# Metrics based on fuzzy similarities between lower dimensional features for intercomparison of reanalyses

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## Definition of Lower Dimensional Features

- Lower dimensional features in climate models, are subsets of subspaces of striclty less dimensionality compared to the space where the model is designed and running.
- These lower dimensional features very often carry climate information of high importance for decision makers.
- A reliable and useful climate model has to be able to reproduce important lower dimensional features in an accurate way.

## Example

These can be for example curves in a two dimensional spatial domain:

- sea ice boundary
- sea ice leads
- the boundary of the dry zone
- Gulf stream

## Example

Existence, inexistence or in a fuzzy case, the degree of occurrence of a phenomenon

- Double ITCZ
- atmospheric rivers
- number of tropical storms

## Defficiency of RMSE

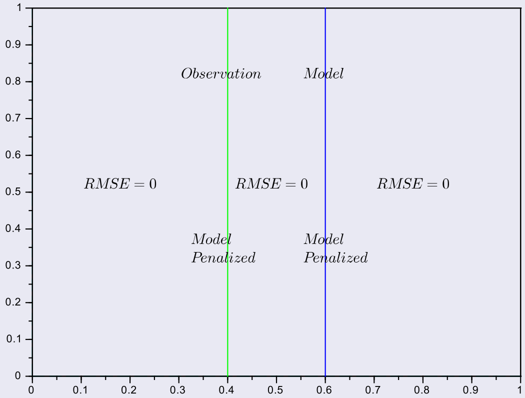


Figure: RMSE penalises models that are not accurate in the location twice.

## Fuzzification avoids this drawback

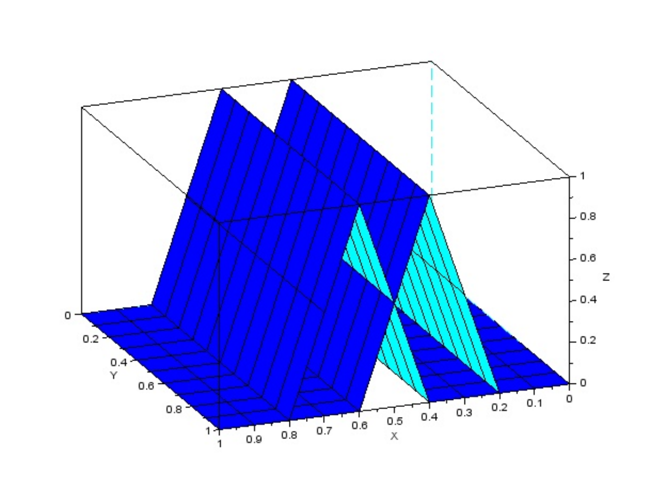


Figure: Fuzzification.

## Definition of a fuzzy set

Intuitively, a fuzzy set is a set with uncertain boundaries and it can be mathematically represented as a function

$$A : X \to [0, 1],$$

with the following interpretation: $A(x)$ represents the membership grade of element $x$ in the fuzzy set $A$, where $A(x) = 1$ means complete membership of $x$ in the fuzzy set $A$, $A(x) = 0$ means complete non-membership of $x$ in $A$, while intermediate values show partial membership of $x$ in $A$.

## Fuzzy sets and linguistic variables

A fuzzy set is able to model linguistic concepts.

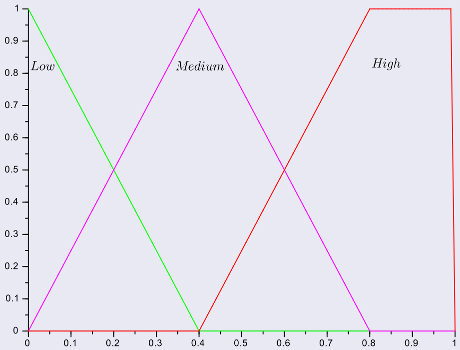


Figure: Examples of fuzzy sets.

## Fuzzy Intersection

Intersection of two fuzzy sets can be performed by taking the minimum of their membership grades.

$$(A \wedge B)(x) = \min\{A(x), B(x)\}$$

## Fuzzy Union

Union of fuzzy sets is obtained by taking the maximum of their membership grades.

$$(A \vee B)(x) = \max\{A(x), B(x)\}$$

## Fuzzy Complement

Fuzzy Complement has the membership grade complement of the original membership grade with respect to one.

$$\bar{A}(x) = 1 - A(x)$$

## Properties of Fuzzy Similarities

A fuzzy similarity is a function $Sim(A, B) \in [0, 1]$ with the interpretation as the degree of similarity between $A$ and $B$.

1. An empty model versus non-empty observation of a lower dimensional feature, should produce 0 (or nearly 0) similarity.
2. A model that makes the lower dimensional feature distributed everywhere has score 0 or very low.
3. A model compared with its complement should score 0 or very low.
4. A model compared with itself should receive a perfect score of 1.
5. A similarity measure should be sensitive enough to elucidate significant climatic differences/similarities.
6. Dual of triangle inequality .

