Bathymetry-aware mesoscale eddy parameterizations

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Joint Work with:

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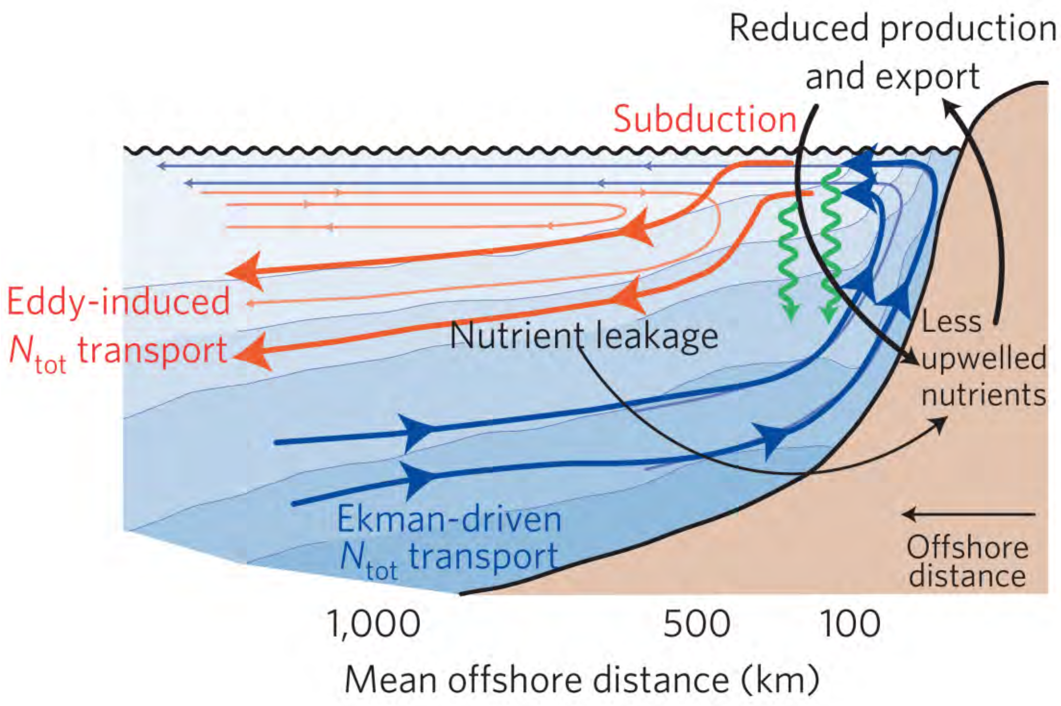
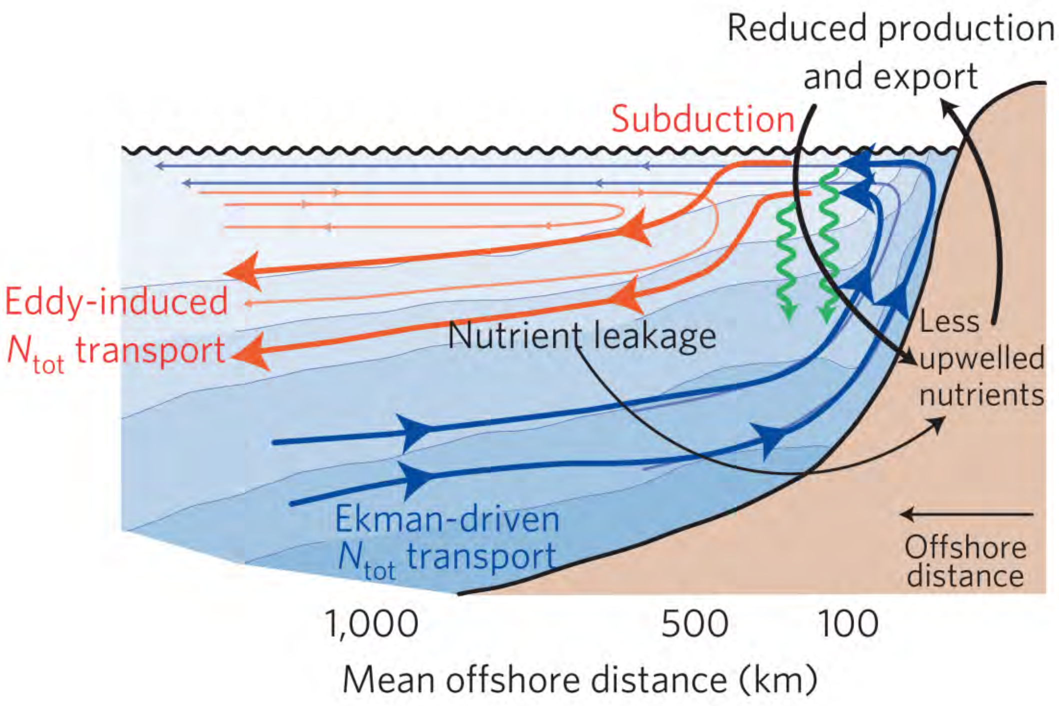
Eddies across continental margins



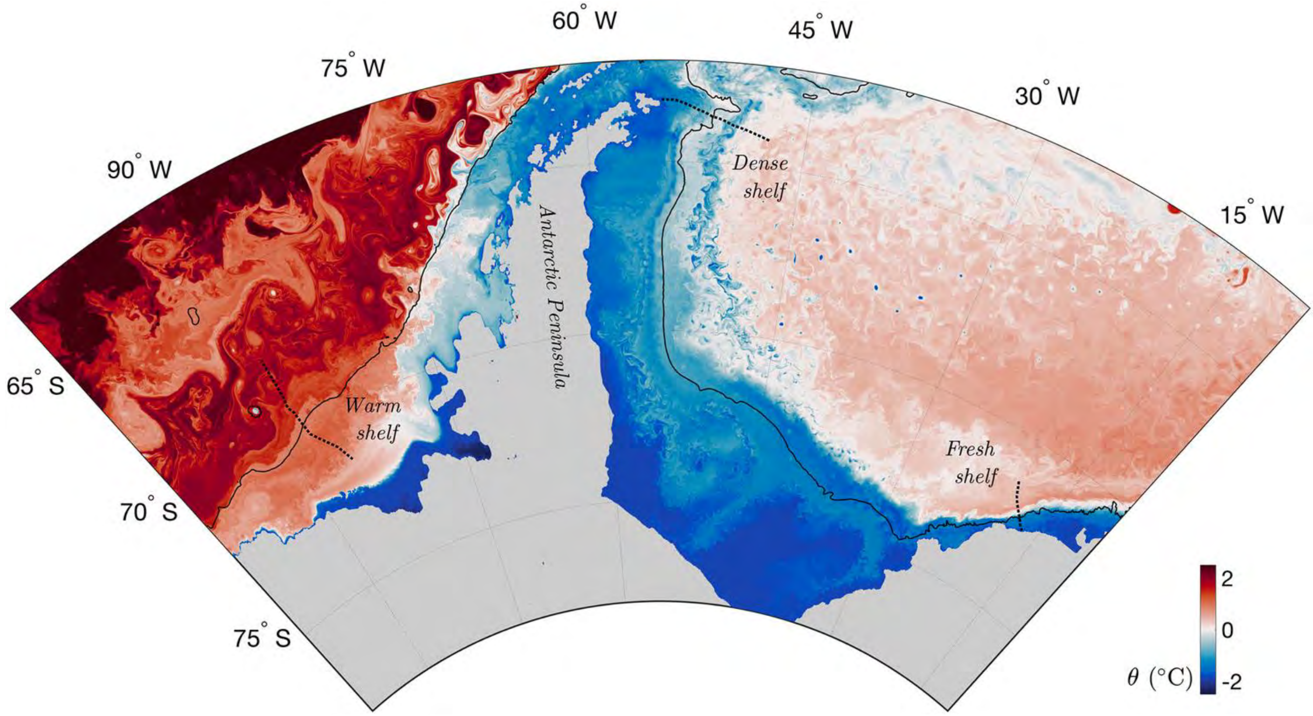
Eddies export nutrients off the Eastern Boundary Upwelling System, thus limiting marine productivity (Gruber et al. 2011).

Eddies across continental margins

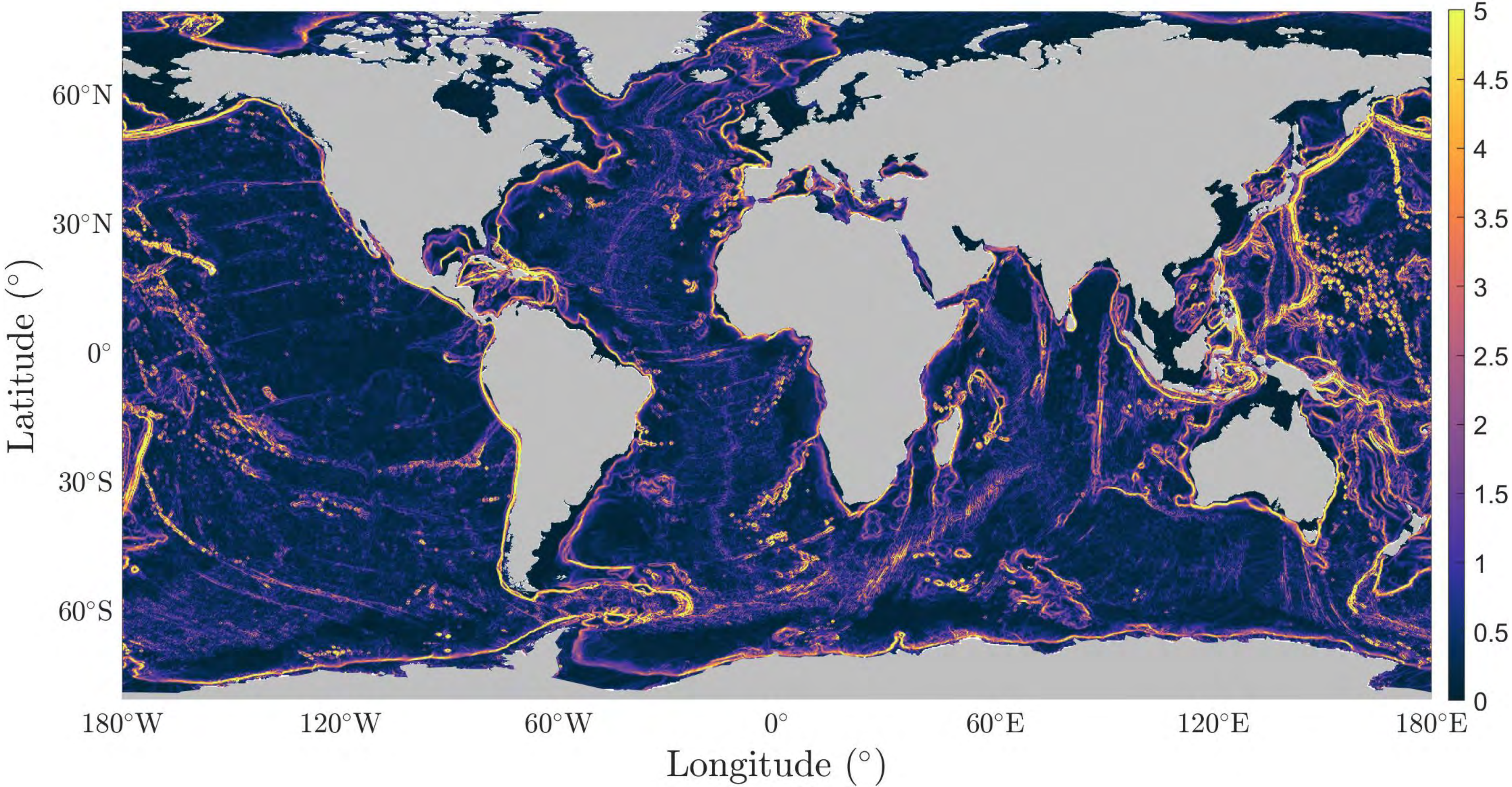


Eddies export nutrients off the Eastern Boundary Upwelling System, thus limiting marine productivity (Gruber et al. 2011).

Eddies drive heat fluxes toward the ice shelves of Antarctica, shaping the abyssal MOC (Thompson et al. 2018).



Eddies across continental margins



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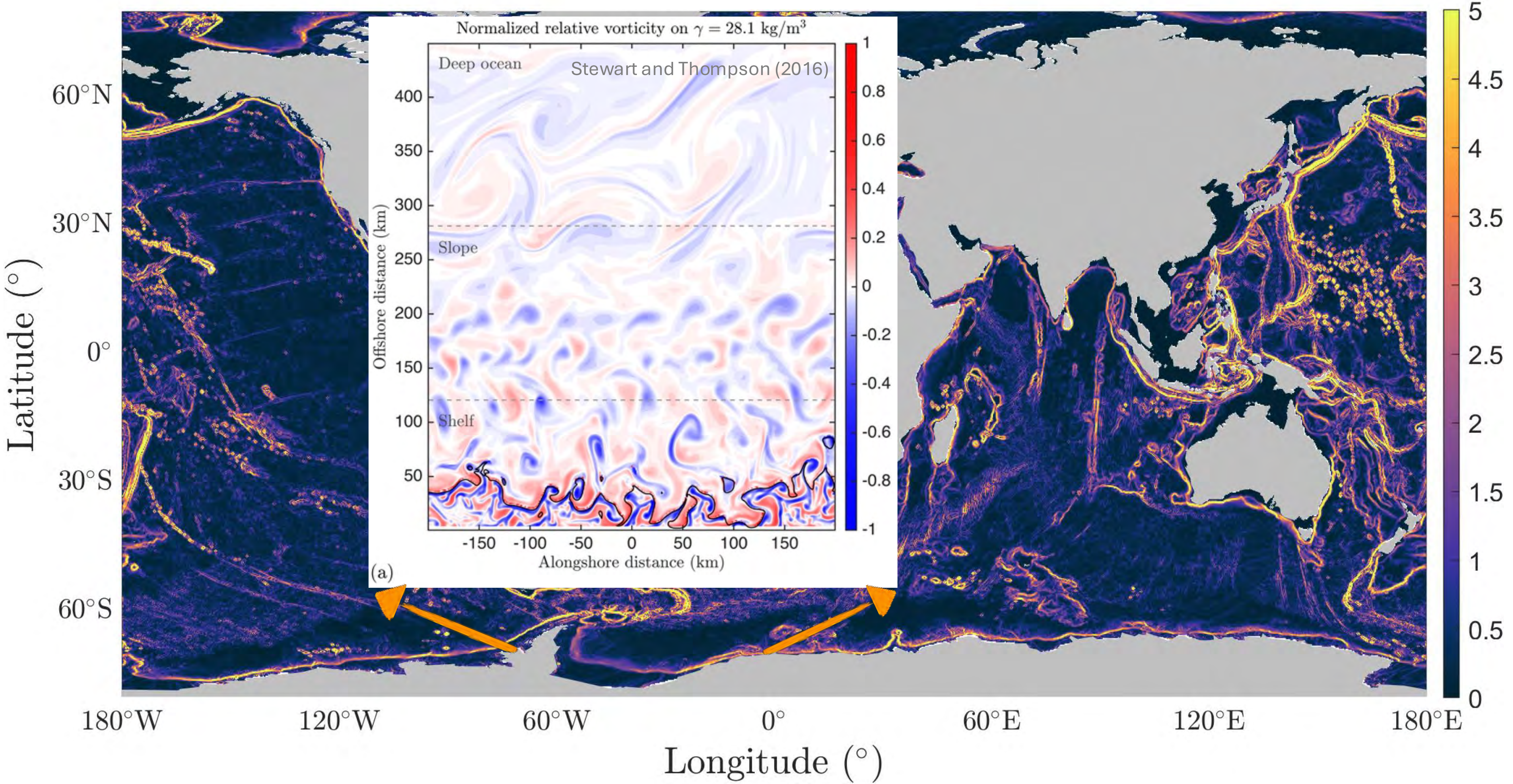
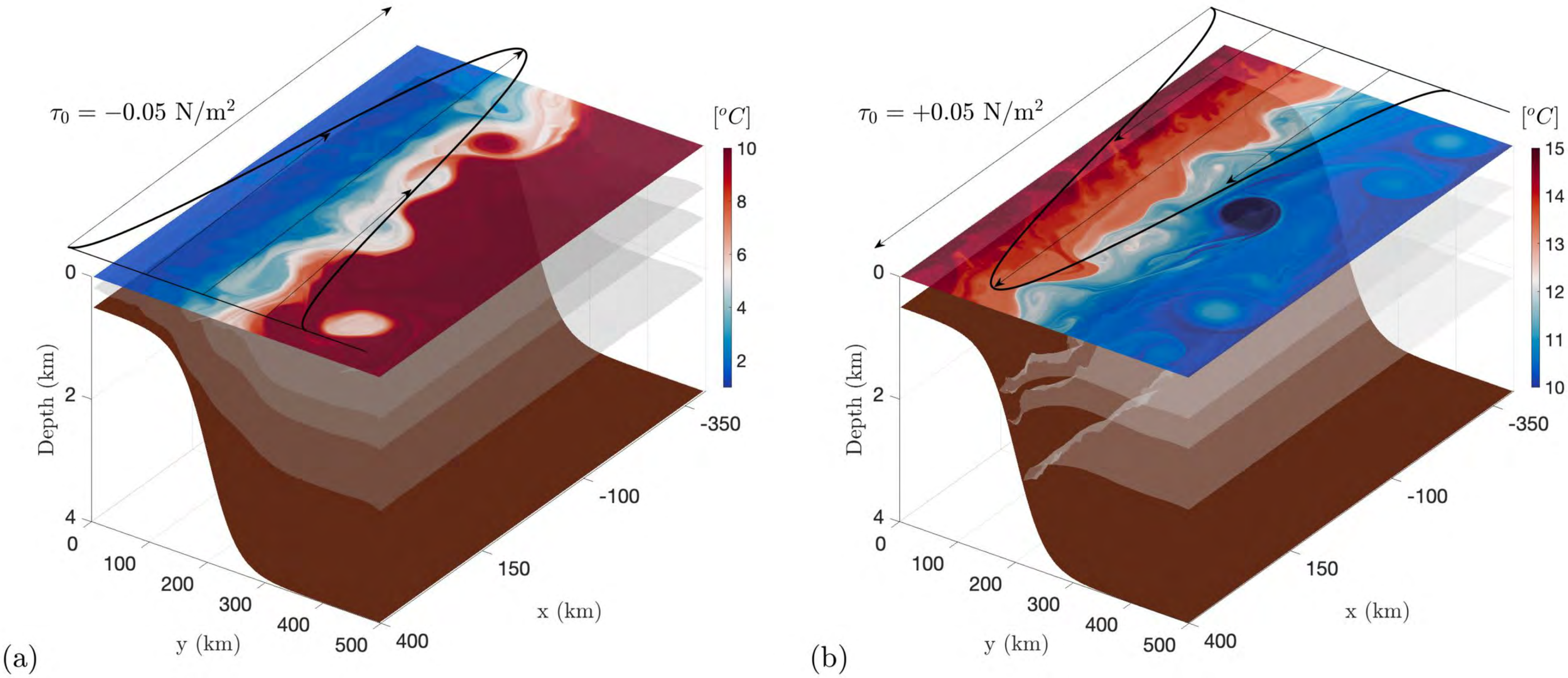


