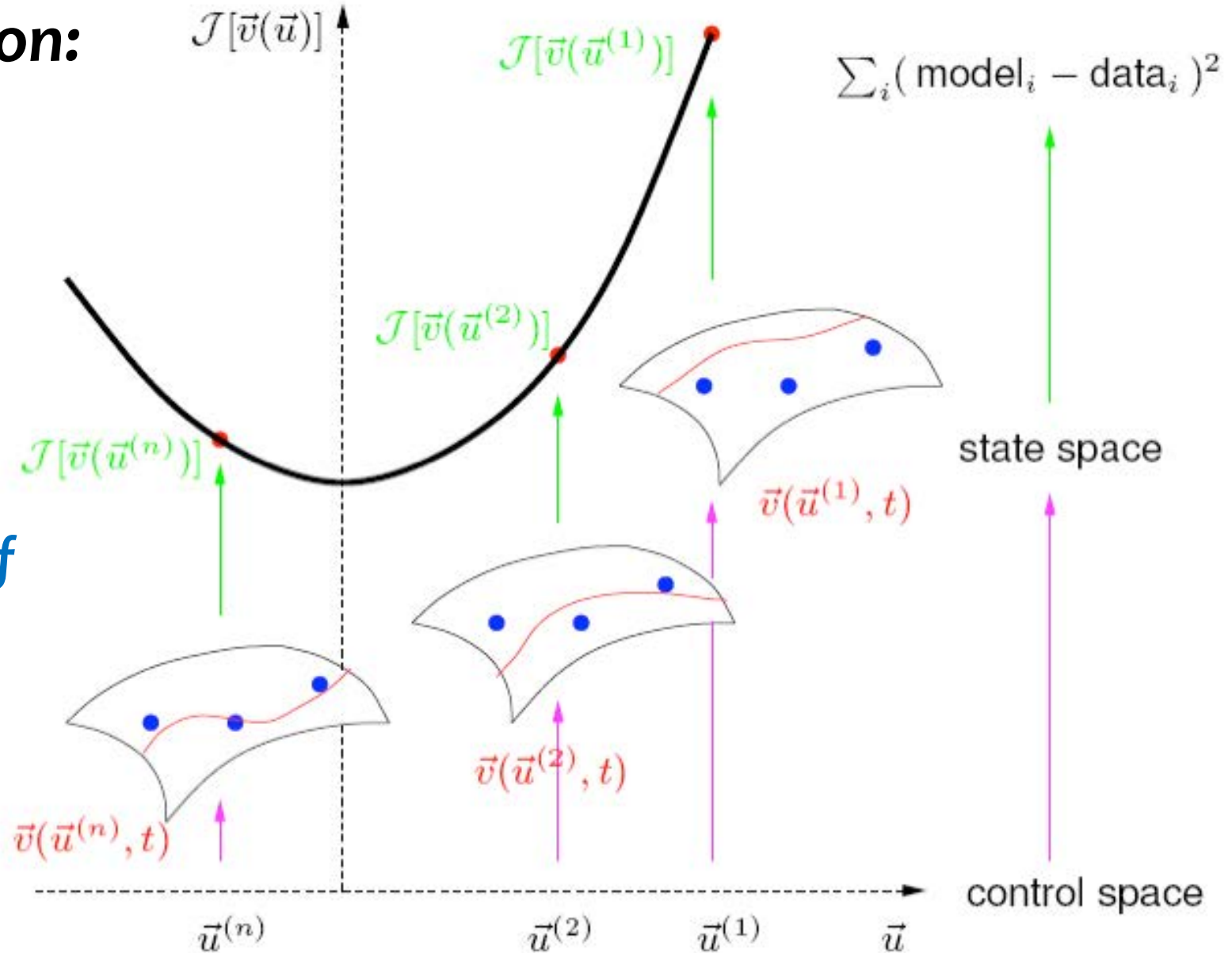
A key unifying computational framework of “learning from data”

Gradient-based optimization:

- inversion (physical models)
 - seek uncertain input / control variables / parameters
- training (neural networks)
 - seek uncertain weights of NN representation

Adjoint / backpropagation

essential tool for computing high-dimensional gradients!



