The atmosphere varies on many time scales (temperature, precipitation, winds).

But the ocean response is a combination of forced and internal variability so it is difficult to sort out mechanisms or to predict how the ocean (temperature, salinity, MOC) will respond to changes in the atmosphere.
Objective: Develop a basic understanding of how a convective basin responds to changes in atmospheric temperature ($T_A$) (changes in precipitation have also been considered with Yuki Yasuda)

Specifically, how does the temperature and salinity of the convective water mass, meridional heat transport, meridional overturning circulation vary in amplitude and phase compared to changes in $T_A$?

Derivation VERY brief, focus on general approach and results
Consider a conceptual model of a convective marginal sea

1. The interior is defined by closed geostrophic contours, mean flow is weak, water is well mixed
2. Exchange with the open ocean by a cyclonic bdy current
3. Exchange between the boundary current and interior by (parameterized) baroclinic instability
4. Mass, heat, and salt budgets yield (nondimensional):

\[
\Delta T_t = -\Delta T(\Delta T - \Delta S) + \frac{2\mu}{\varepsilon} (1 - \Delta T + T' \sin \alpha)
\]

\[
\Delta S_t = -\Delta S(\Delta T - \Delta S) - \gamma
\]

\[
MOC = \frac{\varepsilon \Delta \rho}{2} + \frac{PL}{A} \frac{\mu}{\Delta \rho} (1 - T' \sin \alpha)
\]

\[
\varepsilon = \frac{cP}{L} \quad \mu = \frac{A\Gamma f_0}{\alpha T g C_p H_s T^*} \quad \gamma = \frac{8A\rho_0 f_0 S_0 \alpha_S E_0}{g H_s^2 \alpha_T^2 T^{*2}} \quad T^* = T_{Bin} - T_A
\]

Time is scaled by \( \tau = \frac{2H_0 A\rho_0 f_0 L}{g c P H_s^2 \alpha_T T^{*2}} \)

(similar time scale to Straneo 2006)
A linearized, decoupled analytic solution is possible

\[ \delta T_t + C_1 \delta T - \frac{2\mu}{\varepsilon} T' \sin \omega t = 0 \quad \delta S_t + C_2 \delta S + \Delta S_0 \delta T = 0 \]

\( \delta T \) is forced by the atmosphere, \( \delta S \) is forced by \( \delta T \) via eddy fluxes

solution is:

\[ T(t) = \frac{2\mu / \varepsilon T'}{(C_1^2 + \omega^2)^{1/2}} \sin(\omega t + \phi_T) \quad \phi_T = \tan^{-1}(-\omega / C_1) \]

with

\[ C_1 = 2\Delta T_0 - \Delta S_0 + 2\mu / \varepsilon \]

also a solution for \( S(t) \) of similar form (but messier)

Compares well with full equations over most parameter space but not discussed further today
Consider a simple example of sinusoidal variation in the temperature of the atmosphere over the marginal sea with period 20 years and amplitude $1^\circ C$.

The marginal sea is roughly Labrador Sea-like in terms of size, latitude, depth, mean thermohaline state.

The warmest LS water is found about 4 years after the warmest $T_A$ but before the atmosphere becomes cold.

The freshest LS water is found when the temperature of LS water is transitioning from warm to cold.

Note that there is no change in $E-P$, the changes in salinity arise as a result of the changes in $T_{LS}$ and the influence on eddy fluxes.

Define $\delta T$ and $\delta S$ as amplitude of oscillations in $\Delta T$ and $\Delta S$. 

Compare theory with results from an idealized, eddy-resolving numerical model (MITgcm)

Domain 2000 km x 1000 km, 2000 m deep
5 km horizontal resolution

Forced with:
Cyclonic wind stress curl (white vectors)
Restoring to atmospheric temperature (colors)
Uniform E-P=-2x10^{-8} m/s north of 1200 km latitude
T and S restored to warm/salty south of 200 km
(parameterize rest of ocean)

Oscillate T_A north of 1200 km latitude with amplitude 1°C and frequency ω

Topography:
Sill at 1200 km latitude and 100 km wide topographic slope around basin

Run the model for 50 years, solution is statistically equilibrated, then add oscillation for three periods.

(Configuration is similar to broad survey of equilibrated solutions in Spall, 2012 JPO)
Compare numerical model with theory, 20 year period forcing, 1° C anomaly in $T_A$

Convective water mass changes by about 0.1° C and 0.002 salinity
MOC (at exit of marginal sea) varies by about 0.2 Sv, or 10% of the mean

Amplitude and phase of $T$ and MOC are well predicted by the theory (mean MOC off a little)
Forced variability in salinity is smaller than the natural variability in the system so the forced response only explains 10% of the model variability
Now compare model and theory over a range of frequencies (400 yrs to 2 yrs)

<table>
<thead>
<tr>
<th>Temperature: solid line and circles</th>
<th>Salinity: dashed line and squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOC: dot-dashed line and triangles</td>
<td></td>
</tr>
</tbody>
</table>

General trends and frequency dependence well predicted by the theory

**Temperature:**
- Variability peaks at intermediate frequencies (salinity feedback)
- In phase at low frequencies, lags by 90° at high frequencies

**Salinity:**
- Theory over predicts amplitude at low frequencies
- Correctly predicts phase transition (driven by eddy fluxes)

**MOC:**
- Amplitude is (surprisingly) independent of frequency due to dominance of bdy term
- Always close to 180°C out of phase with $T_A$ (warm atmosphere, weak MOC, no lag)
Summary

- Objective was to develop a basic understanding of how the ocean responds to variations in atmospheric temperature (should be similar to variations in ocean temperature)

- Derived a reduced system of equations to represent a marginal sea and its exchange with the open ocean (+analytic linear solutions)

- Theory compares well with a series of numerical model calculations in terms of amplitude and phase of T, S, MOC

- They key parameter is $2\mu/\varepsilon$, which measures eddy response time

- Simple approach allows for physical understanding of the role of eddies, the feedback between T and S, and what drives changes in the MOC
Now summarize response as a function of forcing parameter $2\mu/\varepsilon$ and frequency $\omega$

$2\mu/\varepsilon >> 1$ atmosphere strongly damps
(strong bdy current; cold salty interior)

$2\mu/\varepsilon << 1$ LW becomes warm and fresh

Amplitude of temperature oscillation depends
strongly on the mean state and the frequency
of the atmospheric variability. The transition
in frequency is very broad, factor of 10.

Salinity anomaly is typically much weaker than
the temperature anomaly except low frequency
and weak atmospheric forcing

A linearized, decoupled analytic solution is also
available which reproduces
the basic elements of the
full solution (lower right)

$$T(t) = \frac{2\mu/\varepsilon T'}{(C_1^2 + \omega^2)^{1/2}} \sin(\alpha t + \phi_T)$$  \hspace{1cm} \phi_T = \tan^{-1}(-\omega/C_1)

$$C_1 = 2\Delta T_0 - \Delta S_0 + 2\mu/\varepsilon$$
Temperature: in phase for low frequency forcing \((\omega < 2\mu/\epsilon)\)

lags by 90° for high frequency forcing

(still cooling at end of cool period because eddies can not respond that quickly)

Salinity: 180° out of phase for low frequency forcing

in phase for high frequency forcing