Retrograde versus Prograde

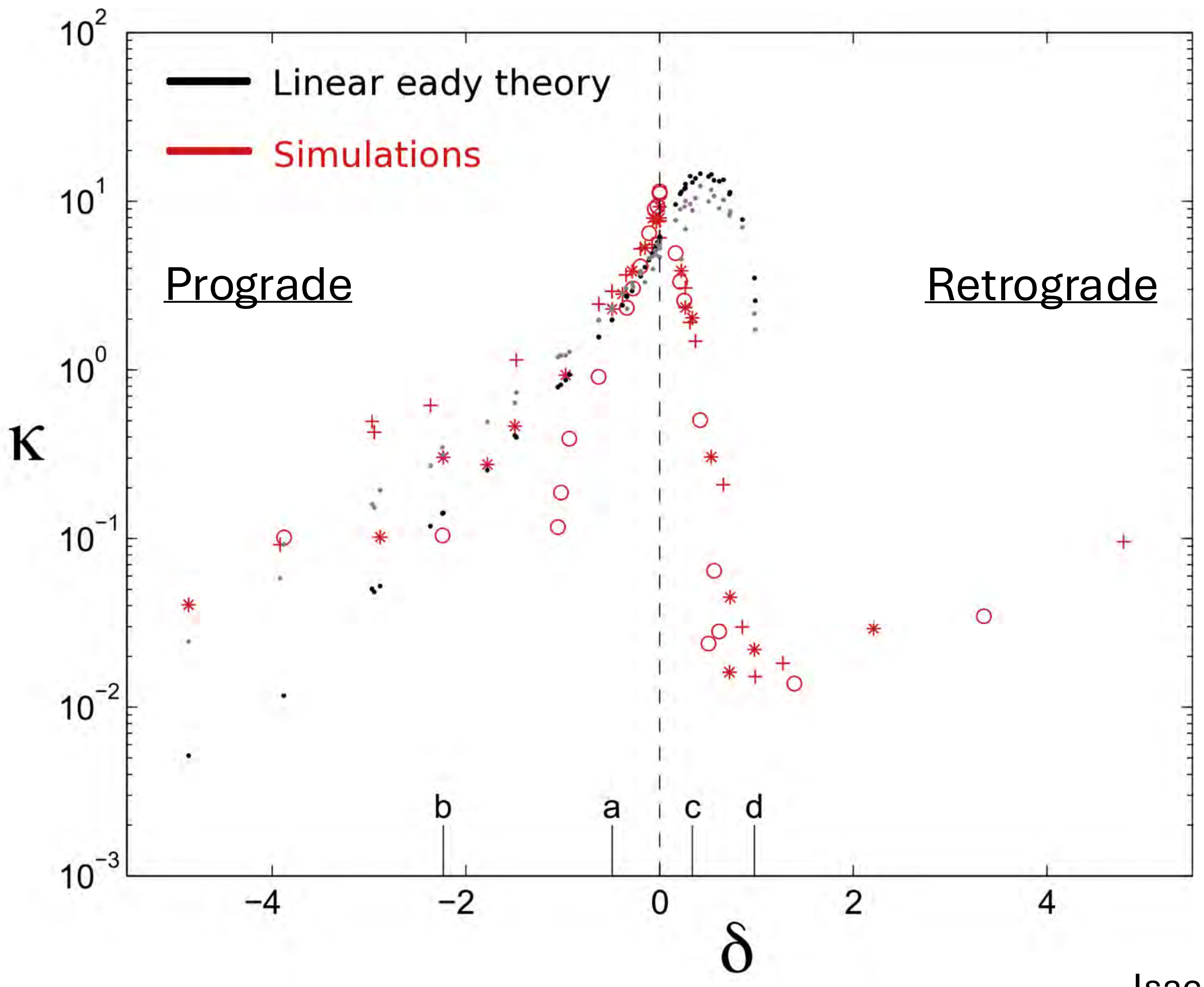


<u>Retrograde</u> or upwelling

<u>Prograde</u> or downwelling

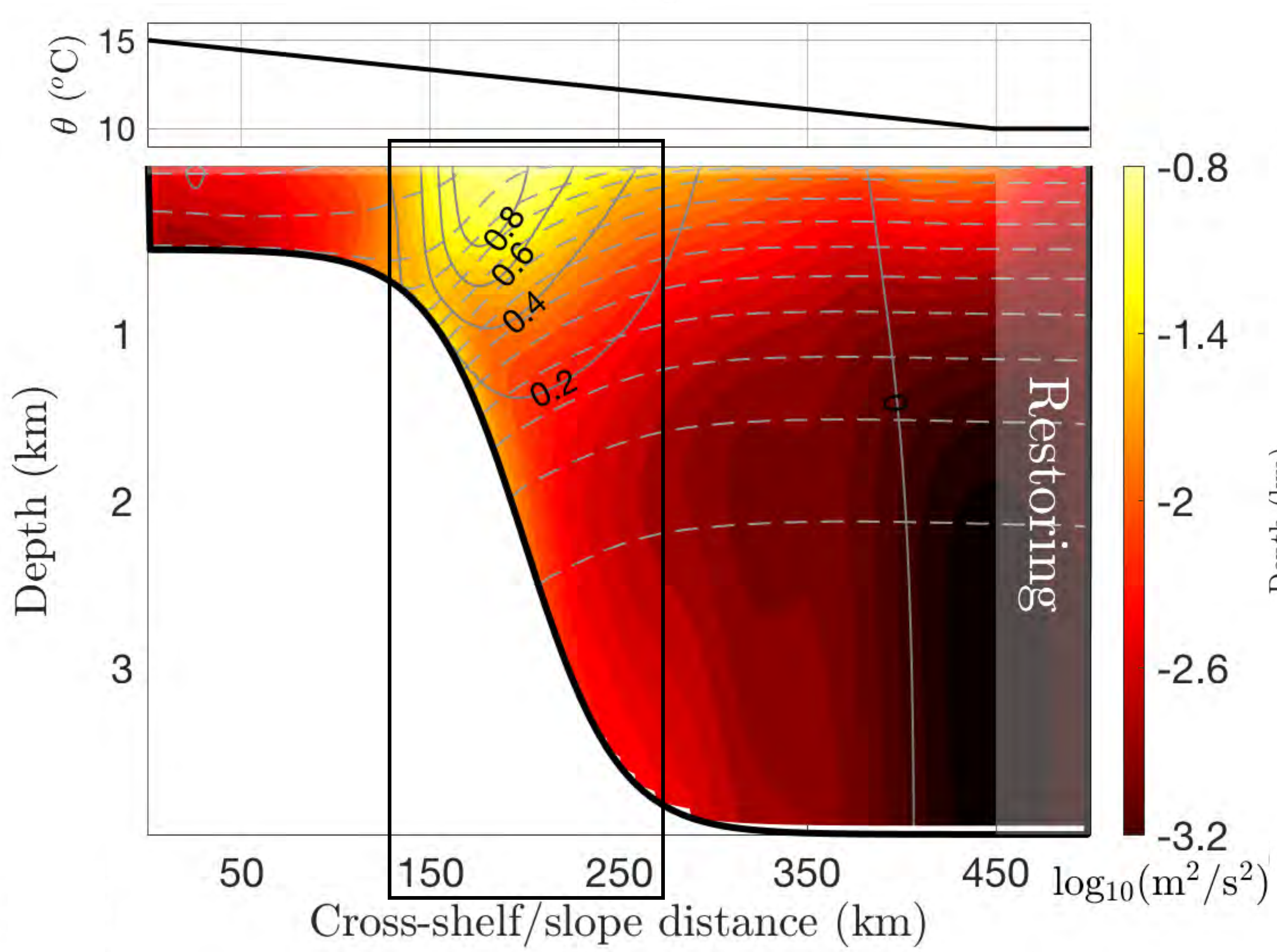
Wang and Stewart (2020); Wei, Wang, and Mak (2024)

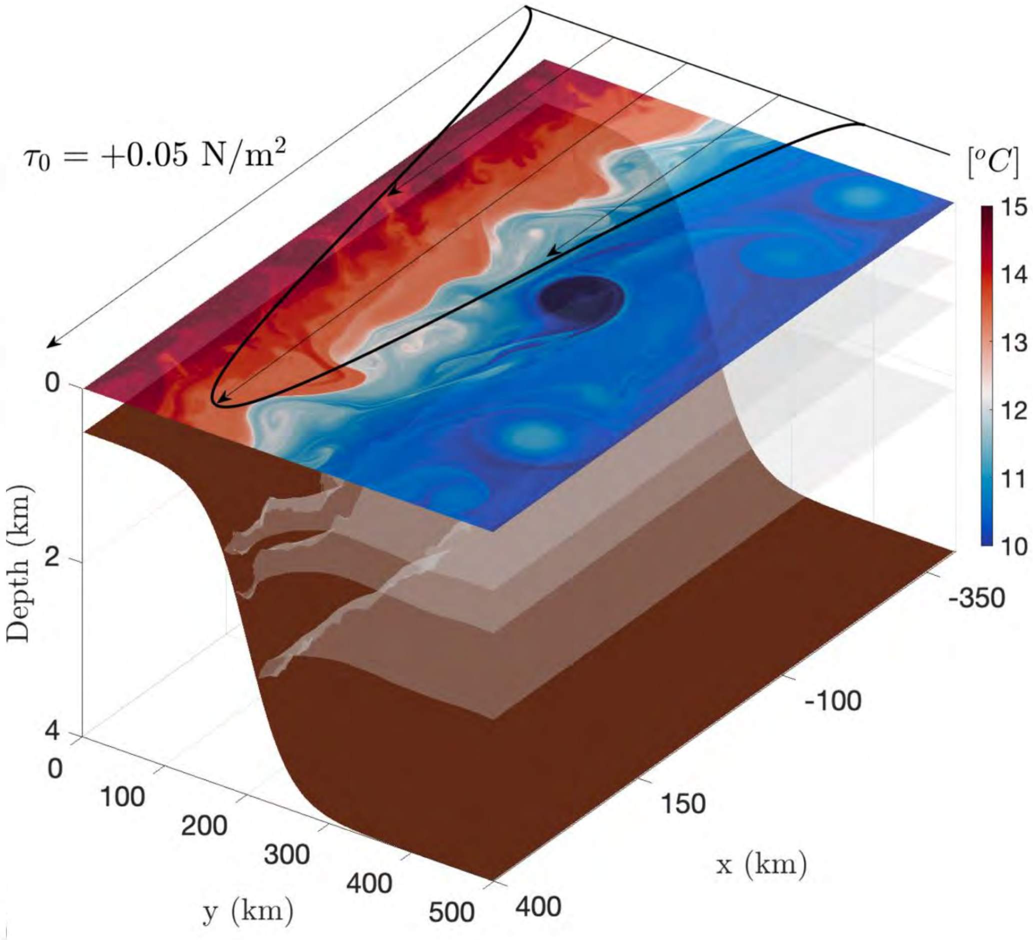
Retrograde versus Prograde



Isachsen (2011)

Eddy and mean properties: Prograde





Prograde or downwelling

Wei, Wang, Mak, and Stewart (2022); Wei, Wang, and Mak (2024)

Scaling of eddy buoyancy diffusivity: Prograde

$$K_{\mathrm{GEOM}} = \alpha \frac{\sqrt{Ri}}{f_0} E,$$

The "GEOMETRIC" theory
(e.g. Marshall et al. 2012; Mak et al. 2022)

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↓

Nonlinear eddy growth rate

$$\frac{f_0^2}{Ri} \cdot K_{\mathrm{GEOM}} = \alpha \cdot \boxed{\sigma_{\mathrm{E}}} \cdot E \equiv \boxed{2\sigma} \cdot E,$$

Normalized Eady growth rate: $\sigma_{\mathrm{E}} = f_0/\sqrt{Ri}$

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Normalized Eady growth rate: $\sigma_{\mathrm{E}} = f_0/\sqrt{Ri}$

$$\sigma / [0.31 \cdot \sigma_{\mathrm{E}}] \sim \mathcal{F}_{\mathrm{GEOM}}(\mathcal{S})$$

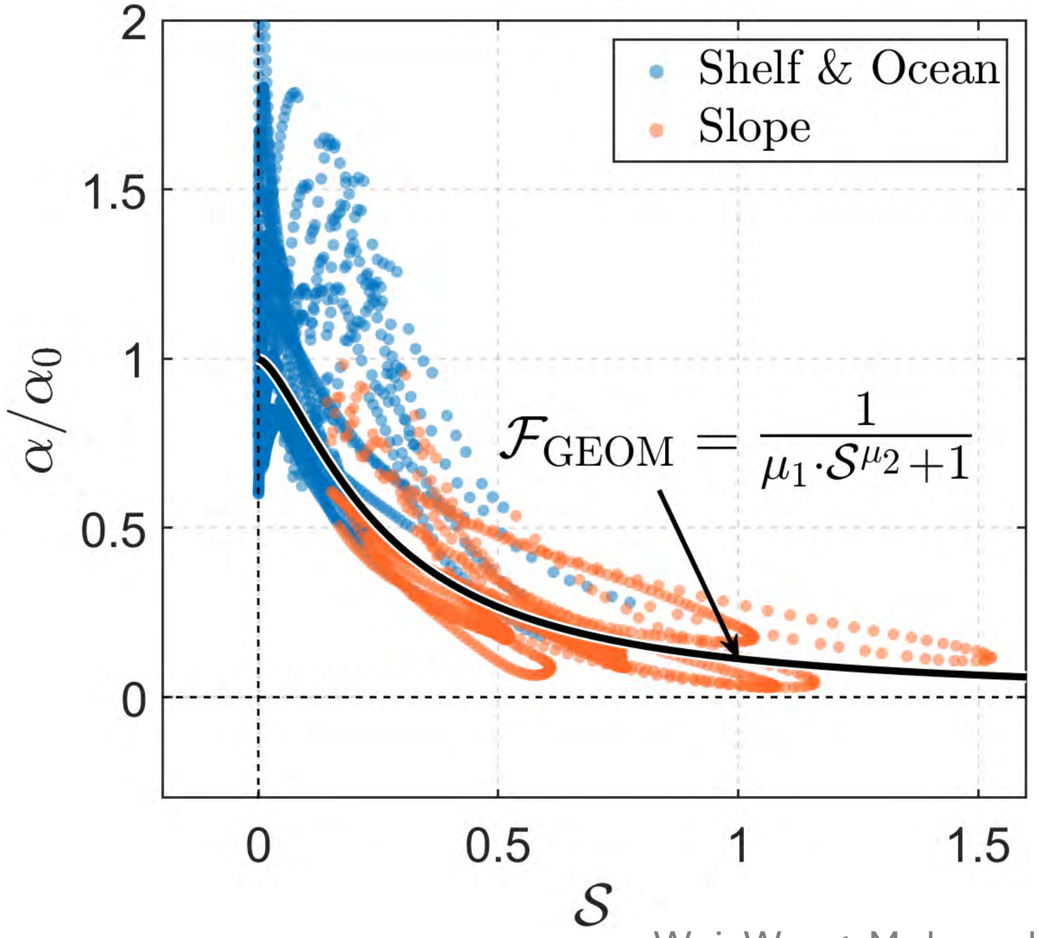
$$\mathcal{S} = \left\langle \left| \frac{\partial H}{\partial y} \right| \frac{1}{|H|} \int_{-|H|}^{0} \frac{N}{f_0} \, \mathrm{d}z \right\rangle$$

The slope Burger number dependence
(e.g. Brink, 2012; 2016; Brink and Cherian, 2013; Hetland, 2017; Chen et al, 2020).

Scaling of eddy buoyancy diffusivity: Prograde

$$\alpha = \alpha_0 \cdot \mathcal{F}_{\mathrm{GEOM}}(\mathcal{S}),$$

$$\mathcal{F}_{\mathrm{GEOM}}(\mathcal{S}) = \frac{1}{\mu_1 \cdot \mathcal{S}^{\mu_2} + 1},$$



Wei, Wang, Mak, and Stewart (2022)

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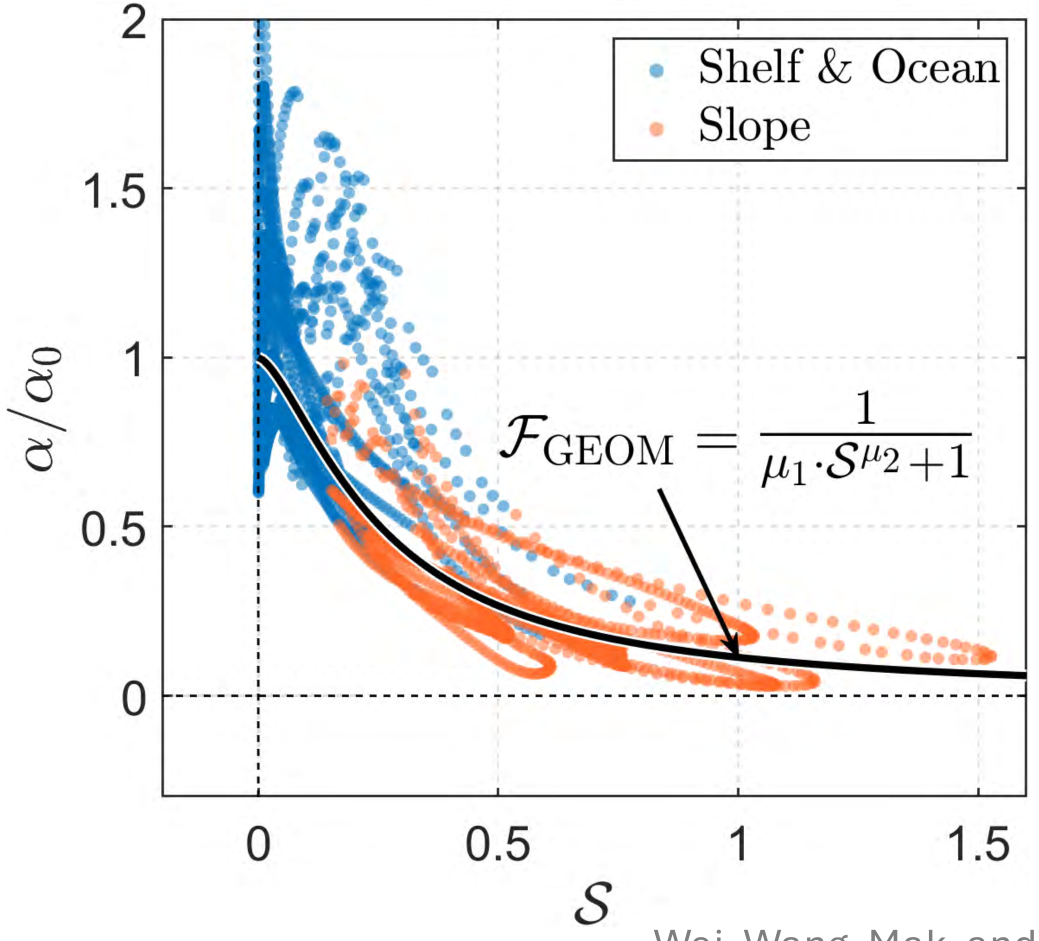

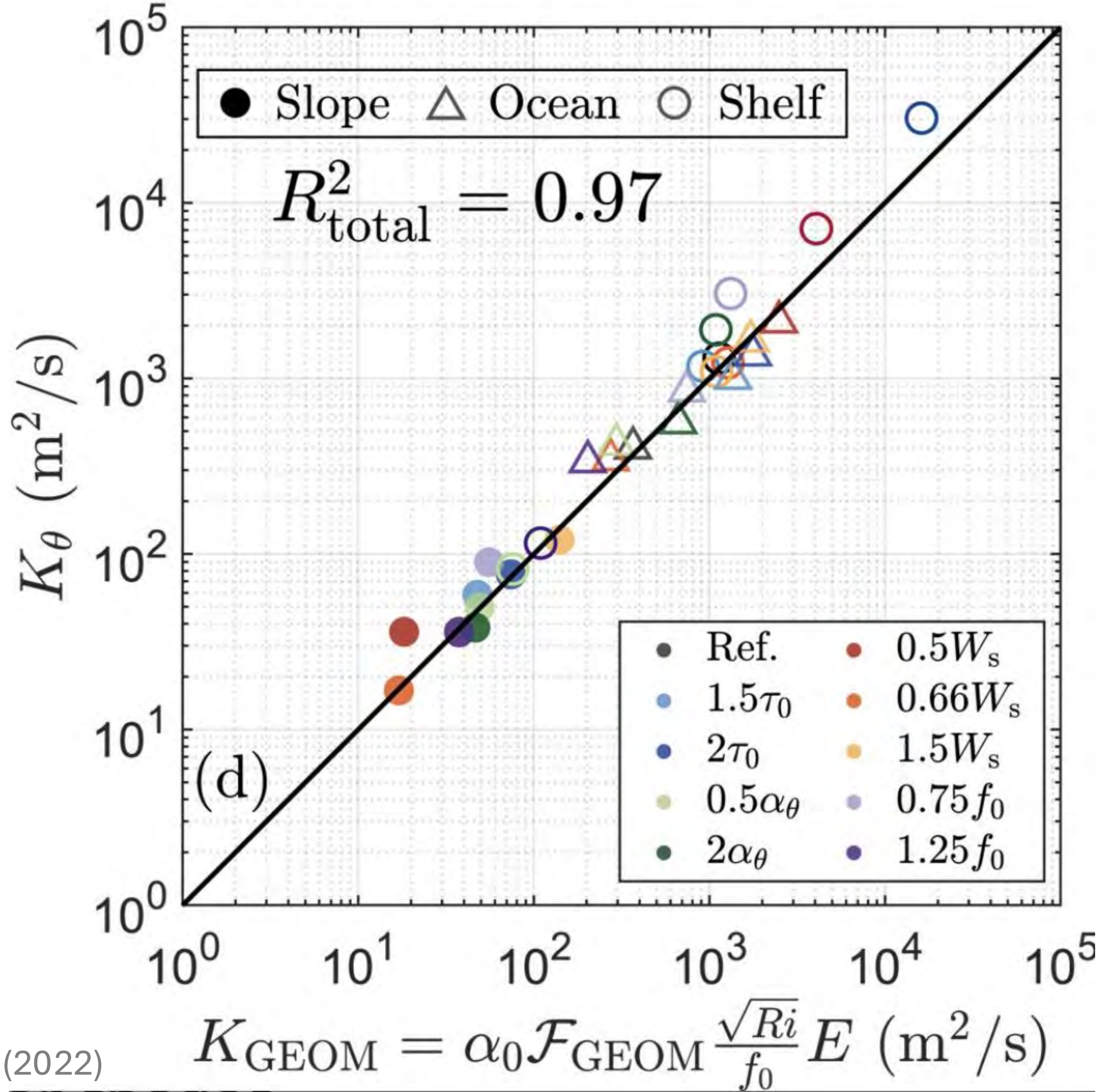

Wei, Wang, Mak, and Stewart (2022)

Convert scaling into eddy closure: Prograde

$$\underbrace{\frac{\partial}{\partial t}\hat{E}}_{\text{tendency}} + \underbrace{\frac{\partial}{\partial y}(\langle v \rangle \hat{E})}_{\text{advection}} = \underbrace{\int_{-|H|}^{0} \mathcal{K}_{\mathrm{gm}} \frac{M^4}{N^2}\, dz}_{\text{source}} - \underbrace{\tau_E^{-1}\hat{E}}_{\text{dissipation}} + \underbrace{\eta_E \frac{\partial^2}{\partial y^2}\hat{E}}_{\text{diffusion}},$$

Wei, Wang, and Mak (2024)

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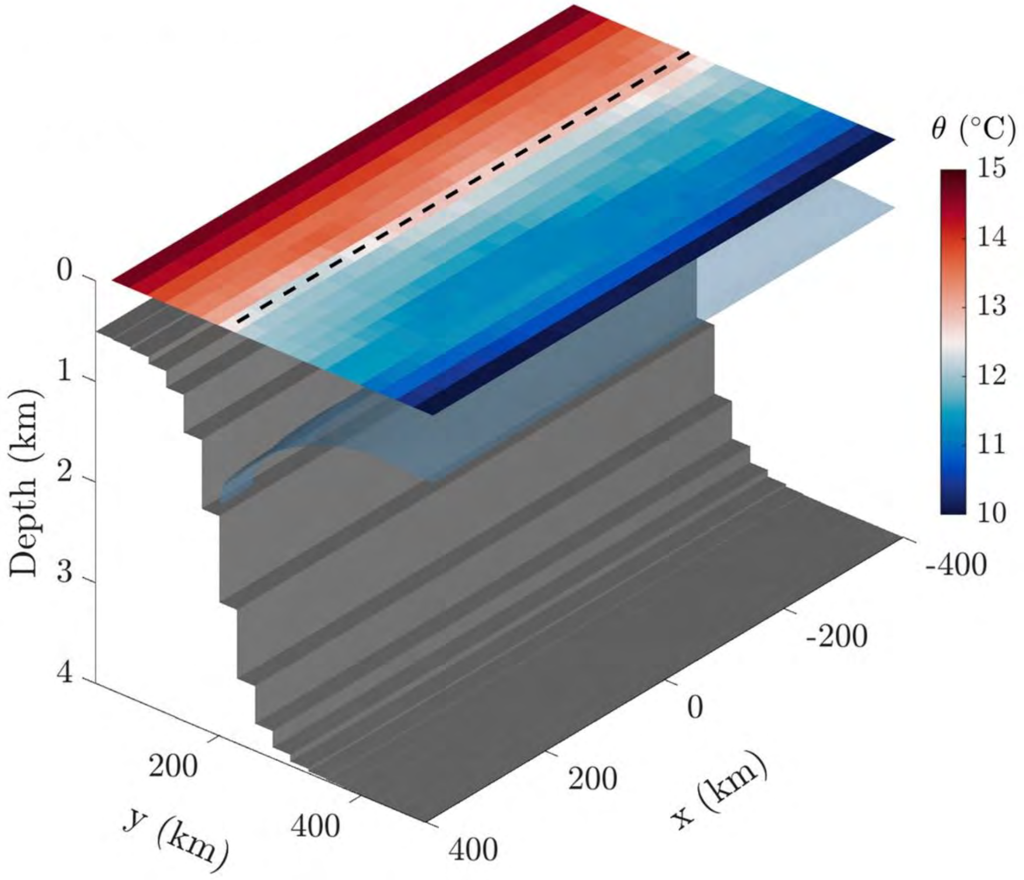
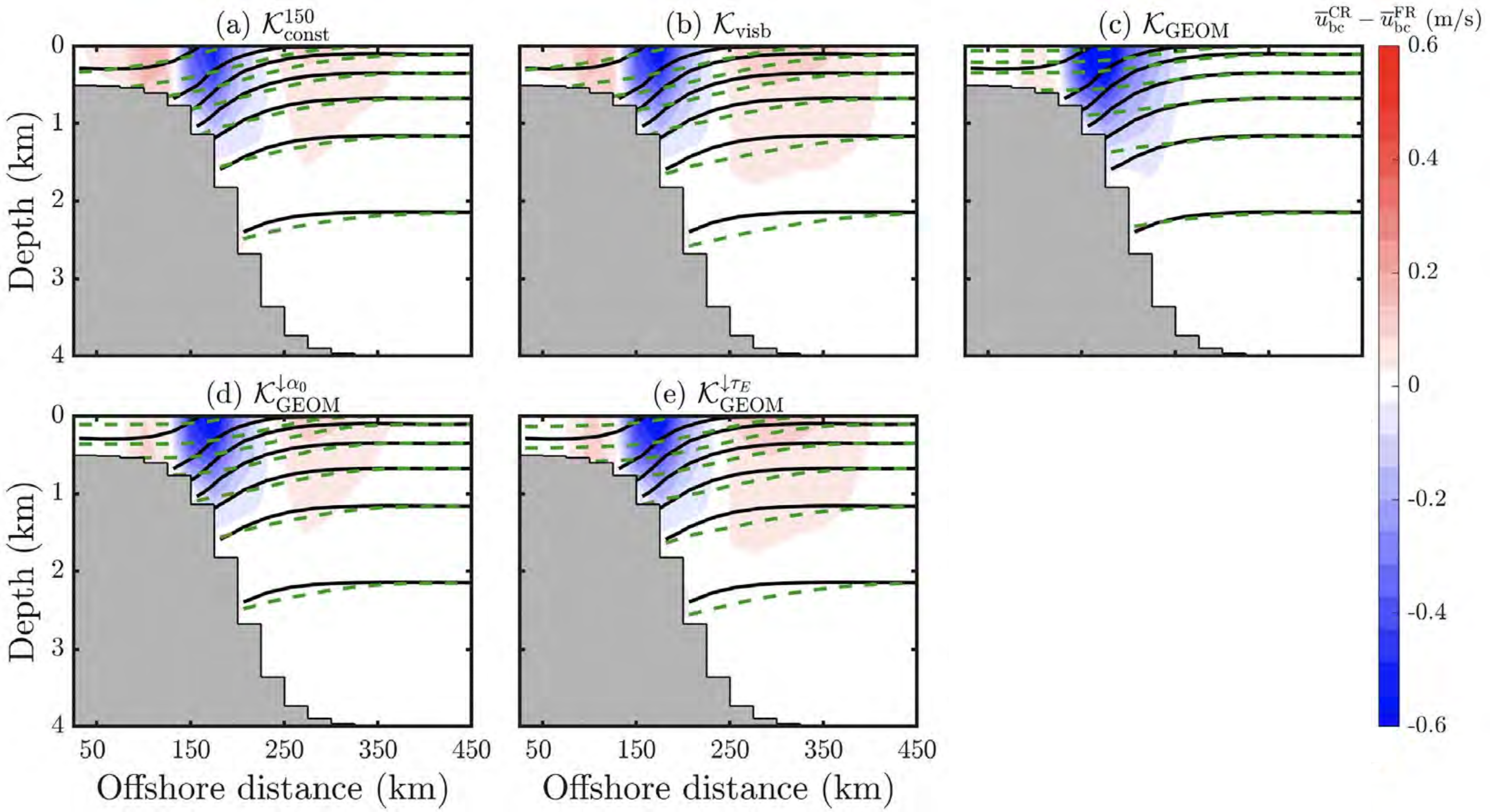