Full-model learning

Can we integrate the
surrogate model training
within full-model calibration

Home

Welcome to Differentiable programming in Julia for Earth system modeling (DJ4Earth)

Cyberinfrastructure for Sustained Scientific Innovation (CSSI)

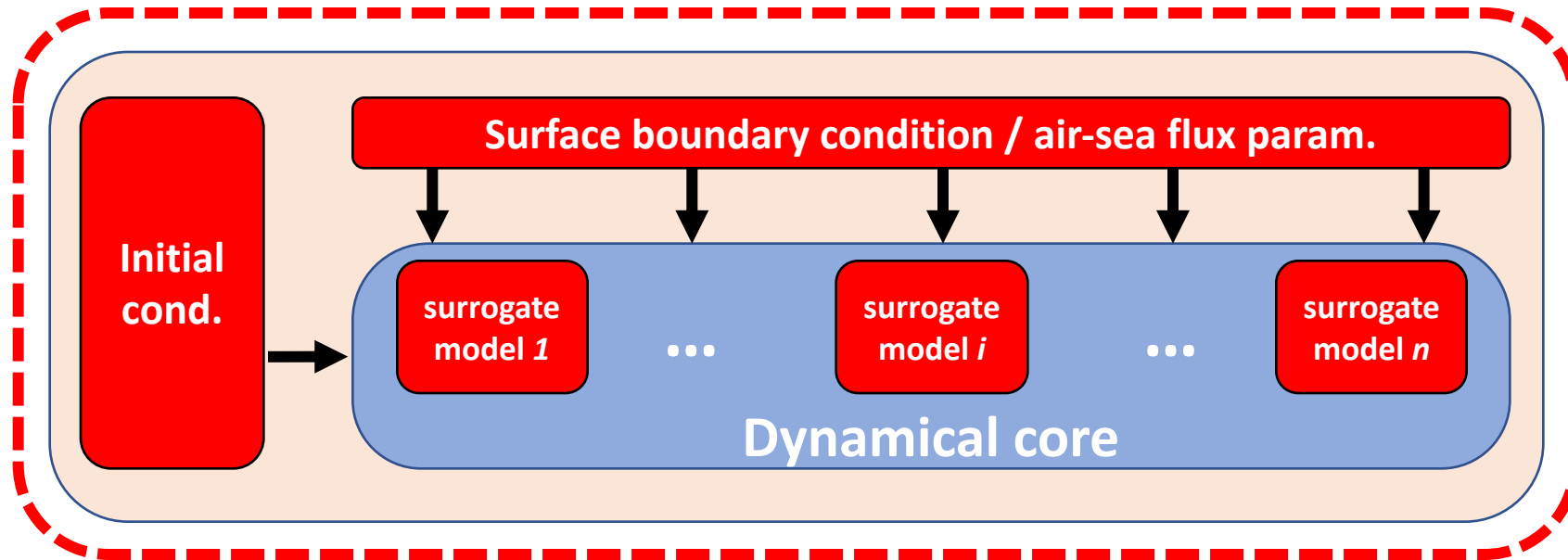


<https://DJ4Earth.github.io>

NSF CSSI: **DJ4Earth**

Convergence of Bayesian inverse methods and scientific machine learning through universal differentiable programming

An end-to-end adjoint enables full-model calibration & initialization



Here: use of full-model [differentiable programming](#) to

- replace parts of model by appropriate surrogates
- use all available observations to train/calibrate all uncertain variables
- combines inverse modeling and ML in [end-to-end learning](#)

relies on general-purpose automatic differentiation (AD)

Since 2023 the idea of differentiable programming has taken off ...

