h-NUMO

A multi-layer shallow water equation model with high-order Discontinuous Galerkin method

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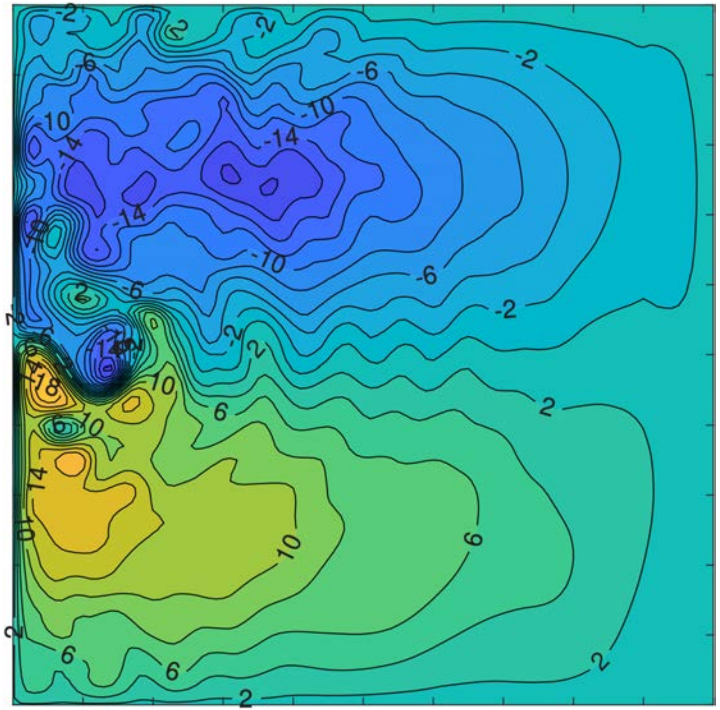
Collaborators

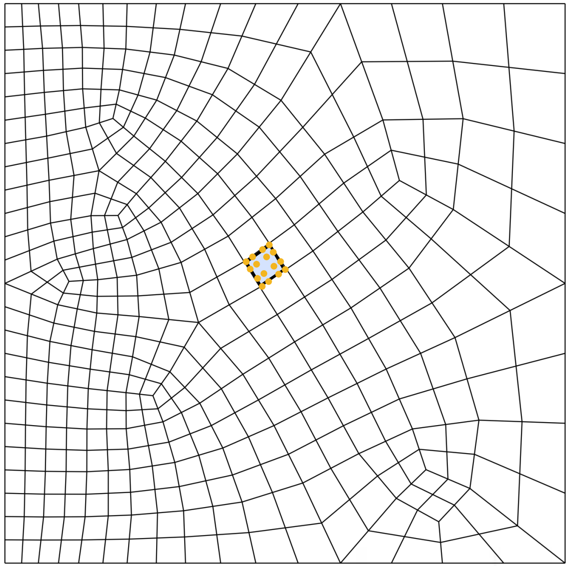


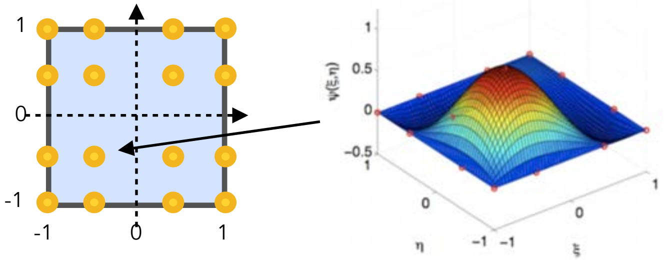
Work done in collaboration with

- Eric Chassignet, COAPS
- Alan Wallcraft, COAPS
- Alexandra Bozec, COAPS
- Robert Higdon, Oregon State

h-NUMO model

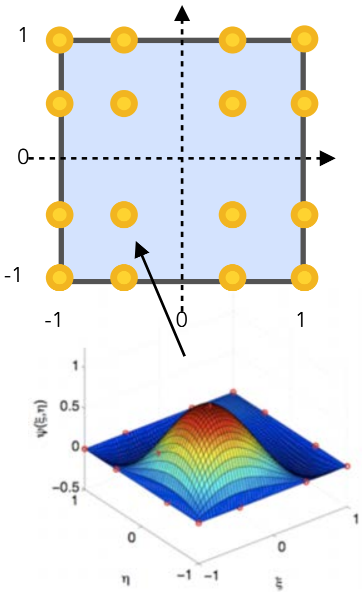
- Part of the NUMA family (F. Giraldo, NPS)
- Multi-layer shallow-water equations
- Discontinuous Galerkin (DG) method
- Quadrilateral unstructured mesh
- Arbitrarily high-order solution approximation
- Barotropic-baroclinic splitting (Higdon, 2015)





Element-based DG methods

Nodal points (LGL)



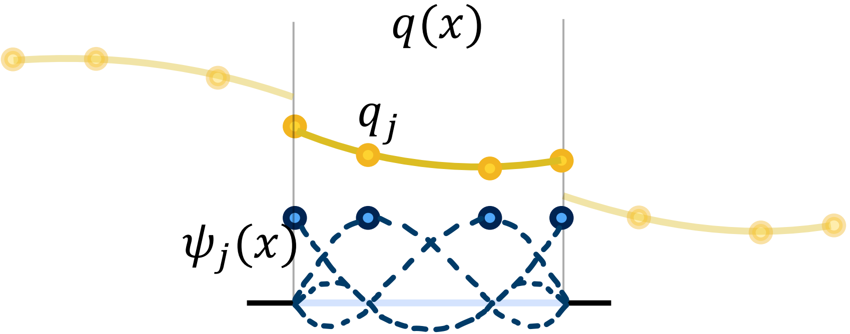
$\psi_j(\mathbf{x})$ basis functions - Lagrange polynomials