Wei, Wang, and Mak (2024)

Parameterization: Prograde



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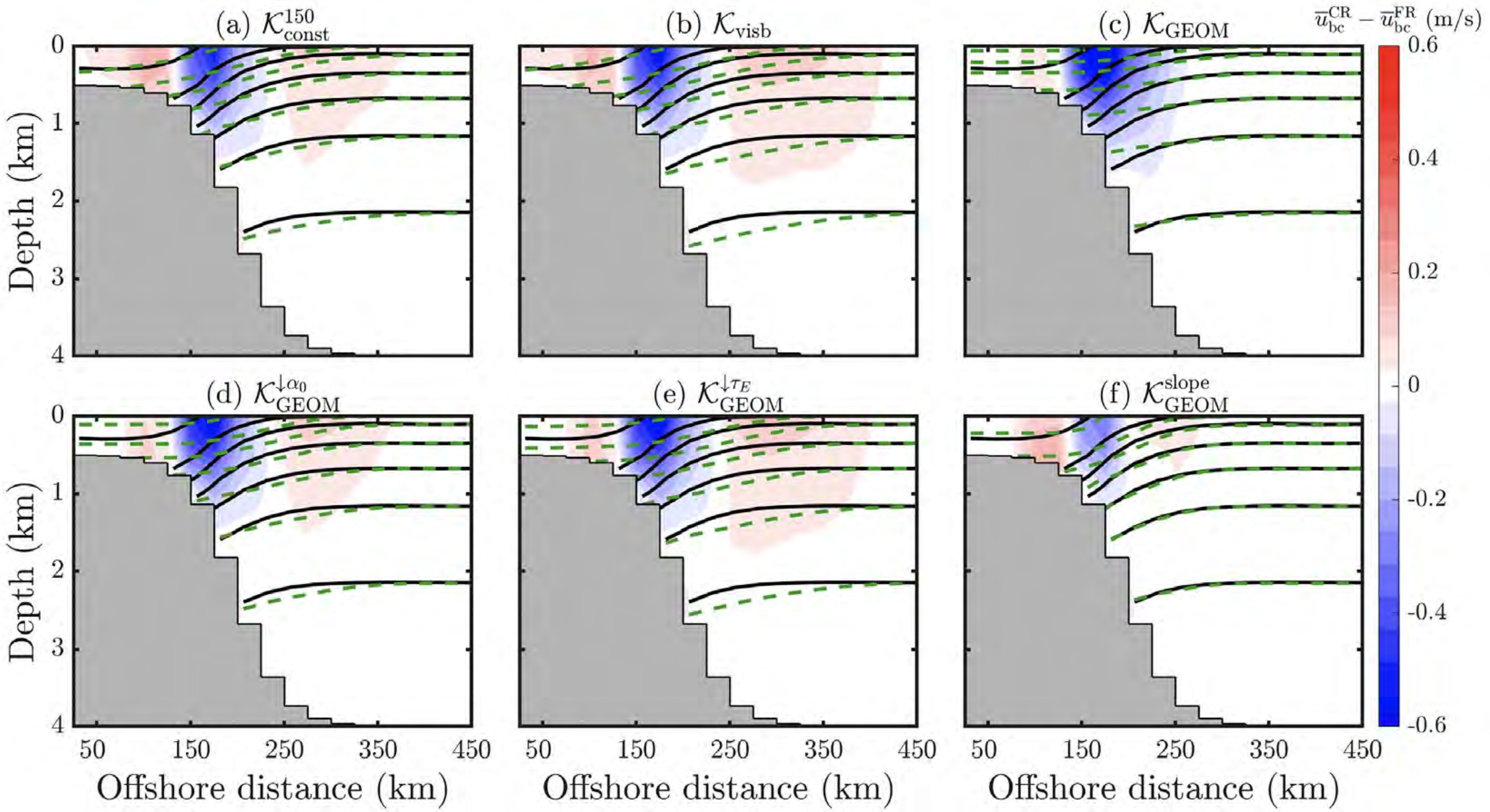

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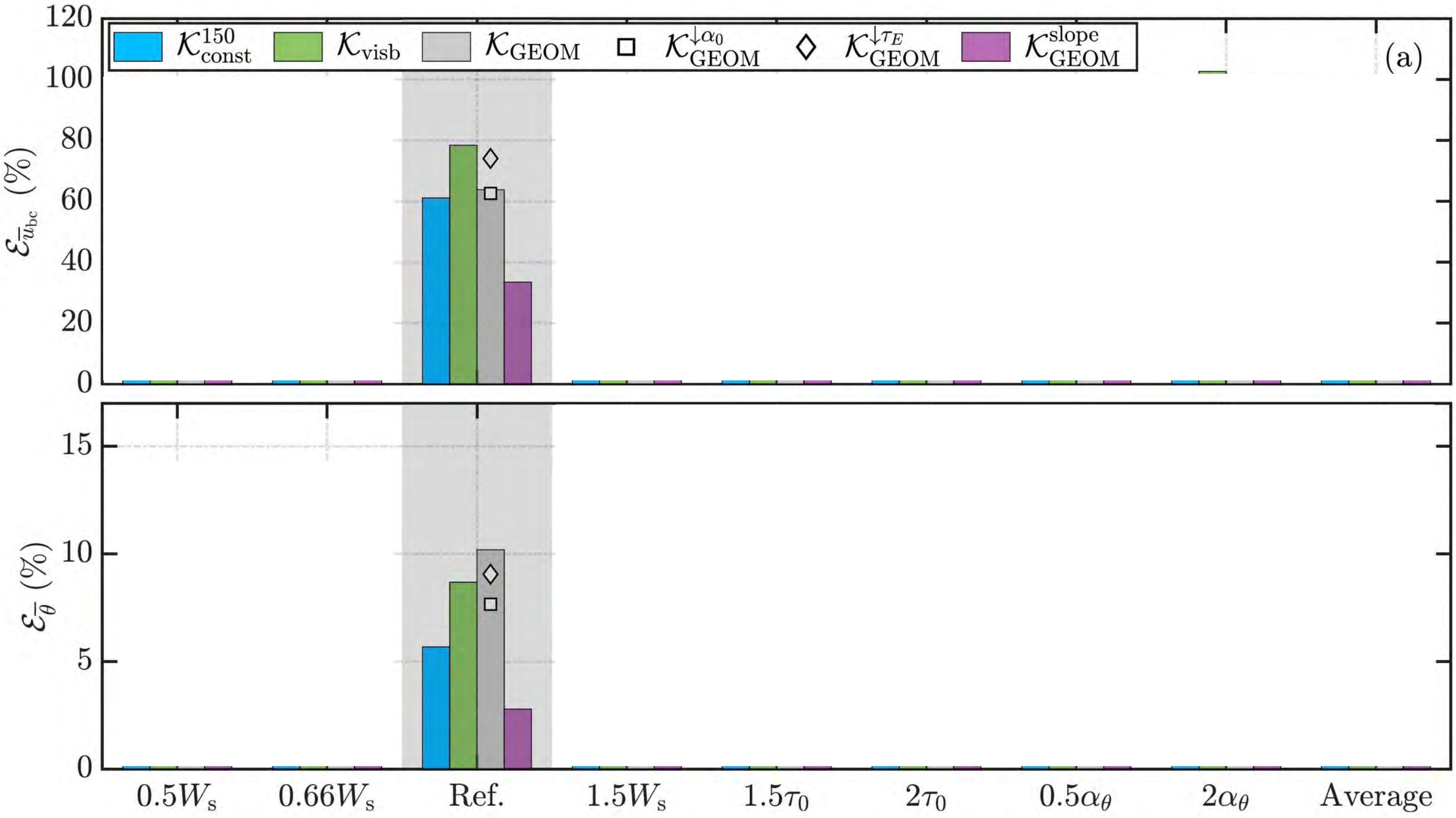

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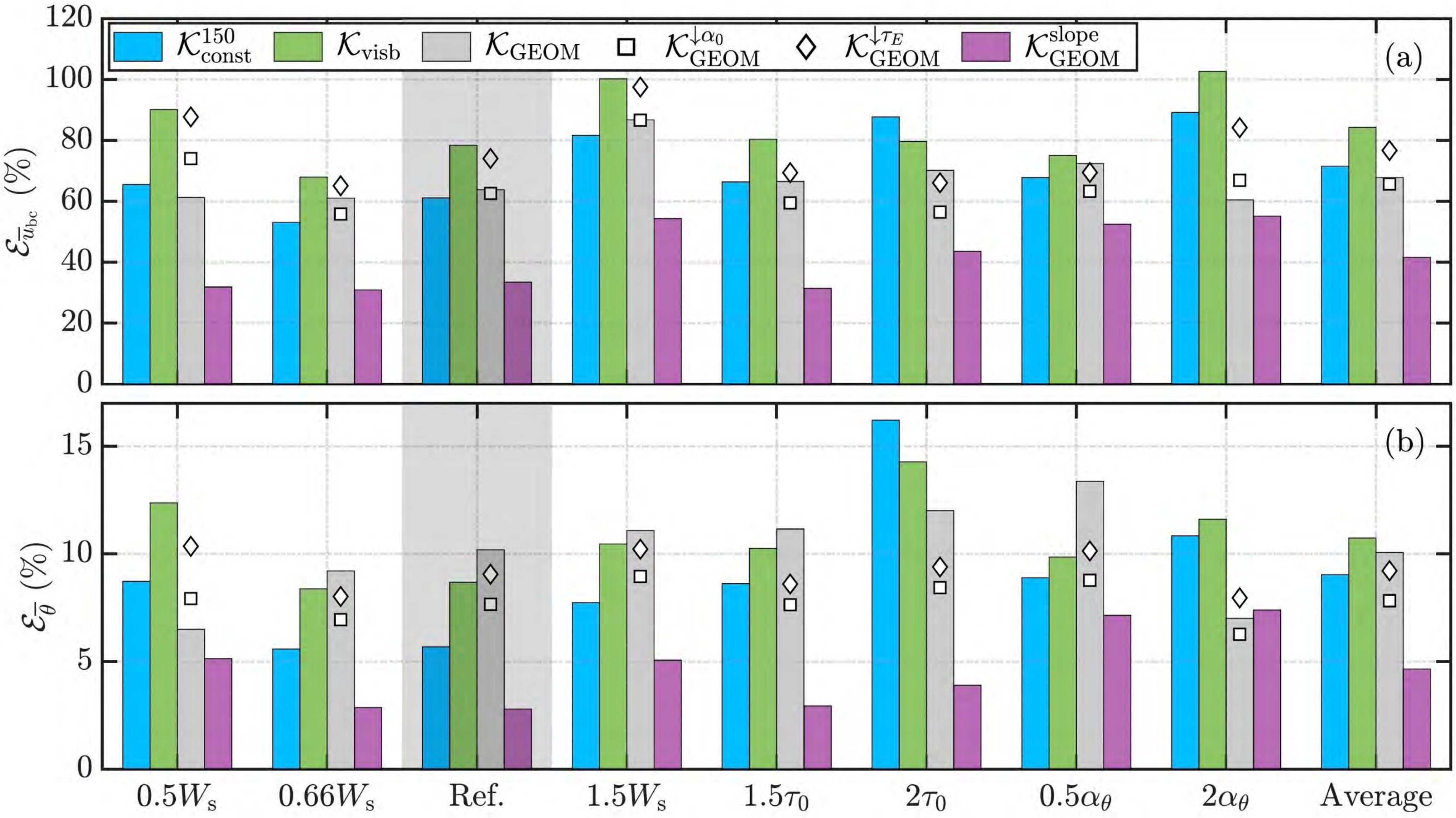
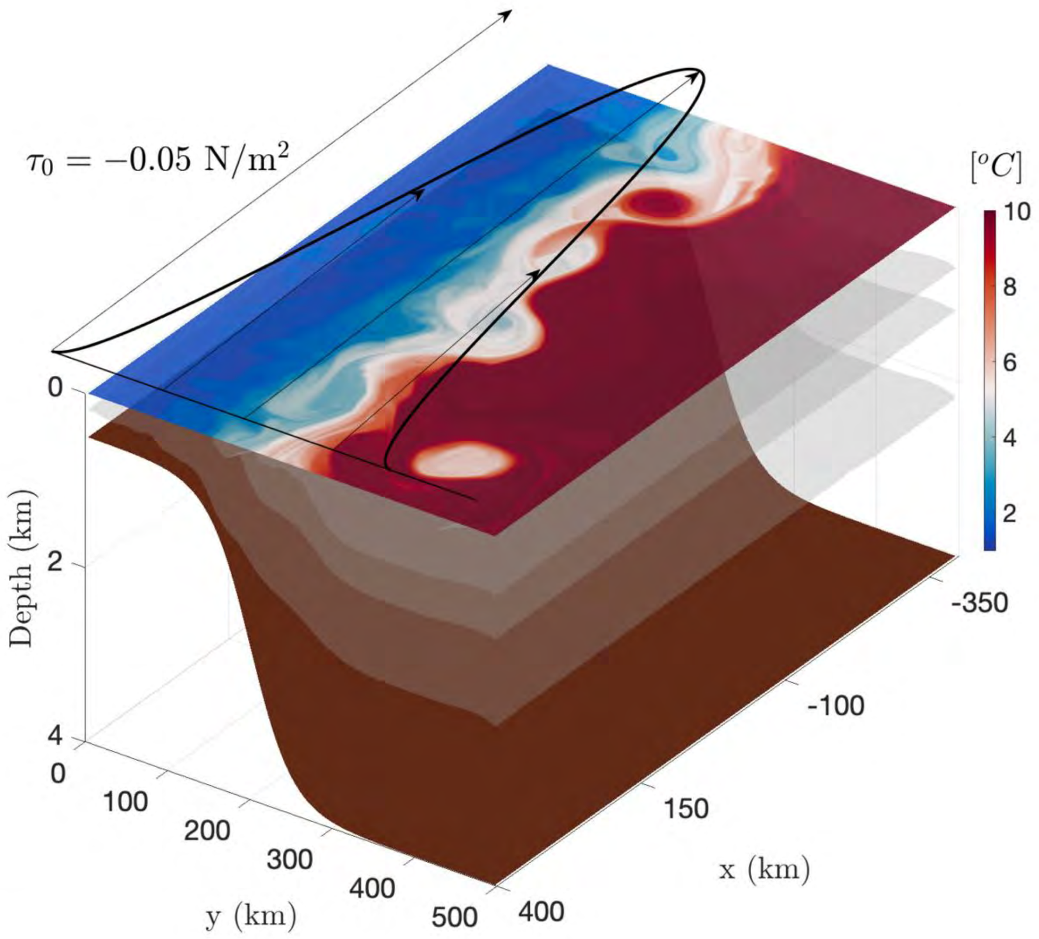
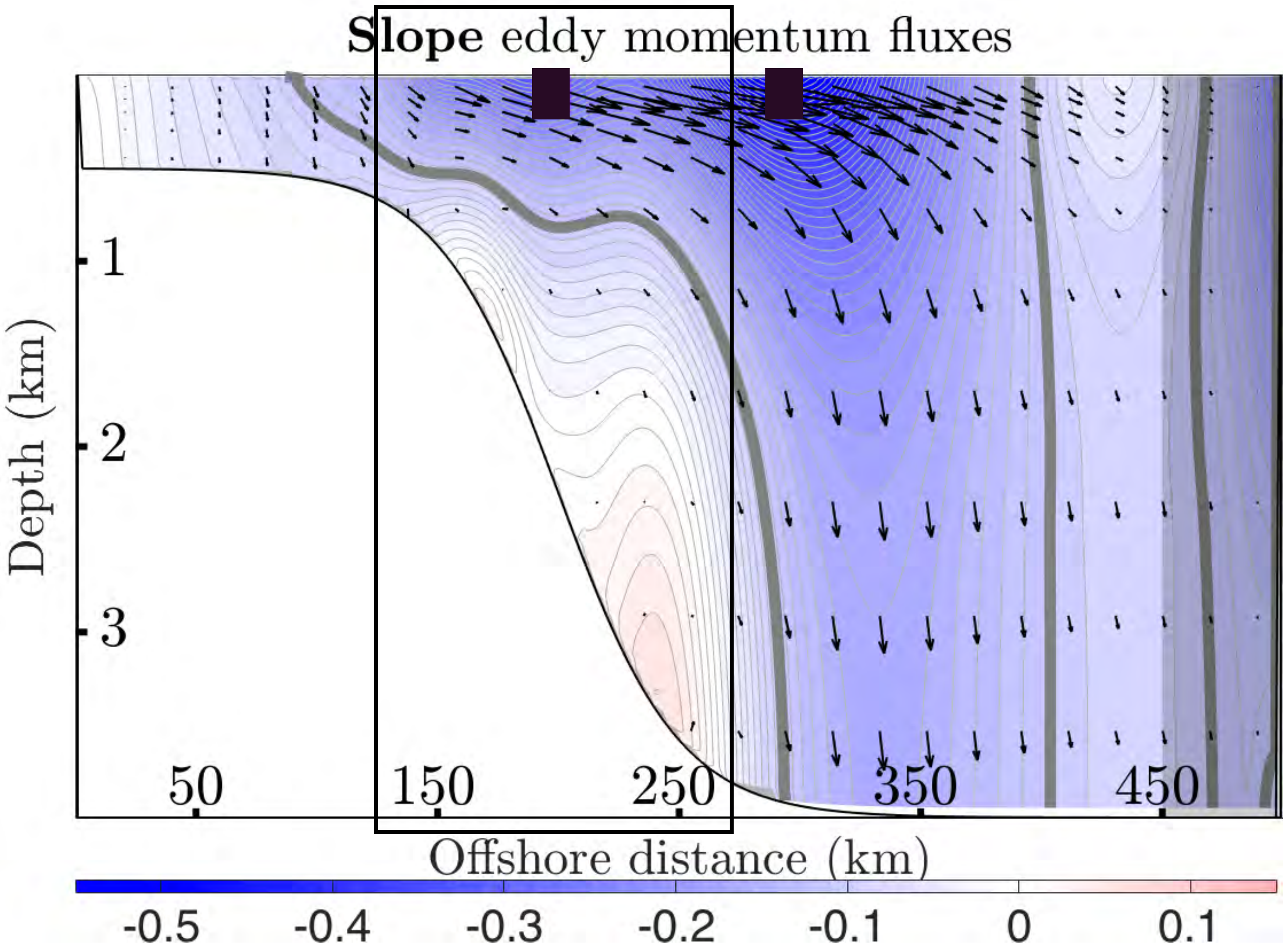


Wei, Wang, and Mak (2024)

Eddy impact on the mean: Retrograde





Wang and Stewart (2018; 2020)

