Derivation of an energetically consistent Eddy-Diffusivity Mass-Flux scheme for oceanic convection

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Context : ”consistent” approach to the parameterization problem

[Gross et al. 2018, Eden & Iske 2019, Lauritzen et al. 2022]

Explicit assumptions + scale-separation operator

→ achieve resolution-aware parameterizations

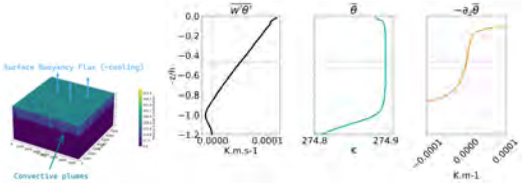
→ consistent Energy Budgets

Case study : parameterization of the vertical flux $\overline{w'\varphi'}$ in the boundary layer under the combined effect of local turbulence and convective plumes

Inaccuracy of the eddy-diffusion approach ($\overline{w'\theta'} \overset{\text{param}}{=} -K_\theta \partial_z \overline{\theta}$)

For convection:

$$\cancel{\overline{w'\theta'} \overset{\text{param}}{=} -K_\theta \partial_z \overline{\theta}}$$



$\Rightarrow$ Transport by dense plumes not directly related to the local gradients

$\Rightarrow$ Need of ’**non-local**’ behavior [Deardorff 1966]

Alternatives beyond eddy-diffusivity

$\longrightarrow$ Use a ”**gradient correction factor**” (a.k.a. *countergradient* term) : $\overline{w'\theta'} \overset{\text{param}}{=} -K_\theta(\partial_z \overline{\theta} - \gamma_\theta)$

$\longrightarrow$ **Non-Penetrative Convective adjustment schemes** [Killworth, 1989; Madec et al., 1991]

$\longrightarrow$ **Coherent structures modeling** : Mass-flux schemes & Multi-fluid modeling

- Atmosphere : [Hourdin et al., 2002; Siebesma et al., 2007; Tan et al., 2018; Thuburn et al., 2018 ...]
- Ocean : proof of concept [Giordani et al., 2021]

Combined Eddy-Diffusivity Mass-Flux Approach:

- Eddy-diffusivity (ED): small scale isotropic turbulence
- Mass-flux (MF): direct contribution of coherent structures (= convective plumes) to fluxes

Outline :

1. From Two-Fluid modeling to Mass-Flux Schemes
2. Horizontally averaged energy budgets (ED & MF coupling)
3. Practical implementation
4. Numerical experiments

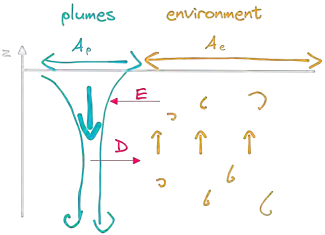
1

Derivation of EDMF schemes starting from first principles

Two-domain decomposition

Decompose horizontal cell average into plume and environment averages

$$\overline{\phi} = a_p \phi_p + a_e \phi_e$$



a_p, a_e: fractional areas of subdomains

w_p, w_e: vertical velocities

ϕ_p, ϕ_e: advected fields (u, v, θ, S)

E, D: horizontal entrainment/detrainment

Decomposition of fluxes and plume averaged equations

① Apply this multi-fluid averaging to fluxes $\overline{w'\phi'}$

$$\overline{w'\phi'} = \underbrace{a_p(w_p - \overline{w})(\phi_p - \overline{\phi})}_{\text{coherent structures}} + \underbrace{a_p\overline{w'_p\phi'_p}^{p}}_{\text{subdomain turbulence}} + \underbrace{a_e(w_e - \overline{w})(\phi_e - \overline{\phi})}_{\text{coherent structures}} + \underbrace{a_e\overline{w'_e\phi'_e}^{e}}_{\text{subdomain turbulence}}$$

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② Assume Eddy-Diffusivity in isotropic environment:

$$\overline{w'\phi'} \overset{\text{param}}{=} \underbracket{-K_\phi\partial_z\overline{\phi}}_{\text{subdomain turbulence}} + \underbracket{\frac{a_p}{1-a_p}w_p(\phi_p - \overline{\phi})}_{\text{coherent structures}}$$