Expand the solution in basis $\psi_j(\mathbf{x})$

$$\mathbf{q}(\mathbf{x},t) \approx \sum_{j=1}^{N_p} \mathbf{q}_j(t)\,\psi_j(\mathbf{x})$$

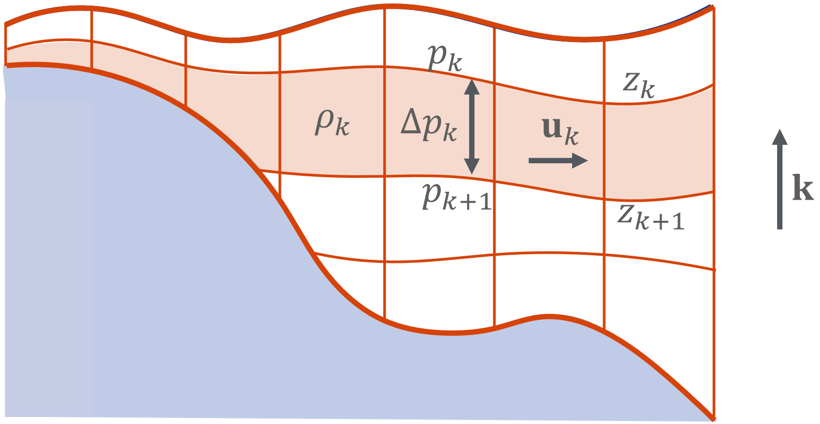
Conservative equation $\dfrac{\partial \mathbf{q}}{\partial t} + \nabla \cdot \mathbf{F}(\mathbf{q}) = S(\mathbf{q})$ becomes

$$\int_{\Omega_e} \psi_i \frac{\partial \mathbf{q}}{\partial t} d\Omega_e + \int_{\Gamma_e} \psi_i \mathbf{n} \cdot \mathbf{F}(\mathbf{q}) d\Gamma_e - \int_{\Omega_e} \nabla\psi_i \cdot \mathbf{F}(\mathbf{q}) d\Omega_e = \int_{\Omega_e} \psi_i S(\mathbf{q}) d\Omega_e$$



Multi-layer SWE

- Density constant within the layer
- Layer velocity constant vertically



$$\frac{\partial \Delta p_k}{\partial t} + \nabla \cdot (\Delta p_k \mathbf{u}_k) = 0$$

$$\frac{\partial (\Delta p_k \mathbf{u}_k)}{\partial t} + \nabla \cdot (\Delta p_k \mathbf{u}_k \otimes \mathbf{u}_k) = -\nabla H_k + g(p_k \nabla z_k - p_{k+1} \nabla z_{k+1}) - f\mathbf{k} \times (\Delta p_k \mathbf{u}_k)$$

$$H_k = g \int_{z_k}^{z_{k+1}} p dz$$

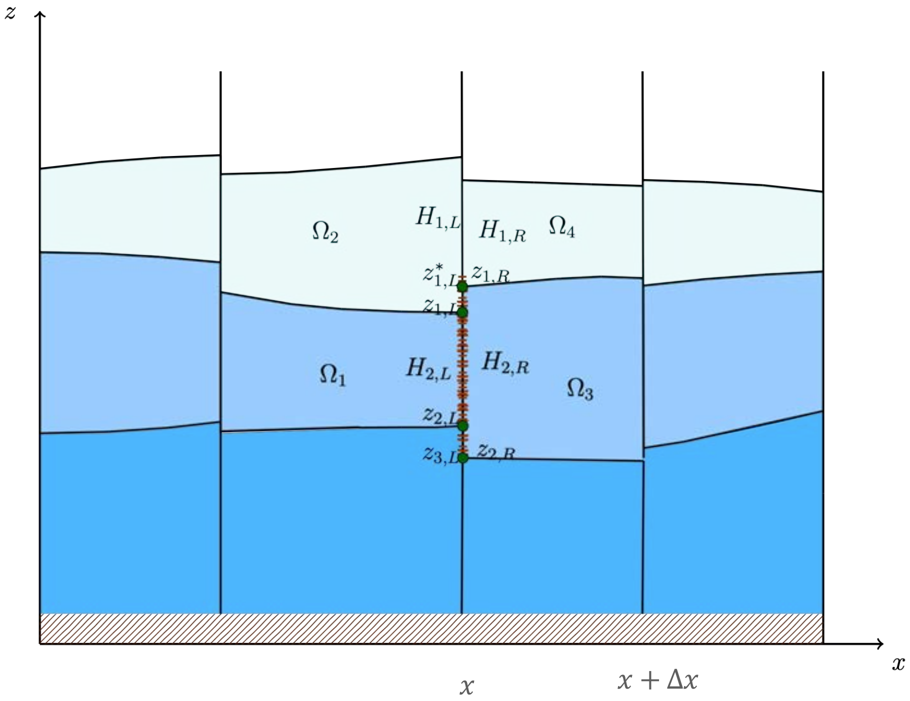
Higdon, R., (2015) *J. Comp. Phys., 295* (230-260)

Barotropic-baroclinic splitting

Higdon (2015)

We kept

- Equation and variable splitting
- Pressure treatment at element interfaces
- Baroclinic predictor-corrector time integration



Barotropic-baroclinic splitting

Higdon (2015)