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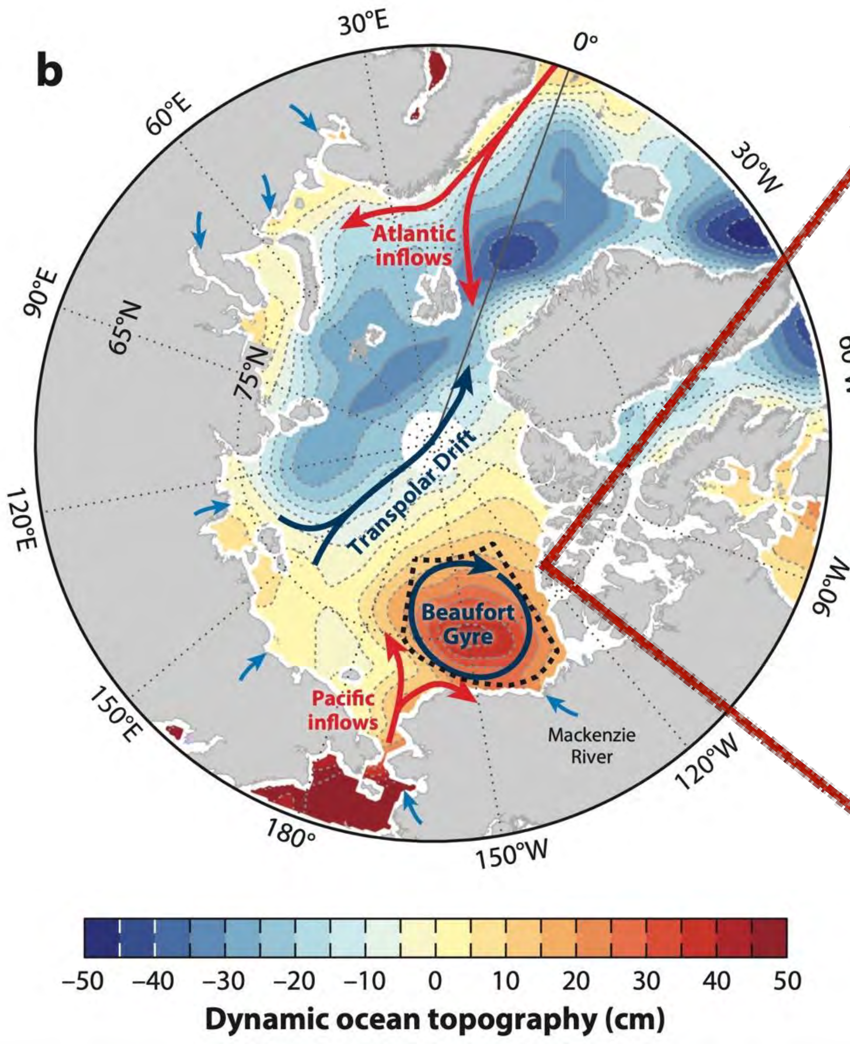
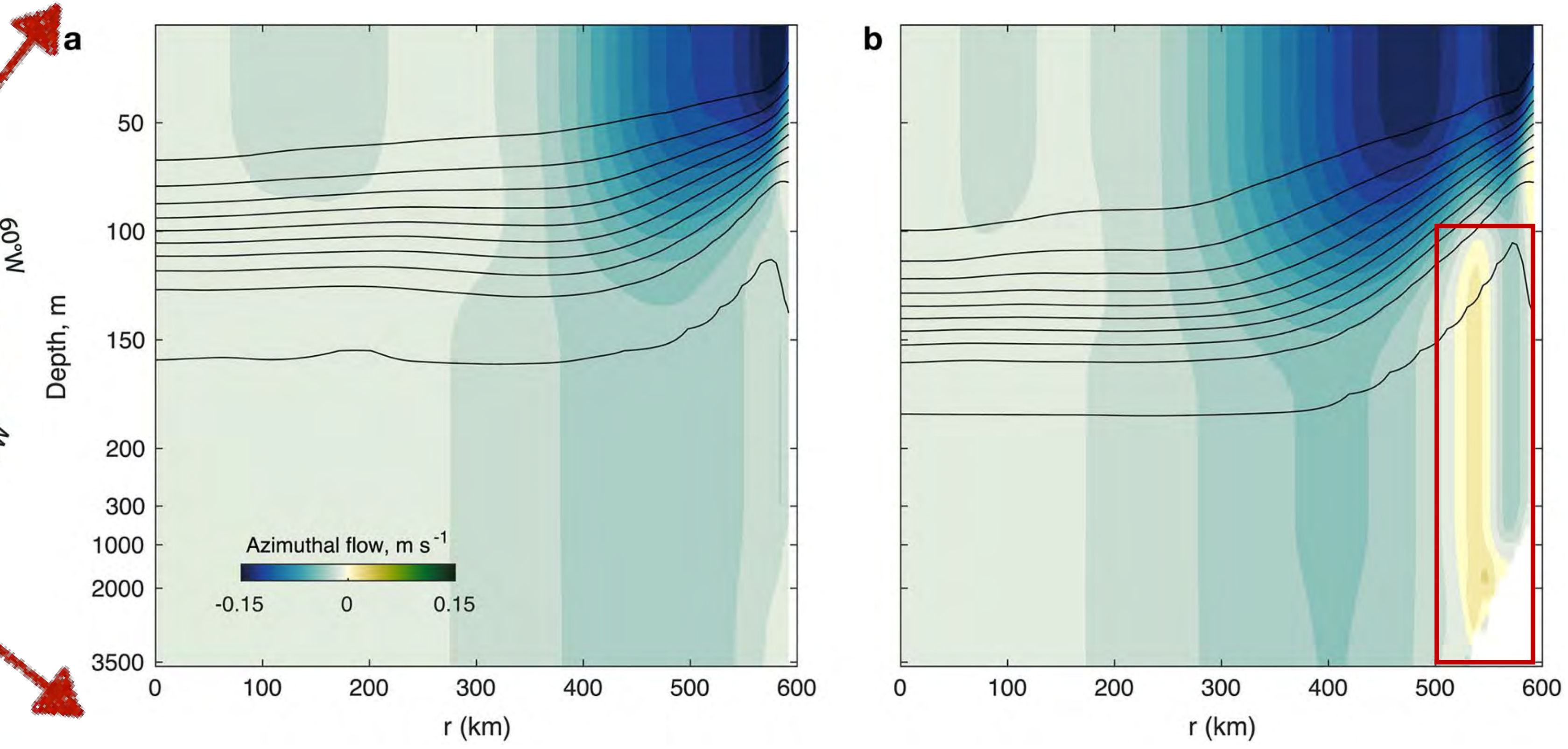


Timmermans and Toole (2019)



Manucharyan and Isachsen (2019)

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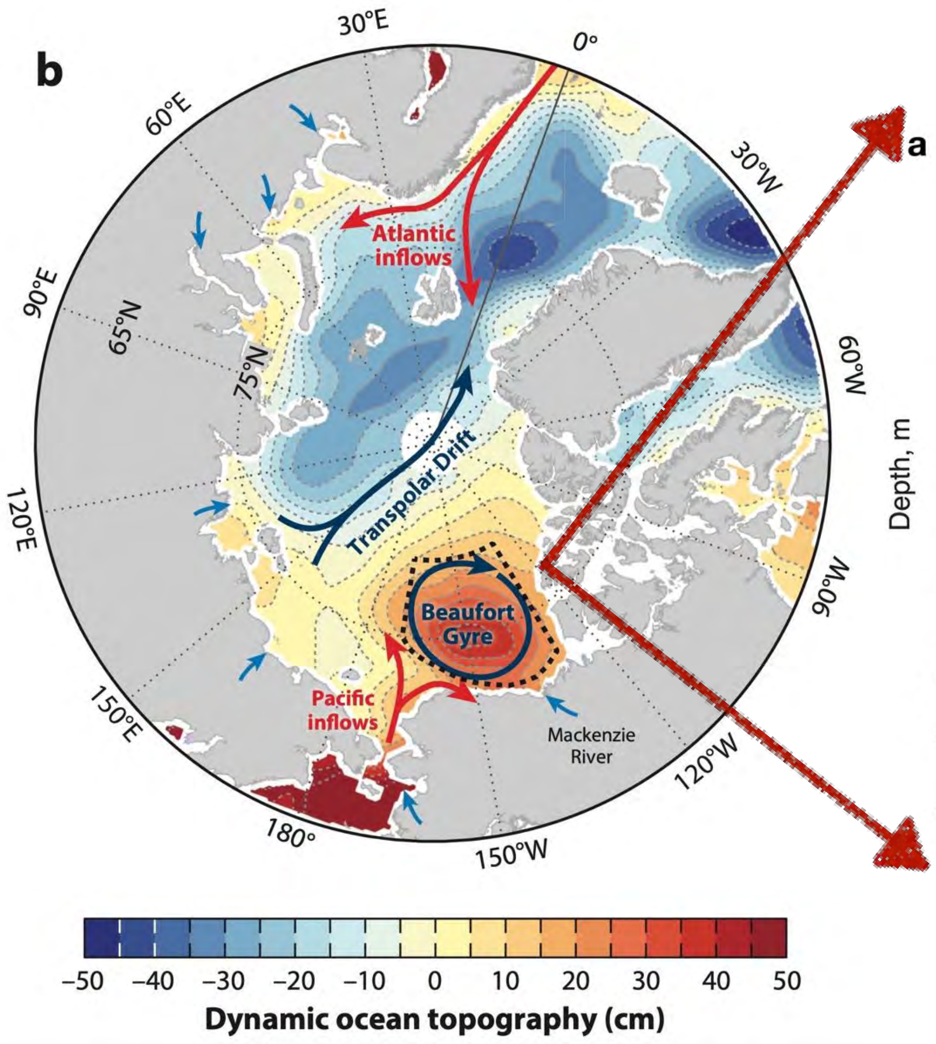
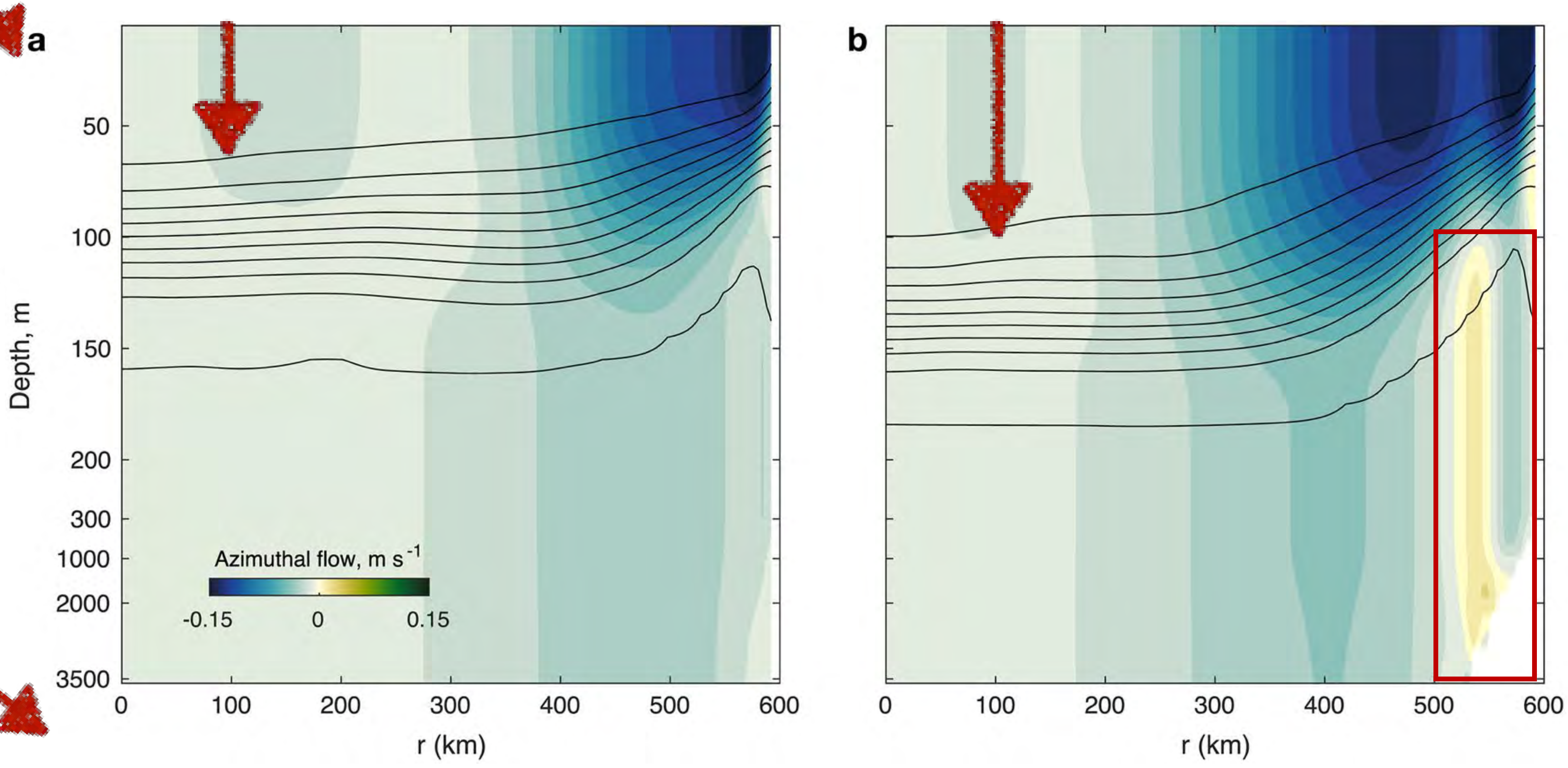


Timmermans and Toole (2019)

Freshwater content difference



Manucharyan and Isachsen (2019)

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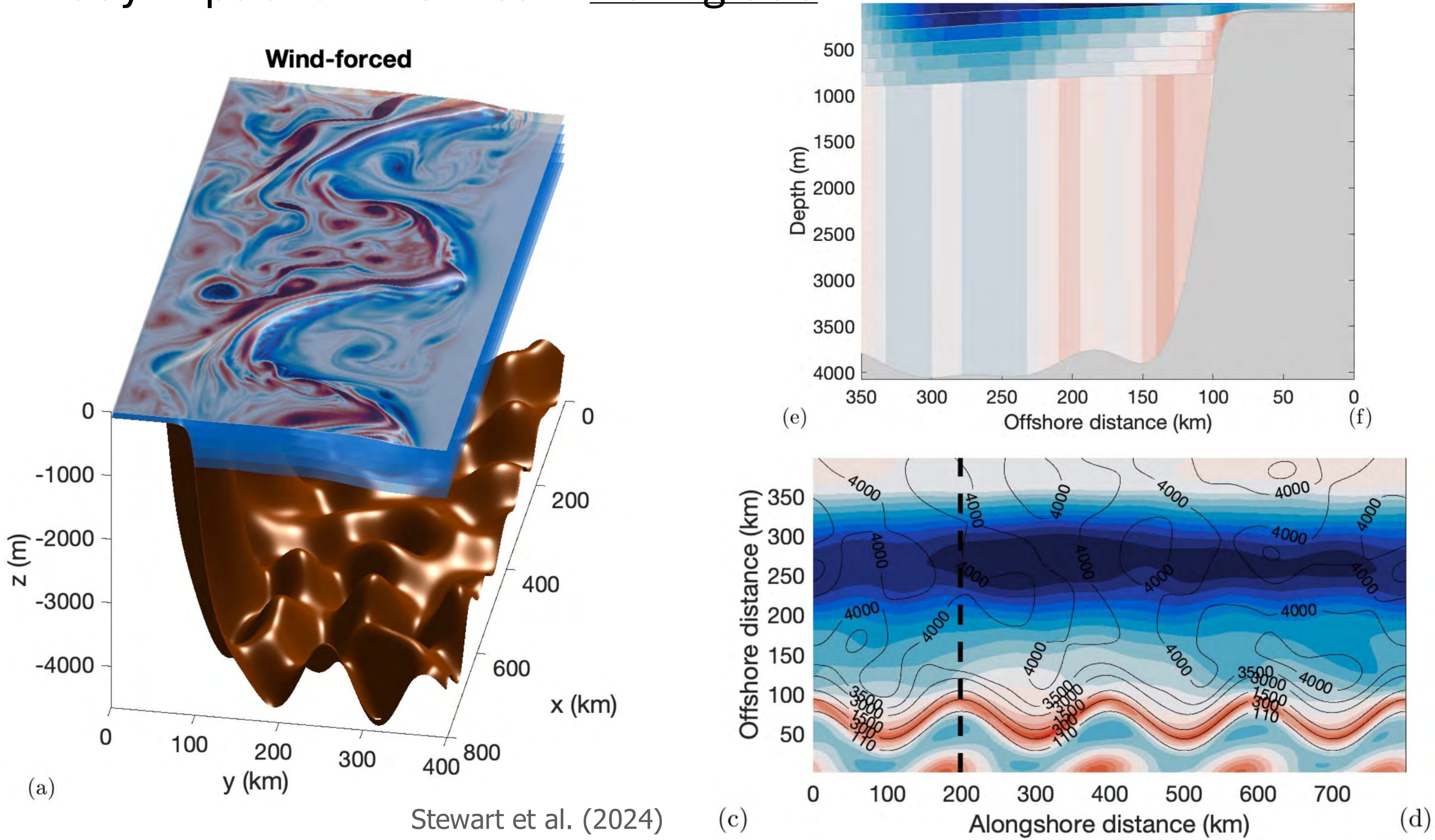
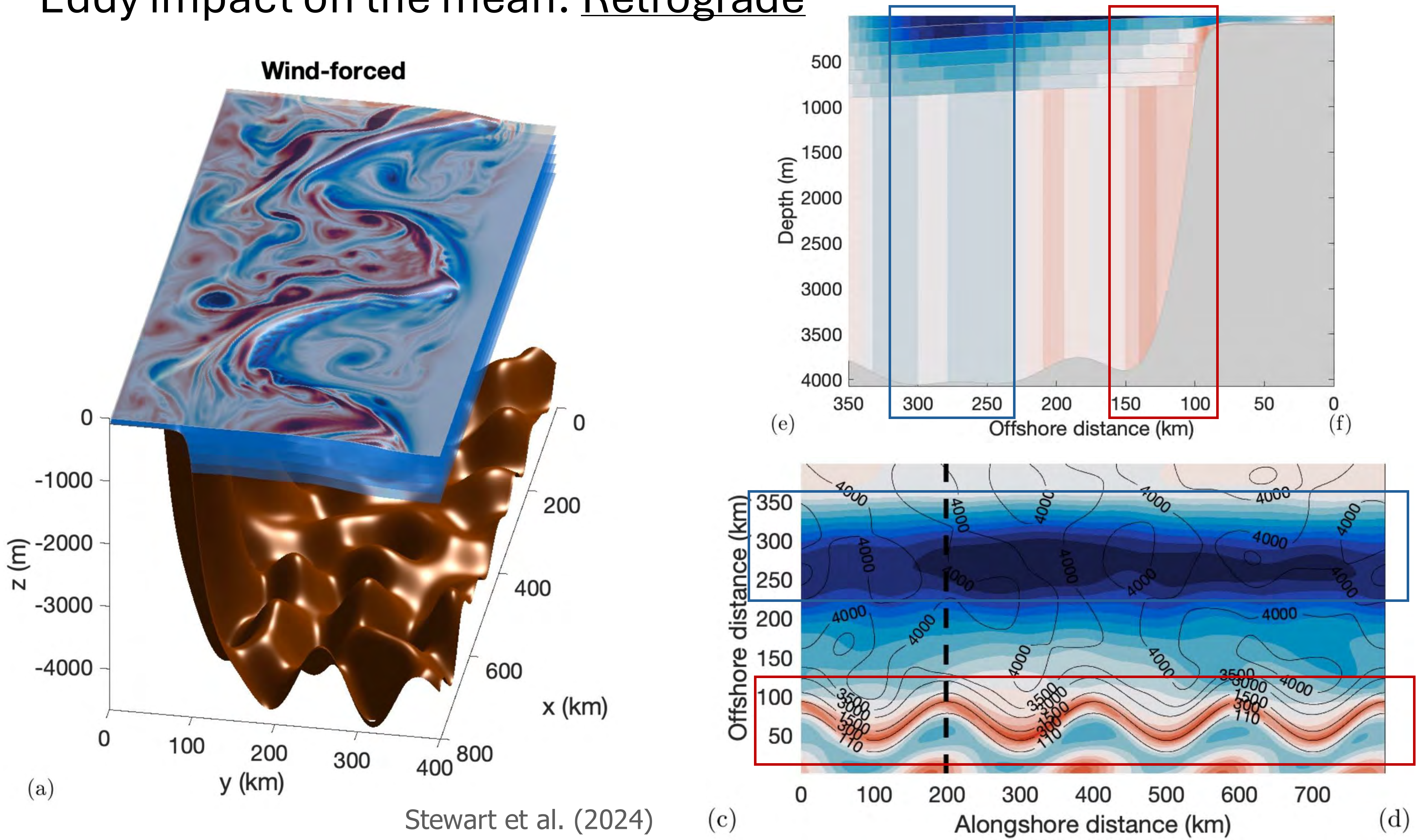


Stewart et al. (2024)

Eddy impact on the mean: <u>Retrograde</u>



Stewart et al. (2024)

Eddy impact on the mean: Retrograde

$$\frac{\partial}{\partial t}\overline{u} \simeq \mathrm{Forcing} + f_0\frac{\partial}{\partial z}\left(\frac{\overline{v'b'}}{\overline{b}_z}\right) - \frac{\partial}{\partial y}\overline{v'u'}$$

Eddy impact on the mean: Retrograde

Eddy form stress transfers momentum *downward*

$$\frac{\partial}{\partial t}\overline{u} \simeq \mathrm{Forcing} + \boxed{f_0 \frac{\partial}{\partial z}\left(\frac{\overline{v'b'}}{\overline{b}_z}\right)} - \boxed{\frac{\partial}{\partial y}\overline{v'u'}}$$

Eddy Reynolds stress transfers momentum *offshore*

Eddy impact on the mean: Retrograde

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Eddy Reynolds stress transfers momentum *offshore*

Depth-integral gives (under equilibrium)

$$\psi_{\mathrm{EMF}} + \psi_{\mathrm{GM}} \simeq \psi_{\mathrm{Ekman}} + \psi_{\mathrm{Residual}}$$

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Eddy Reynolds stress transfers momentum *offshore*

Depth-integral gives (under equilibrium)

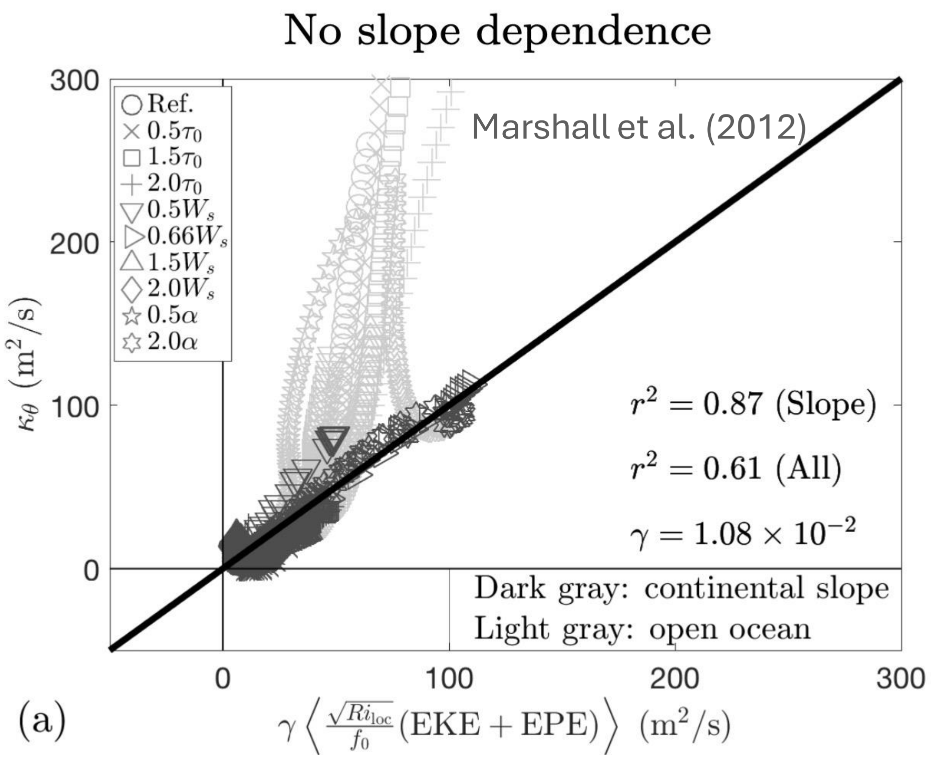
Two-way balance of the ACC
(e.g. Marshall and Radko, 2003)