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③ Subgrid plume model (steady plume hypothesis):

$$\frac{D}{Dt}\phi = \text{sources} \xrightarrow[\text{stationarity}]{\overline{(\cdot)}^{p}} \partial_z(a_p w_p \phi_p) = E\phi_e - D\phi_p + a_p \text{ sources}$$

Decomposition of fluxes and plume averaged equations

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④ Small area limit (?):

$$\partial_z(a_p w_p \phi_p) = E\overline{\phi} - D\phi_p + a_p \text{ sources}, \qquad \overline{w'\phi'} \overset{\text{param}}{=} K_\phi \partial_z \overline{\phi} + a_p w_p(\phi_p - \overline{\phi})$$

Plume averaged equations

$\widetilde{\alpha}$	$= \dfrac{1}{1-a_p}$	Rescaling coefficient
$\overline{w'\phi'}$	$= \widetilde{\alpha} a_p w_p(\phi_p - \overline{\phi}) - K_\phi \partial_z \overline{\phi}$	Vertical turbulent flux for component ϕ
$\overline{w'\mathbf{u}'_h}$	$= \widetilde{\alpha} a_p w_p(\mathbf{u}_{h,p} - \overline{\mathbf{u}}_h) - K_m \partial_z \overline{\mathbf{u}}_h$	Vertical turbulent momentum flux
$\partial_z(a_p w_p)$	$= E - D$	Plume area conservation equation
$a_p w_p \partial_z \phi_p$	$= \widetilde{\alpha} E(\overline{\phi} - \phi_p)$	Plume equation for component ϕ
$a_p w_p \partial_z \mathbf{u}_{h,p}$	$= \widetilde{\alpha} E(\overline{\mathbf{u}}_h - \mathbf{u}_{h,p}) + a_p w_p C_u \partial_z \overline{\mathbf{u}}_h$	Plume horizontal momentum equation
$a_p w_p \partial_z w_p$	$= -(\widetilde{\alpha} b)E w_p + a_p \left\{ aB_p + \widetilde{\alpha}(b'/h) w_p^2 \right\}$	Plume vertical velocity equation
B_p	$= b_{\mathrm{eos}}(\phi_p) - b_{\mathrm{eos}}(\overline{\phi})$	Buoyancy forcing term

- $\widetilde{\alpha} = 1 \Leftrightarrow$ small area limit → standard mass flux scheme

M. Perrot, F. Lemarié & T. Dubos. *Energetically consistent Eddy-Diffusivity Mass-Flux schemes. Part I: Theory and Models*, 2024. `https://hal.science/hal-04439113v3`, under review in JAMES

Eddy-Diffusivity and Turbulent Kinetic Energy

$K_\phi \propto l\sqrt{k}$ (one-equation turbulence models)

'local' ED closure: $\overline{w'\phi'} = -K_\phi \partial_z \overline{\phi}$ $\nearrow$ $\searrow$

$K_\phi \propto k^2/\epsilon$ (second-order turbulence models)

with Turbulent Kinetic Energy (TKE): $k = \frac{1}{2}\overline{\mathbf{u}' \cdot \mathbf{u}'}$

MF terms lead to energy exchanges

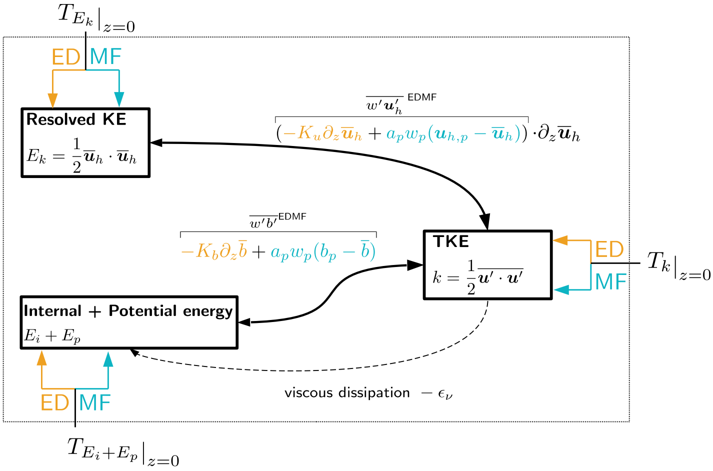
→ ED and MF are **not independent!**

Energy budgets enable coherent coupling

2

Horizontally Averaged Energy budgets

Bulk and boundary energy fluxes within EDMF closure



$T_{E_k}\big|_{z=0}$

ED MF

Resolved KE

$E_k = \frac{1}{2}\overline{\boldsymbol{u}}_h \cdot \overline{\boldsymbol{u}}_h$

$\overbrace{\left(-K_u\partial_z\overline{\boldsymbol{u}}_h + a_p w_p(\boldsymbol{u}_{h,p} - \overline{\boldsymbol{u}}_h)\right)}^{\overline{w'\boldsymbol{u}'_h}^{\text{EDMF}}} \cdot\partial_z\overline{\boldsymbol{u}}_h$