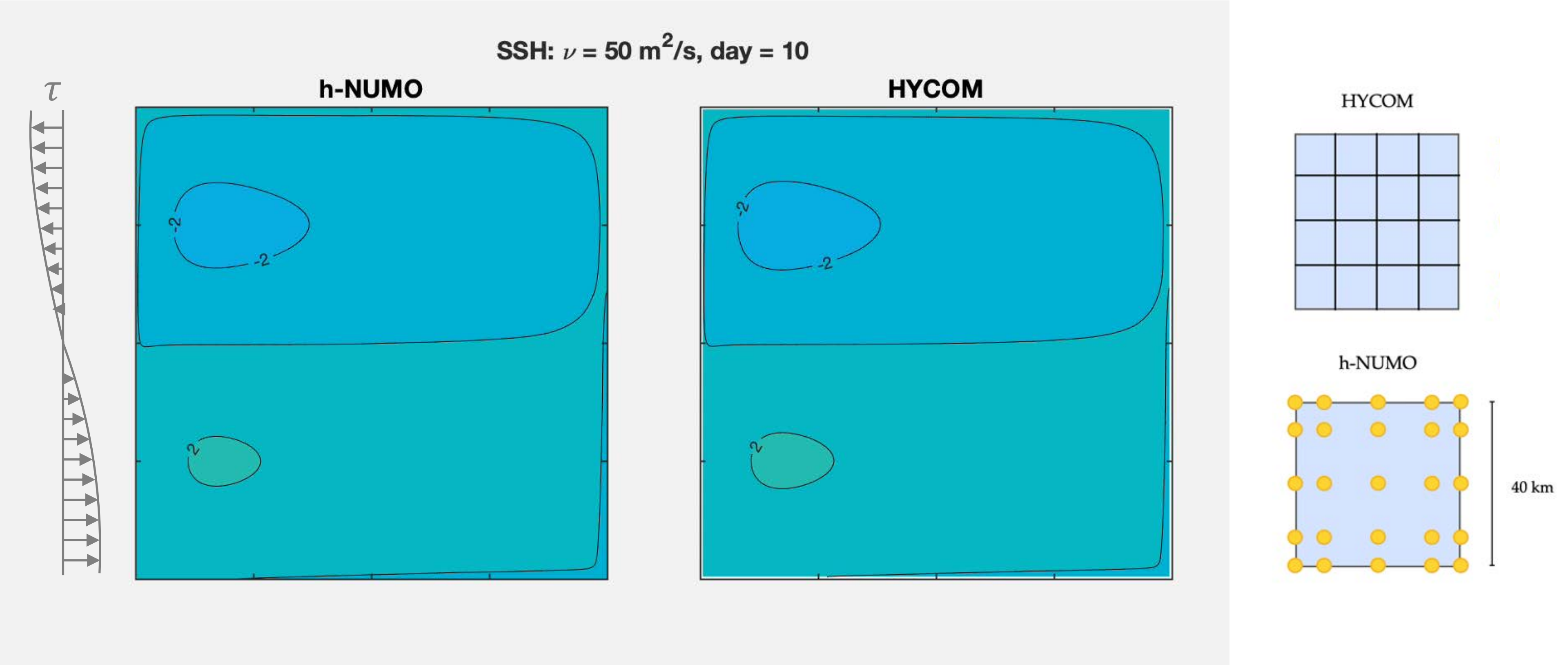
We kept

- Equation and variable splitting
- Pressure treatment at element interfaces
- Baroclinic predictor-corrector time integration

We changed

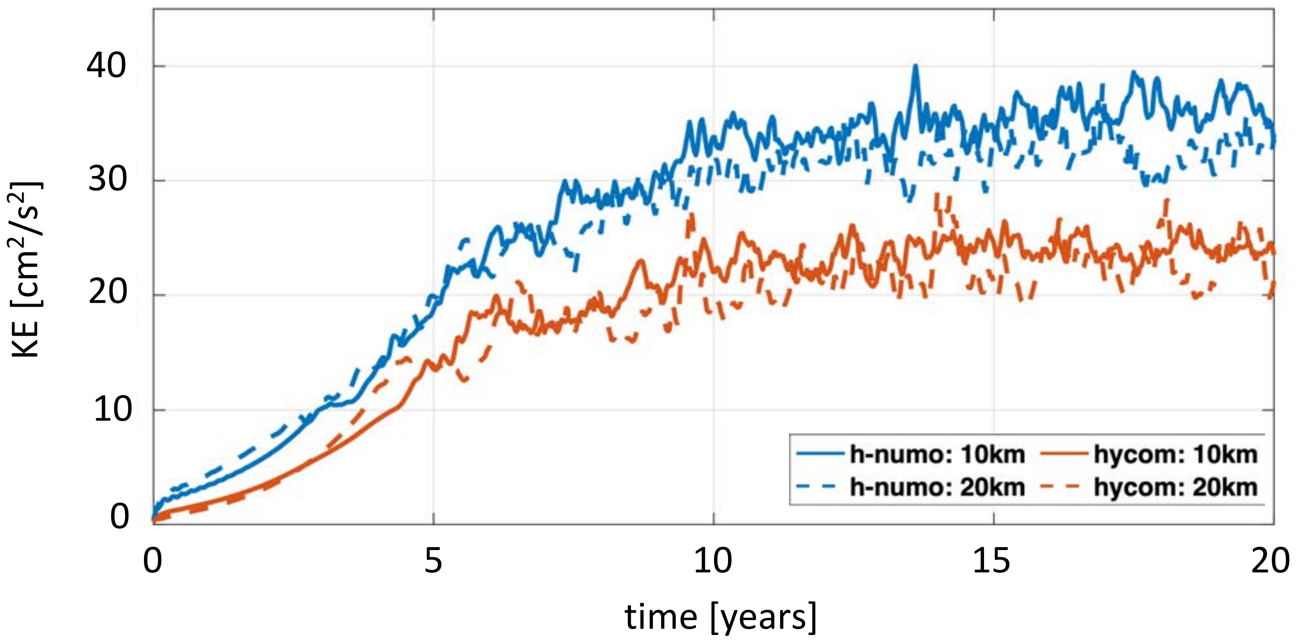
- Nodal DG implementation
- 2 horizontal directions
- RK35 time integration of barotropic equations