Presumably adiabaticity

$$\boxed{\underbrace{\psi_{\mathrm{EMF}}}_{\frac{1}{f_0}\int_{\hat{z}}^{0}\frac{\partial}{\partial y}\overline{v'u'}\,dz}} + \boxed{\underbrace{\psi_{\mathrm{GM}}}_{\left.\frac{\overline{v'b'}}{\overline{b}_z}\right|_{z=\hat{z}}} \simeq \underbrace{\psi_{\mathrm{Ekman}}}_{\frac{\tau}{\rho_0 f_0}}} + \cancel{\psi_{\mathrm{Residual}}}$$

Jet-shifter
(e.g. Stewart et al, 2024)

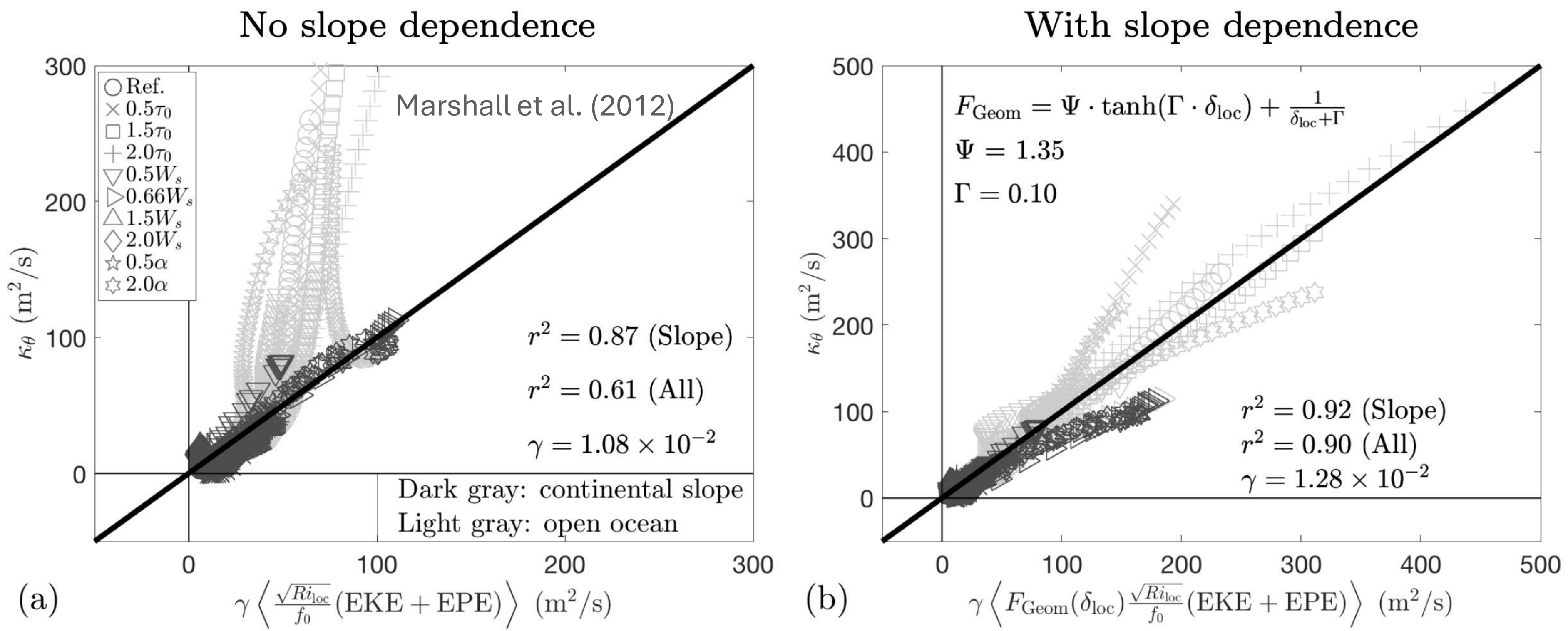
Scaling(s) of eddy buoyancy diffusivity: Retrograde



$$\kappa_{\mathrm{Geom}} = \gamma_{\mathrm{Geom}} \frac{\sqrt{Ri_{\mathrm{loc}}}}{f_0}(\mathrm{EKE} + \mathrm{EPE}),$$

Wang and Stewart (2020)

Scaling(s) of eddy buoyancy diffusivity: Retrograde



(a) Original and (b) slope-aware forms of the GEOMETRIC scaling vs κ_θ.

$$\kappa_{\text{Geom}} = \gamma_{\text{Geom}} \frac{\sqrt{Ri_{\text{loc}}}}{f_0}(\text{EKE} + \text{EPE}),$$

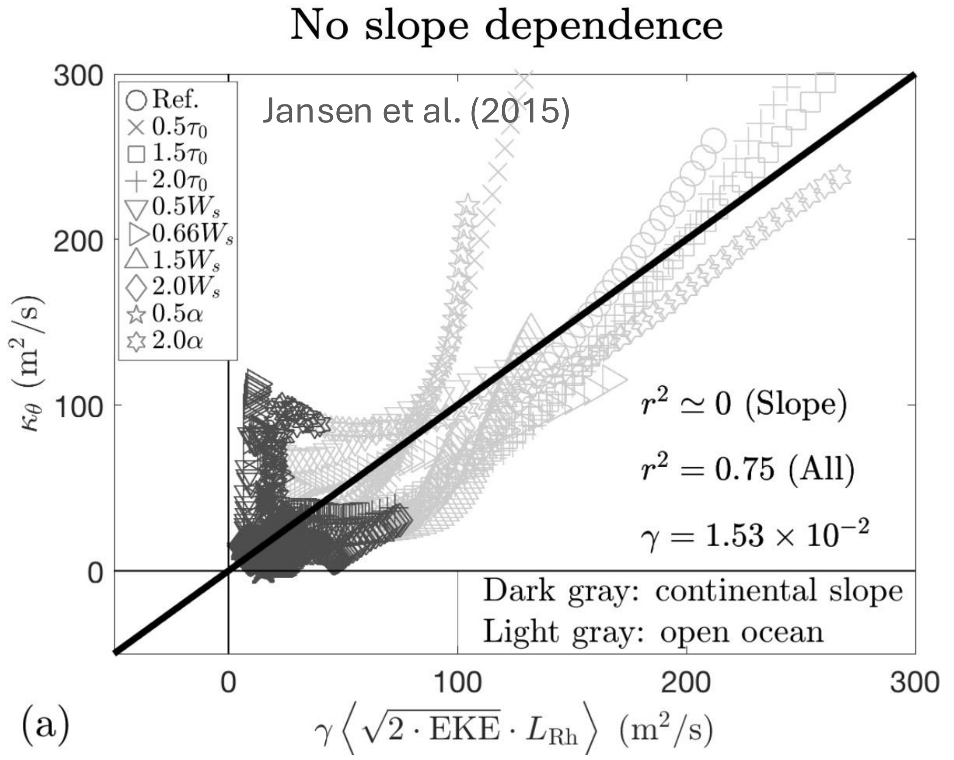
$$\delta = \frac{\text{bottom slope}}{\text{isopycnal slope}}$$

(e.g. Isachsen, 2011)

$$\gamma_{\text{Geom}} = \gamma \left[\Psi \cdot \tanh(\Gamma \cdot \delta_{\text{loc}}) + \frac{1}{\delta_{\text{loc}} + \Gamma} \right]$$

Wang and Stewart (2020)

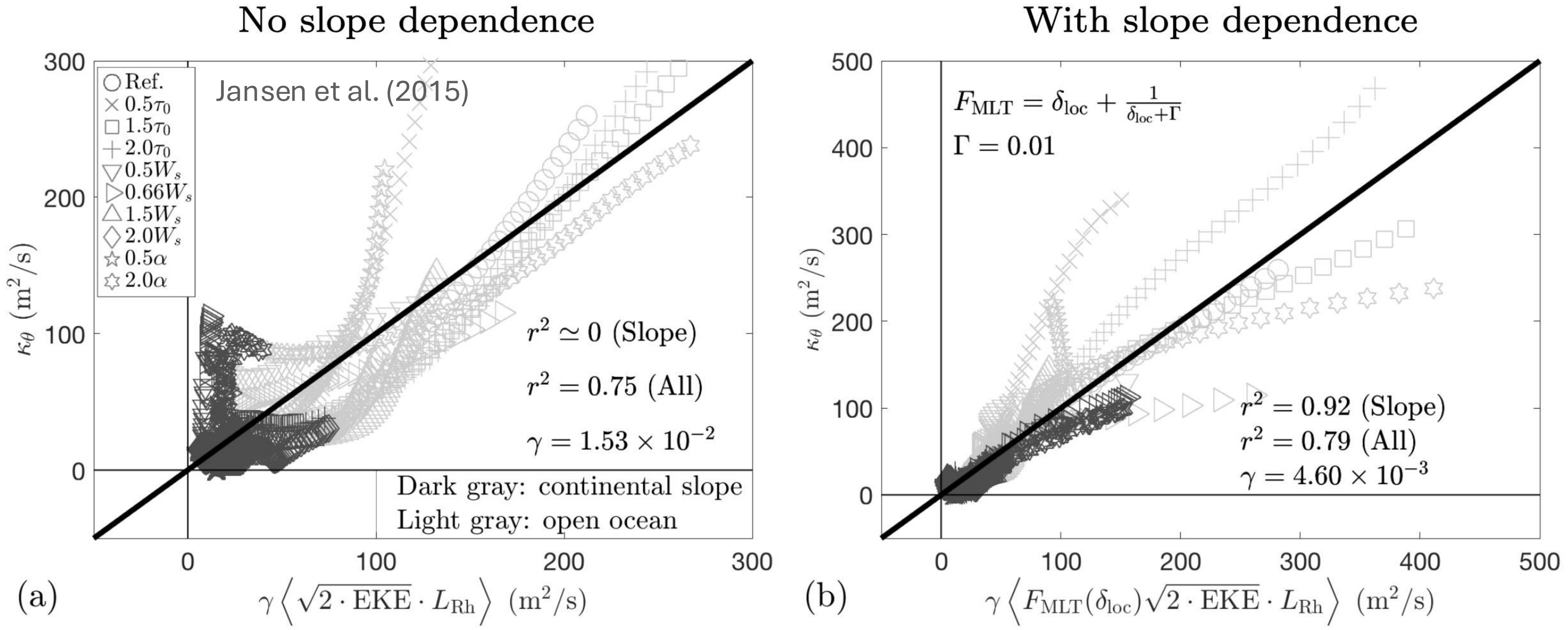
Scaling(s) of eddy buoyancy diffusivity: Retrograde



$$\kappa_{\mathrm{MLT}} = \gamma_{\mathrm{MLT}} \sqrt{2 \cdot \mathrm{EKE}} \cdot L_{\mathrm{Rh}},$$

Wang and Stewart (2020)

Scaling(s) of eddy buoyancy diffusivity: Retrograde



(a) Original and (b) slope-aware forms of the MLT-based scaling vs κ_θ.

$$\kappa_{\mathrm{MLT}} = \gamma_{\mathrm{MLT}} \sqrt{2 \cdot \mathrm{EKE}} \cdot L_{\mathrm{Rh}},$$

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(e.g. Isachsen, 2011)

Wang and Stewart (2020)

Scaling of eddy momentum fluxes: <u>Retrograde</u>

$$\frac{\partial}{\partial y}\overline{v'u'} \sim \mathcal{K}_q \cdot \beta_{\text{topog.}}, \quad \mathcal{K}_q \equiv C_{\text{eddy}} \left.\frac{\text{EKE}}{f_0}\right|_{\text{Barotropic}}$$

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$$\Downarrow$$

$$\overline{v'u'} \sim \text{EKE}$$

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$$\downarrow$$

$$\overline{v'u'} \sim \text{EKE}$$

Consistent with barotropic "GEOMETRIC"
(Hoskins et al., 1983; Marshall et al., 2012):

$$M = \frac{\overline{v'^2 - u'^2}^z}{2}, \quad N = \overline{u'v'}^z,$$

$$M^2 + N^2 \leq \text{EKE}$$

$$N = \gamma_m E \sin 2\phi_m \cos^2\lambda,$$

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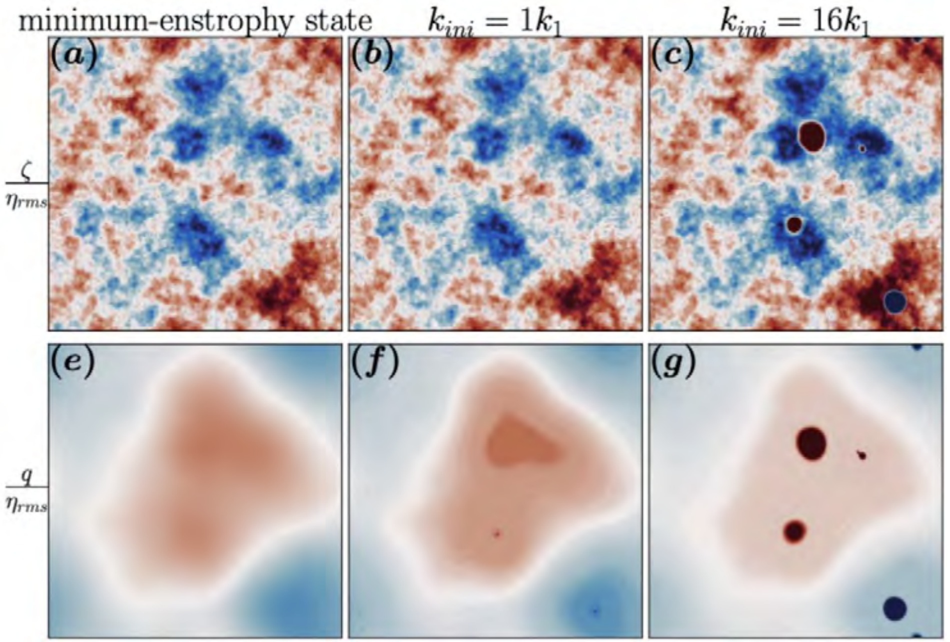
Resembling "selective decay" of 2D topographic turbulence (Bretherton and Haidvogel, 1976):



He and Wang (2024)

Scaling(s) of eddy momentum fluxes: Retrograde

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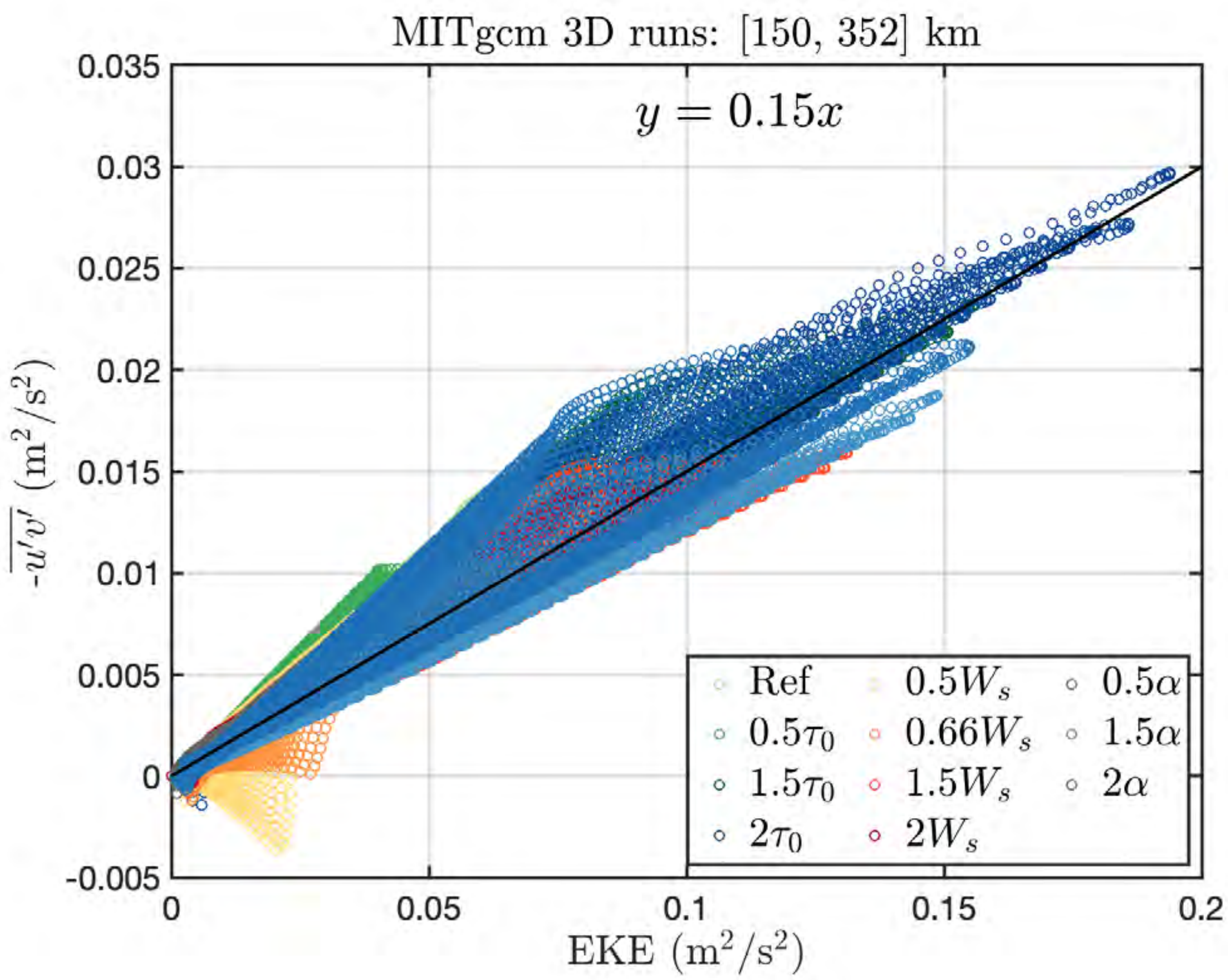

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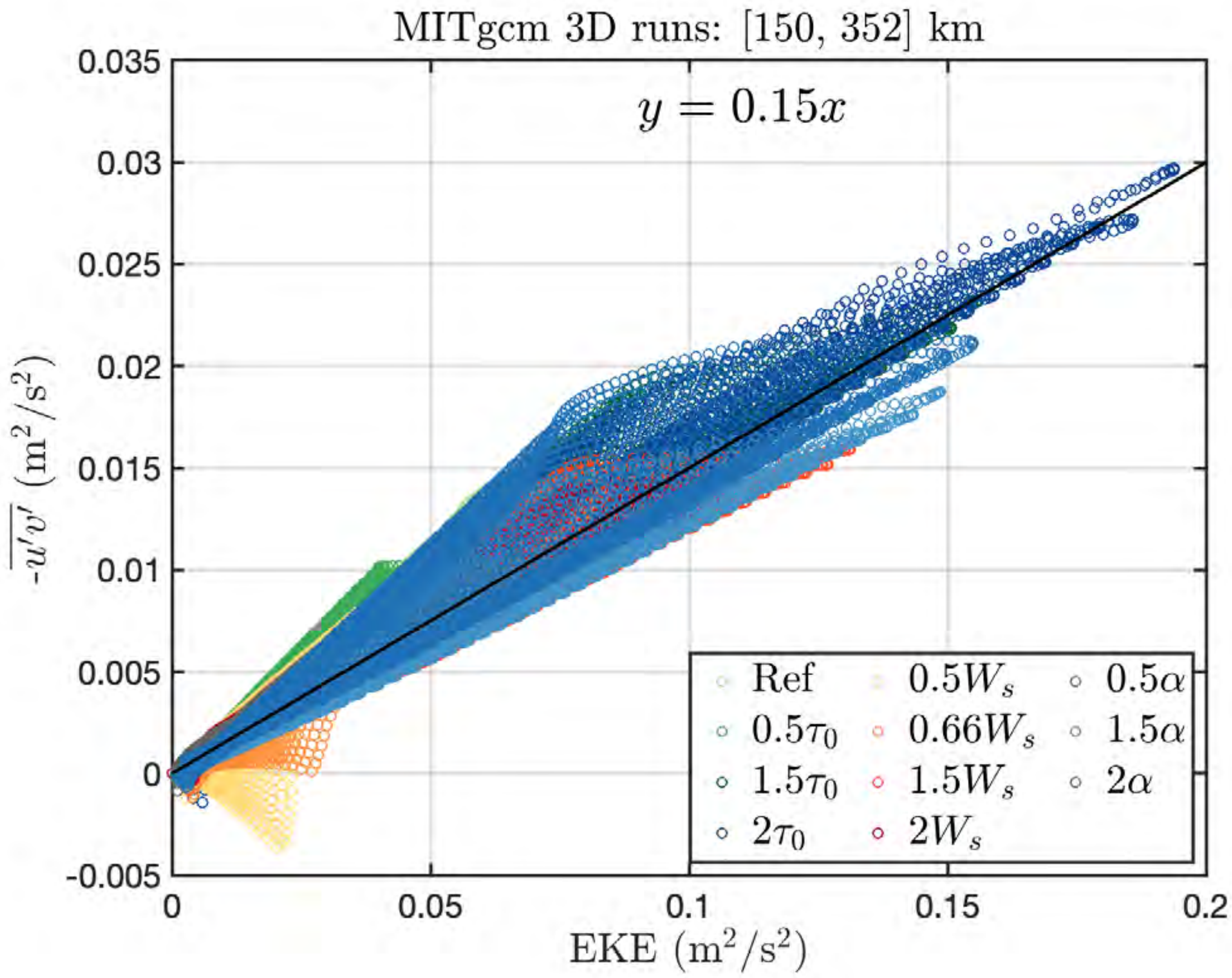
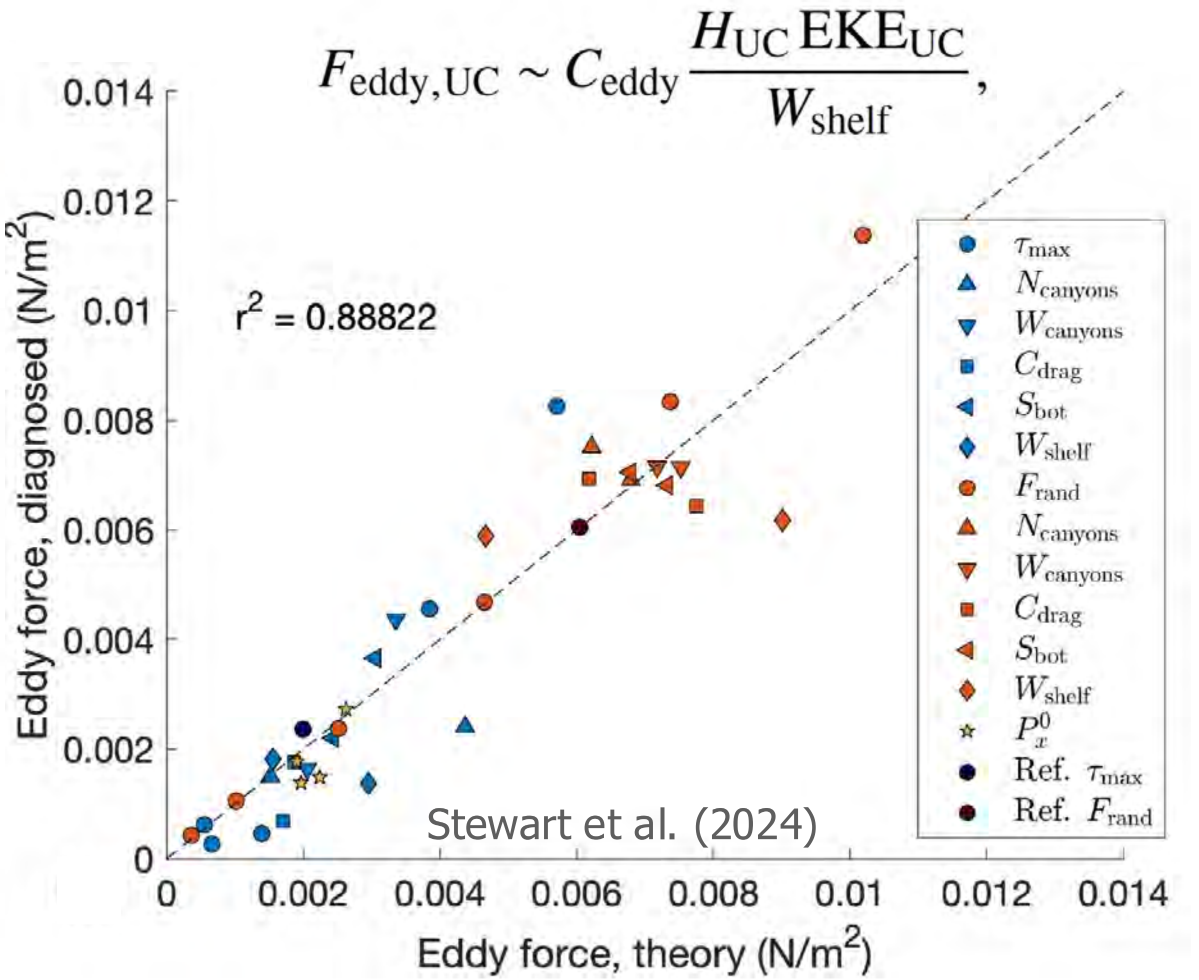
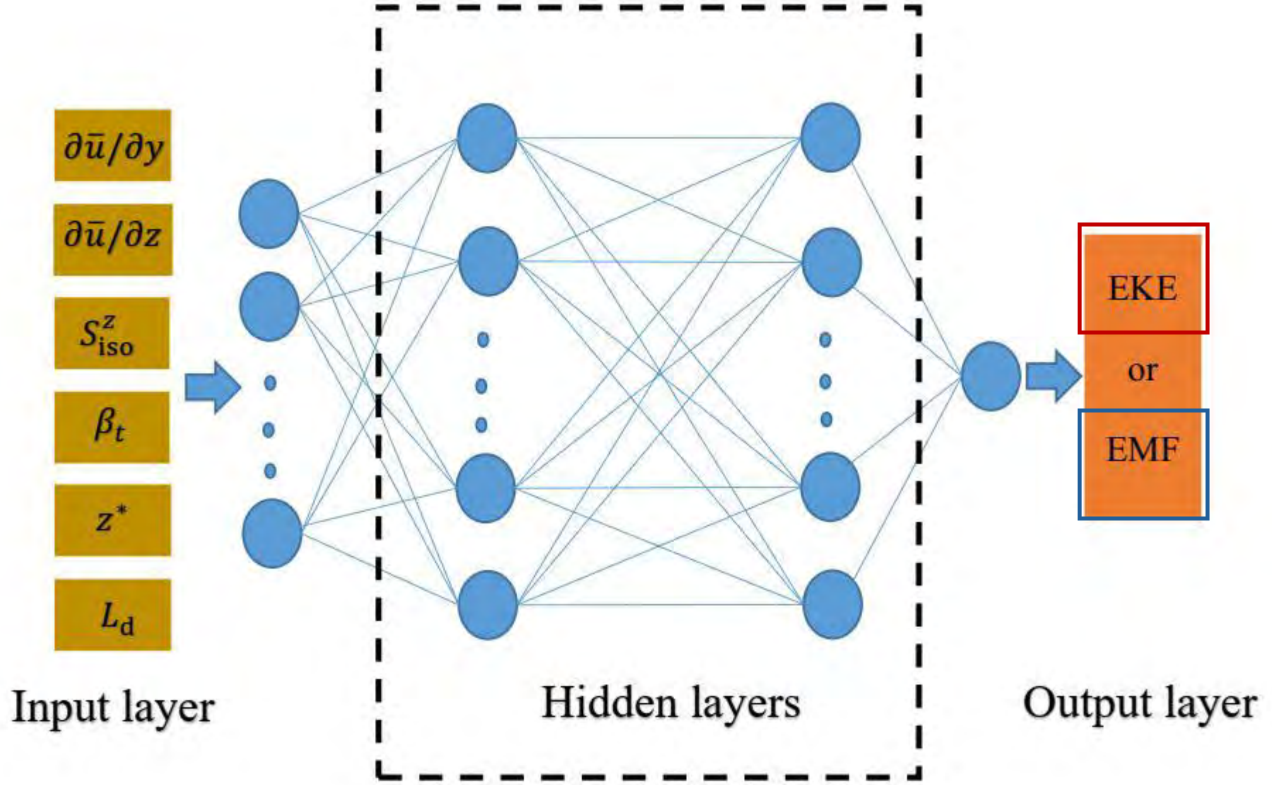