TKE

$k = \frac{1}{2}\overline{\boldsymbol{u}'\cdot\boldsymbol{u}'}$

ED MF

$T_k\big|_{z=0}$

$\overbrace{-K_b\partial_z\overline{b} + a_p w_p(b_p - \overline{b})}^{\overline{w'b'}^{\text{EDMF}}}$

Internal + Potential energy

$E_i + E_p$

viscous dissipation $-\epsilon_\nu$

ED MF

$T_{E_i+E_p}\big|_{z=0}$

Consistent energy budgets

Novelty for ocean (and atmosphere) EDMF: consistent TKE equation

$\partial_t k - \partial_z \left(K_k \partial_z k \right)$	$= K_u (\partial_z \overline{\mathbf{u}}_h)^2 - K_\phi \partial_z \overline{b}$	ED related TKE production terms
	$- a_p w_p (\mathbf{u}_{h,p} - \overline{\mathbf{u}}_h) \cdot \partial_z \overline{\mathbf{u}}_h + a_p w_p (b_p - \overline{b})$	MF related TKE production terms
	$- \partial_z \left(a_p w_p \left[k_p - k + \frac{1}{2} \lVert \mathbf{u}_p - \mathbf{u} \rVert^2 \right] \right)$	MF related TKE transport term
	$- \overline{\epsilon}_\nu$	TKE dissipation
$a_p w_p \partial_z k_p$	$= E \left(k - k_p + \frac{1}{2} \lVert \mathbf{u}_p - \mathbf{u} \rVert^2 \right) - a_p (\epsilon_\nu)_p$	Plume related TKE
K_k	$= c_k l_m \sqrt{k}$	TKE eddy-diffusivity

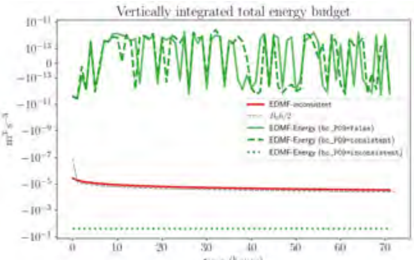
- Goes beyond existing literature (Witek et al., 2011a-b; Han & Bretherton, 2019) not motivated by considerations of energetic consistency

Consistent energy budgets

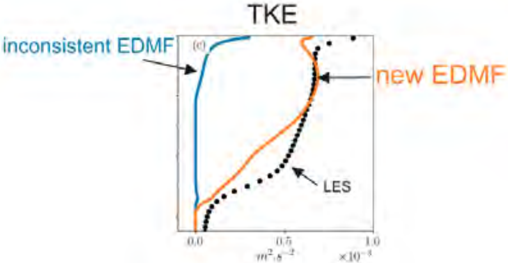
$$
\begin{aligned}
\partial_t k - \partial_z \left(K_k \partial_z k \right) &= K_u (\partial_z \overline{\mathbf{u}}_h)^2 - K_\phi \partial_z \overline{b} \\
&- a_p w_p (\mathbf{u}_{h,p} - \overline{\mathbf{u}}_h) \cdot \partial_z \overline{\mathbf{u}}_h + a_p w_p (b_p - \overline{b}) \\
&- \partial_z \left(a_p w_p \left[k_p - k + \frac{1}{2} \|\mathbf{u}_p - \mathbf{u}\|^2 \right] \right) \\
&- \overline{\epsilon}_\nu
\end{aligned}
$$

ED related TKE production terms
MF related TKE production terms
MF related TKE transport term
TKE dissipation

→ Energy conservation



→ Significantly reduced TKE bias vs LES



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Practical implementation

Discretization aspects

- Extension of the energy-conserving discretization by Burchard (2002) to the EDMF context
- Adapted for handling any unstable part of the density profile

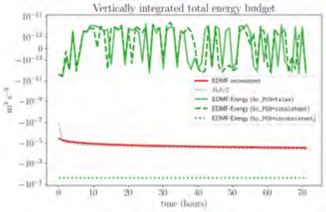


Figure: Time series of the vertically integrated energy budget for the case W005_C500.