## Examples of a Fuzzy Similarities

- $Sim_1(A, B) = \frac{V(A \wedge B)}{V(A \vee B)}$
- $Sim_2(A, B) = \frac{V(A \cdot B)}{V(A \vee B)}$
- $Sim_3(A, B) = \frac{V(A \wedge B)}{V(A) \vee V(B)}$
- $Sim_4(A, B) = \frac{V(\bar{A} \vee \bar{B})}{V(\bar{A} \wedge \bar{B})}$
- $Sim_5(A, B) = \frac{V(A \cdot B)}{V(A + B - AB)}$
- $Sim_6(A, B) = \frac{1}{1 + RMSE(A, B)}$

## Observation to Model Comparison

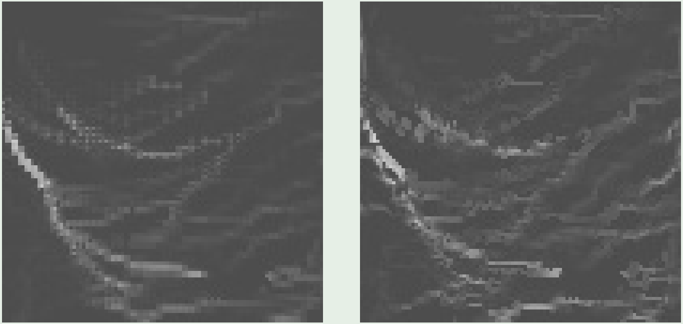

Figure: Illustration of sea-ice (black) and leads (white) of a selected area in the Beaufort Sea. Lower dimensional features (leads) extracted from (left) kinematic interpretation of the RADARSAT Geophysical Processor System (RGPS) data and (right) model simulation (Model 1) on 26 February 2004 (see Levy et al., 2008, 2010).

## Fuzzifications of the observation and simulation

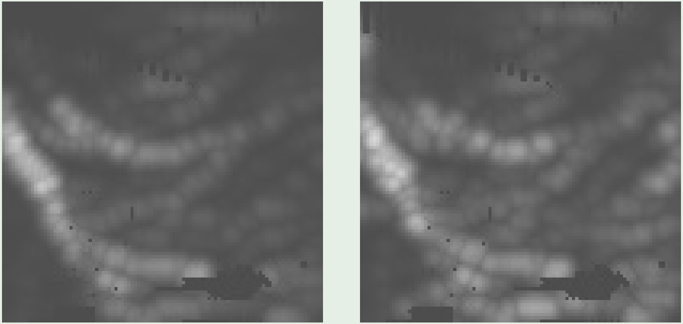

Figure: Fuzzifications of sea-ice observation (left), and fuzzyfication of the sea-ice prediction given by Model 1 (right).

## Fuzzy intersection and union

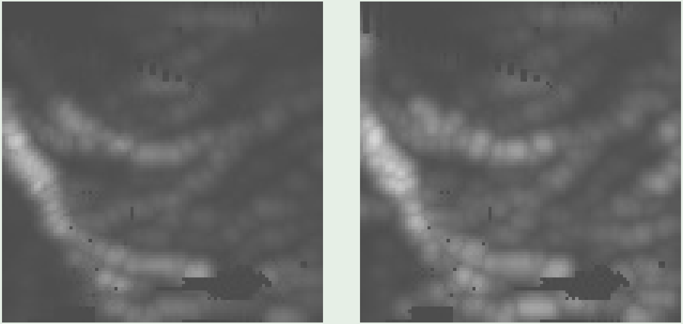

Figure: Fuzzy intersection (left) and fuzzy union (right), of the fuzzifications of observation and model 1 output used in the computation of fuzzy similarity measures.

## Other models used in comparison

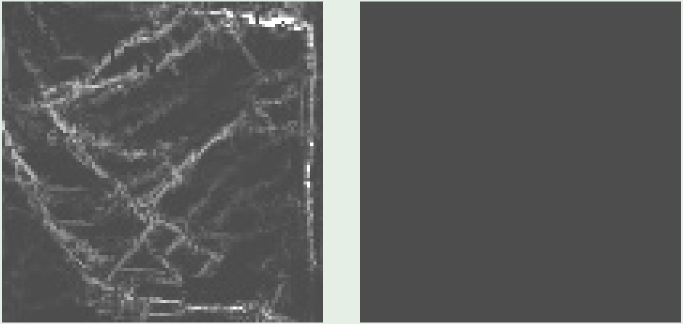

Figure: Comparison of two different predictions of sea-ice (black) and leads (white) of a selected area in the Beaufort Sea. A model simulation that overpredicts the leads (Model 2) (left), and a hypothetical lead- free prediction (Model 3) (right)

## Observation to Model Comparison

We will compare using RMSE and fuzzy metrics the performance of two models of ice leads

| Similarity | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| $Sim_1$ | 0.6700731 | 0.3160257 | 0 |
| $Sim_2$ | 0.1167839 | 0.0712802 | 0 |
| $Sim_3$ | 0.6950285 | 0.3313794 | 0 |
| $Sim_4$ | 0.9689203 | 0.8751084 | 0.9374443 |
| $Sim_5$ | 0.0751849 | 0.0572648 | 0 |
| $Sim_6(RMSE)$ | 0.9534698 | 0.905803 | 0.951054 |

Table: Comparison of various models using various fuzzy similarities and RMSE-based similarity.

## New metrics

How can we define new metrics for comparing performances of different models in predicting lower dimensional features?

## Uncertainties

How can we better quantify uncertainties, for meaningful comparison of climate models?

## Standard deviation

We saw that RMSE is not the best when we are dealing with lower dimensional features. Can we use different measures, other than standard deviation for uncertainty quantification?

## Further mathematical ideas

How can we combine new mathematical ideas at the boundaries of traditional disciplines in order to improve climate models?