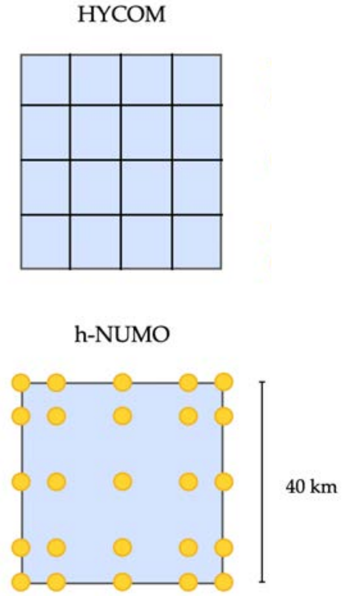
Double gyre



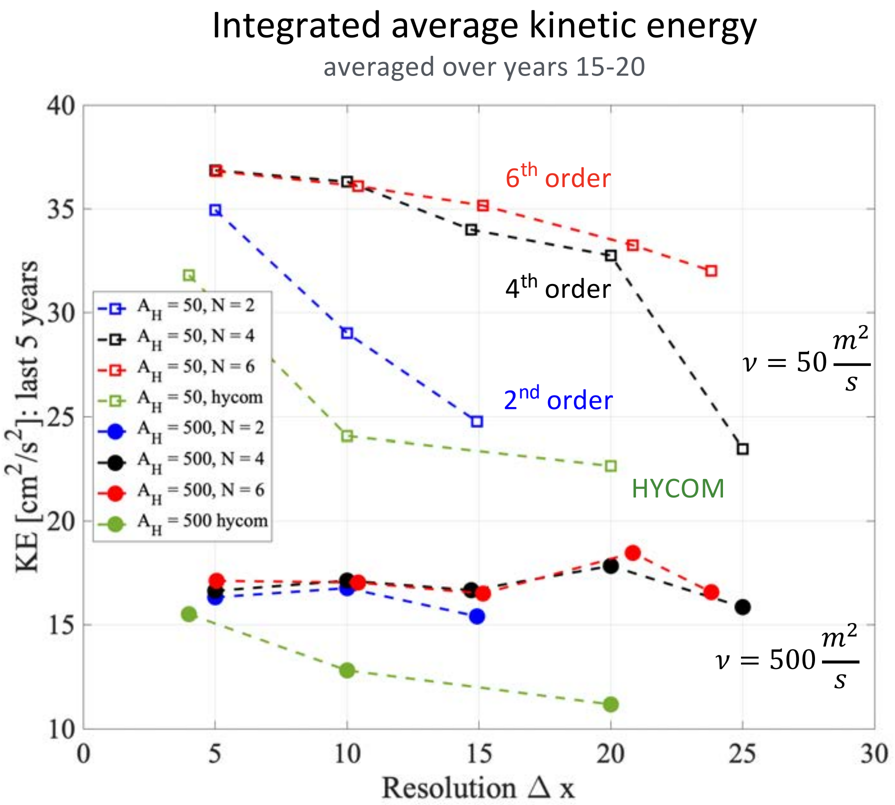
Double gyre

Integrated kinetic energy
weighted by pressure thickness

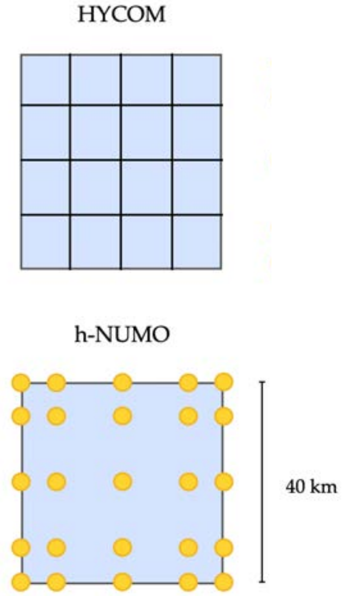




Double gyre

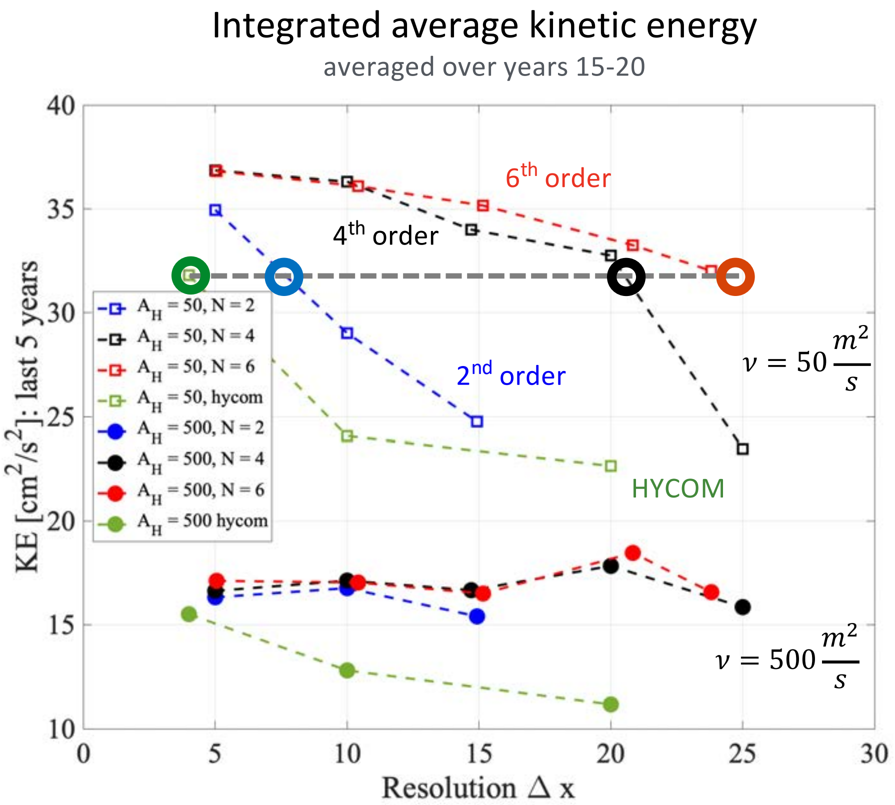


Integrated average kinetic energy
averaged over years 15-20
KE [cm^2/s^2]: last 5 years
Resolution Δ x
A_H = 50, N = 2
A_H = 50, N = 4
A_H = 50, N = 6
A_H = 50, hycom
A_H = 500, N = 2
A_H = 500, N = 4
A_H = 500, N = 6
A_H = 500 hycom
6th order
4th order
2nd order
HYCOM
$\nu = 50\,\frac{m^2}{s}$
$\nu = 500\,\frac{m^2}{s}$