"boundary layer-then-convection strategy"

$$
\begin{aligned}
&\text{ED step} \\
\phi^{n+1,\star} &= \phi^n + \Delta t \partial_z \left(K_\phi(k^n, b^n) \partial_z \phi^{n+1,\star} \right) \\
\mathbf{u}_h^{n+1,\star} &= \mathbf{u}_h^n + \Delta t \partial_z \left(K_{\mathbf{u}}(k^n, b^n) \partial_z \mathbf{u}_h^{n+1,\star} \right) \\
b^{n+1,\star} &= b_{\text{eos}}(\phi^{n+1,\star}) \\
&\text{MF step} \\
[a_p, w_p, \phi_p, \mathbf{u}_{h,p}, k_p, B_p] &= \text{MF}(b^{n+1,\star}, \mathbf{u}_h^{n+1,\star}) \\
\phi^{n+1} &= \phi^{n+1,\star} - \Delta t \partial_z \left(a_p w_p (\phi_p - \phi^{n+1,\star}) \right) \\
\mathbf{u}_h^{n+1} &= \mathbf{u}_h^{n+1,\star} - \Delta t \partial_z \left(a_p w_p (\mathbf{u}_{h,p} - \mathbf{u}_h^{n+1,\star}) \right) \\
&\text{TKE update} \\
k^{n+1} &= k^n + \Delta t \partial_z \left(K_k \partial_z k^{n+1} \right) + \mathcal{F}_k(b^{n+1}, \mathbf{u}_h^{n+1}, \mathbf{u}_h^n, \ldots)
\end{aligned}
$$

M. Perrot & F. Lemarié. *Energetically consistent Eddy-Diffusivity Mass-Flux schemes. Part II: implementation and evaluation in an oceanic context*, 2024. `https://hal.science/hal-04666049`, under review in JAMES

4

Numerical experiments

A collection of LES cases

In collaboration with B. Deremble, IGE, Grenoble (TRACCS project)

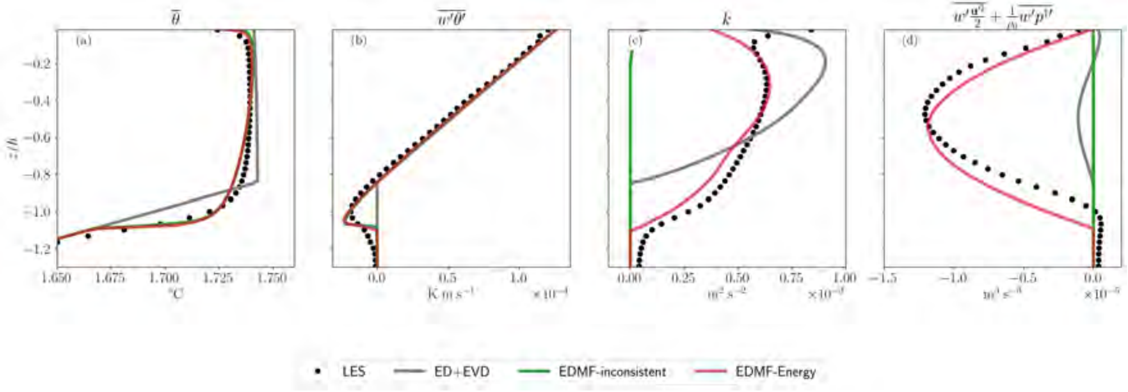
Case	**heat flux** W m^{-2}	**swrad** W m^{-2}	**wind** Pa	**E-P** mm day^{-1}	**Coriolis** s^{-1}	**Ini. ML** -	**Model** -
FC075	-75	0	0	0	0	no	NCAR-LES
FCML075	-75	0	0	0	0	yes	NCAR-LES
FC500	-500	0	0	0	0	no	Meso-NH/Croco
W05_C500	-500	0	0.5	0	0	no	Meso-NH
W005_C500	-500	0	0.05	0	0	no	Meso-NH/Croco
W01_EC75	-75	0	0.1	1.37	10^{-4}	no	NCAR-LES
DC	-75	235	0.	1.37	0	no	NCAR-LES
WANG1	-111	0	0	0	1.26×10^{-4}	yes	Meso-NH
W05_C100	-100	0	0.5	0	1.06×10^{-4}	no	Meso-NH/Basilisk
W05_C200	-200	0	0.5	0	1.06×10^{-4}	no	Meso-NH/Basilisk
W081_W150	150	0	0.81	0	1.06×10^{-4}	no	Meso-NH/Basilisk