Geosci. Model Dev., 16, 3123–3135, 2023

<https://doi.org/10.5194/gmd-16-3123-2023>

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Geoscientific
Model Development



Differentiable programming for Earth system modeling

Maximilian Gelbrecht^{1,2}, Alistair White^{1,2}, Sebastian Bathiany^{1,2}, and Niklas Boers^{1,2,3}

¹Earth System Modelling, School of Engineering and Design, Technical University of Munich, Munich, Germany

²Potsdam Institute for Climate Impact Research, Potsdam, Germany

³Department of Mathematics and Global Systems Institute, University of Exeter, Exeter, UK

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<https://doi.org/10.5194/gmd-16-3123-2023>

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Geoscientific
Model Development

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<https://doi.org/10.1038/s43017-023-00450-9>



Differentiable programming

Maximilian Gelbrecht^{1,2}, Alistair White^{1,2}, Sebastian

¹Earth System Modelling, School of Engineering and De

²Potsdam Institute for Climate Impact Research, Potsdam,

³Department of Mathematics and Global Systems Institute,

nature reviews earth & environment

Perspective

Differentiable modelling to unify machine learning and physical models for geosciences

A list of authors and their affiliations appears at the end of the paper

Since 2023 the idea of differentiable programming has taken off ...

Geosci
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135, 2023

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<https://doi.org/10.1038/s43017-023-00450-9>

Check for updates

arXiv > math > arXiv:2406.09699

Mathematics > Numerical Analysis

[Submitted on 14 Jun 2024]

Differentiable Programming for Differential Equations: A Review

Facundo Sapienza, Jordi Bolibar, Frank Schäfer, Brian Groenke, Avik Pal, Victor Boussange, Patrick Heimbach, Giles Hooker, Fernando Pérez, Per-Olof Persson, Christopher Rackauckas

A list of authors and their affiliations

Maximilian
¹Earth System Modelling
²Potsdam Institute for Climate
³Department of Mathematics and Global

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physical

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Model Development



<https://doi.org/10.1038/s43017-023-00450-9>

Check for updates

Article

Neural general circulation models for weather and climate

<https://doi.org/10.1038/s41586-024-07744-y>

Received: 13 November 2023

Accepted: 15 June 2024

Published online: 22 July 2024

Dmitrii Kochkov^{1,6}✉, Janni Yuval^{1,6}✉, Ian Langmore^{1,6}, Peter Norgaard^{1,6}, Jamie Smith^{1,6}, Griffin Mooers¹, Milan Klöwer², James Lottes¹, Stephan Rasp¹, Peter Düben³, Sam Hatfield³, Peter Battaglia⁴, Alvaro Sanchez-Gonzalez⁴, Matthew Willson⁴, Michael P. Brenner^{1,5} & Stephan Hoyer^{1,6}✉

A list of authors and their affiliations

...nge, Patrick Heimbach,

Why Julia?