HYCOM
h-NUMO
40 km

Double gyre



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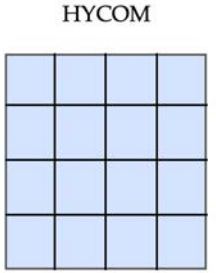
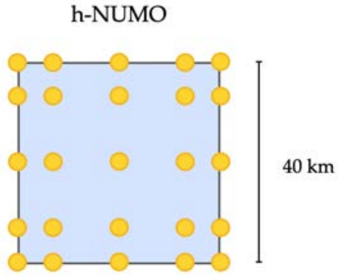


HYCOM

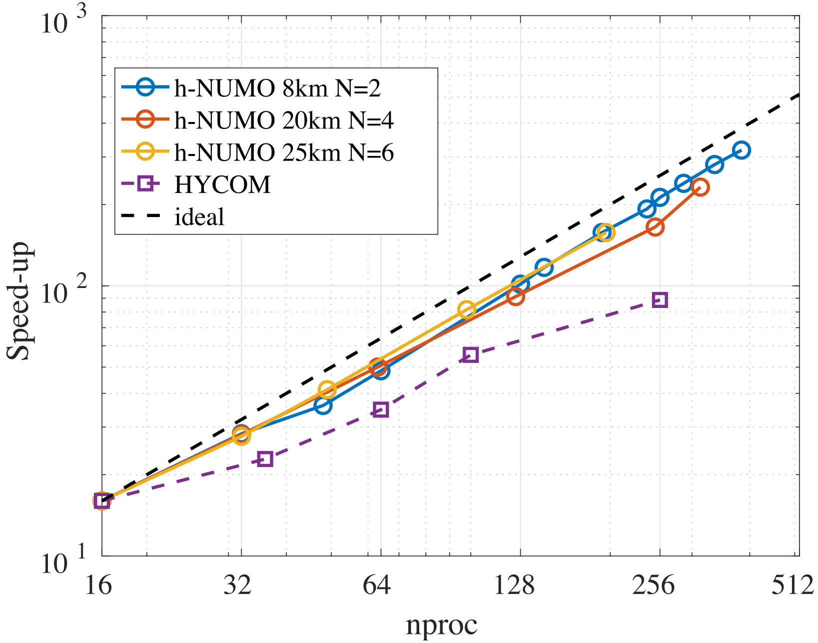


h-NUMO
40 km

Parallel performance

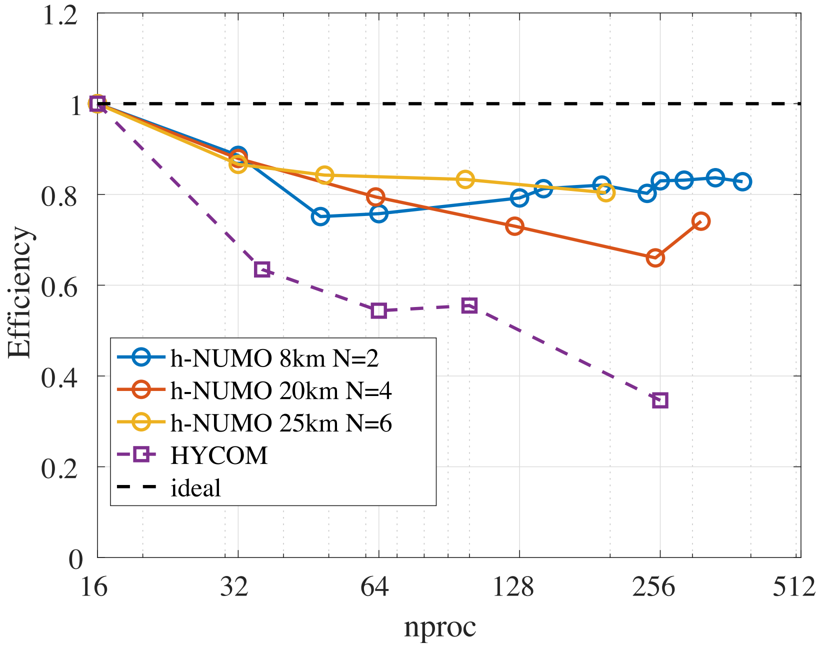
Speed-up

as a function of number of MPI ranks



Parallel efficiency

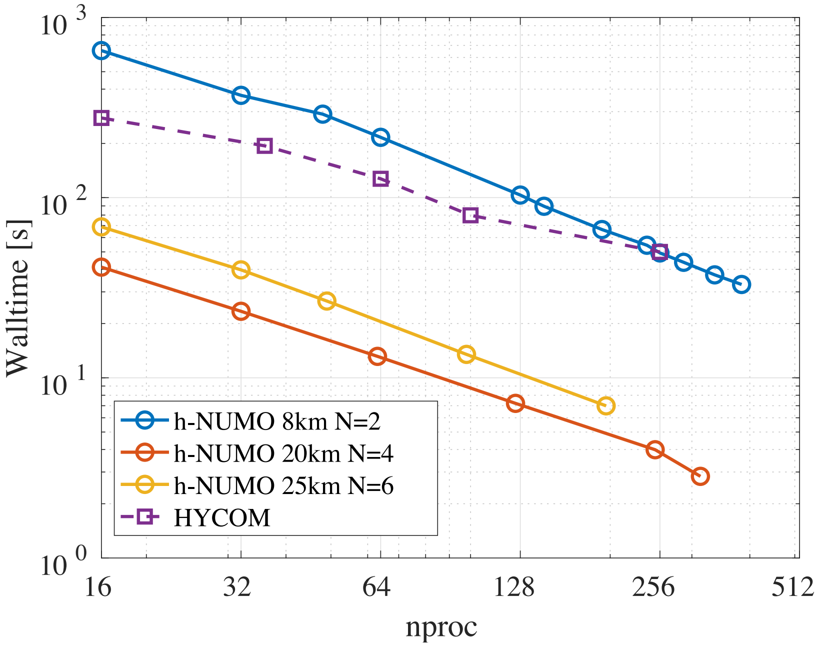
as a function of number of MPI ranks



Parallel performance