Stewart et al. (2024)

Convert scaling(s) into eddy closure(s): Retrograde



Xie, Wei, and Wang (2024)

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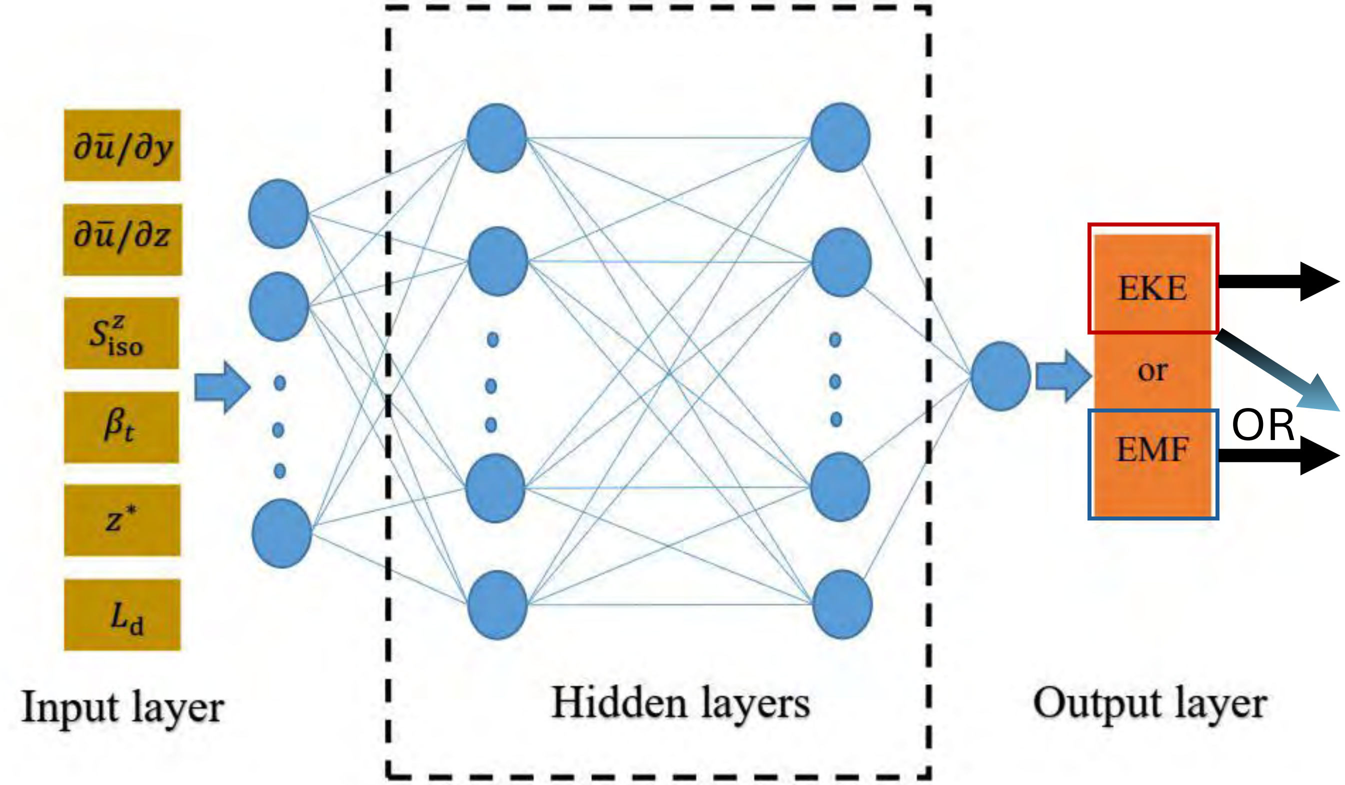


Model tracer equation(s)

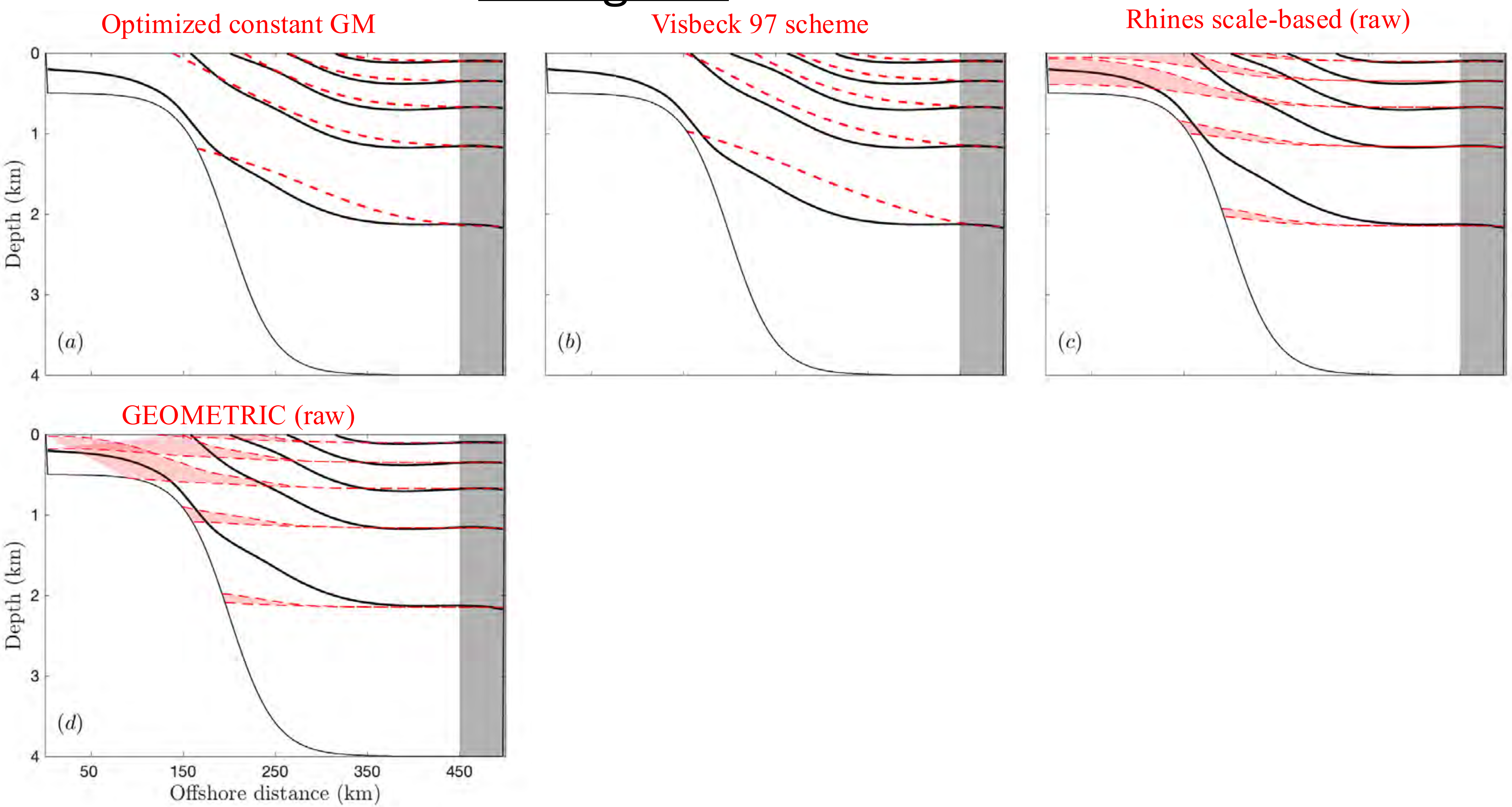
$$\mathcal{K}^{\text{slope}}_{\text{GEOM}} = \gamma_{\text{geom}} \cdot \mathcal{F}_{\text{geom}}(\delta) \cdot \frac{\sqrt{\langle N^2 \rangle}}{\left|\langle \bar{b}_y \rangle\right|} \cdot \langle E \rangle,$$

Imposing global momentum constraint (Green 1970) online

Model momentum equation(s)

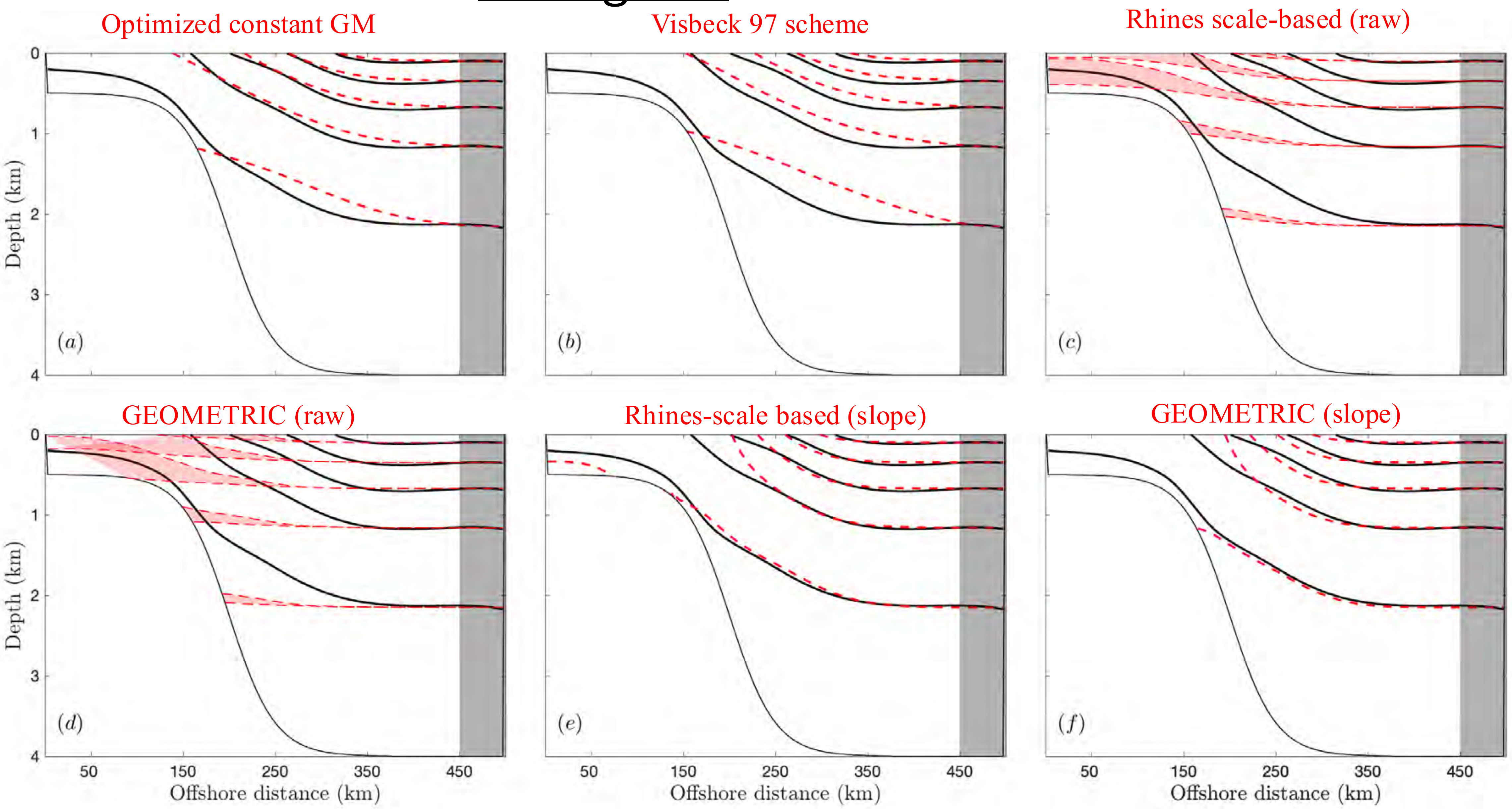
Xie, Wei, and Wang (2024)

Parameterization: Retrograde



Xie, Wei, and Wang (2024)

Parameterization: Retrograde



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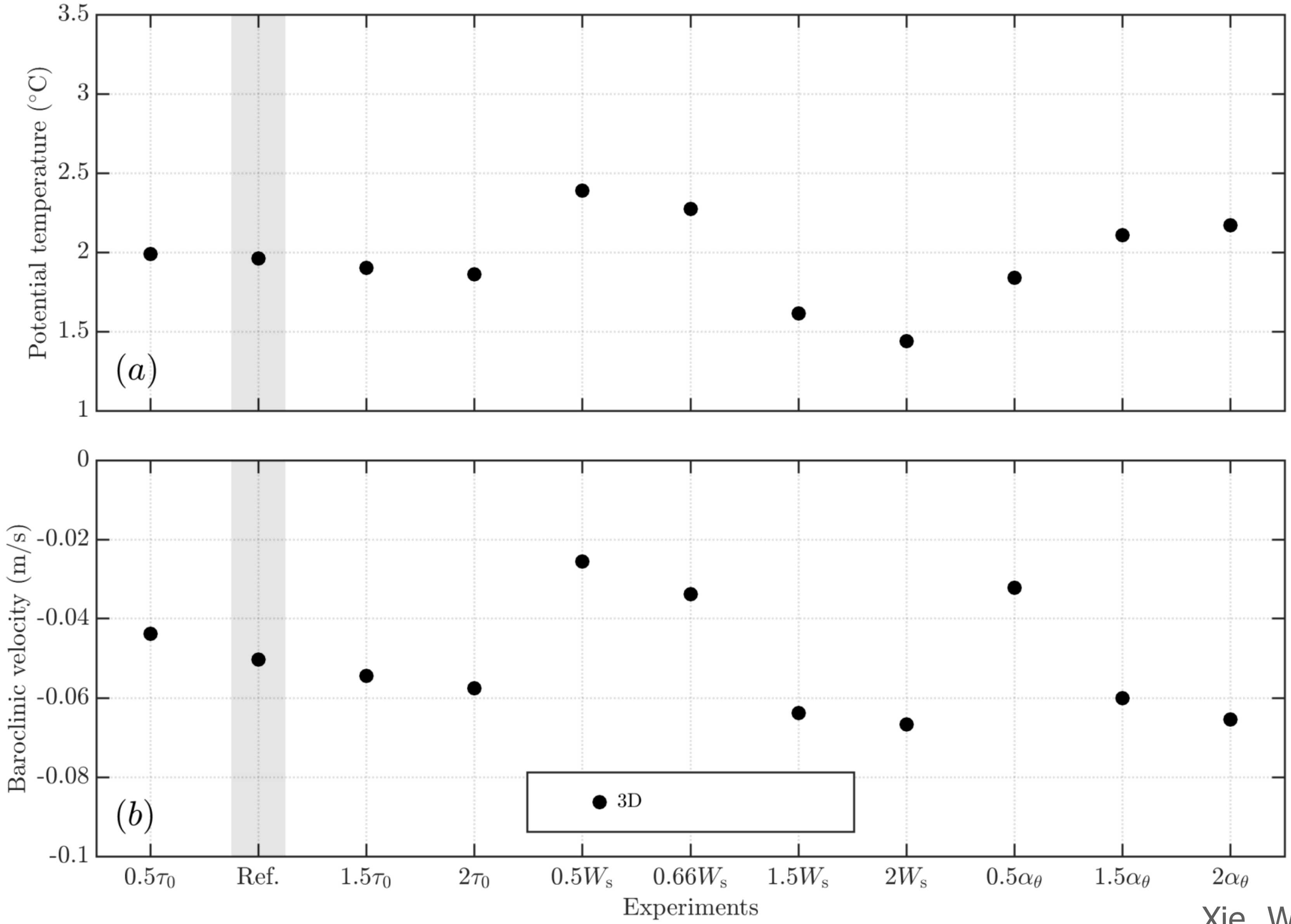
Parameterization: Retrograde



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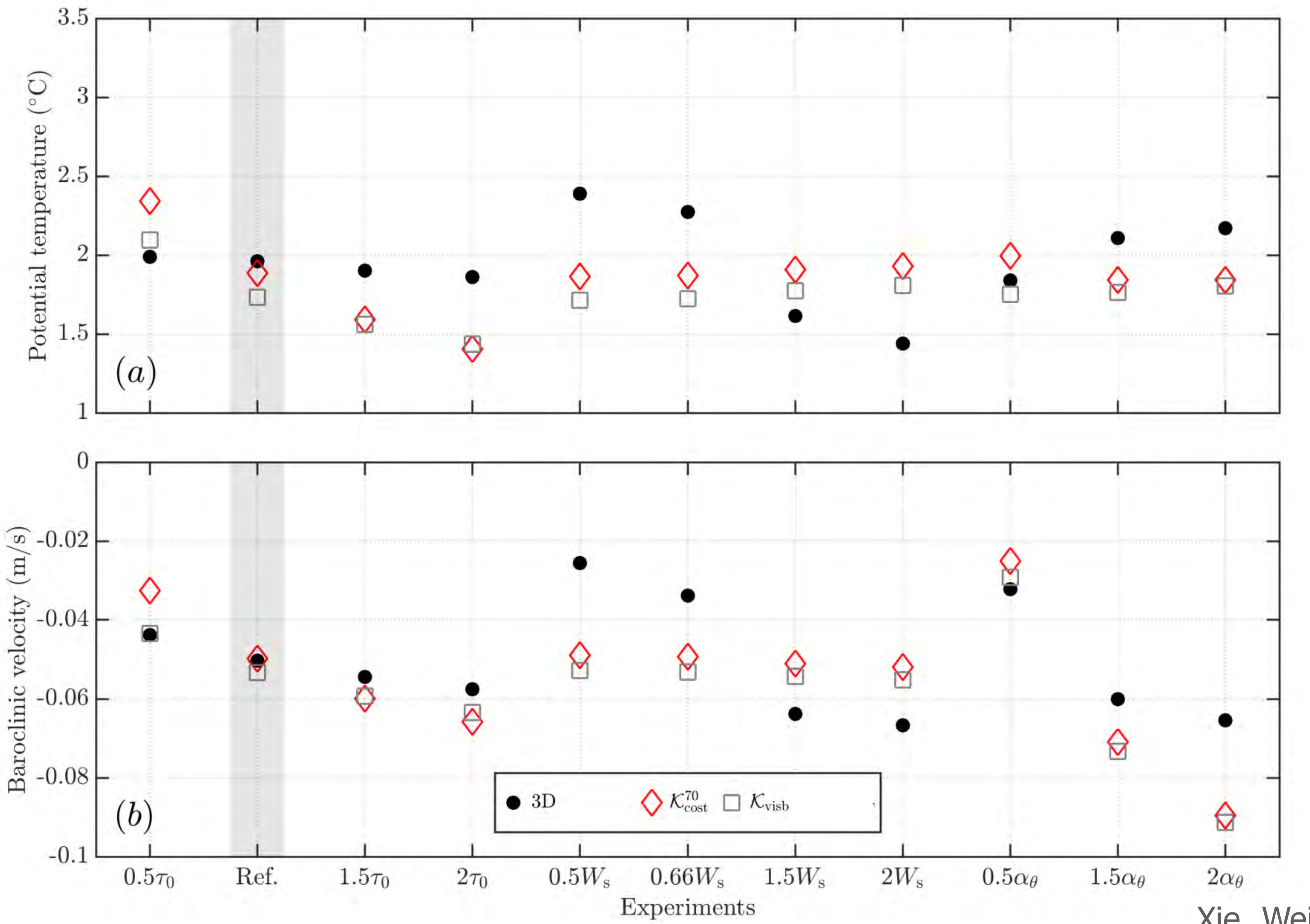