1/ Building on *Climate Modeling Alliance (CiMA)*

2/ Serious efforts in AD, *differentiable programming*

3/ Harness next-gen. *compute architecture*

DJ4Earth



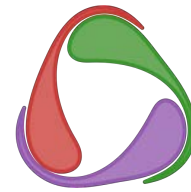
CLIMATE MODELING ALLIANCE

A NEW APPROACH TO CLIMATE MODELING

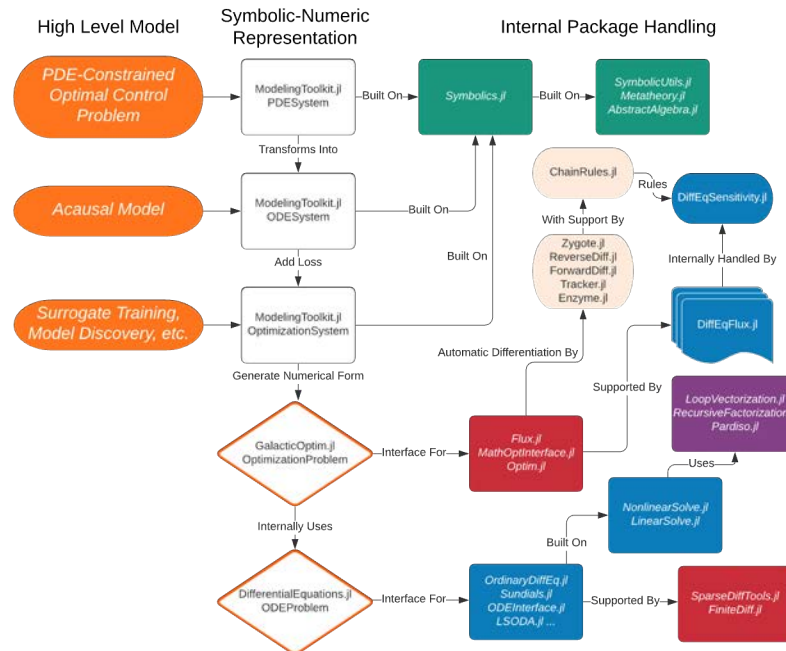
CLIMATE MACHINE

SCALABLE PLATFORM

OPEN HUB



The SciML Common Interface, Oversimplified



TOOLBOX

JULIA: COME FOR THE SYNTAX, STAY FOR THE SPEED

Researchers often find themselves coding algorithms in one programming language, only to have to rewrite them in a faster one. An up- and-coming language could be the answer.

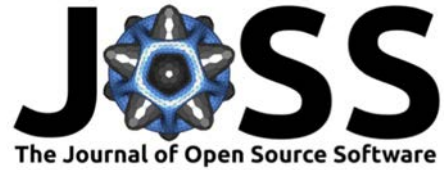
1 AUGUST 2019 | VOL 572 | NATURE

SIAM REVIEW
Vol. 59, No. 1, pp. 65–98

Julia: A Fresh Approach to Numerical Computing*

ClimaOcean.jl:

Ocean model component of the *Climate Model Alliance (CliMA)* model

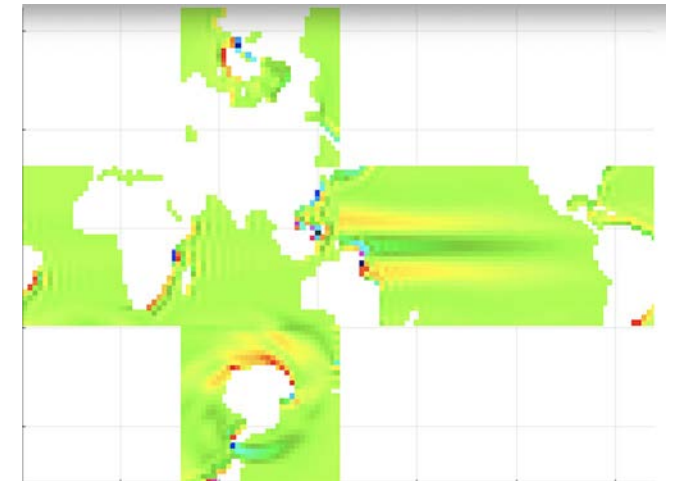
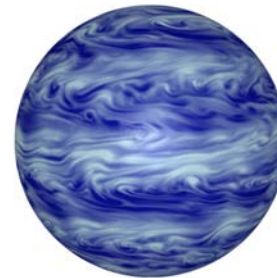
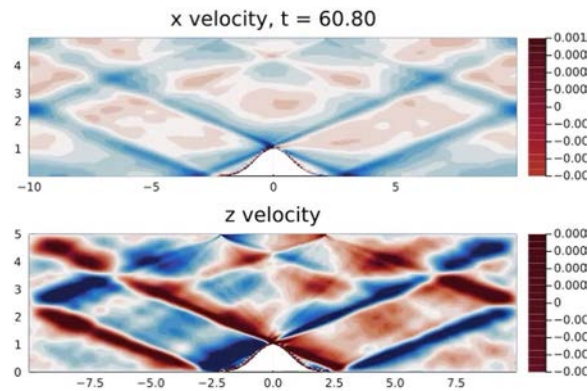
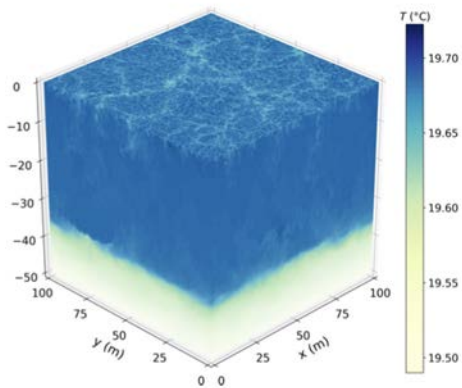


Oceananigans.jl: Fast and friendly geophysical fluid dynamics on GPUs

Ali Ramadhan¹, Gregory LeClaire Wagner¹, Chris Hill¹, Jean-Michel Campin¹, Valentin Churavy¹, Tim Besard², Andre Souza¹, Alan Edelman¹, Raffaele Ferrari¹, and John Marshall¹

¹ Massachusetts Institute of Technology ² Julia Computing, Inc.