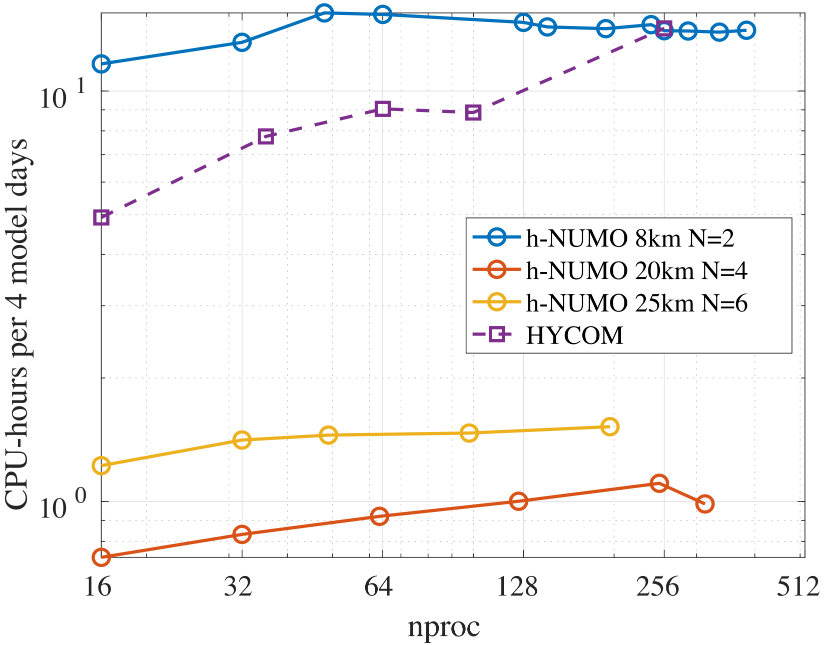
Time to solution

as a function of number of MPI ranks



CPU-hours

as a function of number of MPI ranks

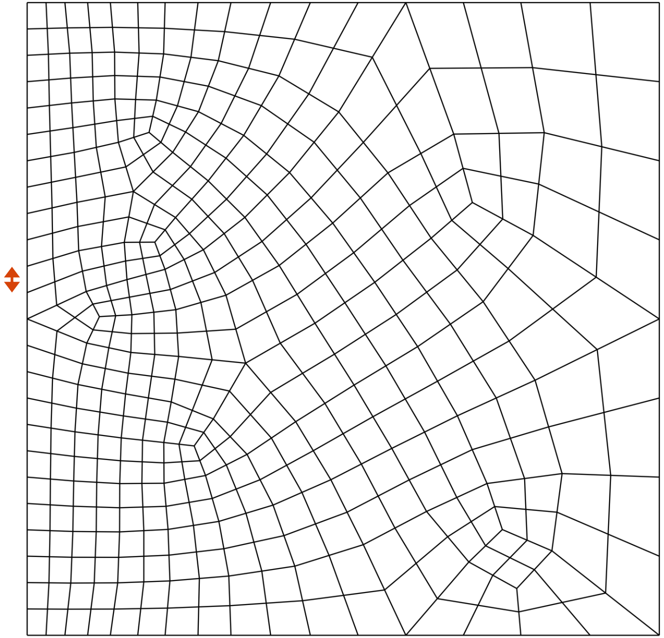


Double gyre

Unstructured grid

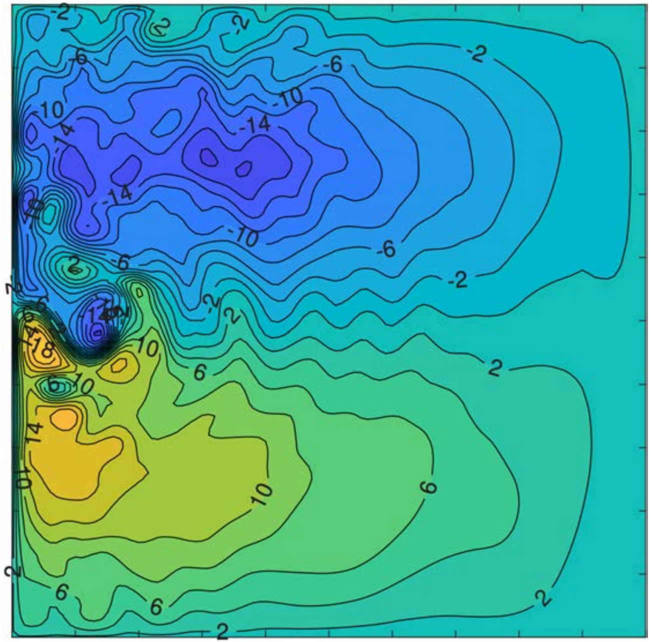
elements shown without nodal points

80 km

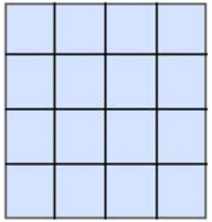


Sea surface height

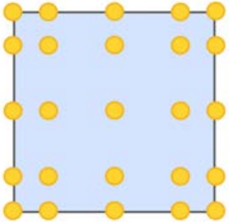
at 20 years simulation time



HYCOM



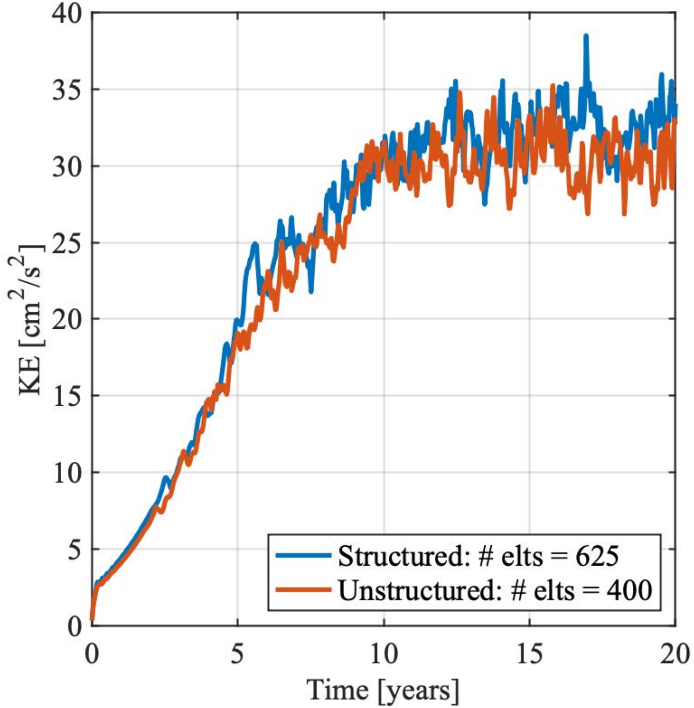