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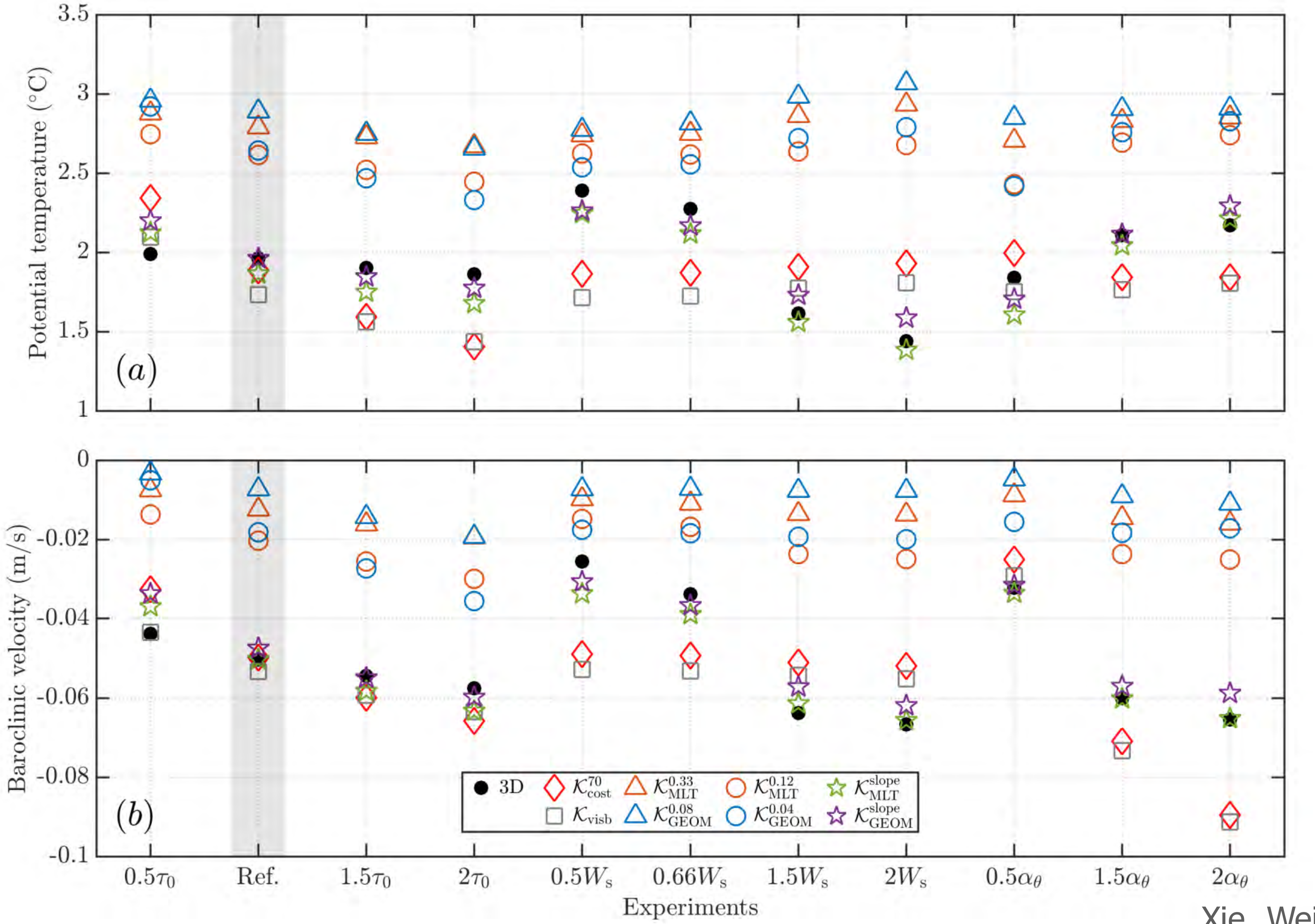
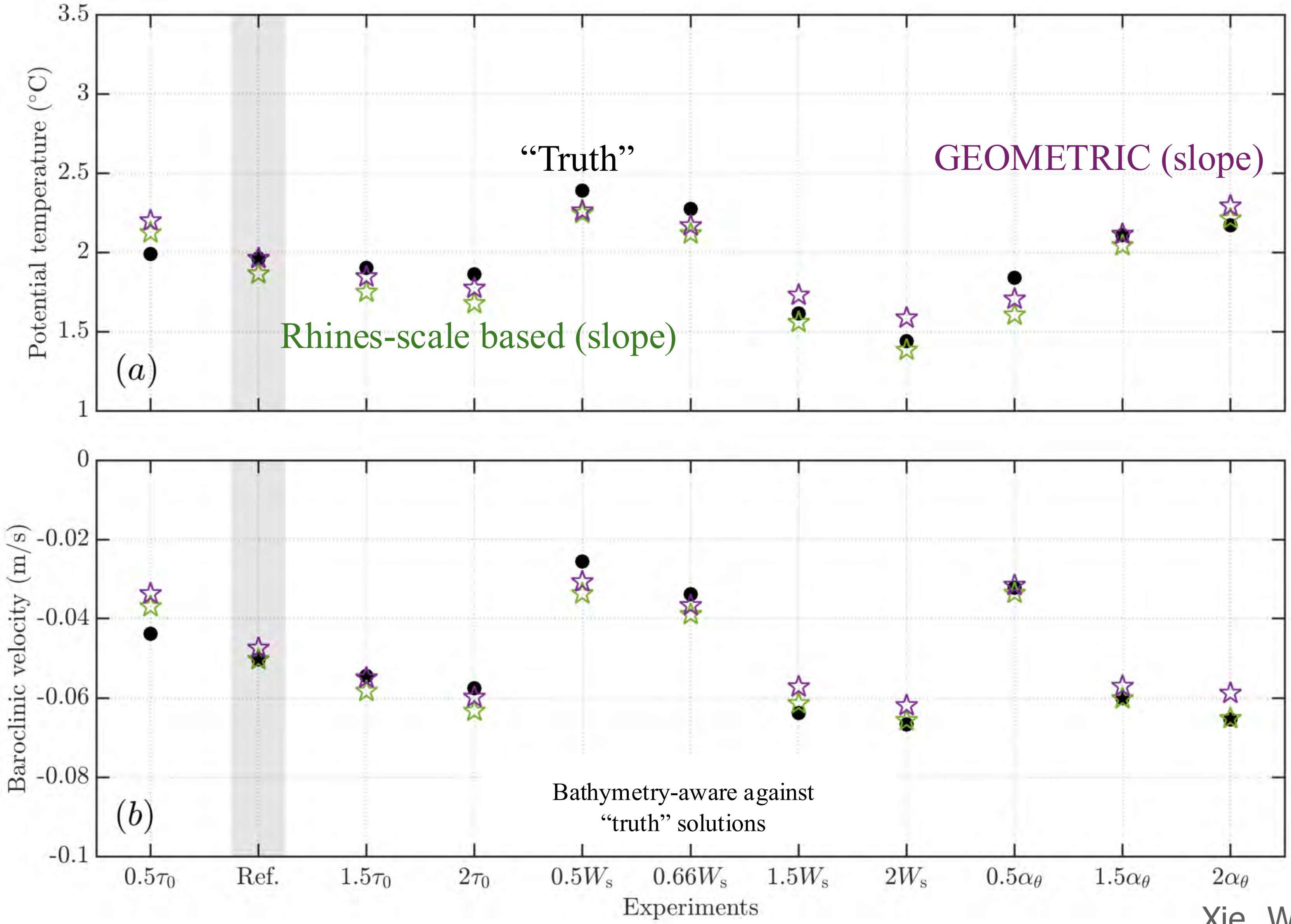
Parameterization: Retrograde



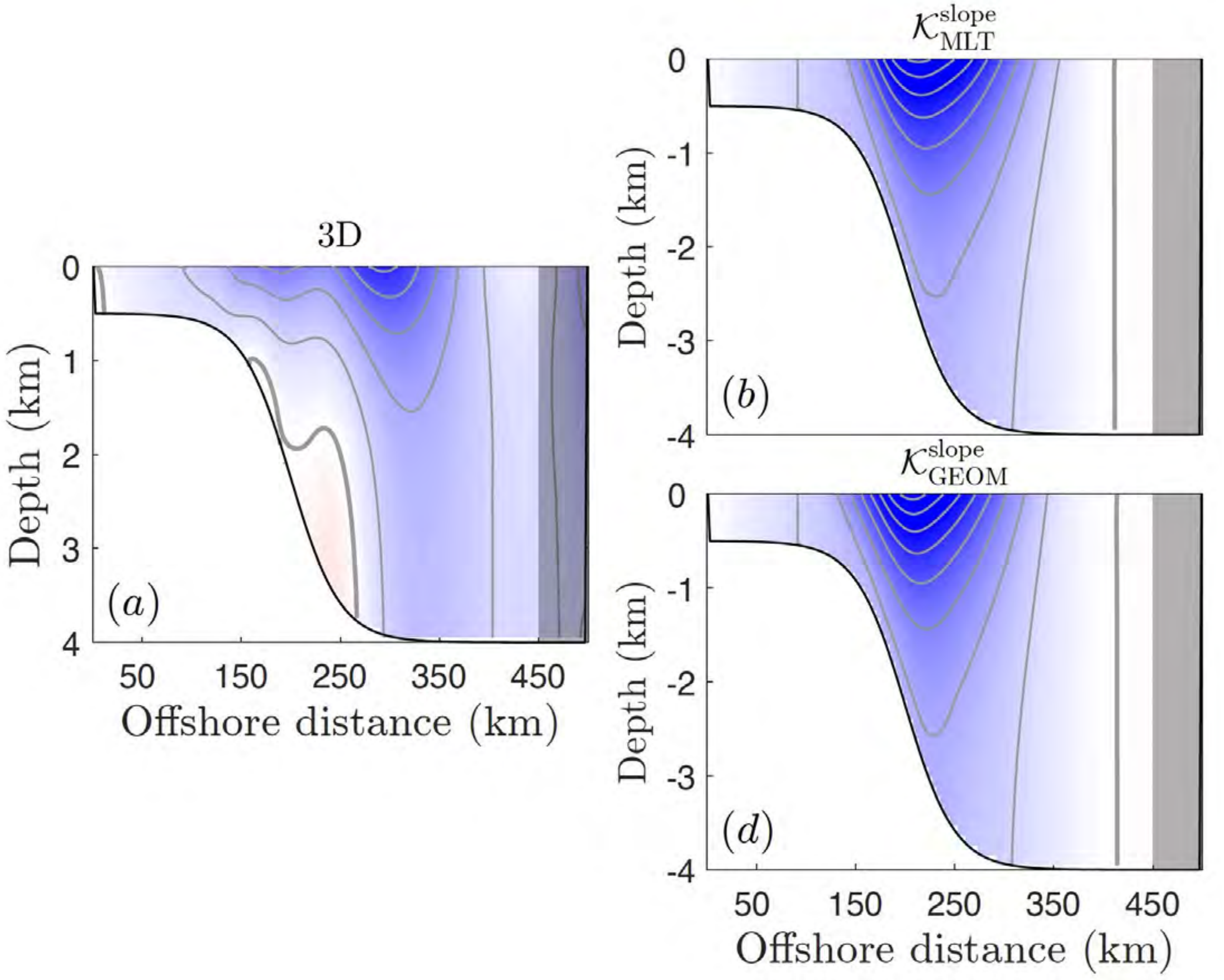
Xie, Wei, and Wang (2024)

Parameterization: Retrograde



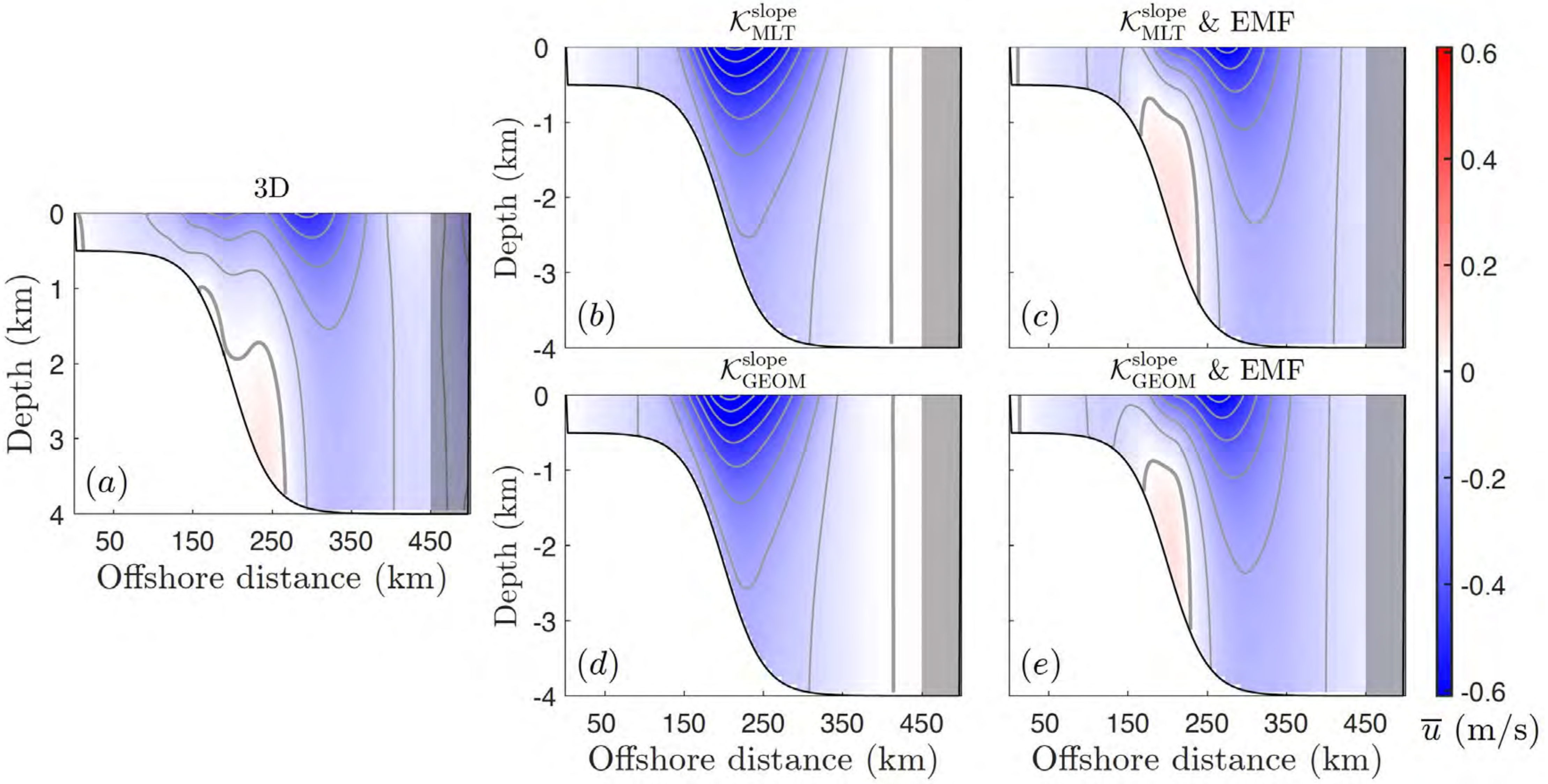
Xie, Wei, and Wang (2024)

Impact of parameterized eddy momentum forcing: Retrograde



Xie, Wei, and Wang (2024)

Impact of parameterized eddy momentum forcing: Retrograde



Xie, Wei, and Wang (2024)

Physics-based eddy closure(s): Retrograde

$$\underbrace{\frac{\partial}{\partial t}\hat{E}}_{\text{tendency}} + \underbrace{\frac{\partial}{\partial y}\left(\langle v\rangle \hat{E}\right)}_{\text{advection}} = \underbrace{\int_{-|H|}^{0} \mathcal{K}_{\mathrm{gm}} \frac{M^4}{N^2}\,\mathrm{d}z}_{\text{baroclinic source}} - \underbrace{\tau_E^{-1}\hat{E}}_{\text{dissipation}} + \eta_E \underbrace{\frac{\partial^2}{\partial y^2}\hat{E}}_{\text{diffusion}}$$

$$\mathcal{K}_{\mathrm{GEOM}}^{\mathrm{slope}} = \gamma_{\mathrm{geom}} \cdot \mathcal{F}_{\mathrm{geom}}(\delta) \cdot \frac{\sqrt{\langle N^2\rangle}}{\left|\left\langle \overline{b}_y \right\rangle\right|} \cdot \langle E\rangle, \longrightarrow \overline{v'u'}$$

Calculating convergence;
Imposing global momentum constraint (Green 1970) online

Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): Retrograde

$$\underbrace{\frac{\partial}{\partial t}\boxed{\hat{E}}}_{\text{tendency}} + \underbrace{\frac{\partial}{\partial y}\left(\langle v\rangle \hat{E}\right)}_{\text{advection}} = \underbrace{\int_{-|H|}^{0} \boxed{\mathcal{K}_{\mathrm{gm}}} \frac{M^4}{N^2}\,\mathrm{d}z}_{\text{baroclinic source}} - \underbrace{\tau_E^{-1}\hat{E}}_{\text{dissipation}} + \underbrace{\eta_E \frac{\partial^2}{\partial y^2}\hat{E}}_{\text{diffusion}} - \overbrace{\int \left\langle \boxed{\overline{v'u'}} \frac{\partial}{\partial y}\overline{u} \right\rangle dz}^{\text{barotropic source or sink}}$$

$$\boxed{\mathcal{K}_{\mathrm{GEOM}}^{\mathrm{slope}}} = \gamma_{\mathrm{geom}} \cdot \mathcal{F}_{\mathrm{geom}}(\delta) \cdot \frac{\sqrt{\langle N^2\rangle}}{\left|\left\langle \overline{b}_y \right\rangle\right|} \cdot \boxed{\langle E\rangle}, \longrightarrow \boxed{\overline{v'u'}}$$

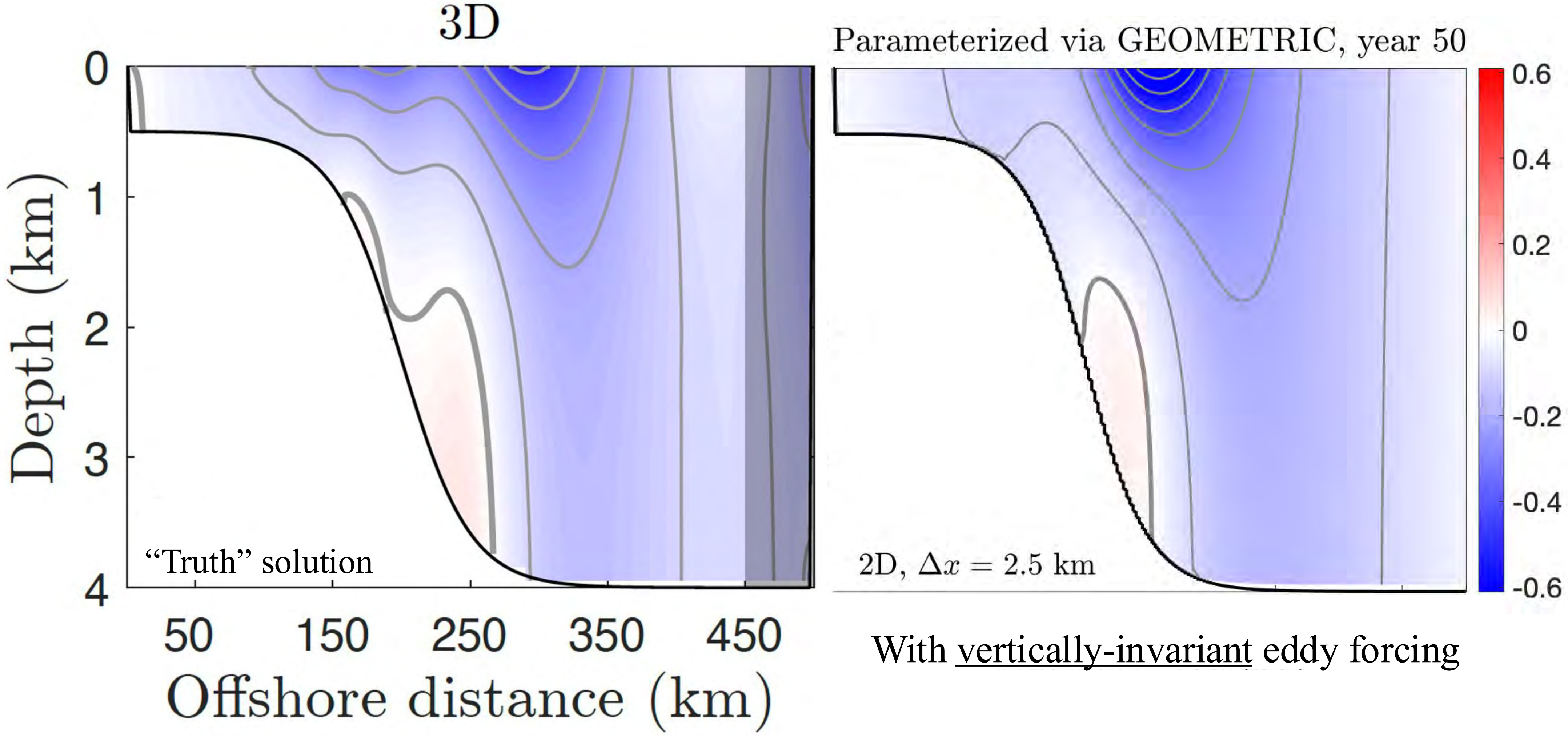
Calculating convergence;
Imposing global momentum
constraint (Green 1970) online

Model tracer equation(s)

Model momentum equation(s)

Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): Retrograde 2D runs



With vertically-invariant eddy forcing

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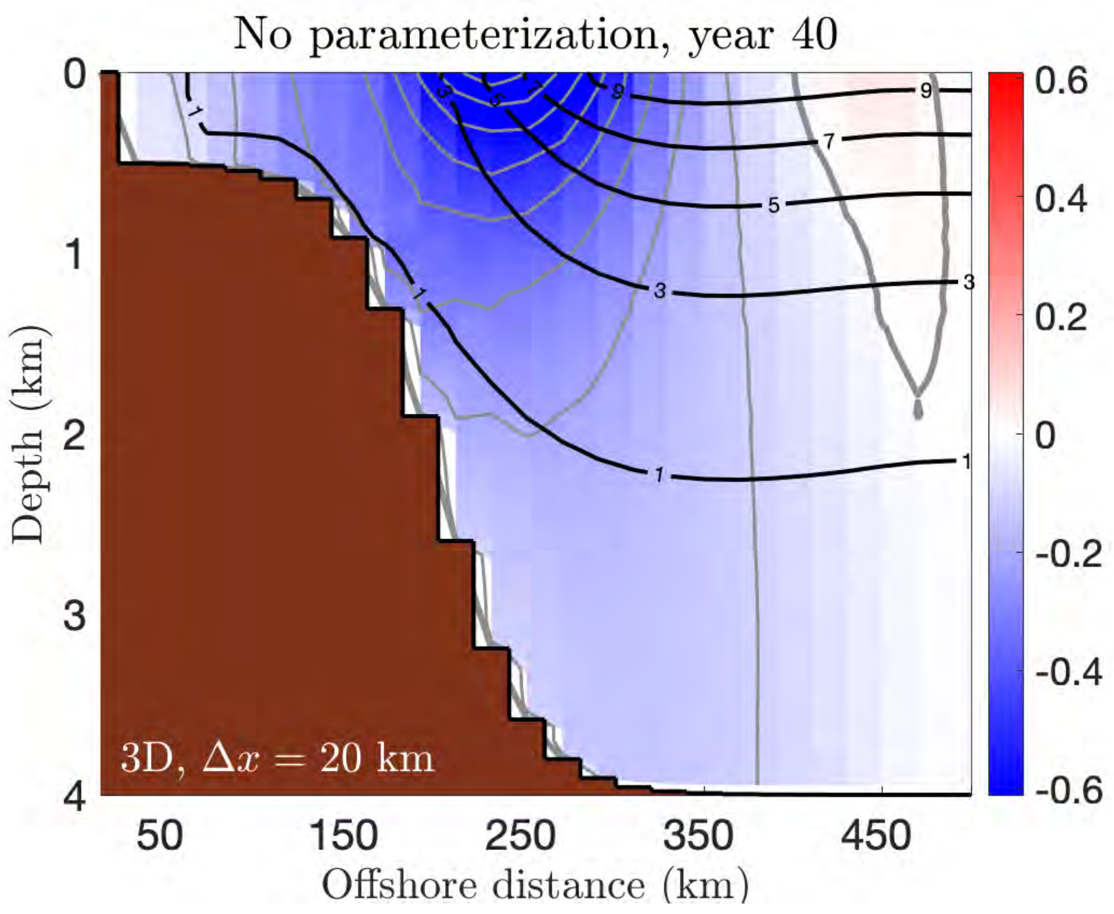
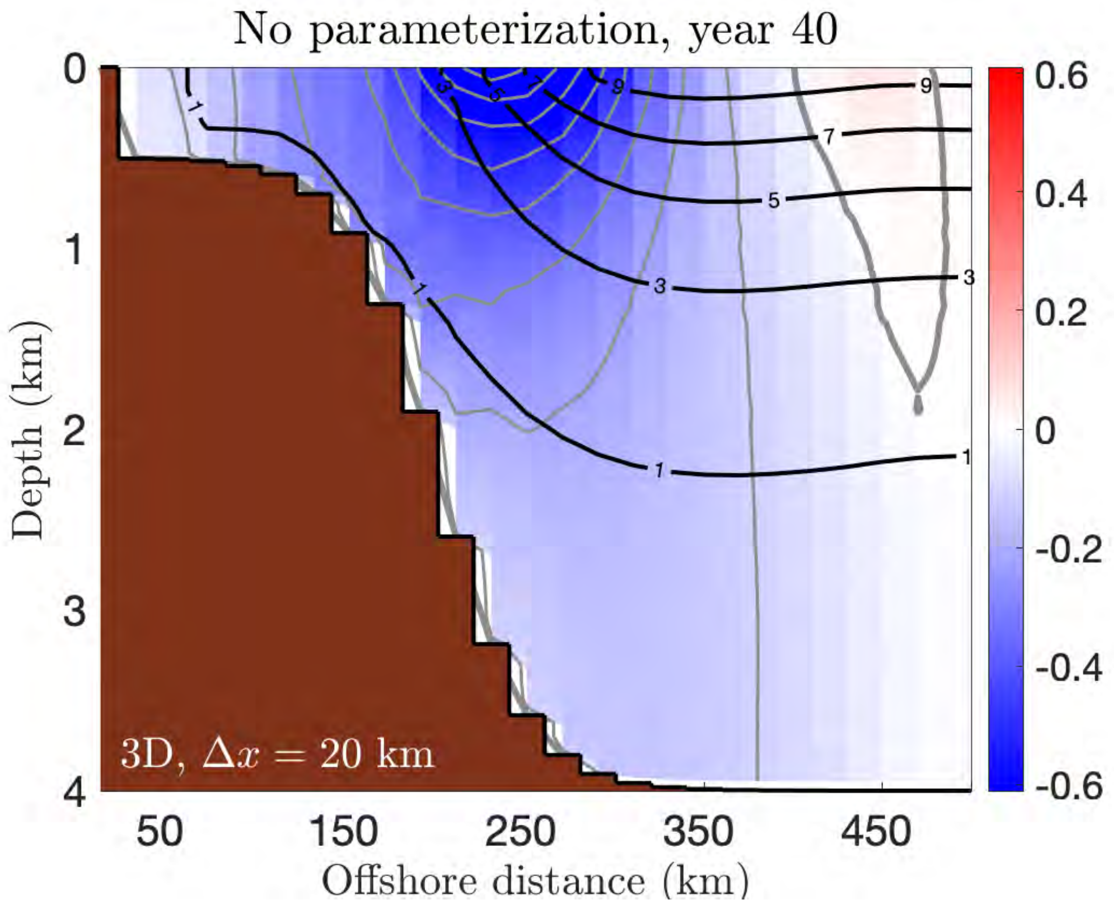
Physics-based eddy closure(s): 3D coarse-grid “raw” flow

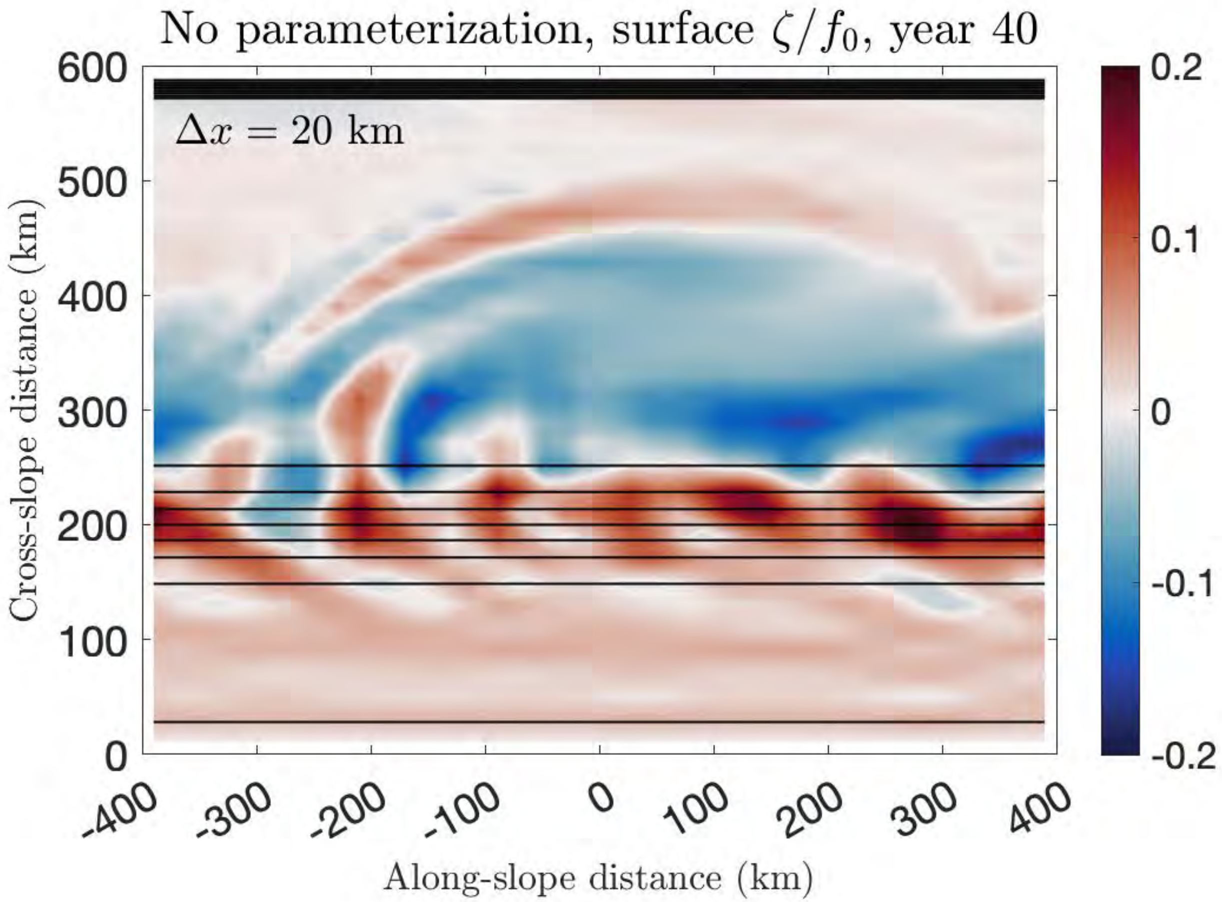


Acceptable stratification, wrong jet structures.

Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): 3D coarse-grid “raw” flow





Acceptable stratification, wrong jet structures.

Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): Retrograde 3D runs

$$\underbrace{\frac{\partial}{\partial t}\hat{E}}_{\text{tendency}} + \underbrace{\frac{\partial}{\partial y}\left(\langle v\rangle \hat{E}\right)}_{\text{advection}} = \underbrace{\int_{-|H|}^{0} \mathcal{K}_{\mathrm{gm}} \frac{M^4}{N^2}\, \mathrm{d}z}_{\text{baroclinic source}} - \underbrace{\tau_E^{-1}\hat{E}}_{\text{dissipation}} + \underbrace{\eta_E \frac{\partial^2}{\partial y^2}\hat{E}}_{\text{diffusion}} - \int \left\langle \overline{v'u'} \frac{\partial}{\partial y}\overline{u} \right\rangle dz$$

Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): Retrograde 3D runs

$$\underbrace{\frac{\partial}{\partial t}\hat{E}}_{\text{tendency}} + \underbrace{\frac{\partial}{\partial y}\left(\langle v\rangle \hat{E}\right)}_{\text{advection}} = \underbrace{\cancel{\int_{-|H|}^{0} \mathcal{K}_{\mathrm{gm}} \frac{M^4}{N^2}\,\mathrm{d}z}}_{\text{baroclinic source}} - \underbrace{\tau_E^{-1}\hat{E}}_{\text{dissipation}} + \underbrace{\eta_E \frac{\partial^2}{\partial y^2}\hat{E}}_{\text{diffusion}} \overbrace{- \int \left\langle \boxed{\overline{v'u'}} \frac{\partial}{\partial y}\overline{u} \right\rangle dz}^{\text{barotropic source or sink}}$$

$\boxed{\overline{v'u'}}$ Calculating convergence; Imposing global momentum constraint (Green 1970) online

Model momentum equation(s)

Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): Retrograde 3D runs

$$\underbrace{\frac{\partial}{\partial t}\hat{E}}_{\text{tendency}} + \underbrace{\frac{\partial}{\partial y}\left(\langle v\rangle \hat{E}\right)}_{\text{advection}} = \underbrace{\cancel{\int_{-|H|}^{0} \mathcal{K}_{\text{gm}} \frac{M^4}{N^2}\, \mathrm{d}z}}_{\text{baroclinic source}} - \underbrace{\tau_E^{-1}\hat{E}}_{\text{dissipation}} + \underbrace{\eta_E \frac{\partial^2}{\partial y^2}\hat{E}}_{\text{diffusion}} \overbrace{- \int \left\langle \boxed{\overline{v'u'}} \frac{\partial}{\partial y}\bar{u} \right\rangle dz}^{\text{barotropic source or sink}}$$

Subgrid entry as energy source

Grid-dissipated KE of alongshore anomalous flow

$\boxed{\overline{v'u'}}$

Calculating convergence; Imposing global momentum constraint (Green 1970) online

Model momentum equation(s)

Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): Retrograde 3D runs

$$\underbrace{\frac{\partial}{\partial t}\hat{E}}_{\text{tendency}} + \underbrace{\frac{\partial}{\partial y}\left(\langle v\rangle \hat{E}\right)}_{\text{advection}} = \cancel{\underbrace{\int_{-|H|}^{0} \mathcal{K}_{\mathrm{gm}}\frac{M^4}{N^2}\,\mathrm{d}z}_{\text{baroclinic source}}} - \underbrace{\tau_E^{-1}\hat{E}}_{\text{dissipation}} + \underbrace{\eta_E \frac{\partial^2}{\partial y^2}\hat{E}}_{\text{diffusion}} \overbrace{-\int \left\langle \boxed{\overline{v'u'}}\, \frac{\partial}{\partial y}\overline{u}\right\rangle dz}^{\text{barotropic source or sink}}$$

Subgrid entry as energy source

Grid-dissipated KE of alongshore anomalous flow

Along-isobath averaging and calculating energetic rate online;

A special form of eddying/mean "flow-splitting" (Mak et al. 2023).

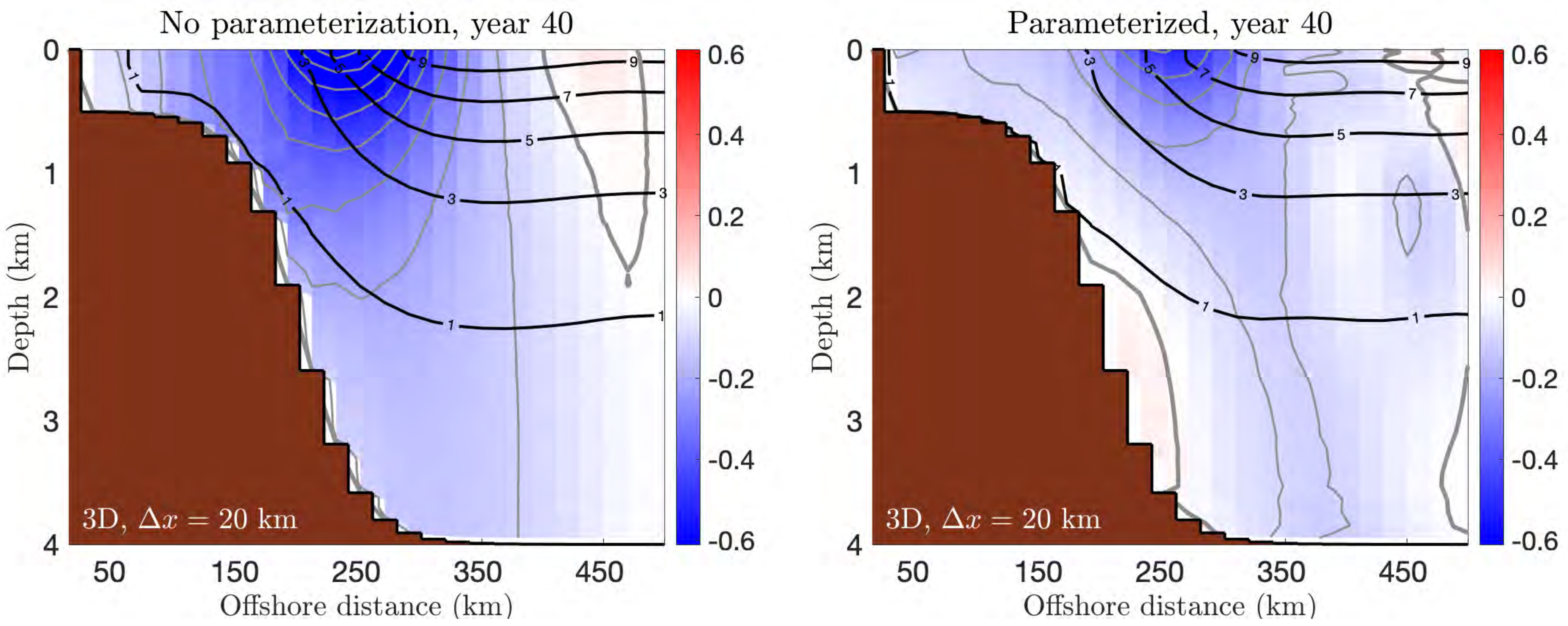
$\boxed{\overline{v'u'}}$

Calculating convergence; Imposing global momentum constraint (Green 1970) online

Model momentum equation(s)

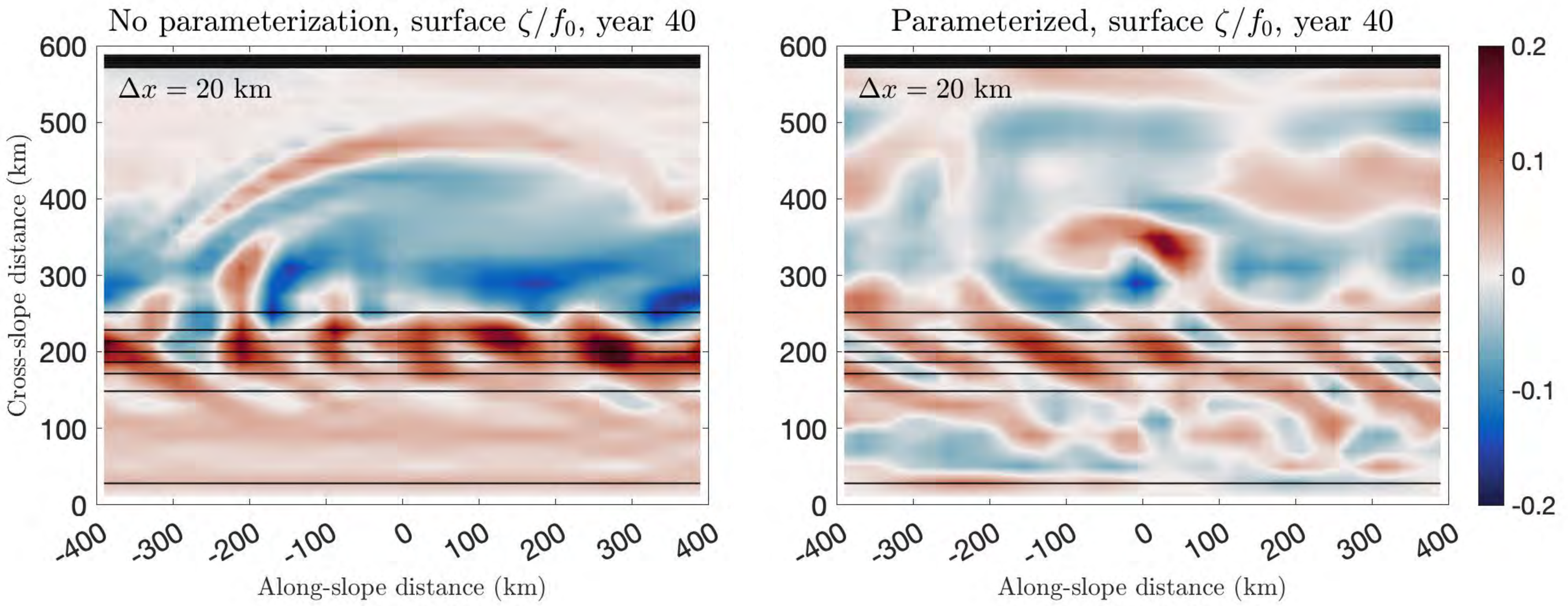
Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): 3D coarse-grid parameterized flow



Ongoing work with Julian Mak and Andrew Stewart

Physics-based eddy closure(s): 3D coarse-grid parameterized flow



Ongoing work with Julian Mak and Andrew Stewart

Summary

- Prograde frontal systems are theorized and parameterized via GEOMETRIC adapted by an analytical function of the slope Burger number controlling efficiency of eddy buoyancy fluxes.

- Retrograde frontal systems are forced jointly by eddy buoyancy and momentum fluxes; the former can be quantified using a range of GM-based scalings, adapted via analytical functions of the topographic slope parameter.

- Eddy momentum fluxes across retrograde fronts depend linearly on eddy energy, and echo with barotropic eddy PV fluxes theorized in 2D topographic turbulence, driving prograde undercurrents.

- Machine learning approaches can augment physics-based, bathymetry-aware mesoscale eddy parameterizations by constraining eddy energy or/and forcing online.

- Scale separation may exist between eddy buoyancy and momentum forcing, which alludes to muting GM-based schemes but utilizing numerically-dissipated energy for driving subgrid-scale eddy momentum forcing in eddy permitting regimes across continental margins (*ongoing*).

Bathymetry-aware recipe (to be cont.):

[GM] Y. Wang, and A. L. Stewart, 2020, "Scalings for eddy buoyancy transfer across continental slopes under retrograde winds", Ocean Modelling, 147, 101579.

[Redi] H. Wei, and Y. Wang, 2021, "Full-depth scalings for isopycnal eddy mixing across continental slopes under upwelling-favorable winds", Journal of Advances in Modeling Earth Systems, 13, e2021MS002498.

[GM] H. Wei, Y. Wang, A. L. Stewart, and J. Mak, 2022, "Scalings for eddy buoyancy fluxes across prograde shelf/slope fronts", Journal of Advances in Modeling Earth Systems, 14, e2022MS00322.

[Redi] C. Xie, H. Wei, and Y. Wang, 2023a, "Impact of parameterized isopycnal diffusivity on shelf-ocean exchanges under upwelling-favorable winds: offline tracer simulations augmented by artificial neural network", Journal of Advances in Modeling Earth Systems, 15, e2022MS00342.

[GM + PV] C. Xie, H. Wei, and Y. Wang, 2023b, "Bathymetry-aware mesoscale eddy parameterizations across upwelling slope fronts: A machine learning-augmented approach", Journal of Physical Oceanography, 53, 2861–289.

[GM] H. Wei, Y. Wang, and J. Mak, 2024, "Parameterizing eddy buoyancy fluxes across prograde shelf/slope fronts using a slope-aware GEOMETRIC closure", Journal of Physical Oceanography, 54, 359–37.

[PV] J. He, and Y. Wang, In Press, "Multiple states of two-dimensional turbulence above topography", Journal of Fluid Mechanics, DOI: 10.1017/jfm.2024.633.

[GM + Energy] P. Deng, and Y. Wang, In Press, "Distinct impacts of topographic versus planetary PV gradients on baroclinic turbulence", Journal of Physical Oceanography, Early Online Release: https://doi.org/10.1175/JPO-D-24-0014.1.