+ preliminary tests with **TLab** (J. P. Mellado et al.) and **Oceananigans**

+ Realistic case : Hymex/ASICS-MED campaign (Giordani et al., 2021)

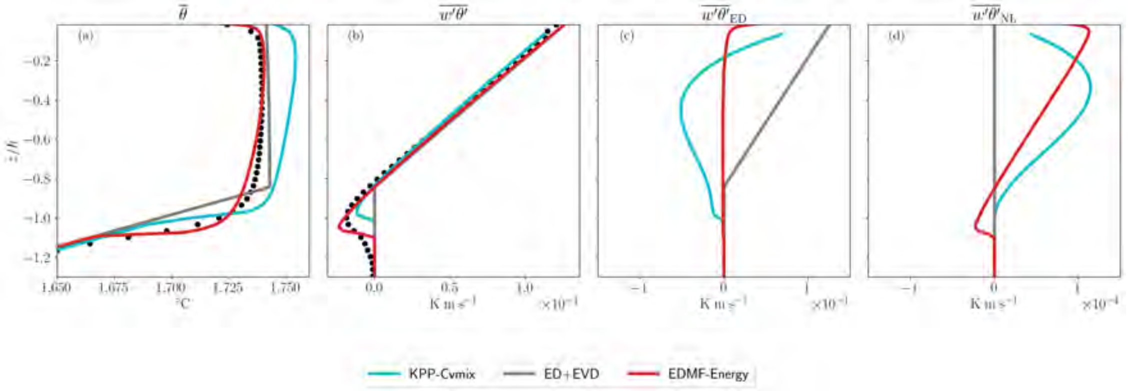
Example 1 : $\mathrm{FC500}$ **case**

Instantaneous profiles after 72 hours

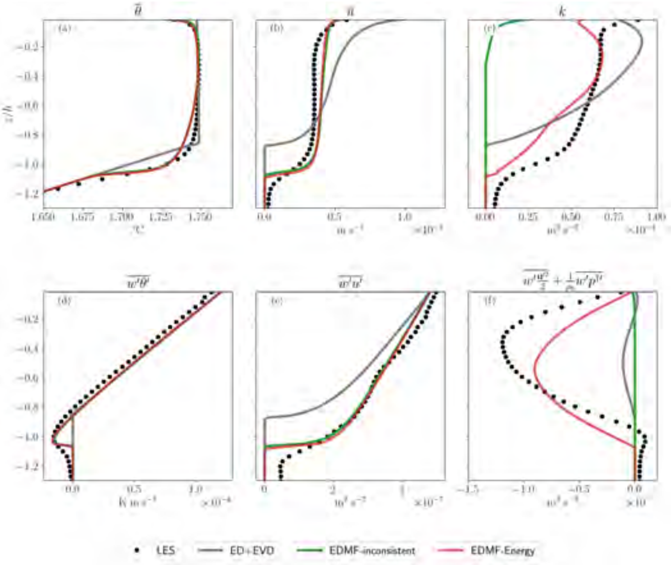


Example 1 : FC500 case

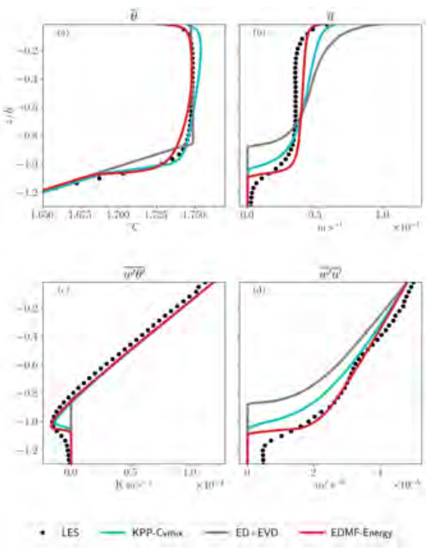
Instantaneous profiles after 72 hours

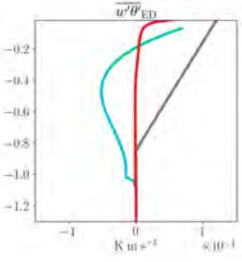


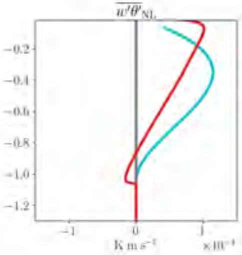
Example 2 : W005_C500 case (Instantaneous profiles after 72 hours)



Example 2 : W005_C500 **case (Instantaneous profiles after** 72 **hours)**

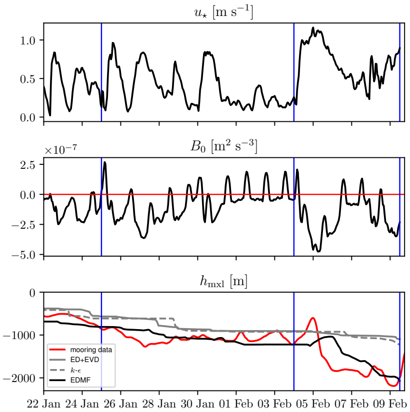


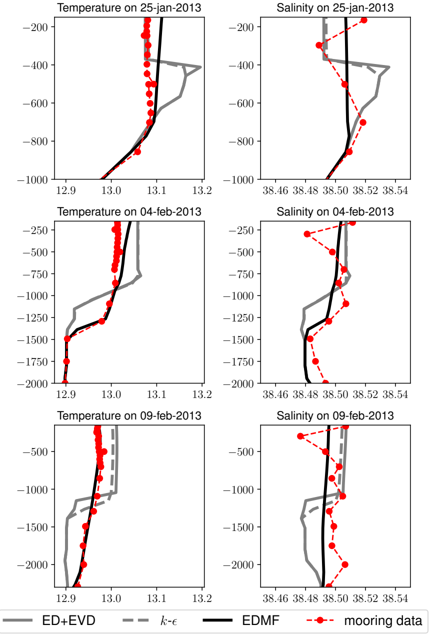




HyMeX/ASICS-MED experiment (LION buoy)

Sequence of strong convective events in the Northwestern Mediterranean during the winter 2013.

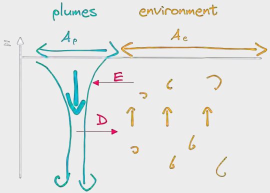




Summary

Eddy-Diffusivity Mass-Flux (EDMF) parameterization for convection