h-NUMO



Double gyre

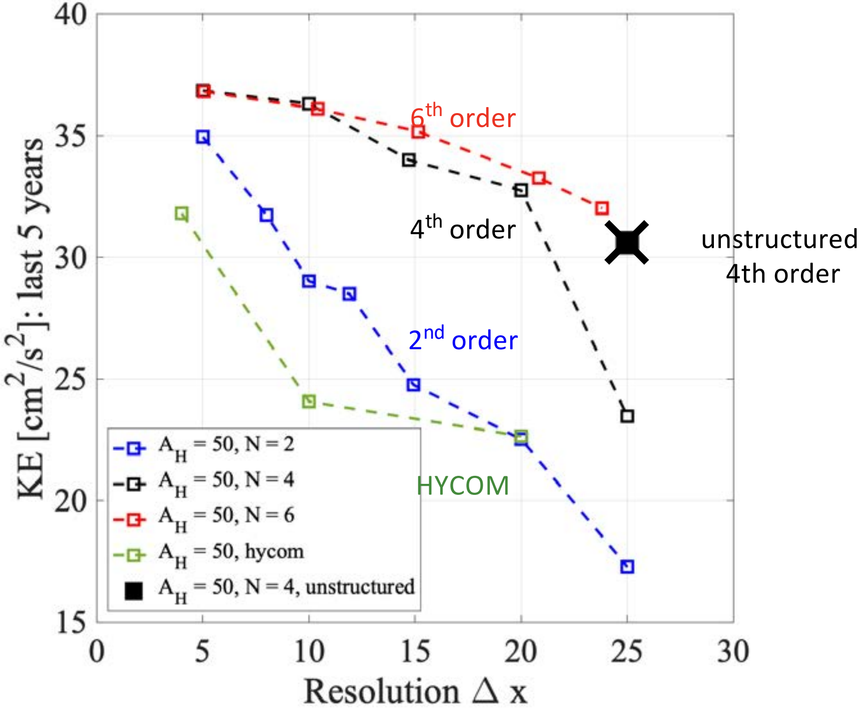
Kinetic energy

as a function of time



Average kinetic energy

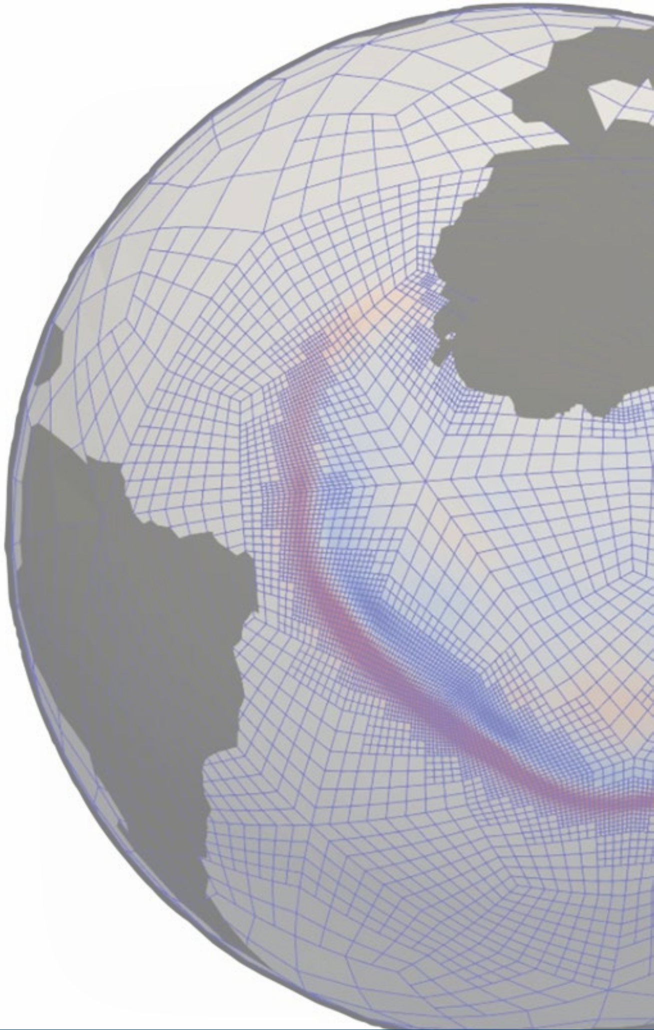
as a function of resolution



Conclusions

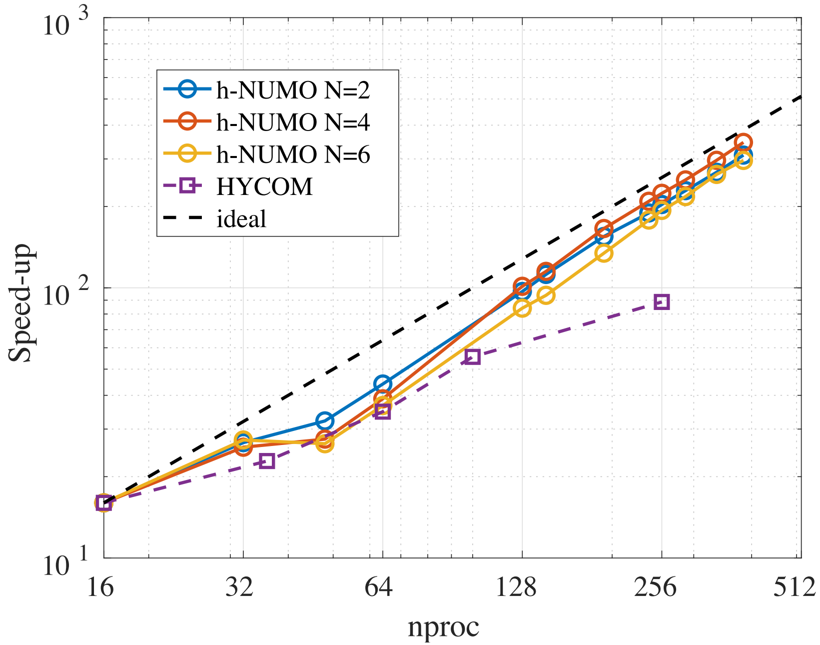
- **h-NUMO delivers the same result as HYCOM**
  - with smaller „resolution", i.e. fewer DOFs
  - faster and with fewer CPU-hours (high-order)
  - with better parallel efficiency

- **Unstructured mesh helps, AMR possible**