- Finite volume, rotating, stratified fluids model for geophysical fluid dynamics (GFD).
- Written from scratch in Julia
- Multiple simulation options.
- GPU and CPU via kernel abstractions
- Parallelize using MPI.jl and multi-threading



<https://github.com/clima/Oceananigans.jl>

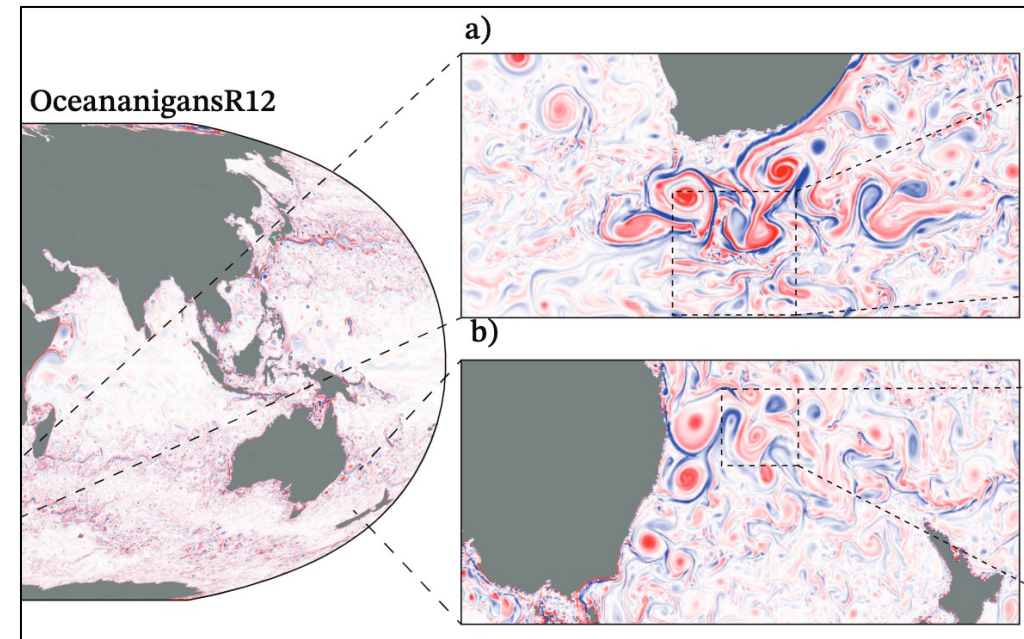
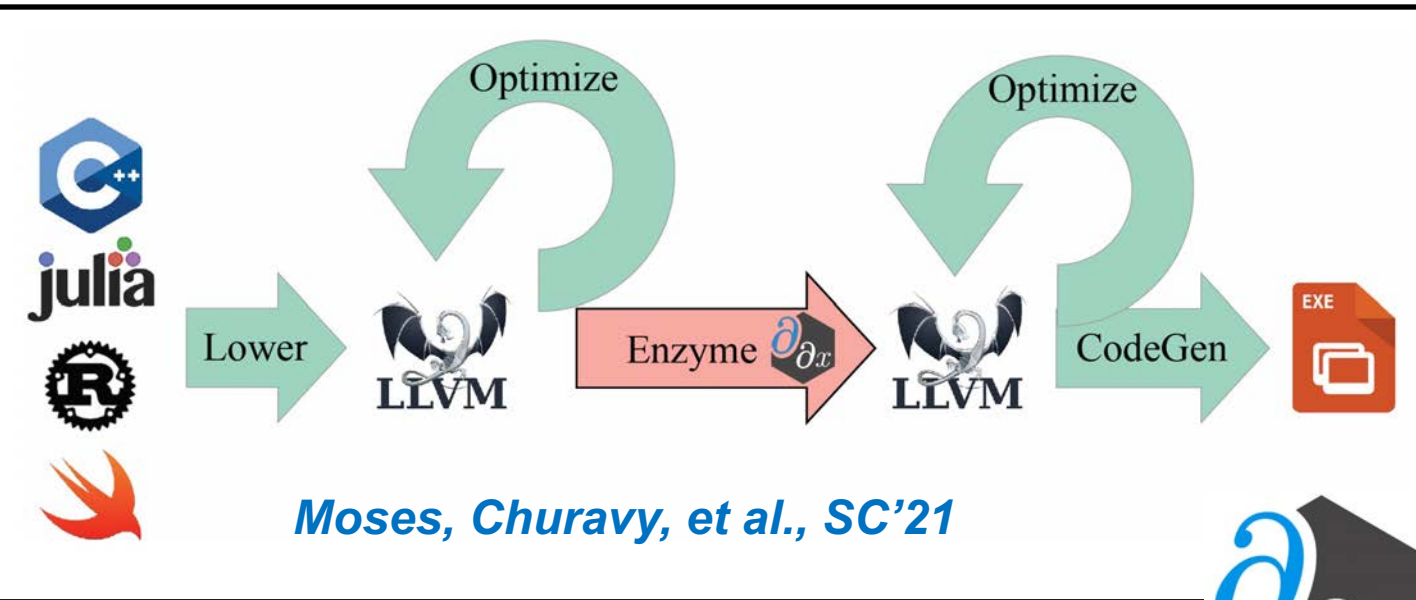
Differentiable programming for full-model / end-to-end learning