$$\overline{w'\phi'} \overset{\text{param}}{=} \underbrace{-K_\phi \partial_z \overline{\phi}}_{\text{local turbulent mixing}} + \underbrace{a_p w_p (\phi_p - \overline{\phi})}_{\text{nonlocal transport by coherent structures}}$$



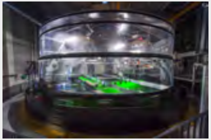
- ✓ PDE-based derivation and explicit assumptions
- ✓ Flexible multi-fluid averaging framework
- ✓ Fully-consistent Subgrid+Resolved energy budgets
- ✓ Evaluation in a SCM against LES simulations and observational data.

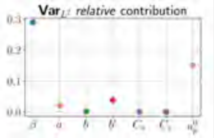
M. Perrot, F. Lemarié & T. Dubos. *Energetically consistent Eddy-Diffusivity Mass-Flux schemes. Part I: Theory and Models*, 2024. under review in JAMES

M. Perrot & F. Lemarié. *Energetically consistent Eddy-Diffusivity Mass-Flux schemes. Part II: implementation and evaluation in an oceanic context*, 2024. under review in JAMES

In progress

- ▷ Sensitivity analysis
- ▷ Parameter estimation (w. O. Zahm et al.)
- ▷ Reference data : LES simulations & laboratory experiments (PLUME project)





- ▷ Implementation in Croco (Med. Sea operational configuration)
- ▷ Implementation in MesoNH (atmosphere)
- ▷ Coupling with parameterization of restratification processes