- **Need to demonstrate efficiency on global mesh with coastlines**

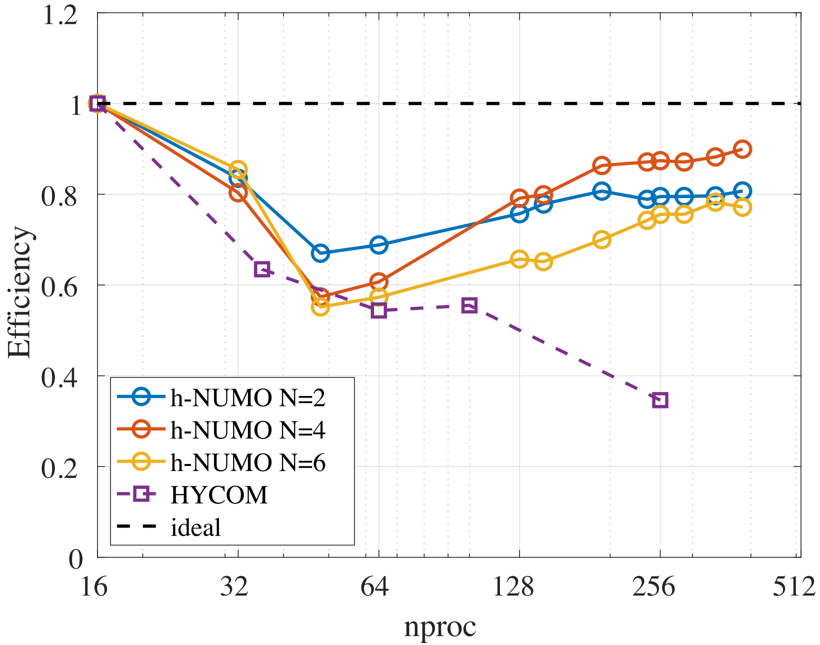


Parallel performance – 4 km resolution

Speed-up
as a function of number of MPI ranks



Parallel efficiency
as a function of number of MPI ranks



Parallel performance – 4km resolution

Time to solution

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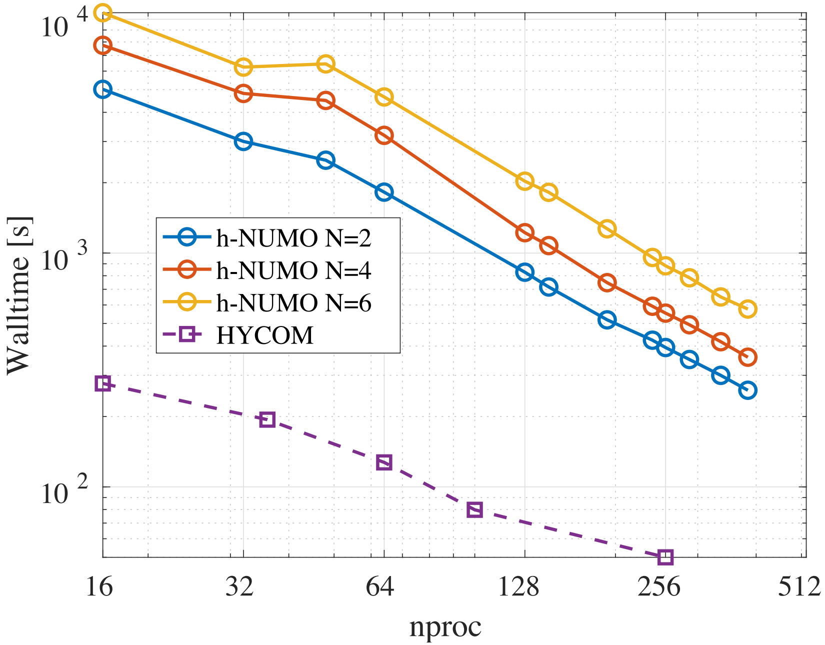

CPU-hours

as a function of number of MPI ranks