Differentiating GPU-enabled ocean model in Julia via the AD tool *Enzyme.jl*



Oceananigans.jl

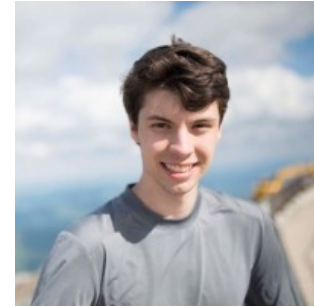
(Silvestri et al., arXiv, 2023, 2024)





Enzyme: Fast, Parallel, and Rewrite-Free Derivatives

- Derivatives are ubiquitous in machine learning (training neural networks, Bayesian inference), scientific computing (uncertainty quantification, simulation)
- Enzyme synthesizes derivatives of arbitrary code within the compiler
 - Differentiate code in any LLVM-based language (C/C++, Julia, Rust, Swift, Fortran, Python, etc) *without rewriting it!*
 - Operating after and alongside program optimization generates asymptotically and empirically faster derivatives
 - First automatic differentiation tool to handle arbitrary GPU kernels



W. Moses



V. Churavy



M. Schanen



S H K Narayanan

Three initial Earth system applications

Ocean

Sea ice

Ice sheets

S. Williamson

J. Kump

N. Loose

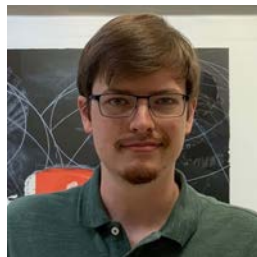
S. Silvestri

G. Wagner

C. Hill

M. Morlighem

C. Gong



- Bringing together concepts from ...
 - ... **big data science** & **sparse data science**
 - ... **computer science** & **computational science**
 - ... **scientific machine learning** & **simulation-based science**
- Sensitivity/gradient information is a powerful ingredient; obtained via
 - **differentiable programming / simulators**
 - **general-purpose automatic differentiation (AD)**

Minitutorials during *SIAM Mathematics for Planet Earth 2024*

<https://github.com/DJ4Earth/MPE24>



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SIAM Conference on Mathematics of Planet Earth (MPE24)

Minitutorials

Differentiable Earth System Models in Julia

Joseph L. Kump, University of Texas, U.S.

Sarah M. Williamson, University of Texas, U.S.

Gong Cheng, Dartmouth College, U.S.

Differentiable Programming in Julia with Enzyme

Valentin Churavy, Massachusetts Institute of Technology, U.S.

William Moses, University of Illinois Urbana-Champaign, U.S.

Michel Schanen, Argonne National Laboratory, U.S.

DJ4Earth